Perspectives on $b \to c \tau \bar{\nu}$

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Many open questions about flavor

- Flavor structure and *CP* violation are major pending questions
- Related to Yukawa couplings, scalar sector, maybe connected to hierarchy puzzle
 Know only that Higgs field is responsible for (bulk of) the heaviest fermion masses
- Important cosmological implications (Baryogenesis)
- Sensitive to new physics at high scales, beyond LHC direct search reach Establishing any of the flavour anomalies would set upper bound on NP scale
- Experiment: expect huge improvements, many new measurements
- Theory: Progress and new directions both in SM calculations and model building





CKM fit: plenty of room for new physics

- SM dominates CP viol. \Rightarrow KM Nobel
- The implications of the consistency often overstated
- Much larger allowed region if the SM is not assumed to hold
- Tree-level (mainly V_{ub} & γ) vs. loopdominated measurements crucial



• In loop (FCNC) processes NP/SM $\sim 20\%$ is still allowed (mixing, $B \to X\ell^+\ell^-$, $X\gamma$, etc.)





R(D) and $R(D^*) - 4\sigma$ tension with SM

• BaBar, Belle, LHCb: enhanced τ rates, $R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu})}{\Gamma(B \to D^{(*)}l\bar{\nu})}$ $(l = e, \mu)$ Notation: $\ell = e, \mu, \tau$ and $l = e, \mu$







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Another look at the data

• Separate R(D) and $R(D^*)$ measurements — all central values above SM:



• Not yet decisive, consistent with both an emerging signal or fluctuations





Roadmap: 1981

- Fig. 3. A Program to Understand B Decay
- Search for exotic B decays. 1.

If found, explore details: -otherwise-

- 2. Search for flavor changing neutral currents. If found, measure $(b \rightarrow dZ^{0})/(b \rightarrow sZ^{0})$; -otherwise-
- 3. Measure semileptonic decay branching ratio.
- 4. Measure ratio $(b \neq uW)/(b \neq cW)$.
- Measure ev: µv: tv ratio in semileptonic decay.

Non-b-Decay Features of B Decay

- 6. Look for lifetime difference between B^{\pm} and B° .
- 7. Look for $B^{O}-\overline{B}^{O}$ mixing.
- [8. CP violation?]



 \Rightarrow dark sector searches? violating symmetries?

- \Rightarrow big part of the program
- \Rightarrow big part of the program
- $\Rightarrow |V_{ub}/V_{cb}|$: essential to constrain NP
- \Rightarrow Prophecy of $R(D^{(*)})$?

- \Rightarrow Seems less important now
- \Rightarrow Was the first item accomplished
- \Rightarrow Became a central focus of the field







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Reasons not to take the tension seriously

- Measurements with τ leptons are difficult
- Need a large tree-level contribution, SM suppression only by m_{τ} NP was expected to show up in FCNCs — need fairly light NP to fit the data
- Strong constraints on concrete models from flavor physics, as well as high- p_T

Reasons to take the tension seriously

- Results from BaBar, Belle, LHCb are consistent
- Often when measurements disagreed in the past, averages were still meaningful
- If Nature were as most theorist imagined (until ~ 10 years ago), then the LHC (Tevatron, LEP, DM searches) should have discovered new physics already





Exciting future

• LHCb and Belle II: increase $pp \rightarrow b\bar{b}$ and $e^+e^- \rightarrow B\bar{B}$ data sets by factor ~ 50



Belle II (50/ab, at SM level): $\delta R(D) \sim 0.005 \ (2\%)$ $\delta R(D^*) \sim 0.010 \ (3\%)$

Measurements will improve a lot! (Even if central values change, plenty of room for establishing deviations from SM)

- Competition, complementarity, cross-checks between LHCb and Belle II
- I'll focus on the 3 modes that are expected to be most precise in the long trem



THEORETICAL PHYSICS



Some key questions — now and in the future

- Can it be a theory issue? not at the current level
- Can it be an experimental issue? someone else's task
- Can [reasonable] models fit the data? yes [depends on your definition]
- What is the smallest deviation from SM in $R(D^{(*)})$ that can be established as NP? TBD: we know how to make progress
- Which channels are most interesting? (To establish deviation from SM / understand NP?) $B_{(s)} \rightarrow D_{(s)}^{(*,**)} \ell \bar{\nu}, \ \Lambda_b \rightarrow \Lambda_c^{(*)} \ell \bar{\nu}, \ B_c \rightarrow \psi \ell \bar{\nu}, \ B \rightarrow X_c \ell \bar{\nu}, \text{ etc.}$
- Which calculations can be made most robust (both continuum and LQCD)?
- What else can we learn from studying these anomalies?





I will not talk much about model building

Excellent summaries last week at Implications of LHCb measurements and future prospects by Monika Blanke and Toni Pich + several talks at this workshop

• I am (currently) most interested in:

What is the smallest deviation from SM, that can be unambiguously established? What are the best observables / distributions to utilize huge increases in data?

- Importance of many cross-checks
 - Measure several hadronic channels, both leptonic and hadronic τ decays
 - Consistent treatment of signals and main backgrounds, also due to $b \to c \ell \bar{\nu}$ (Partly theory, $B \to D^{**} \ell \bar{\nu}$, etc.)





SM predictions — mesons

- No clearly right way how to assign theory uncertainties (maybe except LQCD stat.)
- [strong interaction] model independent
 - \equiv theor. uncertainty suppressed by small parameters
 - ... so theorists argue about $\mathcal{O}(1) \times$ (small numbers) instead of $\mathcal{O}(1)$ effects Well defined starting point is crucial to claim a deviation from SM
- Most progress have come from expanding in $\Lambda_{
 m QCD}/m_Q$ and $lpha_s(m_Q)$
 - Estimating higher orders in α_s by scale variation is not fail-safe
 - Can get unlucky (e.g., in some cases $\Lambda_{\rm QCD}/m_c$ expansion might not work well) Need experimental guidance: $f_{\pi} \sim 140 \,{\rm MeV}, \ m_{\rho} \sim 770 \,{\rm MeV}, \ m_{K}^2/m_s \sim 2 \,{\rm GeV}$
- Consequently: pdf interpretation of theory uncertainties are fraught with peril





$B o D^{(*)} \ell ar{ u}$ or $\Lambda_b o \Lambda_c \ell ar{ u}$ decay

- In the $m_{b,c} \gg \Lambda_{\text{QCD}}$ limit, configuration of brown muck only depends on the fourvelocity of the heavy quark, but not on its mass and spin
- On a time scale $\ll \Lambda_{\text{QCD}}^{-1}$ weak current changes $b \to c$ i.e.: $\vec{p_b} \to \vec{p_c}$ and possibly $\vec{s_Q}$ flips

In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, brown muck only feels $v_b \rightarrow v_c$

Form factors independent of Dirac structure of weak current \Rightarrow all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$



Contains all nonperturbative low-energy hadronic physics

- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all
- Same holds for $\Lambda_b \to \Lambda_c \ell \bar{\nu}$, different Isgur-Wise fn, $\xi \to \zeta$ [also satisfies $\zeta(1) = 1$]





$$B
ightarrow D^{(*)} \ell ar{
u}$$
 and HQET

• Only Lorentz invariance: 6 functions of q^2 , only 4 measurable with e, μ final states

$$\langle D | \bar{c} \gamma^{\mu} b | \overline{B} \rangle = f_{+}(q^{2})(p_{B} + p_{D})^{\mu} + \left[f_{0}(q^{2}) - f_{+}(q^{2}) \right] \frac{m_{B}^{2} - m_{D}^{2}}{q^{2}} q^{\mu}$$

$$\langle D^{*} | \bar{c} \gamma^{\mu} b | \overline{B} \rangle = -ig(q^{2}) \epsilon^{\mu\nu\rho\sigma} \varepsilon_{\nu}^{*} (p_{B} + p_{D^{*}})_{\rho} q_{\sigma}$$

$$\langle D^{*} | \bar{c} \gamma^{\mu} \gamma^{5} b | \overline{B} \rangle = \varepsilon^{*\mu} f(q^{2}) + a_{+}(q^{2}) (\varepsilon^{*} \cdot p_{B}) (p_{B} + p_{D^{*}})^{\mu} + a_{-}(q^{2}) (\varepsilon^{*} \cdot p_{B}) q^{\mu}$$

The a_- and $f_0 - f_+$ form factors $\propto q^\mu = p^\mu_B - p^\mu_{D^{(*)}}$ do not contribute for $m_l = 0$

• HQET: 1 Isgur-Wise function in heavy quark limit + 3 more at $O(\Lambda_{QCD}/m_{c,b})$

• Constrain all 4 functions from $B \to D, D^* l\bar{\nu} \Rightarrow \mathcal{O}(\Lambda^2_{\text{QCD}}/m^2_{c,b}, \alpha^2_s)$ uncertainties [Bernlochner, ZL, Papucci, Robinson, 1703.05330]

• Observables: $B \to Dl\bar{\nu}$: $d\Gamma/dw$ (Only Belle published fully corrected distributions) $B \to D^* l\bar{\nu}$: $d\Gamma/dw$ and $R_{1,2}(w)$ form factor ratios





Explored 7 fit scenarios

• Our fits:

 ;+	QCDSR	Lattice QCD			Rollo Data
ГЦ		$\mathcal{F}(1)$	$f_{+,0}(1)$	$f_{+,0}(w > 1)$	
$L_{w=1}$		+	+		+
$L_{w=1}+SR$	+	+	+	—	+
NoL	—	—	—	—	+
NoL+SR	+				+
$L_{w \ge 1}$	—	+	+	+	+
$L_{w\geq 1}{+}SR$	+	+	+	+	+
th:L $_{w\geq 1}$ +SR	+	+	+	+	

• Role of QCD SR in CLN: $R_{1,2}(w) = \underbrace{R_{1,2}(1)}_{\text{fit}} + \underbrace{R'_{1,2}(1)}_{\text{fixed}}(w-1) + \underbrace{R''_{1,2}(1)}_{\text{fixed}}(w-1)^2/2$ In HQET: $R_{1,2}(1) = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ $R_{1,2}^{(n)}(1) = 0 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ Same parameters determine $R_{1,2}(1) - 1$ (fit) and $R_{1,2}^{(n)}(1)$ (rely on QCDSR)

Sometimes calculations using QCD sum rule predictions for $\Lambda_{
m QCD}/m_{c,b}$ corrections are called the HQET predictions





SM predictions for $R(D^{(*)})$

• Small variations: heavy quark symmetry & phase space leave little wiggle room

Reference (Scenario)	R(D)	$R(D^*)$	Correlation
Data [HFLAV]	0.407 ± 0.046	0.306 ± 0.015	-20%
Lattice [HFLAV]	0.300 ± 0.008		_
Fajfer et al. '12		0.252 ± 0.003	
Bernlochner <i>et al.</i> '17 ($L_{w\geq 1}$)	0.298 ± 0.003	0.261 ± 0.004	19%
Bernlochner <i>et al.</i> '17 ($L_{w\geq 1}+SR$)	0.299 ± 0.003	0.257 ± 0.003	44%
Bigi, Gambino '16	0.299 ± 0.003	—	
Bigi, Gambino, Schacht '17		0.260 ± 0.008	—
Jaiswal, Nandi, Patra '17 (case-3)	0.302 ± 0.003	0.262 ± 0.006	14%
Jaiswal, Nandi, Patra '17 (case-2)	0.302 ± 0.003	0.257 ± 0.005	13%

• Light-cone QCD SR & HQET QCD SR inputs are model dependent

HFLAV SM expectation neglects correlations present in any theoretical framework

• None of these are "ultimate" results — can be improved in coming years





SM predictions — baryons

No $R(\Lambda_c)$ measurement yet — maybe soon?

Ancient knowledge: baryons simpler than mesons

• Used to be well known — forgotten by experimentalists and well known theorists...

VOLUME 75, NUMBER 4PHYSICAL REVIEW LETTERS24 JULY 1995

Form Factor Ratio Measurement in $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$

G. Crawford,¹ C. M. Daubenmier,¹ R. Fulton,¹ D. Fujino,¹ K. K. Gan,¹ K. Honscheid,¹ H. Kagan,¹ R. Kass,¹ J. Lee,¹

[CLEO]

element $|V_{cs}|$ is known from unitarity [1]. Within heavy quark effective theory (HQET) [2], Λ -type baryons are more straightforward to treat than mesons as they consist of a heavy quark and a spin and isospin zero light diquark.





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Combine LHCb measurement of $d\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})/dq^2$ shape [1709.01920] with LQCD results for (axial-)vector form factors [1503.01421] — what can we learn?

[Bernlochner, ZL, Robinson, Sutcliffe, 1808.09464 to appear in PRL; 1810.????]





$$\Lambda_b o \Lambda_c \ell ar{
u}$$

• Ground state baryons are simpler than mesons: brown muck in (iso)spin-0 state

SM: 6 form factors, functions of w = v · v' = (m²_{Λb} + m²_{Λc} - q²)/(2m_{Λb}m_{Λc}) (Λ_c(p', s')|c̄γ_νb|Λ_b(p, s)) = ū_c(v', s') [f₁γ_µ + f₂v_µ + f₃v'_µ]u_b(v, s) (Λ_c(p', s')|c̄γ_νγ₅b|Λ_b(p, s)) = ū_c(v', s') [g₁γ_µ + g₂v_µ + g₃v'_µ]γ₅u_b(v, s) Heavy quark limit: f₁ = g₁ = ζ(w) Isgur-Wise fn, and f_{2,3} = g_{2,3} = 0 [ζ(1) = 1]
Include α_s, ε_{b,c}, α_sε_{b,c}, ε²_c: m_{Λb,c} = m_{b,c} + Λ̄_Λ + ..., ε_{b,c} = Λ̄_Λ/(2m_{b,c}) (Λ̄_Λ ~ 0.8 GeV larger than Λ̄ for mesons, enters via eq. of motion ⇒ expect worse expansion?)

$$\Lambda_{\Lambda} \sim 0.8 \,\text{GeV}$$
 larger than Λ for mesons, enters via eq. of motion \Rightarrow expect worse expansion?)

$$f_{1} = \zeta(w) \left\{ 1 + \frac{\alpha_{s}}{\pi} C_{V_{1}} + \varepsilon_{c} + \varepsilon_{b} + \frac{\alpha_{s}}{\pi} \left[C_{V_{1}} + 2(w-1)C_{V_{1}}' \right] (\varepsilon_{c} + \varepsilon_{b}) + \frac{b_{1} - b_{2}}{4m_{c}^{2}} + \dots \right\}$$

• No $\mathcal{O}(\Lambda_{\text{QCD}}/m_{b,c})$ subleading Isgur-Wise function, only 2 at $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_c^2)$

• Can do more using HQET than for meson decays In $B \to D^{(*)} \ell \bar{\nu}$ decay, there are 6 sub-subleading Isgur-Wise functions at $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_c^2)$





Fits and form factor definitions

Standard HQET form factor definitions: $\{f_1, g_1\} = \zeta(w) \left[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$ $\{f_{2,3}, g_{2,3}\} = \zeta(w) \left[0 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$

Form factor basis in LQCD calculation: $\{f_{0,+,\perp}, g_{0,+,\perp}\} = \zeta(w) \left[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$

LQCD results published as fits to 11 or 17 BCL parameters, including correlations All 6 form factors computed in LQCD \sim Isgur-Wise fn \Rightarrow despite good precision, limited constraints on subleading terms and their *w* dependence

• Only 4 parameters (and m_b^{1S}): { $\zeta', \zeta'', \hat{b}_1, \hat{b}_2$ } $\zeta(w) = 1 + (w - 1)\zeta' + \frac{1}{2}(w - 1)^2\zeta'' + \dots \qquad b_{1,2}(w) = \zeta(w)(\hat{b}_{1,2} + \dots)$

(Expanding to quadratic order in w - 1 or in conformal parameter, z, makes no difference)

Current LHCb and LQCD data do not yet allow constraining ζ''' and/or $\hat{b}_{1,2}'$





Fit to lattice QCD form factors and LHCb (1)

• Fit 6 form factors w/ 4 parameters: $\zeta'(1)$, $\zeta''(1)$, \hat{b}_1 , \hat{b}_2 [LQCD: Detmold, Lehner, Meinel, 1503.01421]







Fit to lattice QCD form factors and LHCb (2)

• Our fit, compared to the LQCD fit to LHCb:

• Obtain: $R(\Lambda_c) = 0.324 \pm 0.004$

A factor of ~ 3 more precise than LQCD prediction — data constrains combinations of form factors relevant for predicting $R(\Lambda_c)$



Our results will make their way into Hammer Advantage

[Bernlochner, Duell, ZL, Papucci, Robinson, soon]





The fit requires the $1/m_c^2$ terms

- E.g., fit results for g_1 blue band shows fit with $\hat{b}_{1,2} = 0$
- Find: $\hat{b}_1 = -(0.46 \pm 0.15) \,\mathrm{GeV}^2$... of the expected magnitude

Well below the model-dependent estimate: $\hat{b}_1 = -3\bar{\Lambda}_{\Lambda}^2 \simeq -2\,{
m GeV}^2$ [Falk & Neubert, hep-ph/9209269]

• Expansion in Λ_{QCD}/m_c appears well behaved (contrary to some claims in literature)







Spinoffs, byproducts, etc.

Has $|V_{cb}|$ been settled?

- $|V_{cb}|$ important to assess if there is an ε_K problem, predict $K \to \pi \nu \bar{\nu}$, $B \to \mu^+ \mu^-$
- The $b \to c \tau \bar{\nu}$ data will make $|V_{cb}|$ much better understood are we there yet? To understand the τ mode thoroughly, must understand the e, μ modes better
- Inclusive / exclusive tension resolved? Fits to Belle $B \to D^* l \bar{\nu}$ data (all good χ^2): Bigi, Gambino, Schacht, 1703.06124, $|V_{cb}|_{BGL} = (41.7^{+2.0}_{-2.1}) \times 10^{-3}$ Grinstein & Kobach, 1703.08170, $|V_{cb}|_{BGL} = (41.9^{+2.0}_{-1.9}) \times 10^{-3}$ Belle, 1702.01521, $|V_{cb}|_{CLN} = (38.2 \pm 1.5) \times 10^{-3}$
- Besides BGL, CLN, we considered 2 other frameworks to "interpolate" [1708.07134]

form factors	BGL	CLN	CLNnoR	noHQS
axial $\propto \epsilon_{\mu}^{*}$	b_0, b_1	$h_{A_1}(1), \ \rho_{D^*}^2$	$h_{A_1}(1), \ \rho_{D^*}^2$	$h_{A_1}(1),\;\rho_{D^*}^2,\;c_{D^*}$
vector	a_0, a_1	$\int_{B_1(1)} B_2(1)$	$\int R_1(1), \ R'_1(1)$	$\int R_1(1), \ R'_1(1)$
${\cal F}$	c_1, c_2	$\int n_1(1), n_2(1)$	$R_2(1), R'_2(1)$	$R_2(1), R'_2(1)$





Understanding $|V_{cb}|$

• Besides FNAL, JLQCD is also calculating the $B \rightarrow D^* \ell \bar{\nu}$ form factors Independent formulations: staggered vs. Mobius domain-wall actions



[T. Kaneko, JLQCD poster at Lattice 2018]

• No qualitative difference between the LQCD calculation at w = 1 or slightly above





Importance of lepton flavor violation searches

- Quark sector: If TeV-scale NP couples to quarks, some mechanism is needed to align couplings with SM Yukawas in order not to generate too large FCNCs
- Lepton sector: New lepton non-universal interaction would in general yield lepton flavor violation (LFV) at some level

• Many LFV searches became more interesting, not previously of high profile: E.g.: $B \to K^{(*)}e^{\pm}\mu^{\mp}$, $B \to K^{(*)}e^{\pm}\tau^{\mp}$, $B \to K^{(*)}\mu^{\pm}\tau^{\mp}$, also in D & K decay

$$\mu \to e\gamma, \ \mu \to eee, \ \mu + N \to e + N^{(\prime)},$$

 τ decays: $\tau \rightarrow \mu \gamma$, $\mu \mu \mu$, *eee*, $\mu \mu e$, etc. Belle II: improve 2 orders of magnitude

 Any discovery ⇒ broad program to map out the detailed structure







ATLAS & CMS: extend high p_T searches

- In some sense unusual & unexpected models: mediator masses, couplings, generation (non-)universality patterns differ from NP signals expected years ago
- Even just extending existing searches can be interesting (allowed regions of masses & couplings in strange models can be ... strange)
 - Extend \tilde{t} and \tilde{b} searches to higher production cross section
 - Search for $t \to b \tau \bar{\nu}$, $c \tau^+ \tau^-$ nonresonant decays
 - Search for states on-shell in t-channel, but not in s-channel
 - Search for $t\tau$ resonances
 - Discussed in more detail in other talks







Conclusions

- Measurable NP contribution to $b \rightarrow c \ell \bar{\nu}$ would imply NP at a fairly low scale
- HQET is a model independent framework, improvable with more e, μ data
- The $\Lambda_{\rm QCD}/m_c$ terms are important, no evidence for bad behavior of expansion
- Measurements will improve in the next decade by nearly an order of magnitude (Even if central values change, plenty of room for significant deviations from SM)
- New directions: model building, high- p_T searches, lepton flavor violation searches
- Best case: discover new physics
 Worst case: better SM tests, better CKM determination, better NP sensitivity
- We will find out: more data + improved predictions





Ultimately, data will tell



"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong." [Feynman]



Extra slides

Heavy quark symmetry 101

- $Q \overline{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{QCD}$ limit
- $Q \overline{q}$: wave function of the light degrees of freedom ("brown muck") insensitive to spin and flavor of Q

(A B meson is a lot more complicated than just a $b\bar{q}$ pair)

In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity v^{μ} [Isgur & Wise]

SU(2n) heavy quark spin-flavor symmetry at fixed v^{μ} [Georgi]



- Similar to atomic physics: $(m_e \ll m_N)$
 - 1. Flavor symmetry ~ isotopes have similar chemistry [Ψ_e independent of m_N]
 - 2. Spin symmetry ~ hyperfine levels almost degenerate $[\vec{s}_e \vec{s}_N \text{ interaction} \rightarrow 0]$





Spectroscopy of heavy-light mesons

• In $m_Q \gg \Lambda_{\text{QCD}}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

angular momentum conservation: $[\vec{J}, \mathcal{H}] = 0$ heavy quark symmetry: $[\vec{s}_Q, \mathcal{H}] = 0$ $\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$

• For a given s_l , two degenerate states:

$$J_{\pm} = s_l \pm \frac{1}{2}$$

 $\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{QCD})$ — same in *B* and *D* sector

Doublets are split by order $\Lambda^2_{\rm QCD}/m_Q$, e.g.: $m_{D^*} - m_D \sim 140 \,{\rm MeV}$ $m_{B^*} - m_B \sim 45 \,{\rm MeV}$ ratio $\sim m_c/m_b$





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