Perspectives on $b \rightarrow c \tau \bar{\nu}$

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Workshop on high-energy implications of flavor anomalies CERN, Oct 22–24, 2018

Many open questions about flavor

- Flavor structure and CP violation are major pending questions
- Related to Yukawa couplings, scalar sector, maybe connected to hierarchy puzzle Know only that Higgs field is responsible for (bulk of) the heaviest fermion masses
- Important cosmological implications (Baryogenesis)
- Sensitive to new physics at high scales, beyond LHC direct search reach Establishing any of the flavour anomalies would set upper bound on NP scale
- Experiment: expect huge improvements, many new measurements
- Theory: Progress and new directions both in SM calculations and model building

CKM fit: plenty of room for new physics

- SM dominates CP viol. \Rightarrow KM Nobel
- The implications of the consistency often overstated
- Much larger allowed region if the SM is not assumed to hold
- Tree-level (mainly V_{ub} & γ) vs. loopdominated measurements crucial

• In loop (FCNC) processes NP / SM $\sim 20\%$ is still allowed (mixing, $B \to X \ell^+ \ell^-$, $X\gamma$, etc.)

$R(D)$ and $R(D^*) - 4\sigma$ tension with SM

• BaBar, Belle, LHCb: enhanced τ rates, $R(D^{(*)})$ $= \frac{\Gamma(B \to D^{(*)} \tau \bar{\nu})}{\Gamma(B \to D^{(*)} \tau \bar{\nu})}$ $\Gamma(B \to D^{(*)} l \bar{\nu})$ $(l = e, \mu)$ Notation: $\ell = e, \mu, \tau$ and $l = e, \mu$

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Another look at the data

• Separate $R(D)$ and $R(D^*)$ measurements — all central values above SM:

• Not yet decisive, consistent with both an emerging signal or fluctuations

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Roadmap: 1981

Fig. 3. A Program to Understand B Decay

wind ys asbeionno bas , waivisvo va sechi

1. Search for exotic B decays.

If found, explore details: -otherwise-

- Search for flavor changing neutral currents. $2.$ If found, measure $(b + dZ^0)/(b + gZ^0)$; -otherwise-
- 3. Measure semileptonic decay branching ratio.
- 4. Measure ratio $(b + uW)/(b + cW)$.
- ⇒ Measure ev: µv: TV ratio in semileptonic decay.

Non-b-Decay Features of B Decay

- 6. Look for lifetime difference between B^{\pm} and B° .
- 7. Look for $B^0 \overline{B}$ ^o mixing.
- [8. CP violation?]

⇒ dark sector searches? violating symmetries?

 \Rightarrow big part of the program

- \Rightarrow big part of the program
- $\Rightarrow |V_{ub}/V_{cb}|$: essential to constrain NP
- \Rightarrow Prophecy of $R(D^{(*)})$?

- \Rightarrow Seems less important now
- \Rightarrow Was the first item accomplished
- \Rightarrow Became a central focus of the field

Reasons not to take the tension seriously

- Measurements with τ leptons are difficult
- Need a large tree-level contribution, SM suppression only by m_{τ} NP was expected to show up in FCNCs — need fairly light NP to fit the data
- Strong constraints on concrete models from flavor physics, as well as high- p_T

Reasons to take the tension seriously

- Results from BaBar, Belle, LHCb are consistent
- Often when measurements disagreed in the past, averages were still meaningful
- If Nature were as most theorist imagined (until \sim 10 years ago), then the LHC (Tevatron, LEP, DM searches) should have discovered new physics already

Exciting future

● LHCb and Belle II: increase $pp \to b\bar b$ and $e^+e^- \to B\bar B$ data sets by factor $\sim\!50$

 $\delta R(D) \sim 0.005$ (2%) $\delta R(D^*) \sim 0.010~(3\%)$

Measurements will improve a lot! (Even if central values change, plenty of room for establishing deviations from SM)

- Competition, complementarity, cross-checks between LHCb and Belle II
- I'll focus on the 3 modes that are expected to be most precise in the long trem

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Some key questions — now and in the future

- Can it be a theory issue? not at the current level
- Can it be an experimental issue? someone else's task
- Can [reasonable] models fit the data? yes [depends on your definition]
- What is the smallest deviation from SM in $R(D^{(*)})$ that can be established as NP? TBD: we know how to make progress
- Which channels are most interesting? (To establish deviation from SM / understand NP?) $B_{(s)} \to D_{(s)}^{(*, **)}$ ${}^{(*, **)}_{(s)}\ell\bar{\nu},\ \Lambda_b\rightarrow \Lambda_c^{(*)}\ell\bar{\nu},\ B_c\rightarrow \psi \ell\bar{\nu},\ B\rightarrow X_c\ell\bar{\nu},$ etc.
- Which calculations can be made most robust (both continuum and LQCD)?
- What else can we learn from studying these anomalies?

• I will not talk much about model building

Excellent summaries last week at [Implications of LHCb measurements and future](https://indico.cern.ch/event/743635/) [prospects](https://indico.cern.ch/event/743635/) by [Monika Blanke](https://indico.cern.ch/event/743635/contributions/3151571/attachments/1736655/2810355/LHCb2018.pdf) and [Toni Pich](https://indico.cern.ch/event/743635/contributions/3151750/attachments/1737209/2810153/LHCb_2018_FlavourAnomalies_pich.pdf) $+$ several talks at this workshop

• I am (currently) most interested in:

What is the smallest deviation from SM, that can be unambiguously established? What are the best observables / distributions to utilize huge increases in data?

- Importance of many cross-checks
	- **–** Measure several hadronic channels, both leptonic and hadronic $τ$ decays
	- **–** Consistent treatment of signals and main backgrounds, also due to $b \to c \ell \bar{\nu}$ **(Partly theory,** $B \to D^{**} \ell \bar{\nu}$, etc.)

SM predictions — mesons

- No clearly right way how to assign theory uncertainties (maybe except LQCD stat.)
- [strong interaction] model independent
	- \equiv theor. uncertainty suppressed by small parameters

... so theorists argue about $\mathcal{O}(1) \times$ (small numbers) instead of $\mathcal{O}(1)$ effects Well defined starting point is crucial to claim a deviation from SM

- Most progress have come from expanding in $\Lambda_{\rm QCD}/m_Q$ and $\alpha_s(m_Q)$
	- **–** Estimating higher orders in α_s by scale variation is not fail-safe
	- **–** Can get unlucky (e.g., in some cases Λ_{QCD}/m_c expansion might not work well) Need experimental guidance: $f_\pi \sim 140\,{\rm MeV},\; m_\rho \sim 770\,{\rm MeV},\; m_K^2/m_s \sim 2\,{\rm GeV}$
- Consequently: pdf interpretation of theory uncertainties are fraught with peril

$B\to D^{(*)}\ell\bar\nu$ or $\Lambda_b\to \Lambda_c\ell\bar\nu$ decay

- In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, configuration of brown muck only depends on the fourvelocity of the heavy quark, but not on its mass and spin
- On a time scale $\ll \Lambda_{\rm QCD}^{-1}$ weak current changes $b \to c$ i.e.: $\vec{p}_b \rightarrow \vec{p}_c$ and possibly \vec{s}_Q flips

In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, brown muck only feels $v_b \to v_c$

Form factors independent of Dirac structure of weak current \Rightarrow all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$

 \mathcal{C} Contains all nonperturbative low-energy hadronic physics

 \bullet $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

 \uparrow

• Same holds for $\Lambda_b \to \Lambda_c \ell \bar{\nu}$, different Isgur-Wise fn, $\xi \to \zeta$ [also satisfies $\zeta(1) = 1$]

$B\to D^{(*)}\ell\bar{\nu}$ and HQET

• Only Lorentz invariance: 6 functions of q^2 , only 4 measurable with e , μ final states

$$
\langle D|\bar{c}\gamma^{\mu}b|\bar{B}\rangle = f_{+}(q^{2})(p_{B} + p_{D})^{\mu} + \left[f_{0}(q^{2}) - f_{+}(q^{2})\right]\frac{m_{B}^{2} - m_{D}^{2}}{q^{2}}q^{\mu}
$$

$$
\langle D^{*}|\bar{c}\gamma^{\mu}b|\bar{B}\rangle = -ig(q^{2})\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu}^{*}(p_{B} + p_{D^{*}})_{\rho}q_{\sigma}
$$

$$
\langle D^{*}|\bar{c}\gamma^{\mu}\gamma^{5}b|\bar{B}\rangle = \epsilon^{*\mu}f(q^{2}) + a_{+}(q^{2})(\epsilon^{*} \cdot p_{B})(p_{B} + p_{D^{*}})^{\mu} + a_{-}(q^{2})(\epsilon^{*} \cdot p_{B})q^{\mu}
$$

The a_- and f_0-f_+ form factors $\propto q^\mu=p_B^\mu-p_{D^{(*)}}^\mu$ do not contribute for $m_l=0$

• HQET: 1 Isgur-Wise function in heavy quark limit $+3$ more at $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$

- Constrain all 4 functions from $B \to D, D^* l \bar{\nu} \Rightarrow \mathcal{O}(\Lambda_{\rm QCD}^2/m_{c,b}^2, \alpha_s^2)$ uncertainties [Bernlochner, ZL, Papucci, Robinson, 1703.05330]
- **Observables:** $B \to D l \bar{\nu}$: $d\Gamma/dw$ (Only Belle published fully corrected distributions) $B\to D^* l\bar{\nu}\!:\,\,\mathrm{d}\Gamma/\mathrm{d} w$ and $R_{1,2}(w)$ form factor ratios

Explored 7 **fit scenarios**

Our fits:

• Role of QCD SR in CLN: $R_{1,2}(w) = R_{1,2}(1)$ ${\overbrace {\rm fit}}$ $+ R'_1$ $_{1,2}^{\prime}(1)$ fixed $(w-1)+R''_1$ $_{1,2}^{\prime\prime}(1)$ fixed $(w-1)^2/2$ In HQET: $R_{1,2}(1) = 1 + \mathcal{O}(\Lambda_{\rm QCD}/m_{c,b}, \alpha_s)$ $R_{1,2}^{(n)}$ $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{c,b}\,,\alpha_s)$ Same parameters determine $R_{1,2}(1) - 1$ (fit) and $R_{1,2}^{(n)}$ $\binom{n}{1,2}(1)$ (rely on QCDSR)

Sometimes calculations using QCD sum rule predictions for $\Lambda_{\rm QCD}/m_{c,b}$ corrections are called the HQET predictions

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SM predictions for $R(D^{(*)})$

• Small variations: heavy quark symmetry & phase space leave little wiggle room

• Light-cone QCD SR & HQET QCD SR inputs are model dependent

HFLAV SM expectation neglects correlations present in any theoretical framework

• None of these are "ultimate" results — can be improved in coming years

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SM predictions — baryons

No $R(\Lambda_c)$ measurement yet — maybe soon?

Ancient knowledge: baryons simpler than mesons

• Used to be well known — forgotten by experimentalists and well known theorists...

Form Factor Ratio Measurement in $\Lambda_c^+ \to \Lambda e^+ \nu_e$

G. Crawford,¹ C.M. Daubenmier,¹ R. Fulton,¹ D. Fujino,¹ K.K. Gan,¹ K. Honscheid,¹ H. Kagan,¹ R. Kass,¹ J. Lee,¹

[CLEO]

element $|V_{cs}|$ is known from unitarity [1]. Within heavy quark effective theory (HQET) [2], Λ -type baryons are more straightforward to treat than mesons as they consist of a heavy quark and a spin and isospin zero light diquark.

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Combine LHCb measurement of $d\Gamma(\Lambda_b\to\Lambda_c\mu\bar\nu)/\mathrm{d}q^2$ shape [1709.01920] with LQCD results for (axial-)vector form factors [1503.01421] — what can we learn?

[Bernlochner, ZL, Robinson, Sutcliffe, 1808.09464 to appear in PRL; 1810.?????]

$$
\boxed{\Lambda_b \to \Lambda_c \ell \bar{\nu}}
$$

• Ground state baryons are simpler than mesons: brown muck in (iso)spin-⁰ state

• SM: 6 form factors, functions of $w = v \cdot v' = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2)/(2m_{\Lambda_b}m_{\Lambda_c})$ $\langle \Lambda_c(p',s') | \bar c \gamma_\nu b | \Lambda_b(p,s) \rangle \, = \, \bar u_c(v',s') \Big[f_1 \gamma_\mu + f_2 v_\mu + f_3 v_\mu' \Big]$ μ $\big]u_{b}(v,s)$ $\langle \Lambda_{c}(p^{\prime},s^{\prime})|\bar{c}\gamma_{\nu}\gamma_{5}b|\Lambda_{b}(p,s)\rangle\,=\,\bar{u}_{c}(v^{\prime},s^{\prime})\Big[g_{1}\gamma_{\mu}+g_{2}v_{\mu}+g_{3}v_{\mu}^{\prime}\Big]$ μ $\Big]\gamma_5 \, u_b(v,s)$ Heavy quark limit: $f_1 = g_1 = \zeta(w)$ Isgur-Wise fn, and $f_{2,3} = g_{2,3} = 0$ [$\zeta(1) = 1$] • Include α_s , $\varepsilon_{b,c}$, $\alpha_s \varepsilon_{b,c}$, ε_c^2 $c \, \colon \qquad \quad m_{\Lambda_{b,c}} = m_{b,c} + \bar\Lambda_\Lambda + \dots, \ \ \varepsilon_{b,c} = \bar\Lambda_\Lambda/(2 m_{b,c})$

 $(\bar{\Lambda}_{\Lambda} \sim 0.8 \,\text{GeV}$ larger than $\bar{\Lambda}$ for mesons, enters via eq. of motion \Rightarrow expect worse expansion?)

$$
f_1 = \zeta(w) \left\{ 1 + \frac{\alpha_s}{\pi} C_{V_1} + \varepsilon_c + \varepsilon_b + \frac{\alpha_s}{\pi} \Big[C_{V_1} + 2(w-1)C'_{V_1} \Big] (\varepsilon_c + \varepsilon_b) + \frac{\hat{b}_1 - \hat{b}_2}{4m_c^2} + \dots \right\}
$$

• No $\mathcal{O}(\Lambda_{\rm QCD}/m_{b,c})$ subleading Isgur-Wise function, only 2 at $\mathcal{O}(\Lambda_{\rm QCD}^2/m_c^2)$

• Can do more using HQET than for meson decays In $B\to D^{(*)}\ell\bar\nu$ decay, there are 6 sub-subleading Isgur-Wise functions at ${\cal O}(\Lambda_{\rm QCD}^2/m_c^2)$

Fits and form factor definitions

• Standard HQET form factor definitions: $\{f_1, g_1\} = \zeta(w) \left[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$ ${f_{2,3}, g_{2,3}} = \zeta(w) \left[0 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$

Form factor basis in LQCD calculation: $\{f_{0,+,\perp}, g_{0,+,\perp}\} = \zeta(w) \left[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\right]$

LQCD results published as fits to 11 or 17 BCL parameters, including correlations All 6 form factors computed in LQCD \sim Isgur-Wise fn \Rightarrow despite good precision, limited constraints on subleading terms and their w dependence

• Only 4 parameters (and m_b^{1S}): { ζ' , ζ'' , \hat{b}_1 , \hat{b}_2 } $\zeta(w) = 1 + (w - 1)\zeta' + \frac{1}{2}$ $\frac{1}{2}(w-1)^2 \zeta'' + \dots$ $b_{1,2}(w) = \zeta(w) (\hat{b}_{1,2} + \dots)$

(Expanding to quadratic order in $w - 1$ or in conformal parameter, z , makes no difference)

• Current LHCb and LQCD data do not yet allow constraining ζ''' and/or \hat{b}'_1 1,2

Fit to lattice QCD form factors and LHCb (1)

• Fit 6 form factors w/ 4 parameters: $\zeta'(1)$, $\zeta''(1)$, \hat{b}_1 , \hat{b}_2 [LQCD: Detmold, Lehner, Meinel, 1503.01421]

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Fit to lattice QCD form factors and LHCb (2)

• Our fit, compared to the LQCD fit to LHCb:

Obtain: $R(\Lambda_c) = 0.324 \pm 0.004$

A factor of \sim 3 more precise than LQCD prediction — data constrains combinations of form factors relevant for predicting $R(\Lambda_c)$

Our results will make their way into $\text{Hammer} \ll \text{MC}$ [Bernlochner, Duell, ZL, Papucci, Robinson, soon]

The fit requires the $1/m_c^2$ terms

- \blacktriangleright E.g., fit results for g_1 blue band shows fit with $b_{1,2} = 0$
- Find: $\hat{b}_1 = -(0.46 \pm 0.15) \,\text{GeV}^2$... of the expected magnitude

Well below the model-dependent estimate: $\hat{b}_1 = -3\bar{\Lambda}^2_\Lambda \simeq -2\,\mathrm{GeV}^2$ [Falk & Neubert, hep-ph/9209269]

• Expansion in $\Lambda_{\rm QCD}/m_c$ appears well behaved (contrary to some claims in literature)

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Spinoffs, byproducts, etc.

Has $|V_{cb}|$ been settled?

- $|V_{cb}|$ important to assess if there is an ε_K problem, predict $K \to \pi \nu \bar{\nu}$, $B \to \mu^+ \mu^-$
- The $b \to c\tau\bar{\nu}$ data will make $|V_{cb}|$ much better understood are we there yet? To understand the τ mode thoroughly, must understand the e, μ modes better
- Inclusive / exclusive tension resolved? Fits to Belle $B \to D^* l \bar{\nu}$ data (all good χ^2): Bigi, Gambino, Schacht, 1703.06124, $|V_{cb}|_{\rm BGL} = (41.7^{+2.0}_{-2.1})$ $^{+2.0}_{-2.1}$ × 10⁻³ Grinstein & Kobach, 1703.08170, $|V_{cb}|_{\rm BGL} = (41.9^{+2.0}_{-1.9})$ $^{+2.0}_{-1.9}$ × 10^{-3} Belle, 1702.01521, $|V_{cb}|_{\text{CLN}} = (38.2 \pm 1.5) \times 10^{-3}$
- **Besides BGL, CLN, we considered 2 other frameworks to "interpolate"** [1708.07134]

 $$ **ng |** V_{cb} **| ∣** $\overline{}$ *r w r* +− + \blacksquare \blacksquare \blacksquare −+ − () ({ }) ² *O Om ^s b,c* [−] + α+

are not small (≲10%).

• Besides FNAL, JLQCD is also calculating the $B \to D^* \ell \bar \nu$ form factors Independent formulations: staggered vs. Mobius domain-wall actions \overline{M} \overline{M} \overline{M} $R_{\text{L}}, \text{true}$

[T. Kaneko, JLQCD poster at Lattice 2018]

• No qualitative difference between the LQCD calculation at $w = 1$ or slightly above

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Importance of lepton flavor violation searches

- Quark sector: If TeV-scale NP couples to quarks, some mechanism is needed to align couplings with SM Yukawas in order not to generate too large FCNCs
- Lepton sector: New lepton non-universal interaction would in general yield lepton flavor violation (LFV) at some level

• Many LFV searches became more interesting, not previously of high profile: E.g.: $B \to K^{(*)}e^{\pm}\mu^{\mp}$, $B \to K^{(*)}e^{\pm}\tau^{\mp}$, $B \to K^{(*)}\mu^{\pm}\tau^{\mp}$, also in D & K decay

$$
\mu \to e\gamma, \ \mu \to eee, \ \mu + N \to e + N^{(\prime)},
$$

 $τ$ decays: $τ → μγ$, $μμ$, eee, $μμ$ e, etc. Belle II: improve 2 orders of magnitude

• Any discovery \Rightarrow broad program to map out the detailed structure

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ATLAS & CMS: extend high p_T **searches**

- In some sense unusual & unexpected models: mediator masses, couplings, generation (non-)universality patterns differ from NP signals expected years ago
- Even just extending existing searches can be interesting (allowed regions of masses & couplings in strange models can be ... strange)
	- $-$ Extend \tilde{t} and \tilde{b} searches to higher production cross section
	- $-$ Search for $t\to b\tau\bar\nu,\,c\tau^+\tau^-$ nonresonant decays
	- **–** Search for states on-shell in t-channel, but not in s-channel
	- **–** Search for tτ resonances
	- Discussed in more detail in other talks

Conclusions

- Measurable NP contribution to $b \to c \ell \bar{\nu}$ would imply NP at a fairly low scale
- HQET is a model independent framework, improvable with more e, μ data
- The $\Lambda_{\rm QCD}/m_c$ terms are important, no evidence for bad behavior of expansion
- Measurements will improve in the next decade by nearly an order of magnitude (Even if central values change, plenty of room for significant deviations from SM)
- New directions: model building, high- p_T searches, lepton flavor violation searches
- Best case: discover new physics Worst case: better SM tests, better CKM determination, better NP sensitivity
- We will find out: more data $+$ improved predictions

Ultimately, data will tell

"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong." The expression of the summaniment of the summaniment of

Extra slides

Heavy quark symmetry 101

- $Q\overline{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{\rm QCD}$ limit
- \bullet $Q\overline{q}$: wave function of the light degrees of freedom ("brown muck") insensitive to spin and flavor of Q

(A B meson is a lot more complicated than just a $b\bar{q}$ pair)

In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity v^{μ} [Isgur & Wise]

 $SU(2n)$ heavy quark spin-flavor symmetry at fixed v^{μ} [Georgi]

- Similar to atomic physics: $(m_e \ll m_N)$
	- 1. Flavor symmetry \sim isotopes have similar chemistry [Ψ_e independent of m_N]
	- 2. Spin symmetry \sim hyperfine levels almost degenerate $[\vec{s}_e \vec{s}_N]$ interaction $\rightarrow 0$]

Spectroscopy of heavy-light mesons

In $m_Q \gg \Lambda_{\rm QCD}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

> angular momentum conservation: $[\vec{J}, \mathcal{H}] = 0$
heavy quark symmetry: $[\vec{s}_Q, \mathcal{H}] = 0$ \int $\Rightarrow [\vec{s}_l, \mathcal{H}] = 0$

• For a given
$$
s_l
$$
, two degenerate states:

$$
J_{\pm} = s_l \pm \frac{1}{2}
$$

 \Rightarrow $\Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda_{\rm QCD}^2/m_Q$, e.g.: $m_{D^*} - m_D \sim 140 \,\text{MeV}$ $m_{B^*} - m_B \sim 45 \,\text{MeV}$ ratio ~ m_c/m_b

