Bose-Einstein graviton condensate in a Schwarzschild black hole

Jorge Alfaro
Pontificia Universidad Católica de Chile

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Dvali and Gómez

G. Dvali and C. Gomez, Black Hole's 1/N Hair. *Phys. Lett.* **719**, 419 (2013);G. Dvali and C. Gomez, Landau-Ginzburg Limit of Black Hole's Quantum Portrait: Self Similarity and Critical Exponent. Phys. Lett. B **716**, 240 (2012);....

→ BH are quantum objects

→ BH as Bose-Einstein condensates(BEC)

→ Hawking radiation is seen as leakeage from the BEC

New perspectives on:

→ Bekenstein entropy

→ Absence of hair

→ Quantum nature of information storage and the possible information loss in BHs.

The physics of BH can be understood in this picture in terms of a single number N, the number of gravitons contained in the Bose-Einstein condensate (BEC).

- These condensed gravitons have a wave length $\lambda \sim \sqrt{N}\, L_P$, L_P being the Planck length;
- They have a characteristic interaction strength $\alpha_g \sim 1/N$ (reminiscent of meson interactions in large N_c gauge theories)
- The leakage leads to a Hawking temperature of order $T_H \sim 1/(\sqrt{N}\, L_P)$, equal to the inverse of λ.
- The mass of the BH is $M \sim \sqrt{N}\, M_P$ and its Schwarzschild radius therefore is given by $r_s \sim \sqrt{N}\, L_P$, thus agreeing with the Compton wavelength of the quantum gravitons λ, in accordance with the uncertainty principle that dictates $\lambda \simeq r_s$ in the ground state of the quantum system.

Main source of the talk:

Bose-Einstein graviton condensate in a Schwarzschild black hole, J.A.,D. Espriu and L. Gabbaneli, **Class.Quant.Grav. 35 (2018) no.1, 015001**

- $G_{\alpha\beta}$ will be the Einstein tensor: $G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$, where the Ricci tensor and scalar curvature are constructed with the metric $g_{\alpha\beta}$ in the usual way.
- We will denote by $\tilde{g}_{\alpha\beta}$ the background metric that in our case it will invariably be the Schwarzschild metric.
- Perturbations above this background metric will be denoted by $h_{\alpha\beta}$, so $g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$.
- We will use the Minkowskian metric convention $\eta_{\alpha\beta} = diag(-1, 1, 1, 1)$.
- We will leave for later discussion whether the indices of $h_{\alpha\beta}$ have to be raised or lowered with the background metric $\tilde{g}_{\alpha\beta}$ or the full one $g_{\alpha\beta}$. Likewise for the corresponding volume element ($\sqrt{-g}$ versus $\sqrt{-\tilde{g}}$).

The graviton condensate has necessarily to be described by a tensor field that in our philosophy has to be considered as a perturbation of the classical metric. Only spherically symmetric perturbations will be considered to keep things as simple as possible.

We note that the usual Gross-Pitaevskii equation employed to describe Bose-Einstein condensates is

→ a non-linear Schrödinger equation; i.e. an equation of motion that contains self-interactions (hence the non-linearity),

→ a confining potential for the atoms or particles constituting the condensate,

→ a chemical potential that is conjugate to the number of particles or atoms contained in the condensate.

Among all these ingredients, the perturbed Einstein equations already contain most of them. They are already non-linear and while there is no confining potential explicitly included (as befits a relativistic theory) they do confine particles, at least classically, because if the selected background corresponds to a Schwarzschild BH, the strong gravitational field classically traps particles inside the horizon.

- There is one ingredient missing:the equivalent of the chemical potential.

We add to the appropriate action a chemical potential term such as

$$\Delta S_{chem.pot.} = -\frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{\mu}\,h_{\alpha\beta}h^{\alpha\beta} = -\frac{1}{2}\int d\tilde{V}\tilde{\mu}\,h^2\,, \tag{1}$$

that is conjugate to the quantity $h^2 \equiv h_{\alpha\beta}h^{\alpha\beta}$, which should be related to the graviton density of the condensate inside a differential volume element $d\tilde{V}$.

- The full action for $h_{\alpha\beta}$ is

$$S(h) = M_P^2\int d^4x\sqrt{-g}\,R(g) - \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{\mu}\,h_{\alpha\beta}h^{\alpha\beta}\,. \tag{2}$$

- In the first term of (2), we raise and lower indices with $g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$, whereas in the second term of this action we raise and lower indices with $\tilde{g}_{\alpha\beta}$.
- Note that we assume that the chemical potential is r-dependent.

The actual equations of motion derived from the previous action may take slightly different forms depending on the choice of volume elements. For the time being, let us take the simplest possibility

$$G_\mu{}^\nu(\tilde{g}_{\alpha\beta}+h_{\alpha\beta})=\tilde{\mu}\,h_\mu{}^\nu\,. \tag{3}$$

where indices are assumed to be raised with the background metric only.

- In the chemical potential part $\tilde{\mu}$ is considered in the grand canonical ensemble; this implies that this magnitude is an external field and does not vary in the action; in particular, for these equations of motion, it is independent of $h_{\mu\nu}$, so $\delta\,\tilde{\mu}/\delta\,h_{\mu\nu}=0$. Under these considerations this term resembles the chemical potential term of the Gross-Pitaevskii equation.
- The chemical potential satisfies the constraint,which is valid up to every order in perturbation theory.:

$$\nabla^\nu G_{\mu\nu}=0 \quad\Longrightarrow\quad (\mu\,h_{\mu\nu})^{;\nu}=\mu^{,\nu}\,h_{\mu\nu}+\mu\,h_{\mu\nu}{}^{;\nu}=0\,. \tag{4}$$

- ∇^ν in equation (4) is defined using the full metric.

Even after the inclusion of μ there is not other normalizable solution outside the black hole horizon than the trivial solution for the perturbation corresponding to $\mu = 0$.

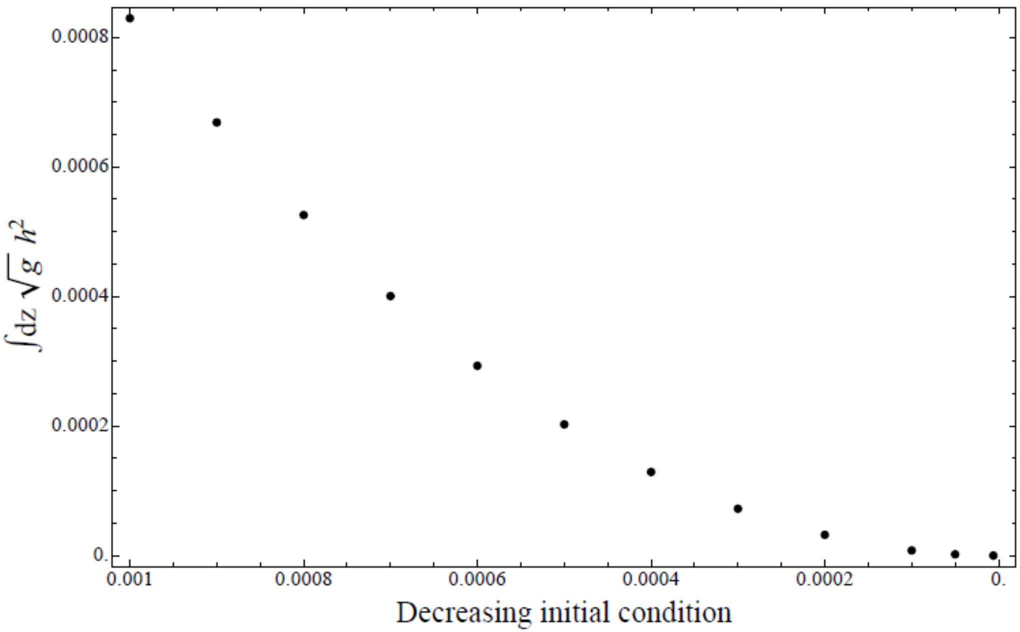

Figure 1. Each point represent a magnitude proportional to the integral of h^2 outside the event horizon. The closer the limit to an asymptotically flat space-time is (i.e. decreasing the initial condition near infinity), the smaller this integral is.

In Figure 2 a plot of the numerical solution reveals that to a very good approximation throughout the interior of the BH horizon

$$h_t{}^t = h_r{}^r = \text{constant}\,. \tag{5}$$

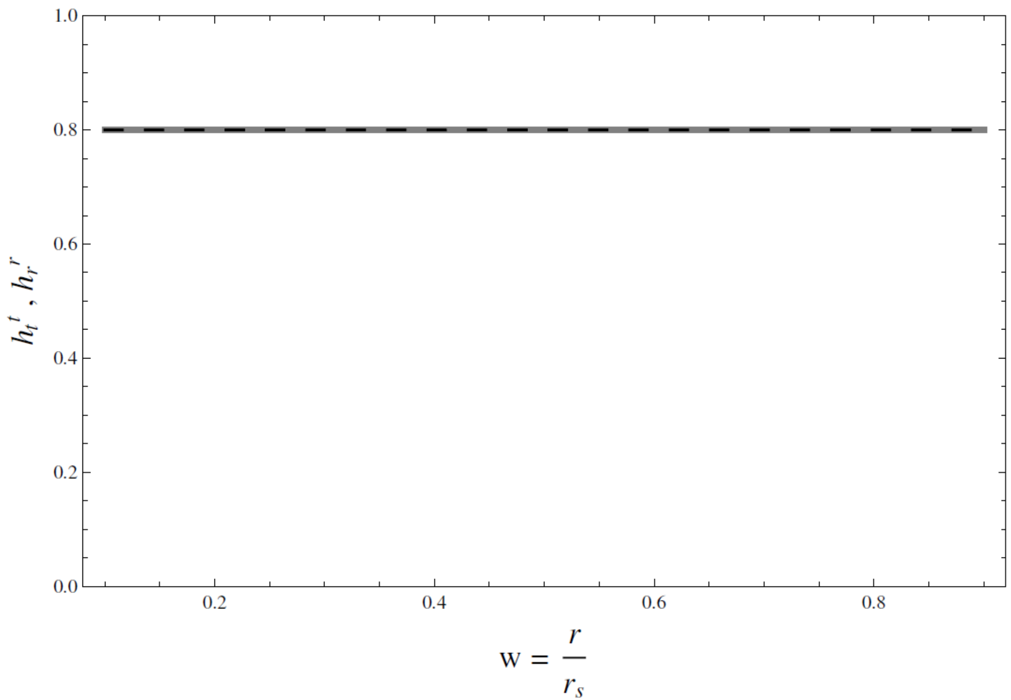

Figure 2. The graph shows that $h_t{}^t$ (represented by a gray solid line) and $h_r{}^r$ (by a black dashed line), as obtained by numerical integration of the $O(h_{\mu\nu}^2)$ equations, are constant and equal.

- Ansatz: $h_t{}^t = h_r{}^r = \varphi$ with φ being a constant.
- In fact, there are several possible solutions depending on how one chooses to treat the separation between background and fluctuation metrics.
- If we decide to raise and lower indices with the full metric, it is natural to keep the volume element as the one given by $\sqrt{-g}$, $g_{\alpha\beta}$ being the full metric. The LHS of the equation of motion now reads

$$G_t{}^t = G_r{}^r = \frac{-\varphi}{r^2} \tag{6}$$

- The resulting equation of motion for a constant φ is now

$$-\frac{\varphi}{r^2} = \mu\,\varphi - \frac{3}{2}\,\mu\,\varphi^2 \tag{7}$$

  that can be interpreted as a mean field-like Gross-Pitaevskii equation for the condensate wave function φ.
- Using the fact that $X \equiv \mu\, r^2$, we (re)obtain that X is a constant and

$$X = -\frac{1}{1-\frac{3}{2}\varphi} \simeq -1 - \frac{3}{2}\,\varphi + \ldots. \tag{8}$$

The left graph in Figure 3 presents the curves for the two covariant components of the perturbation. The dimensionful chemical potential is plotted also for comparison. As $X = \mu\ r^2$ is a constant function, $\mu \propto 1/r^2$ and it is *not null* over the event horizon. Of course it is simpler to represent just φ or h^2, which are just constants.

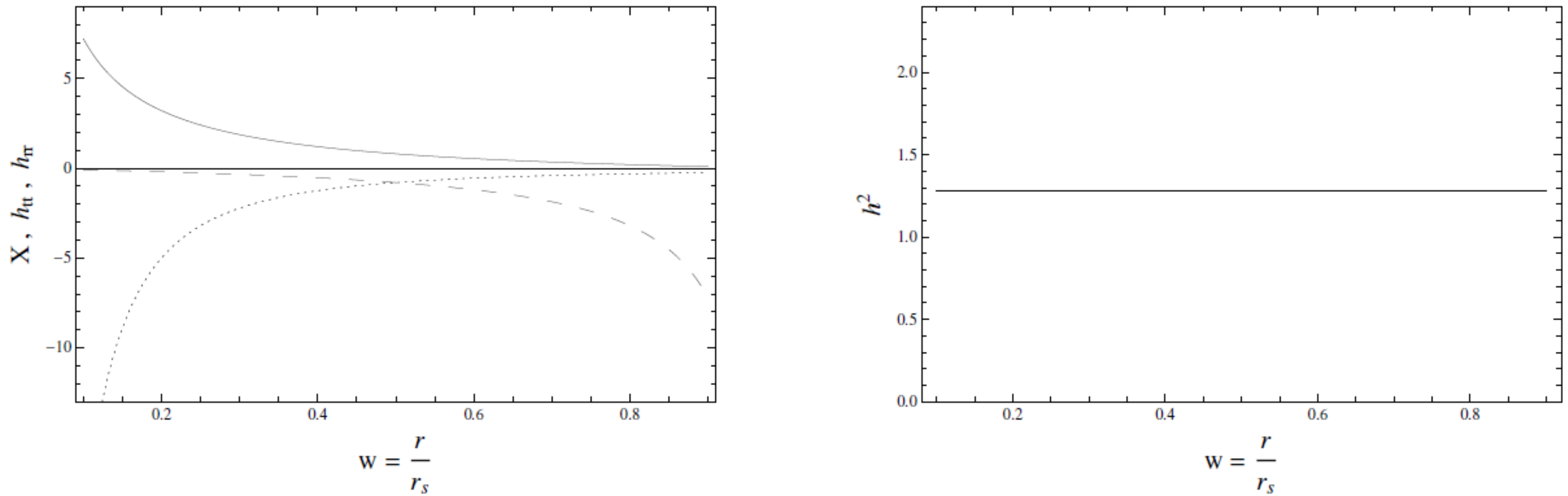

Figure 3. Curves for h_{tt} (black solid), h_{rr} (dashed) line and the dimensionful chemical potential μ (dotted line). On the right plot, h^2 is shown. As already emphasized, a whole family of solutions are obtained, parametrized by a real arbitrary constant. One can simply trade this constant for the value of φ.

Finite Norm

It is immediate to see that the solution found is of finite norm. The integral is

$$\int_0^{r_s} d\,r\; r^2\,[(h_t{}^t)^2 + (h_r{}^r)^2] \simeq h^2\; r_s^3\,. \tag{9}$$

- Then the integral over the interior of the BH of h^2 is given by $\frac{4\,\pi}{3}\,r_s^3\,h^2$.
- This quantity is related to the total number of gravitons of the condensate.
- h^2 would be related to its density in the BH interior that turns out to be constant.

Note that $h_{\alpha\beta}$ is dimensionless and that in order to have a properly normalized kinetic term we have to divide the Einstein-Hilbert action by M_P^2.

Possibly our more striking results are that the dimensionless chemical potential $X(w) = \mu(r)\, r^2$ stays constant and non-zero throughout the interior of the BH and that so does the quantity h^2 previously defined.

It is totally natural to interpret X as the variable conjugate to N, the number of gravitons.

How could we determine the value of N from our solution? Even after using a properly normalized $\hat{h}_{\alpha\beta} = M_P\, h_{\alpha\beta}$, the dimensions of (9) are not appropriate to deliver to us the value of N.

The quantity

$$\rho_{\hat{h}} \equiv \frac{1}{2}\, \hat{h}_{\alpha\beta}\, \frac{1}{\lambda}\, \hat{h}^{\alpha\beta}\,, \tag{10}$$

with λ being the graviton wave length, has the right ingredients to play the role of probability density in the present context. In the BEC $\lambda = r_s$.

- If we assume this, then

$$N = \frac{8\pi}{3} M_P^2 r_s^2 h^2 \qquad \Rightarrow \qquad r_s = \frac{1}{|h|}\sqrt{\frac{3}{8\pi}}\sqrt{N}\, L_P \tag{11}$$

  that agrees nicely with [DG2] under the maximum packaging condition $\lambda = r_s$.

- Recall that $h_\alpha{}^\alpha = \varphi$ is a constant that is entirely determined by the value of the dimensionless chemical potential X. The rest of relations can be basically derived from this.

- Indeed within the present philosophy we assign to each graviton in the BEC an energy $\varepsilon = 1/\lambda = 1/r_s$.

- The total energy of the system will be given by

$$E = \frac{1}{2}\int dV \varepsilon^2 \hat{h}^2 = \int dV \varepsilon\, \rho_{\hat{h}}\,. \tag{12}$$

- If the energy ε is a constant and given by $1/r_s$, then Eq. (9) has to be interpreted as the number of gravitons in the BEC, i.e. the integral of the graviton density in the interior of the BH.

- As seen above the dimensionless chemical potential X has a rather peculiar behaviour.
  - → It has a constant value inside the BH and it appears to be exactly zero outside. This behaviour is summarized in Figure 4.

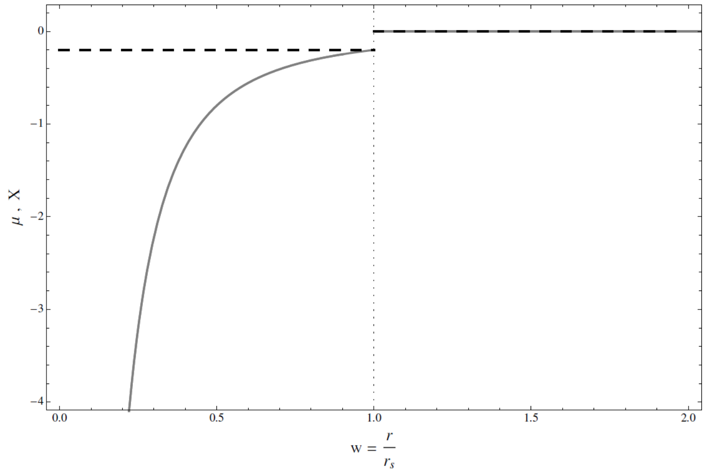

Figure 4. The behaviour of both chemical potentials, μ in solid line and the dimensionless X in dashed line, across the horizon of the BH (dotted line). In this case the jump at the horizon amounts to $\Delta X = \Delta \mu \simeq 0.2$.

- For a moment forget about the geometrical interpretation of BH physics and let us treat the problem as a collective many body phenomenon.
- From this figure it is clear why gravitons are trapped behind the horizon: the jump of the chemical potential at $r = r_s$ would prevent the 'particles' inside to reaching infinity.

BUT *one* of the modes can escape at a time without paying any energy penalty if the maximum packaging condition is verified.

Semiclassical calculation inspired by this picture; using $M \sim M_P \sqrt{N}$:

$$\frac{d\,M}{d\,t} \simeq M_P \frac{1}{2\sqrt{N}} \frac{d\,N}{d\,t} = \frac{1}{2\,r_s} \frac{d\,N}{d\,t}\,. \tag{13}$$

To estimate $d\,N/d\,t$ (which is negative) we can use geometrical arguments to determine the flux.

→ Assume that for a given value of r_s only one mode can get out (as hypothesized above) and that propagation takes place at the speed of light, we get:

$$\frac{d\,M}{d\,t} \simeq -\frac{3}{2} \frac{1}{r_s^2}\,. \tag{14}$$

→ This agrees with the results of [DG2] —for instance Eq. (35)— and yields $T \simeq 1/r_s$. The approximations made in the discussion could only modify the coefficient by a numerical factor of $O(1)$.

To determine the rate of variation of N we have to multiply the surface $(4\,\pi\,r_s^2)$ times the flux; i.e. the density of the mode times the velocity, assumed to be $c = 1$ in our units. Since the density of the mode is constant in the interior, it is just $3/(4\,\pi\,r_s^3)$.

- Charged Black Hole(Reissner-Nordstrom): Two Horizons.
- In the inner region, which is "classical", Is there a BE Condensate?
- We need a criteria to choose from two possible solution of our basic Gross-Pitaevskii equation:
  - Quasilocal Gravitational Energy(QLGE).A. Lundgren et al,PRD75 084026(2007)
- Tentative result(J.A., D. Espriu and L. Gabbanelli): The condensate lower the QLGE in the classical forbidden region, but it increases the QLGE in the classically allowed region.

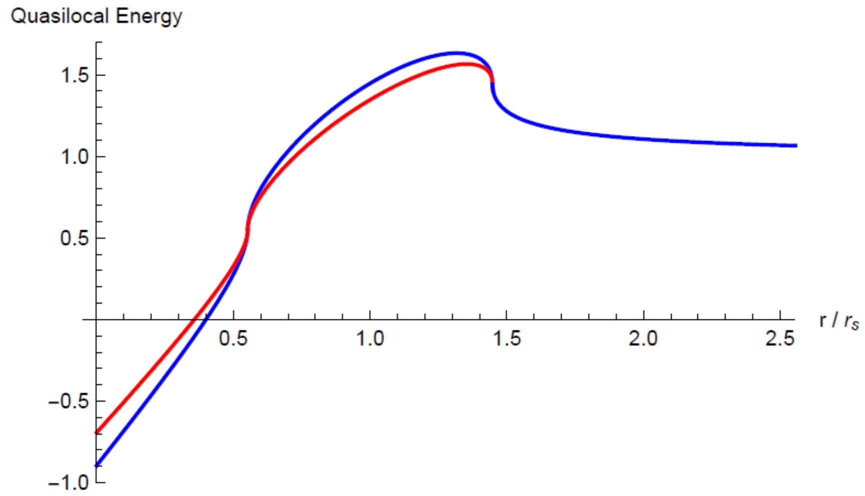

Figure 5. Brown-York QLGE for a Reissner-Nordstrom BH wit a charge $Q^2 = 0.8m^2$ and a wave function $\varphi = 0.4.$(red) and $\varphi = 0$(blue). Both axis are in units of the BH mass.

- We have some insights in the reformulation of a quantum theory of black holes in the language of condensed matter physics.The key point of the theory is to identify the black hole with a Bose-Einstein condensate of gravitons.
- We have conjectured the set of equations that play the role of the Gross-Pitaevskii non-linear equation; they are derived from the Einstein-Hilbert Lagrangian after adding a chemical potential-like term.
- The exterior solution has a zero chemical potential. On the contrary, in the interior we have found a *normalizable* result, which leads to a non-zero chemical potential of the BH that behaves as $1/r^2$. There is a finite jump of the chemical potential at the BH horizon.
- From the existence and knowledge of this solution, most relations obtained in [DG1, DG2] can be rederived.
- The relation between the number of gravitons and the geometric properties of the BH involve an a priori independent and tuneable parameter, the dimensionless chemical potential X (related to the mean-field wave function of the condensate φ).
- Quasi Local Gravitational Energy permits to discriminate between solutions of our Gross-Pitaevskii gravitational equation.
- Our approach is somewhat different from Dvali, Gómez and coworkers. We assume from the start the existence of a classical geometry background that acts as confining potential for the condensate.

- It is quite plausible that condensates of other quantum fields inside the BH horizon could be formed. While we do not expect much of a conceptual difference, it would be very interesting to see the similarities and differences with the case of quantum gravitons.

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