Scale-dependent FLRW Cosmology

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Introduction

$$\underbrace{\text{Gravity} + \text{Quantum Mechanics}}_{\text{Quantum gravity}}$$

↓

$$\underbrace{\text{Observables}}$$

1. Black holes
2. Cosmology

Introduction

$$\text{Approaches} \begin{cases} \text{Perturbative} \\ \text{Non perturbative} \\ \text{String Theory, Spin Foam, ...} \end{cases}$$

All those approaches should induce an effective action (at least in some limit)

$$\Gamma_k = \Gamma_k(G_k\ ,\ \Lambda_k\ ,\ \cdots)$$

Introduction

$$\Gamma_k$$

↓

Quantum corrections

↓

scale dependent couplings

↓

$$\{G_k \; , \; \Lambda_k \; , \; \cdots\}$$

Problems

We want to derive "observables", but two problems appear:

1. The **renormalization scale** k is arbitrary

2. The functional form of couplings **depend** on the approach used to get them.

How can we solve these problems?

Ideas and Techniques

To fix the **first** problem we impose

$$\frac{\delta \Gamma_k}{\delta k} = 0.$$

by use the so-called "Principle of minimal sensibility".

To fix the **second** problem we use symmetry of the problem, e.g.

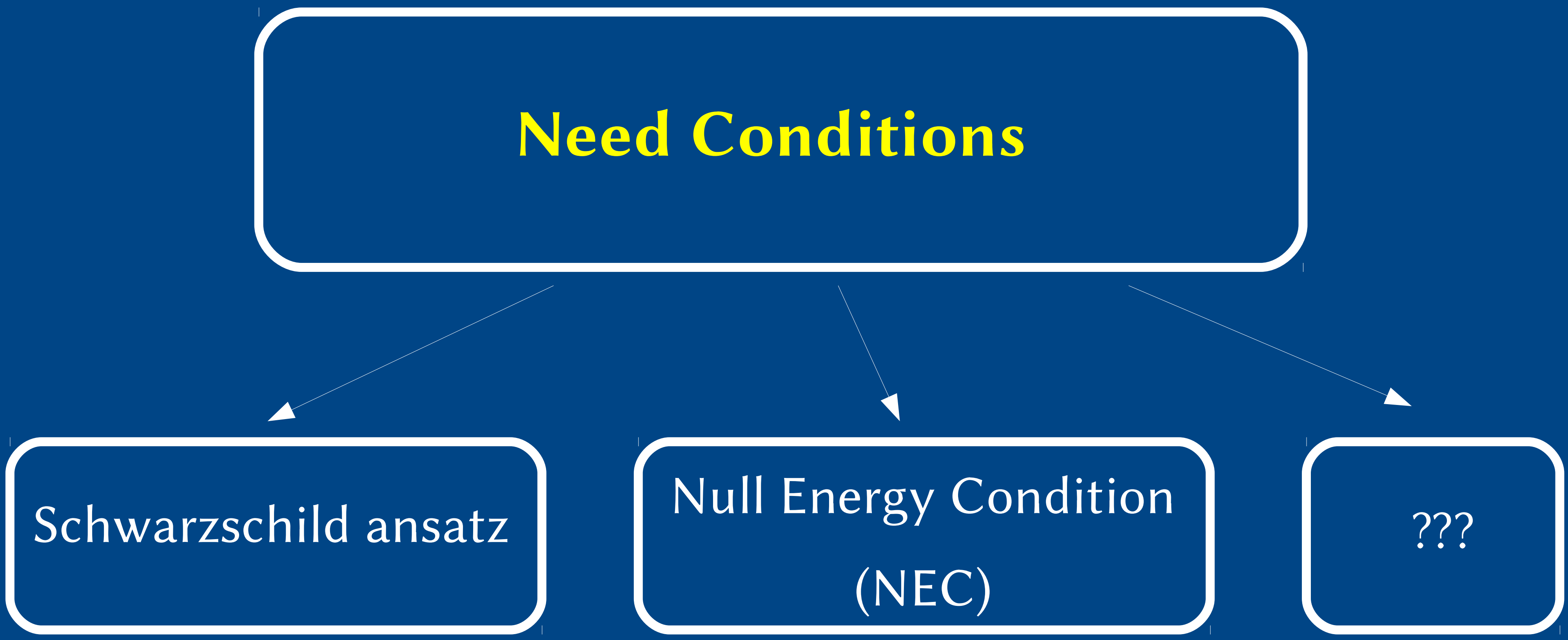
$$k = k(t), \qquad \therefore \qquad G_k \longrightarrow G(t)$$

We have symbolically

$$\mathcal{Q}(k) \longrightarrow \mathcal{Q}(k(t)) \longrightarrow \mathcal{Q}(t)$$

Ideas and Techniques

We have more unknown functions than equations!



Ideas and Techniques

$$\Gamma[g_{\mu\nu}, k]$$

$$\frac{\delta \Gamma_k}{\delta g_{\mu\nu}} = 0 \qquad \frac{\delta \Gamma_k}{\delta k} = 0$$

1 Solution

$$G_{\mu\nu} + \Lambda_k g_{\mu\nu} = 8\pi G_k T_{\mu\nu} \qquad R \frac{\partial}{\partial k}\left(\frac{1}{G_k}\right) = 2 \frac{\partial}{\partial k}\left(\frac{\Lambda_k}{G_k}\right)$$

$$\mathcal{Q}(k) \longrightarrow \mathcal{Q}(k(t)) \longrightarrow \mathcal{Q}(t)$$

missing one eq

Physical conditions

$$\{G(t), \Lambda(t), a(t), \cdots\}$$

Classical Action

The gravitational action in four dimensions is

$$I_0[g_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa_0}\Big(R - 2\Lambda_0\Big)\right],$$

which leads to

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda_0 g_{\mu\nu},$$

being Λ_0 and $\kappa_0 \equiv 8\pi G_0$ are the cosmological constant and the Einstein's constant respectively.

The line element for a FLRW universe looks like:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a_0(t)^2\left[\frac{1}{1-\kappa r^2}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

Classical FLRW Solution

And the corresponding equations are

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{1}{3}\Lambda_0 = 0$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda_0 = 0$$

With the solution given by

$$a_0(t) = \frac{1}{2}\alpha_0 e^{\frac{t}{\tau}}\left[1 + 3\alpha_0^{-2}\frac{\kappa}{\Lambda_0}e^{-2\frac{t}{\tau}}\right]$$

Where we have defined

$$\tau = \sqrt{\frac{3}{\Lambda_0}}$$

Action with Running Couplings

The gravitational action in four dimensions is

$$\Gamma[g_{\mu\nu}, k] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa_k}\Big(R - 2\Lambda_k\Big)\right].$$

Thus, varying with respect to the metric field

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda_k g_{\mu\nu} + \kappa_k T_{\mu\nu},$$

where the **effective** energy-momentum tensor is given by

$$\kappa_k T_{\mu\nu} = \kappa_k T^m_{\mu\nu} - \Delta t_{\mu\nu}$$

being the new term:

$$\Delta t_{\mu\nu} = G_k\Big(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\Big)G_k^{-1}$$

Action with Running Couplings

In the same way, by varying the action with respect to the scale-field $k(x)$ one obtains the equations

$$R \frac{\partial}{\partial k} \left(\frac{1}{\kappa_k} \right) = 2 \frac{\partial}{\partial k} \left(\frac{\Lambda_k}{\kappa_k} \right)$$

However, we don't use it! We prefer to use NEC!

On the other hand, we take into account that:

$$\mathcal{O}(k) \longrightarrow \mathcal{O}(k(t)) \longrightarrow \mathcal{O}(t)$$

And solve with respect to the temporal variable.

Equations with Running Couplings

The metric has a general form:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left[\frac{1}{1-\kappa r^2} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2 \right],$$

And the effective equations become

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{1}{3}\Lambda(t) = \frac{1}{3}\kappa(t)\rho(t),$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -\kappa(t)p(t).$$

Equations with **Running Couplings**

Where the effective fluid parameters are given by

$$\frac{1}{3}\kappa(t)\rho(t) \equiv \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)$$

$$-\kappa(t)p(t) \equiv -2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\ddot{G}}{G}\right) + 2\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)$$

Thus, we have just **two** equations to determine **three** variables. This means we need to add a new ingredient to complete the set.

NEC

The NEC applied to the effective energy-momentum tensor is

$$T_{\mu\nu}^{\text{effec}} \ell^\mu \ell^\nu = -\Delta t_{\mu\nu} \ell^\mu \ell^\nu = C.$$

The vector field ℓ^μ satisfies the geodesic equation, namely

$$\frac{\mathrm{d}\ell^\mu}{\mathrm{d}t} + \Gamma^\mu_{\nu\sigma} \ell^\nu \ell^\sigma = 0$$

A "straightforward" ansatz is

$$\ell^\mu \equiv \{\ell^0(t), \ell^1(t,r), 0, 0\}$$

To obtain:

$$\ell^\mu = C_0 \; a^{-1} \{1, (1-\kappa r^2)^{1/2} \; a^{-1}, 0, 0\}.$$

NEC

Thus, using the result of ℓ^μ we finally become to

$$-2\left(\frac{\dot{G}}{G}\right)^2+\left(\frac{\ddot{G}}{G}\right)-\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)=\frac{C}{C_0}a^2$$

Hereafter, we will focus on the particular case $C=0$.

Scale-dependent FLRW Solution ($C = 0$ and $\kappa = 0$)

The simplest solutions are given below

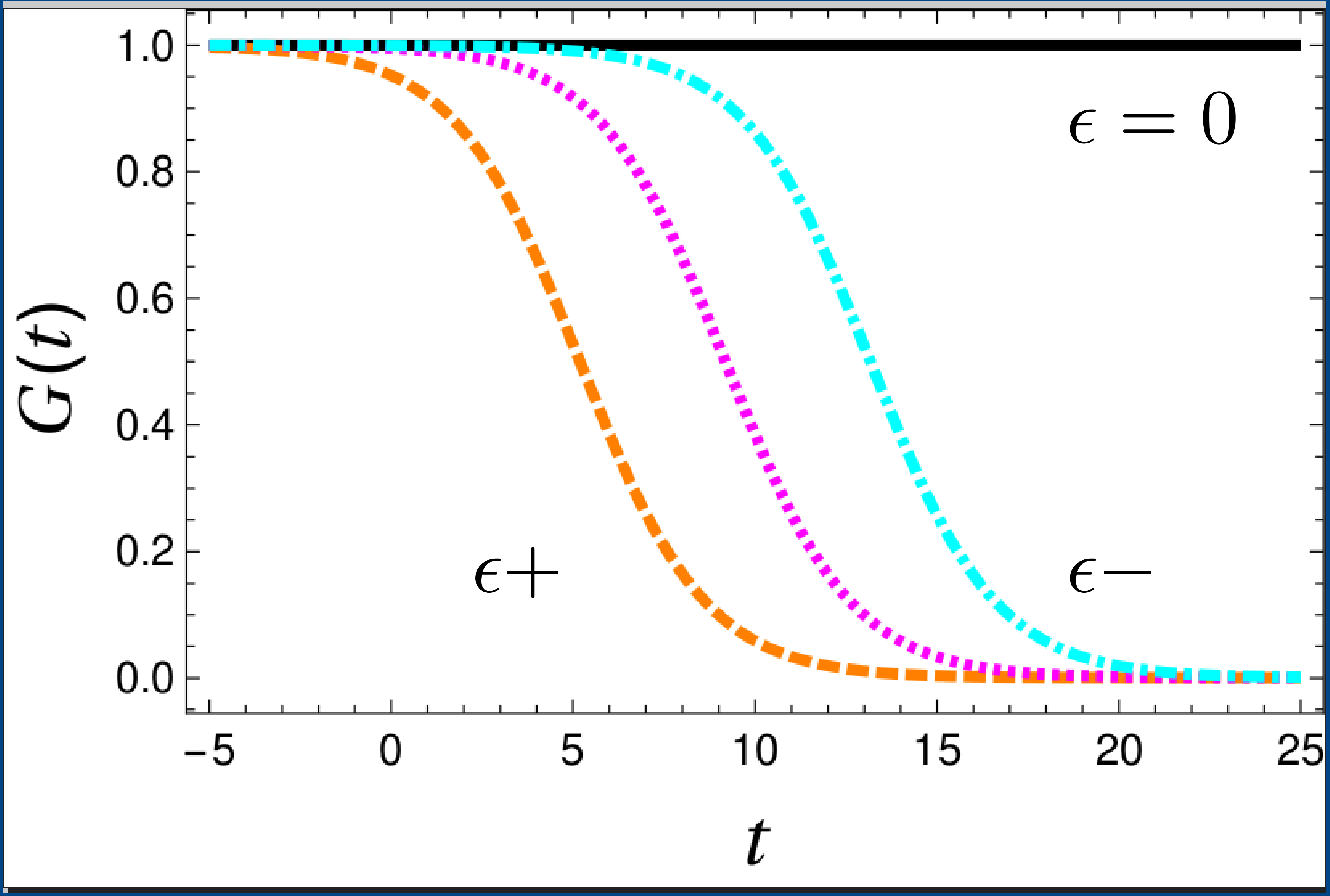
$$a(t) = \frac{1}{2}\alpha_0 e^{t/\tau}$$

$$G(t) = \frac{G_0}{1 + \epsilon a(t)},$$

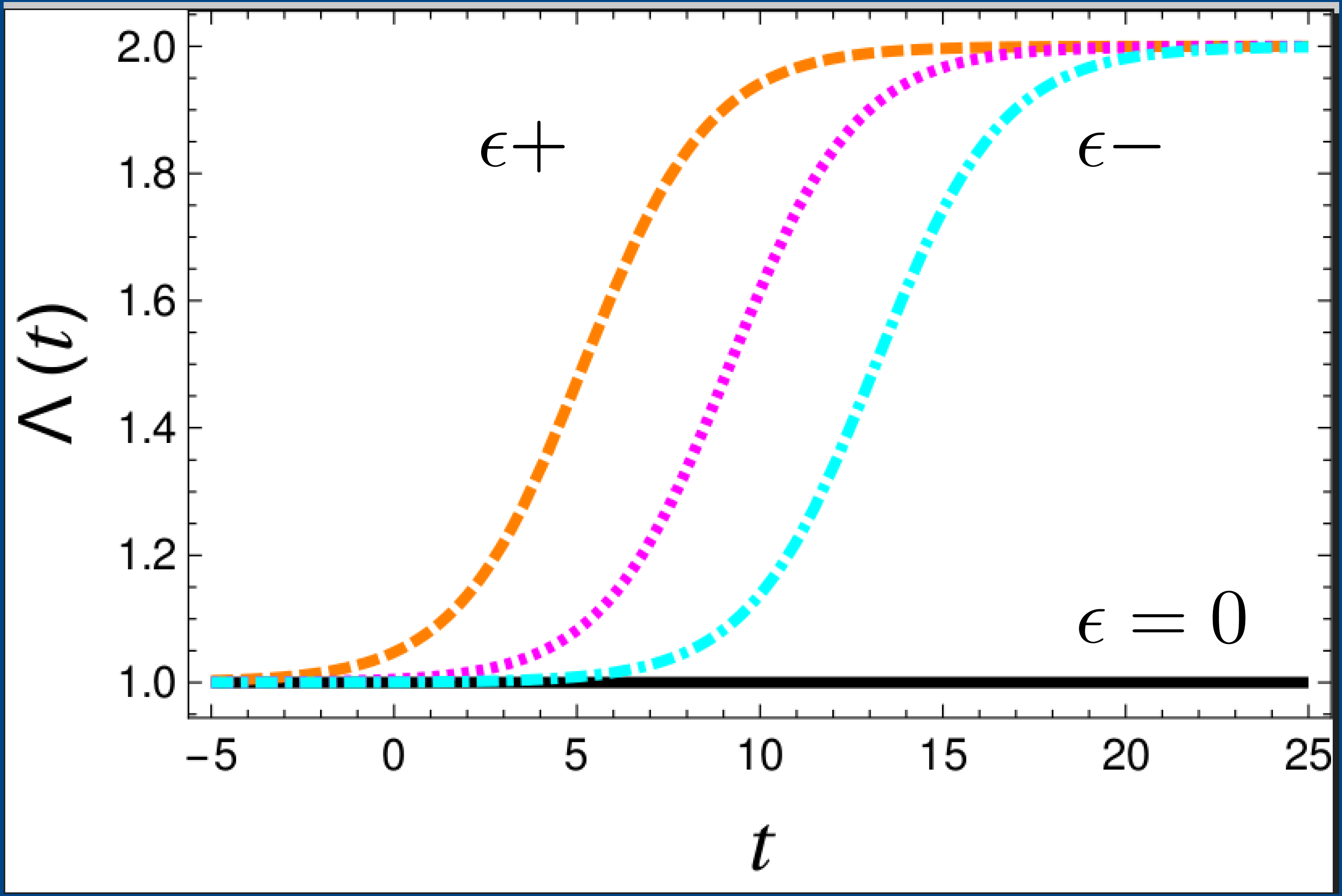
$$\Lambda(t) = \Lambda_0 \left(\frac{1 + 2\epsilon a(t)}{1 + \epsilon a(t)} \right)$$

Where the scale setting was inspired by the classical case.

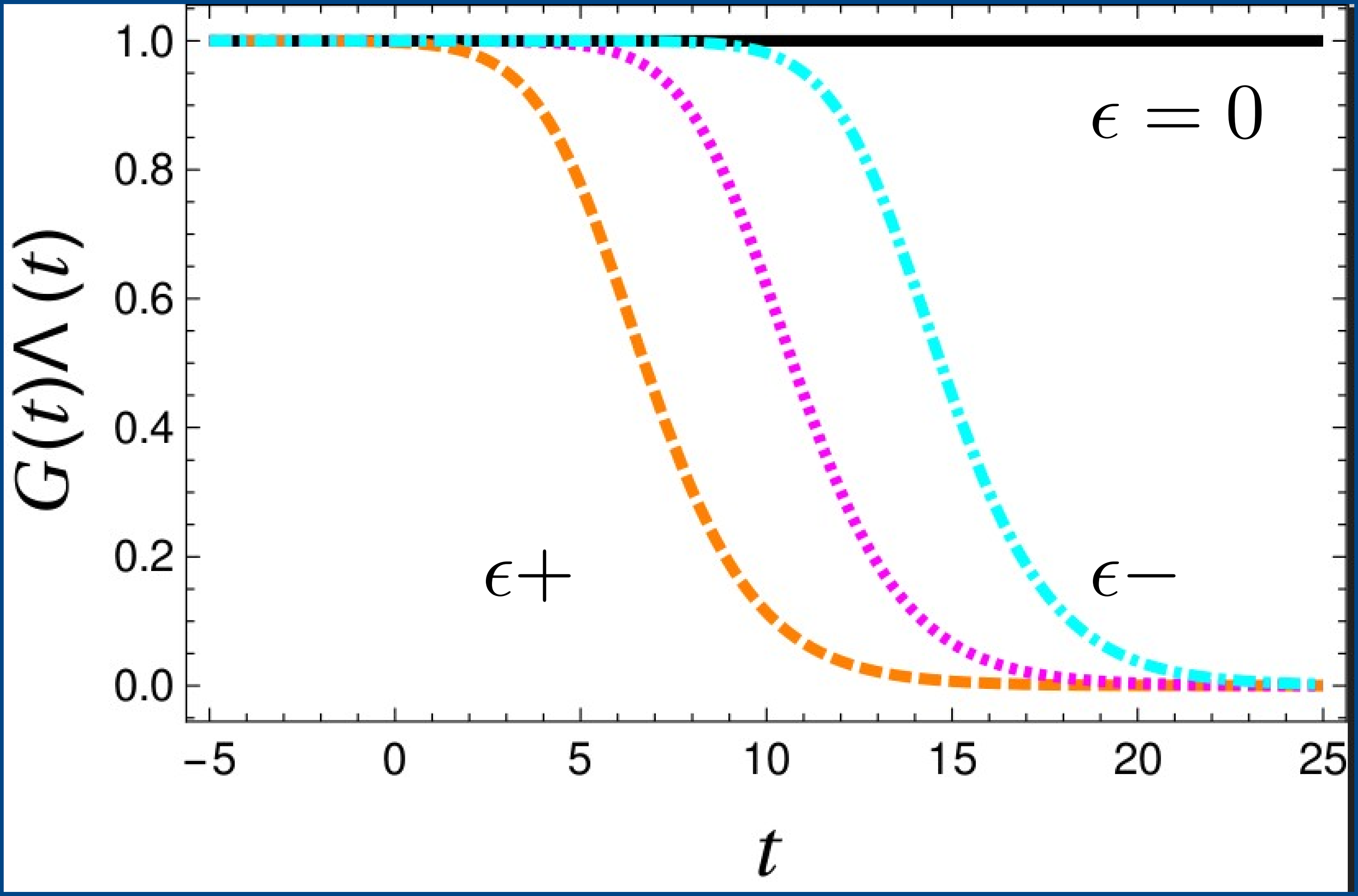
Numerical Results A



Numerical Results B



Numerical Results C



Message

1- The running of the gravitational coupling introduces effective fluid parameters. After combine it with NEC, we obtain an exact analytical solution.

2- Integration constants play a crucial role in this scale dependence approach!

3- Scale-dependence in the cosmological context might help to alleviate the cosmological constant problem!