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Massive stealth fields from deformation method

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Based in the collaboration:

Cristian C. Quinzacara, Paola Meza, Almeira Sampson & M.V. Eur. Phys. J. C (2018) 78. [arXiv:1805.04621]

1 Introduction

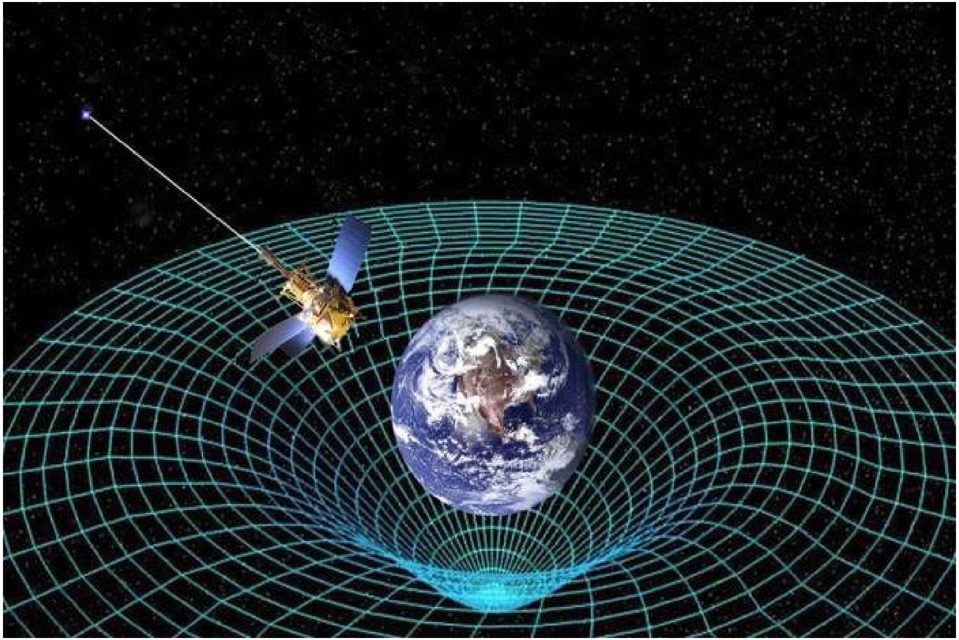
It is generally believed that matter curves the space since given an action principle of pure gravity and matter,

$$S[g,\phi] = S_G[g] + S_M[g,\phi], \tag{1.1}$$

it happens that the equations of motion of the metric tensor g,

$$\frac{\delta S[g,\phi]}{\delta g^{\mu\nu}} = \sqrt{-g}\Big(\underbrace{G_{\mu\nu}[g]}_{\text{Einstein tensor}} - \underbrace{T_{\mu\nu}[g,\phi]}_{\text{Hilbert EM tensor}} \Big) = 0. \tag{1.2}$$

Hence matter curves the spacetime...



However, there are many examples in the literature where this is not the case: Alvarez et al. (2016, 2017), Ayon-Beato et al. (2005, 2006), Ayón-Beato et al. (2013, 2015, 2018), Hassaine (2014), Smolić (2018).

Here the energy momentum tensor vanishes for non-trivial solutions of the matter field equations. In this case, the geometry is not curved in the presence of non-trivial matter fields configuration. These configurations are dubbed *Stealth Fields.*

The purpose of this paper is to **present a method to construct models with massive stealth fields** in arbitrary backgrounds.

$$\underbrace{S_M[\phi]}_{\text{"original action"}} \qquad \rightarrow \qquad \underbrace{S_M^{\theta}[\phi]}_{\text{"deformed action"}}$$

The scalar field action is "deformed" into a new action with deformation parameter θ.

The deformed action possesses:

- a massive stealth mode of mass θ^{-1}
- other modes with rescaled effects on the gravity background

2 A toy model

Let $f(x)$ be any function of $x \in \mathbb{R}$ with a saddle point at $x = 0$,

$$\left.\frac{df}{dx}\right|_{x=0} = 0\,, \tag{2.3}$$

and let $y(x)$ another function which possess, for definiteness, two zeros at $x = 0$ and $x = 1$,

$$y(0) = 0, \qquad y(1) = 0\,. \tag{2.4}$$

We define the composition of functions $F(x) := f(y(x))$, which inherits from the parent functions f and y the properties,

$$\left.\frac{dF}{dx}\right|_{x=0} = \left.\frac{dF}{dx}\right|_{x=1} = 0\,, \tag{2.5}$$

such that it has two saddle points, at $x = 0, 1$. It is straightforward to prove this. We can use the chain rule to evaluate dF/dx at $x = 0, 1$,

$$\left.\frac{dF}{dx}\right|_{x=0,1} = \left.\left(\frac{df(y)}{dy}\frac{dy(x)}{dx}\right)\right|_{x=0,1} = \left.\frac{df(y)}{dy}\right|_{y=0} \left.\frac{dy(x)}{dx}\right|_{x=0,1}\,.$$

Here the $df(y)/dy|_{y=0}$ vanishes because from (2.4) y takes zero-value and from (2.3) the derivative of f vanishes when the argument is zero.

Hence from an arbitrary function $f(x)$ with saddle point at $x = 0$ (2.3) we can construct another arbitrary function $F(x)$ with saddle points at the kernel of the map $y : \mathbb{R} \to \mathbb{R}$, in this example $x = 0, 1$.

3 θ-deformation of scalar field theories

We shall apply now the same logic in the language of functional calculus, with the dictionary:

$$x \to \phi, \qquad y(x) \to \phi^\theta[g,\phi], \qquad f(x) \to S_M[g,\phi], \qquad F(f(x)) \to S_M[g,\phi^\theta[g,\phi]].$$

Here the scalar field ϕ is the analogous of x, ϕ^θ is a funcional map from ϕ, $S_M[g,\phi]$ is the action principle of a scalar field (with saddle points) analogous of f, and $S_M[g,\phi^\theta[g,\phi]]$ is a new action principle obtained from a composition of the functionals S_M and ϕ^θ analogous of $F(f(x))$.

The field transformation to be considered is:

$$\phi^\theta[g,\phi] = (1-\theta^2\Box)\phi, \tag{3.6}$$

where θ is a real-valued parameter. Here

$$\Box\phi := \frac{1}{\sqrt{(-g)}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)\,,$$

is the Laplace-Beltrami operator acting upon ϕ.

Therefore the kernel of the deformation map $\phi \to \phi^\theta[g, \phi]$ consists of the trivial vacuum $\phi = 0$ and the massive configuration $\phi = \phi_m$ with mass $m = \theta^{-1}$:

$$\phi^\theta[g, \phi_m] = (1 - \theta^2 \Box)\phi_m = 0\,, \qquad m = \theta^{-1}\,, \tag{3.7}$$

which is equivalent to the Klein-Gordon equation in curved space.

4 Scalar field action deformation

We shall consider theories of the type,

$$S[g, \phi] = S_G[g] + S_M[g, \phi]. \tag{4.8}$$

$$\delta\phi \qquad \to \qquad \Upsilon[g, \phi] := \frac{\delta S_M[g, \phi]}{\delta\phi} = \frac{\delta S[g, \phi]}{\delta\phi} = 0\,, \tag{4.9}$$

Hence the E.o.M are:

$$G_{\mu\nu}[g] - T_{\mu\nu}[g, \phi] = 0, \qquad \Upsilon[g, \phi] = 0, \tag{4.10}$$

Now we produce a "deformed" field theory,

$$S^\theta[g,\phi] = S_G[g] + S_M[g,\phi^\theta], \qquad \phi^\theta[g,\phi] := (1-\theta^2\Box)\phi \tag{4.11}$$

with E.o.Ms:

$$G_{\mu\nu}[g] - \widetilde{T}_{\mu\nu}[g,\phi] = 0, \qquad \widetilde{\Upsilon}[g,\phi] = 0, \tag{4.12}$$

We claim that:

$$\widetilde{\Upsilon}[g,\phi] = 0, \qquad G_{\mu\nu}[g,\phi] = 0\,, \qquad \widetilde{T}_{\mu\nu}[g] = 0\,, \tag{4.13}$$

for the massive mode of mass $m = \theta^{-1}$.

Indeed, in reference **Cristian C. Quinzacara, Paola Meza, Almeira Sampson & M. V. Eur. Phys. J. C (2018) 78. [arXiv:1805.04621]** we proved that the following statements are true:

- Let $\phi = 0$ (trivial vacuum) be a saddle point of $S_M[g, \phi]$ (i.e. a solution of the equations of motion), then the deformed action $S_M[g, \phi^\theta]$ has a saddle point at $\phi^\theta = 0$, i.e. for massive ϕ of mass $m = \theta^{-1}$.

- The energy momentum tensor of the massive ϕ ($m = \theta^{-1}$) vanishes (ϕ is a stealth field), hence it does not curve the spacetime.

The proof is very general, and it uses the functional generalization of the chain rule,

$$\frac{\delta F[G[f]]}{\delta f(y)} = \int d^D z \frac{\delta F[G[f]]}{\delta G[f](z)} \frac{\delta G[f](z)}{\delta f(y)}\,, \tag{4.14}$$

to be confronted with our toy model.

5 Example 1:

The simplest example we can imagine is:

$$\widetilde{S}[g,\phi] = -\frac{M^2}{2}\int d^Dx\,\sqrt{-g}\,\phi^2\,, \tag{5.15}$$

The equation of motion for the scalar field is, $\phi = 0$, and its energy momentum tensor vanishes for this solution.

The deformed action is:

$$\widetilde{S}[g,\phi^\theta] = -\frac{M^2}{2}\int d^Dx\,\sqrt{-g}(\phi^\theta[g,\phi_m])^2 = -\frac{M^2}{2}\int d^Dx\,\sqrt{-g}\Big(\phi^2-2\theta^2\,\phi\Box\phi+\theta^4\,(\Box\phi)^2\Big)\,, \tag{5.16}$$

where we replaced the original field ϕ by ϕ^θ.

This action can be regarded as a degenerated (single-parameter) case of the two-parametric fourth-order action principle analyzed in Hawking and Hertog (2002).

The equation of motion for ϕ yields:

$$(1-\theta^2\Box)^2\phi = 0\,. \tag{5.17}$$

Clearly satisfied by massive ϕ of mass θ^{-1}. A direct calculation of the energy momentum tensor produces:

$$\widetilde{T}_{\mu\nu}[g,\phi] = -\frac{1}{4}M^2 g_{\mu\nu}\Big((1-\theta^2\Box)\phi\Big)^2 - \frac{1}{2}M^2\theta^2 g_{\mu\nu}\Box\phi(1-\theta^2\Box)\phi$$
$$+\frac{1}{2}M^2\theta^2\Big(\delta^\rho_\mu\delta^\sigma_\nu + \delta^\rho_\nu\delta^\sigma_\mu - g_{\mu\nu}g^{\rho\sigma}\Big)\nabla_\rho\phi\nabla_\sigma\Big((1-\theta^2\Box)\phi\Big)\,, \tag{5.18}$$

which is evidently zero for the massive $\phi = \phi_m$.

6 Example 2: Deformation of the massive field action

In the case of the scalar field ϕ, with mass M the matter action principle is given by:

$$S_M[g,\phi] = -\frac{1}{2}\int d^D x \sqrt{-g}\,\left(\nabla^\mu\phi\nabla_\mu\phi + M^2\phi^2\right). \tag{6.19}$$

With E.o.M:

$$\left(\Box - M^2\right)\phi = 0\,. \tag{6.20}$$

The deformed action principle is

$$S_M[g,\phi] = -\frac{1}{2}\int d^Dx\,\sqrt{-g}\,\left(\nabla^\mu\phi^\theta\nabla_\mu\phi^\theta + M^2(\phi^\theta)^2\right)\,, \qquad \phi^\theta[g,\phi] = (1-\theta^2\Box)\phi. \tag{6.21}$$

We obtain the E.o.M (from variation with respect to ϕ):

$$\left(1-\theta^2\Box\right)^2\left(\Box - M^2\right)\phi = 0\,, \tag{6.22}$$

which has both solutions, of mass θ^{-1} and of mass M.

The energy momentum tensor is given by:

$$\widetilde{T}_{\mu\nu}[g,\phi] = -\frac{1}{2}\left(\frac{1}{2}g_{\mu\nu}\left(\nabla^{\rho}\phi^{\theta}\,\nabla_{\rho}\phi^{\theta} + M^{2}(\phi^{\theta})^{2}\right) - \nabla_{\mu}\phi^{\theta}\,\nabla_{\nu}\phi^{\theta}\right) + \frac{1}{2}\theta^{2}g_{\mu\nu}\Box\phi\,\left(\Box - M^{2}\right)\phi^{\theta}$$
$$- \frac{1}{2}\theta^{2}\left(\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu} + \delta^{\rho}_{\nu}\delta^{\sigma}_{\mu} - g_{\mu\nu}g^{\rho\sigma}\right)\nabla_{\rho}\phi\nabla_{\sigma}\left(\Box - M^{2}\right)\phi^{\theta}\,, \quad (6.23)$$

which vanishes for the field of mass θ^{-1}, because $\phi^{\theta} = (1-\theta^{2}\Box)\phi = 0$.

Hence ϕ of mass θ^{-1} is **stealth**.

Now we analyze the energy-momentum for ϕ_M. First note that:

$$\phi^\theta[g, \phi_M] := (1 - \theta^2 \Box)\phi_M = \lambda \phi_M , \qquad \lambda := 1 - \frac{M^2}{m^2} = 1 - M^2\theta^2 \, , \tag{6.24}$$

Hence the energy-momentum tensor for this solution is (6.23) given by:

$$\widetilde{T}_{\mu\nu}[g, \phi_M] = -\frac{1}{2}\lambda^2 \left(\frac{1}{2} g_{\mu\nu} \left(\nabla^\rho \phi_M \nabla_\rho \phi_M + M^2 \phi_M^2 \right) - \nabla_\mu \phi_M \nabla_\nu \phi_M \right) = \lambda^2 \, T_{\mu\nu}[g, \phi_M] \, , \tag{6.25}$$

where $T_{\mu\nu}[g, \phi_M]$ is the energy momentum tensor provided by the standard massive field action (6.21).

Therefore, the energy-momentum tensor of the original gravity-matter system is rescaled by factor λ^2 in the deformed theory.

This can be interpreted also as a rescaling of the Newton coupling constant by a factor, $G_{\mathtt{N}} \to \lambda^2 G_{\mathtt{N}}$, in the standard nomenclature.

$\rightarrow$ Hence the mass of the stealth field (equivalently the deformation parameter) can be used to smooth or amplify the effects of the massive field of mass M on the gravitational background.

7 Overview and remarks

- We can construct a wide class of scalar field action principles in curved space which admits massive stealth configurations.

- The existence of stealth configurations may produce cosmological effects, by means of the energy-momentum tensor rescaling of regular matter fields.

Thank you !

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