The Evolution of EW Theory

The Weak Mixing Angle

An Ode to $\sin^2\theta_W$

A Personal Perspective

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For up-to-date EW Talks see

Ayres Freitas “Precision Electroweak Theory”

Vincenzo Cirigliano. “Atoms, Molecules, Nuclei”

- Robin Erbacher “Precision EW at Colliders”
- Krishna Kumar “Low Energy EW Probes”

et al.

Happy Anniversaries

50 yrs of Electroweak Unification (S. Weinberg 1967)

40 yrs of PV eD Scattering (C. Prescott et al. 1978)

1979 Nobel Prize: Glashow, Salam & Weinberg

On A More Personal Level

25 yr ago I lectured on polarized electron scattering at SLAC. Met A. Czarnecki (fresh PHD). I said PV e-e Moller Experiment was interesting (based on work with E. Derman 1979) but impossible. Recommended he do 2 loop EW muon g-2.

15 yr ago E158 at SLAC Measured PV Moller Asymmetry

$$A_{LR}(ee) = -131(14)(10)\times 10^{-9}$$

Outline

1) Ancient History
2) Some Early EW Radiative Corrections
3) Beautiful Natural Relations & The Legacy of GUTS
4) Parity Violating Weak Neutral Currents

   Atomic Parity Violation vs Polarized Electron Scattering

5) The Age of Precision

   Z pole & Low Energy Studies: SUSY GUTS

6) Running $\sin^2\theta_W(Q^2)$
7) $\sin^2\theta_W$ Outlook: ***JLAB & MESA***

1) Ancient Electroweak History

- **Glashow** (1961)

Had SU(2)xU(1) Algebra

"No Higgs Mechanism"

m_W & m_Z arbitrary (put in by hand)

$\gamma = B\sin\theta + W^0\cos\theta$ massless photon

$Z = W^0\sin\theta - B\cos\theta$ massive neutral gauge boson

Weak Neutral Currents Required!

a mixing angle appearance

(Given Little Attention)

- **<u>Weinberg</u>** (1967) $SU(2)_L x U(1)_Y$ + <u>Higgs Mechanism</u>

  generates $W^{\pm}$,Z, lepton masses

  spontaneous sym. Breaking

  predicts fundamental scalar H

$$\mathbf{m_W = m_Z \cos\theta_W \ \& \ e = g \sin\theta_W \qquad \tan\theta_W = g'/g}$$

implicit mixing angle

Weinberg speculated that the theory might be renormalizable!

Weak Neutral Currents right around the corner!

Little Attention until ‘tHooft proved renormalizability (1971)

Weak Neutral Currents Discovered (1972) Neutrino scattering!

$\theta \rightarrow \theta_W$ **<u>W</u>einberg - Glasho<u>w</u> or <u>W</u>eak Mixing Angle**

<u>Most Important Electroweak Parameter!</u>

2.) **Early Radiative Corrections**

Muon Anomalous Magnetic Moment: $a_\mu^{EW}=\Delta(g_\mu-2)/2$

Jackiw & Weinberg (1972) Bars & Yoshimura (1972)

Altarelli, Cabibbo & Maiani (1972) Fujikawa,Lee & Sanda (1972)

Bardeen, Gastmans & Lautrup (1972)

Finite because $g_W=2$ three 1 loop diagrams W, Z & H

$a_\mu^{EW}(\text{1 loop})=5G_F m_\mu^2/24(2^{½})\pi^2[1+1/5(1-4\sin^2\theta_W)+O(m_\mu^2/M^2)]=1\underline{95\text{x}10}^{-11}$

non-linear gauge, Dimensional, Regularization renormalization 1678

2 loops (Czarnecki, Krause, WJM 1995) -20% reduction to $154\text{x}10^{-11}$

Currently: **$\Delta a_\mu=a_\mu^{exp}-a_\mu^{SM}=276(73)\text{x}10^{-11}$ (*3.7σ*) deviation**

New Problem: **$a_e^{exp}-a_e^{SM}=-87(36)\text{x}10^{-14}$ (2.4σ) *deviation***

One & two loop (1678 diagrams) for muon g-2

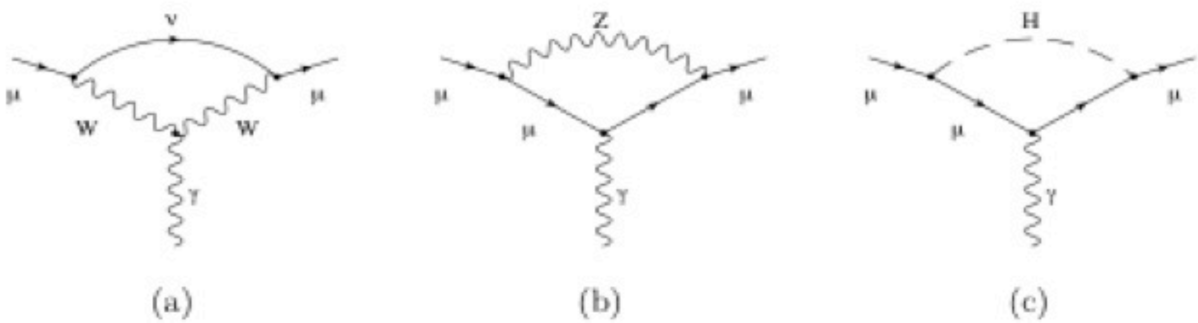


Figure 2: One-loop electroweak radiative corrections to a_μ.

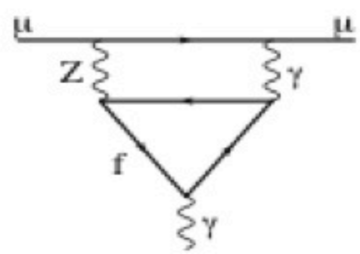


FIG. 3: Effective $Z\gamma\gamma^*$ coupling induced by a fermion triangle, contributing to $a_\mu^{\rm EW}$.

Other Radiative Corrections Examples

Neutron beta Decay vs Muon Decay

A. Sirlin (1973) Finite Prediction

Flavor changing (loop induced) weak neutral currents

M. Gaillard & B. Lee (1974) Applied GIM Mechanism

+ Others (Not so many)

3) ***A Beautiful Relation***

SU(2)xU(1) + Higgs Doublet + Renormalizability

- ***$\sin^2\theta^0_W = 1-(m^0_W/m^0_Z)^2 = (e^0/g^0)^2$ Natural Bare Relation***

Radiative (Loop) Corrections - Finite & Calculable!
Demonstrated by Bollini, Giambiagi & Sirlin (1972)

WJM(1974) Thesis: Finite Parts Calculated

but model incomplete: Charm, Color, 3rd Generation?

time not quite right for full EW Radiative Corrections

Main effect: α=1/137 → $\alpha(m_Z)$~1/128 Large 7% Effect
Increased m_W and m_Z predictions by ~ 3.5%

1974: A Great Year For Unification

1974 Classics

- ***Pati & Salam:***

  Lepton Number as the Fourth Color

  4490 Citations

- ***Georgi & Glashow:***

  Unity of All Elementary Particle Forces

  4594 Citations

- ***Georgi, Quinn & Weinberg:***

  Hierarchy of Interactions in Unified Gauge Theories

  (Running of $\sin^2\theta_W(Q^2)$) 3/8 → 0.2

  1819 Citations

Natural Consequence – Proton Decay!

Grand Unified Theories: SU(5), SO(10), E_6…

$g^0_3 = g^0_2 = g^0_1 = g^0_{GUT}$ For $SU(3)_c x SU(2)_L x U(1)_Y$

$\sin^2\theta^0_W = 3/8$ Finite Rational Number!

Quarks & Leptons: 3 Mixed Families

10 + 5* + 1 of SU(5), 16 of SO(10), 27 of E_6…

Predicted $\sin^2\theta_W^{exp} \approx 0.21$

Provide a natural extension of the Standard Model

Explain: Charge-Color Quantization, quark-lepton unification…

Easily include (suggest) supersymmetry

Superstring connection

Part of the Particle Physics Vernacular

4) Parity Violating Weak Neutral Currents

- **Early Days**: By 1975 the $SU(2)_L xU(1)_Y$ structure of the Glashow-Weinberg-Salam Model was nearly established.

  Predicted Weak Neutral Currents seen in neutrino scattering at CERN! But did the NC have the right coupling?

$$g/\cos\theta_W Z^\mu f\gamma_\mu(T_{3f}-2Q_f\sin^2\theta_W - T_{3f}\gamma_5)f$$

A New Form of Parity Violation Predicted!

Non Maximal but Distinctive & Large

γ-Z Interference → Parity Violation Everywhere!

Atomic Parity Violation (APV)

- **$Q_W(Z,N) = Z(1-4\sin^2\theta_W)-N$** Weak Charge

  θ_W=Weak Mixing Angle

$Q_W(p)=1-4\sin^2\theta_W=0.07$ (Difficult) (Recently measured in ep scattering)
$Q_W(^{209}Bi_{83}) = -43 -332\sin^2\theta_W = -\underline{127}$
Bi Much Larger but Complicated Atomic Physics
Originally APV not seen in Bi (1977)→ SM Ruled Out?

$-29 \le Q_W(^{209}Bi_{83}) \le 16$ (Washington)
$-20 \le Q_W(^{209}Bi_{83}) \le 74$ (Oxford)

Note $-230 \le Q_W(^{209}Bi_{83}) \le -87$ (Novosibirsk 1978)

(Later clearly seen in Tl, Bi, **Cs (Carl Wieman)**…)

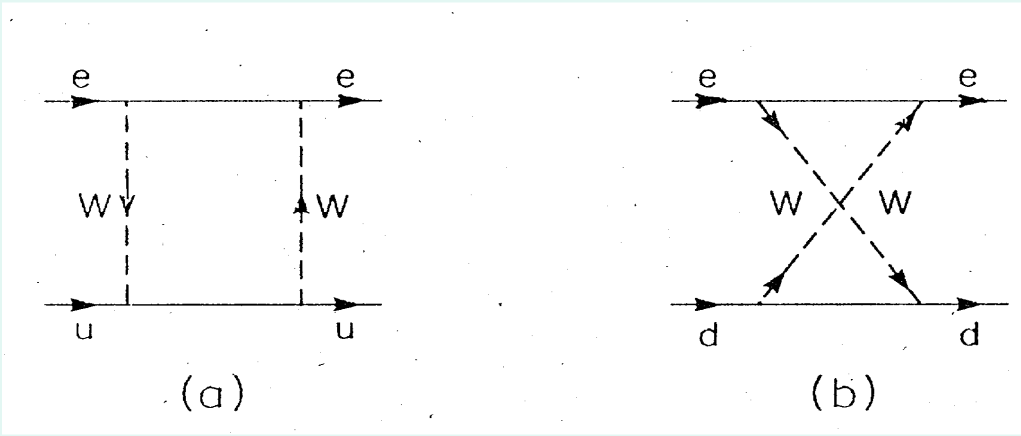
Loop Induced Parity Violation
WJM & A. Sanda (1978)

WW Box diagram relatively large ~ 10% of SM.

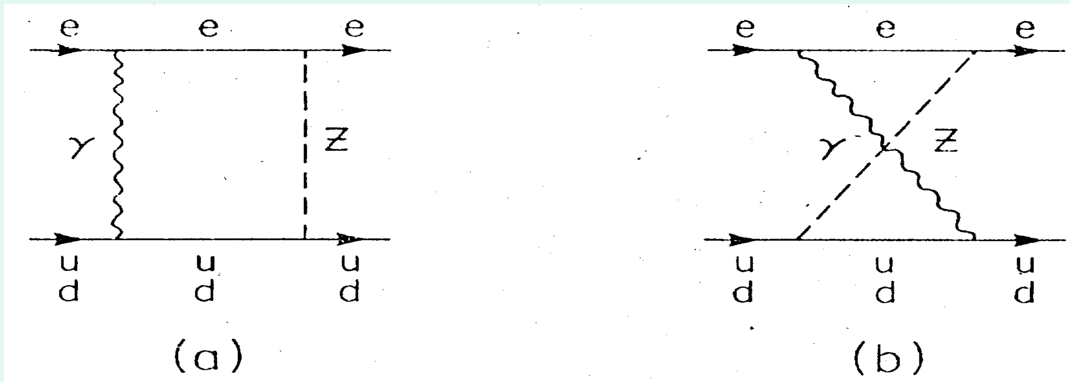
Due to quark coherence effect.

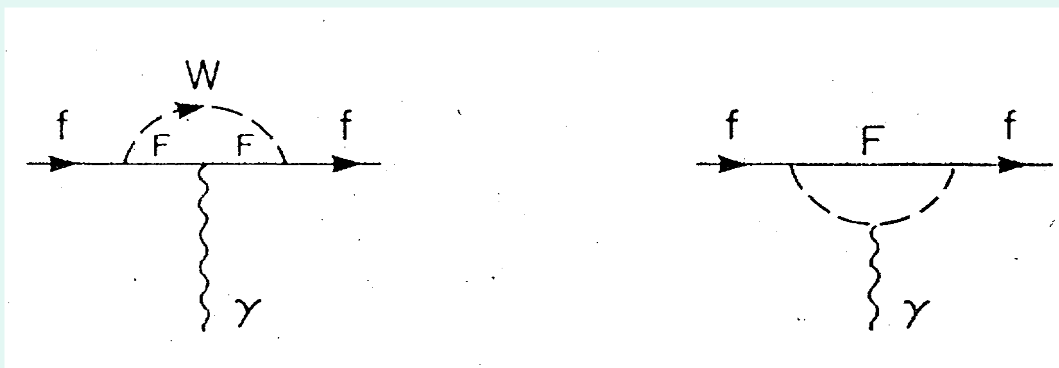
Must be there in PV experiments

Set the stage for complete Radiative Corrections



Other Interesting Loops





- **L. Wolfenstein**: "*Eventually, Atomic Physicists will make extremely precise APV measurements*"

words of encouragement

- 1982-84 A. Sirlin and WJM calculate full radiative corrections to atomic parity violation

Theoretically very clean

Precise Q_W Predictions! ±0.2%!

Wait for Experiment

Carl Wieman $Q_W(Cs)^{exp}$=**-73.16(28)(20)**

1978 SLAC Polarized eD Asymmetry (Charles Prescott, Spokesman)

$e+D \rightarrow e+X$ γ-Z Interference

$A_{RL} = \sigma_R - \sigma_L / \sigma_R + \sigma_L \propto 2\text{x}10^{-4} Q^2 \text{GeV}^{-2} (1-2.5\sin^2\theta_W) \sim 10^{-4}$ Expected

Exp. Gave $A_{RL}^{exp} = 1.5\text{x}10^{-4} \rightarrow \sin^2\theta_W = \underline{0.21(2)}$

Confirmed $SU(2)_L \text{x} U(1)_Y$ SM!

$\pm 10\%$ Determination of $\sin^2\theta_W$ Precision!

Major Discovery - Nobel Prize Material!

Evidence for Grand Unification?

Georgi & Glashow; Georgi, Quinn & Weinberg (1974)

$\sin^2\theta^0_W = 3/8$ Rational Number!

Seemed to agree with GUTS (SU(5), SO(10)…)

$\sin^2\theta_W(\mu) = 3/8$ at unification $\mu = m_X \sim 2\times10^{14}$ GeV

$\sin^2\theta_W(m_Z)_{MS} = 3/8[1 - 109\alpha/18\pi \ln(m_X/m_Z) + \ldots]$

$\approx$0.21! (Great Desert?)

But later, minimal SU(5) ruled out by proton decay

exps $\tau(p \rightarrow e^+\pi^0) > 10^{33}$ yr $\rightarrow m_X > 3\times10^{15}$ GeV

Coupling Unification

Current Values: $\alpha_3(m_Z)=0.1185(6)$

$\alpha_2(m_Z)=0.0338(1)$

$\alpha_1(m_Z)=0.0170(1)$

Come together but do *not* quite unify without an intermediate mass scale(s): m_{susy}, m_R SO(10), m_{scalar}…

Predict $\sin^2\theta_W(m_Z)\approx 0.233$

Generic SUSY GUT ➔ **$M_X\approx(1\text{TeV}/m_{susy})^{2/15}\times 10^{16}$GeV**

(G. Senjanovic & WJM 1982)

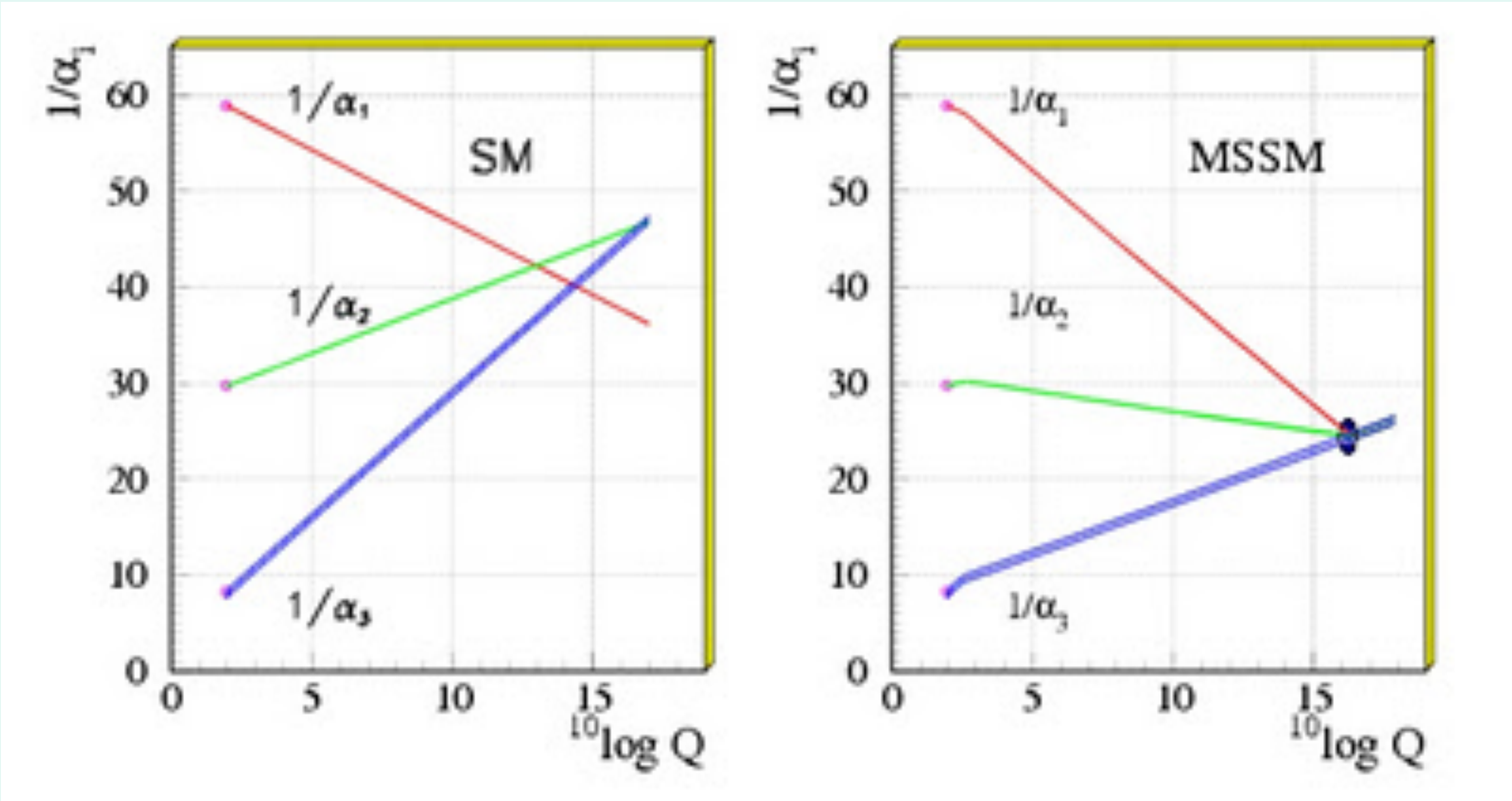
Proton Partial Lifetime:

$\tau(p\rightarrow e^+\pi^0)\approx(1\text{TeV}/m_{susy})^{8/15}\times 10^{35\pm 1}$yr

Uncertainties: Matrix Elements (Lattice), $\alpha_3(m_Z)$, mass splittings…

SUSY GUT Unification

S. Raby PDG



LHC/ Proton Decay Complementarity

Current experimental "hint" of SUSY?

$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 276(73) \times 10^{-11}$ (*3.7σ*)

suggests ***$m_{susy} \approx 100\text{-}500$GeV***

some tension with LHC $m_{susy} \geq 1$ TeV (squarks & gluinos)

SUSY GUTS "prefer" heavier $m_{susy} \approx 3\text{-}10$TeV

Heavier $m_{susy} \rightarrow$ shorter $\tau(p \rightarrow e^+\pi^0) \approx (1\text{TeV}/m_{susy})^{8/15} \times 10^{35\pm1}$yr

Heavier m_{susy} makes $p \rightarrow e^+\pi^0$ easier to observe!

but it makes direct SUSY at the LHC less likely

Together They Squeeze SUSY

5) 1980s - Age of EW Precision

$\sin^2\theta_W$ needed better than ±1% determination

Renormalization Prescription Required

EW Radiative Corrections Computed

DIS $\nu_\mu N$, $\nu_\mu e$, APV, m_Z, m_W (A. Sirlin &WJM)

, Γ_Z, A_{LR}, A_{FB} Z pole observables (Many Others)

Define Renormalized Weak Mixing Angle: $\sin^2\theta_W^R$

$\sin^2\theta^0_W=1-(m^0_W/m^0_Z)^2=(e^0/g^0)^2$ Natural Bare Relation

$\sin^2\theta_W \equiv 1-(m_W/m_Z)^2$ On Shell Definition, Popular in1980s

Induces large $\alpha(m_t/m_W)^2$ corrections

Now Largely Abandoned

$\sin^2\theta_W(\mu)_{MS} \equiv e^2(\mu)_{MS}/g^2(\mu)_{MS}$ Good for GUT running

No Large RC Induced

Theoretically Nice/ But Unphysical

$\sin^2\theta_W^{lep}$ = $Z\mu\mu$ coupling at the Z pole

very popular at LEP

= $\sin^2\theta_W(m_Z)_{MS}$+0.00028 (best feature)

$\sin^2\theta_W(Q^2)$ = Physical Running Angle

Continuous

Incorporates γZ mixing loops: quarks, leptons, $W^{\pm}$

Precision measurements at the Z Pole ($e^+e^-\to Z\to ff$)

Best Determinations

$\sin^2\theta_W(m_Z)_{MS}$ = 0.23070(26) A_{LR} (SLAC)

$\sin^2\theta_W(m_Z)_{MS}$ = 0.23193(29) $A_{FB}(bb)$ (CERN)

(3.2 sigma difference!)

Precision EW Parameters (status):

Quantity	2008 Value		2018 Value	Comment
α^{-1}	137.035999084(51)		***137.035999046(27)***	$\alpha^{-1}(a_e)$ vs α^{-1}(Cs)
G_μ	$1.16637(1)\times10^{-5}GeV^{-2}$		$1.1663787(6)\times10^{-5}GeV^{-2}$	$\tau_{\mu+}$ PSI
m_Z	91 .1875(21)GeV		91.1876(21)GeV	-
*m_t	171.4(2.1)GeV	→	173.0(0.4)GeV	FNAL/LHC
*m_H	>114GeV	→	125.1(0.2)GeV	LHC
m_W	80.410(32)GeV	→	80.382(15)GeV	LEP2/FNAL/LHC
$\sin^2\theta_W(m_Z)$	0.23070(26)		0.23070(26)	SLAC A_{LR}
$\sin^2\theta_W(m_Z)$	0.23193(29)		0.23193(29)	CERN A_{FB}(bb)
	(3 sigma difference?)			
$\sin^2\theta_W(m_Z)_{ave}$	0.23125(16)		0.23125(16)	Z Pole Ave.

Standard Model Predictions Through 2 loops Assuming No New Physics

$\sin^2\theta_W(m_Z)_{MS}=\pi\alpha/\sqrt{2}m_W^2G_\mu(1-\Delta r(m_Z)_{MS})$

$\Delta r(m_Z)_{MS}=0.0693(2)$ ➔ $\sin^2\theta_W(m_Z)_{MS}=\underline{0.23110(9)}$

$\sin^2 2\theta_W(m_Z)_{MS}=2\sqrt{2}\pi\alpha/m_Z^2G_\mu(1-\Delta r'(m_t,m_H))$

$\Delta r'(m_t,m_H)=0.0598(2)$ ➔ $\sin^2\theta_W(m_Z)_{MS}=\underline{0.23124(6)}$
±0.03%

Error Expected to be reduced (improved m_t) to ~ ±0.01%

Corresponds to $m_W=80.362(6)$

Any significant difference with other precise $\sin^2\theta_W$ measurement Implies "New Physics"

Currently $\sin^2\theta_W(m_Z)_{ave}= 0.23125(16)$ Excellent Agreement

Best Off Z Resonance Measurements of $\sin^2\theta_W$

(Not Competitive with Z Pole)

Reaction	$\sin^2\theta_W(m_Z)_{MS}$	<Q>
Cs APV	0.2283(20)	2.5MeV
E158 ee	0.2329(13)	160MeV
Q_{weak} ep	0.2320(9)	160MeV
	0.2310(11)	
6GeV Dis eD	0.2299(43)	1.5GeV
NuTeV ν_μN	0.2356(16)	3-4GeV

Average Low Q^2 Determination

$\sin^2\theta_W(m_Z)_{MS}$=0.23216(64) vs 0.23125(16) Z Pole

E158 at SLAC Pol $ee \rightarrow ee$ Moller)

$E_e \approx 50$GeV on fixed target, $Q^2=0.02$GeV^2

$A_{LR}(ee)=-131(14)(10)\times10^{-9}$ α $(1-4\sin^2\theta_W)$

EW Radiative Corrections ~-40%! (Czarnecki &WJM 1996)

More $\sin^2\theta_W$ Sensitivity!

Measured to ±12% $\rightarrow \sin^2\theta_W$ to ±0.6% (20 to 1)

$\rightarrow \sin^2\theta_W(m_Z)_{MS}$=<u>0.2329(13) slightly high</u>

Was Best Low Q^2 Determination of $\sin^2\theta_W$

Together APV(Cs) & E158, $Q_W(p) \rightarrow \sin^2\theta_W(Q^2)$ running

$\sin^2\theta_W(m_Z)_{MS}$=<u>0.232(1</u>) Good agreement with Z Pole

No Sign of "New Physics"

What about other low energy measurements?

- DIS ν Scattering: $R_\nu \equiv \sigma(\nu_\mu N \rightarrow \nu_\mu X)/\sigma(\nu_\mu N \rightarrow \mu X)$ loops

  $\rightarrow m_t$ heavy & $\sin^2\theta_W(m_Z)_{MS}$=0.233 Higher!

  First Evidence For SUSY GUTS?

  Amaldi, Bohm, Durkin, Langacker, Mann, Marciano, Sirlin and Williams (1987) Global Analysis

Later: NuTeV $\sin^2\theta_W(m_Z)_{MS}$=0.236(2) Even Higher

Inconsistent with Z Pole Measurements (2-3 sigma?)

Atomic Parity Violation Strikes Back

1990 $Q_W(Cs)^{exp}=-71.04(1.38)(0.88)$ C. Wieman et al.

Electroweak RC$\rightarrow Q_W(Cs)^{SM}=\rho_{PV}(-23-220\kappa_{PV}(0)\sin^2\theta_W(m_Z)_{MS})$

$=$ **-73.19(3)**

1999 $Q_W(Cs)^{exp}=-72.06(28)(34)$ Better Atomic Th.

2008 $Q_W(Cs)^{exp}=-72.69(28)(39)\rightarrow\sin^2\theta_W(m_Z)_{MS}=0.2290(22)$

2009 $Q_W(Cs)^{exp}=$ **-73.16(28)(20)** $\rightarrow\sin^2\theta_W(m_Z)_{MS}=0.2312(16)$!

±0.5% → **Major Constraint On "New Physics"**

Later atomic theory shifted again $\sin^2\theta_W(m_Z)_{MS}=0.2283(20)$

Radiative Corrections to APV

$$Q_W(Z,N) = \rho_{PV}(-N+Z(1-4\kappa_{PV}\sin^2\theta_W(m_Z)_{MS})$$

$$\rho_{PV}=1-\alpha/2\pi(1/s^2+4(1-4s^2)(\ln(m_Z/M)^2+3/2)+\ldots.)\approx\underline{0.99}$$

$$\kappa_{PV}(0)=1-\alpha/2\pi s^2((9-8s^2)/8s^2+(9/4-4s^2)(1-4s^2)(\ln(m_Z/M)^2+3/2)$$
$$-2/3\sum(T_{3f}Q_f-2s^2Q_f^2)\ln(m_Z/m_f)^2+\ldots)\approx\underline{1.003}$$

$s^2\equiv\sin^2\theta_W(m_Z)_{MS}=0.23125$, M=Hadronic Mass Scale

<u>Radiative Corrections to APV small and insensitive to hadronic unc.</u>

(Cancellation between γZ mixing & WW box)

Same Corrections Apply to elastic eN scattering as $Q^2\rightarrow 0$, $E_e \ll m_N$

6.) Running $\sin^2\theta_W(Q^2)$

- **Electroweak radiative corrections** ***(γ-Z mixing)*** **cause running of** ***$\sin^2\theta_W(Q^2)$. Shift by about 3% for $0<Q^2<m_Z^2$. [Marciano & Sirlin]***

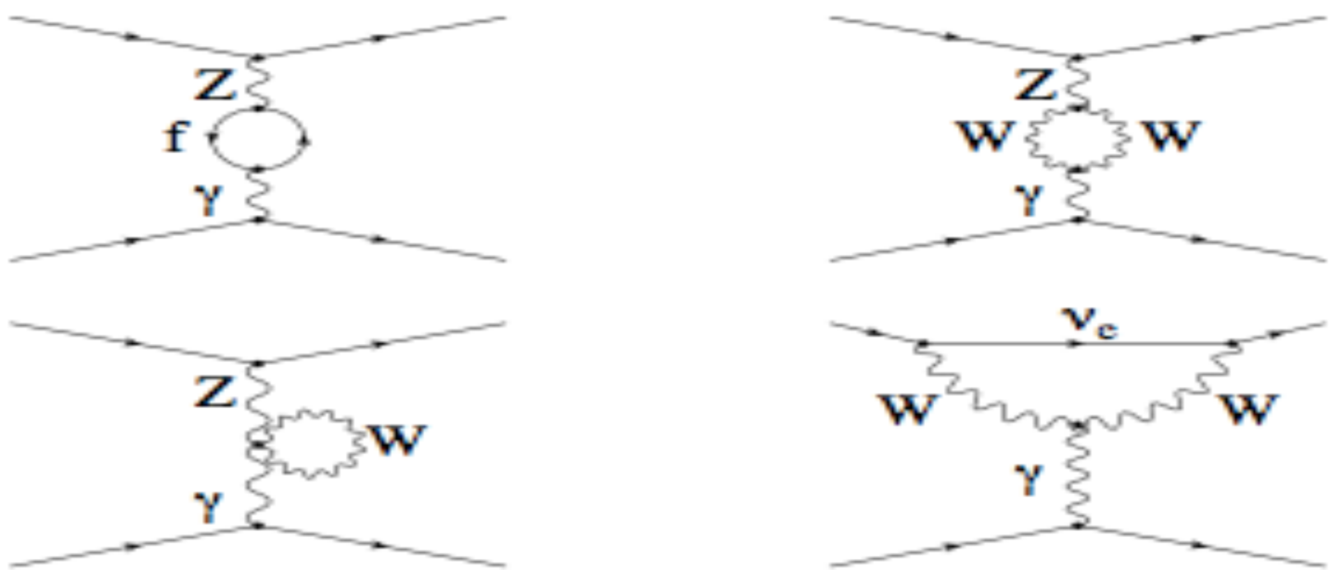
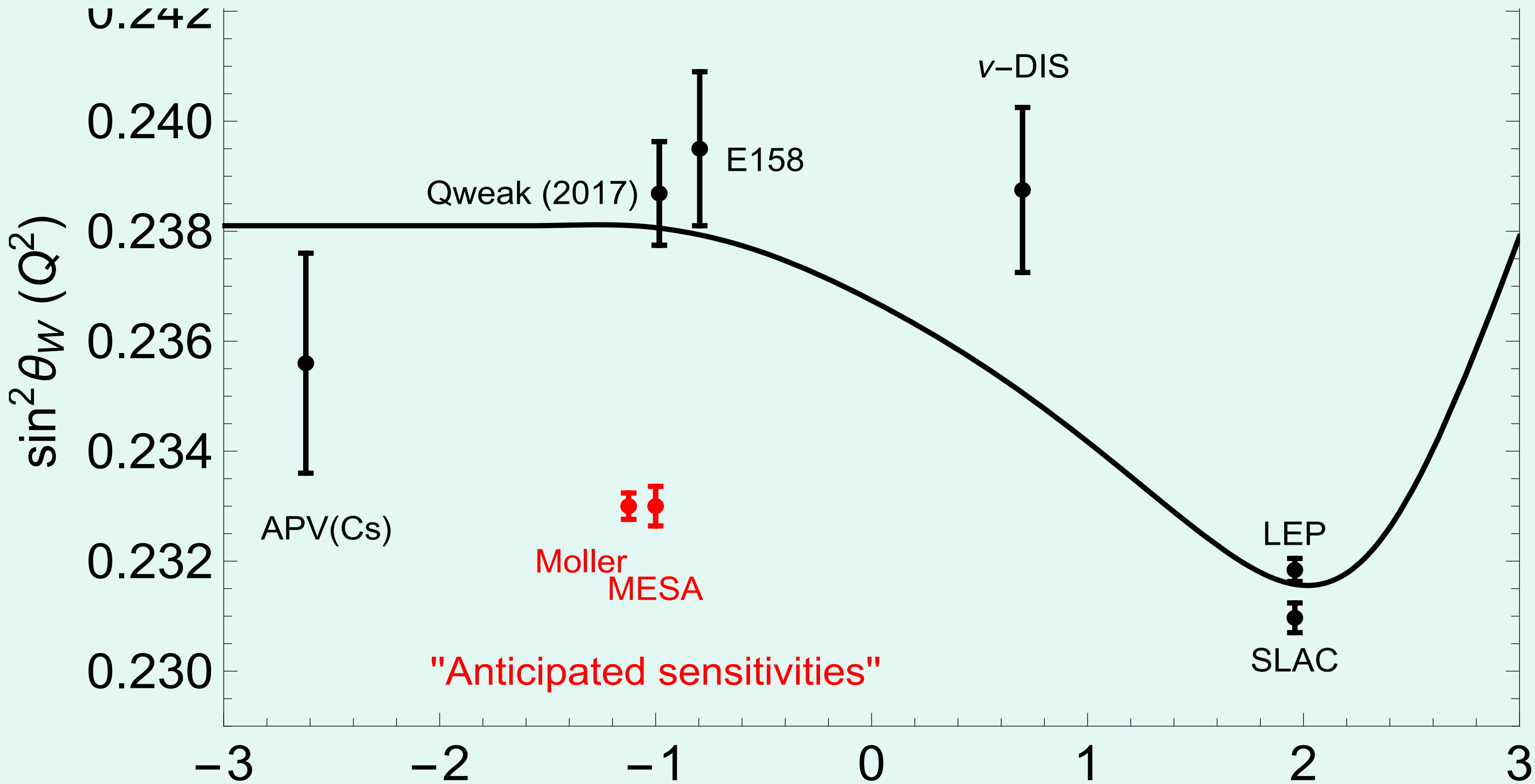
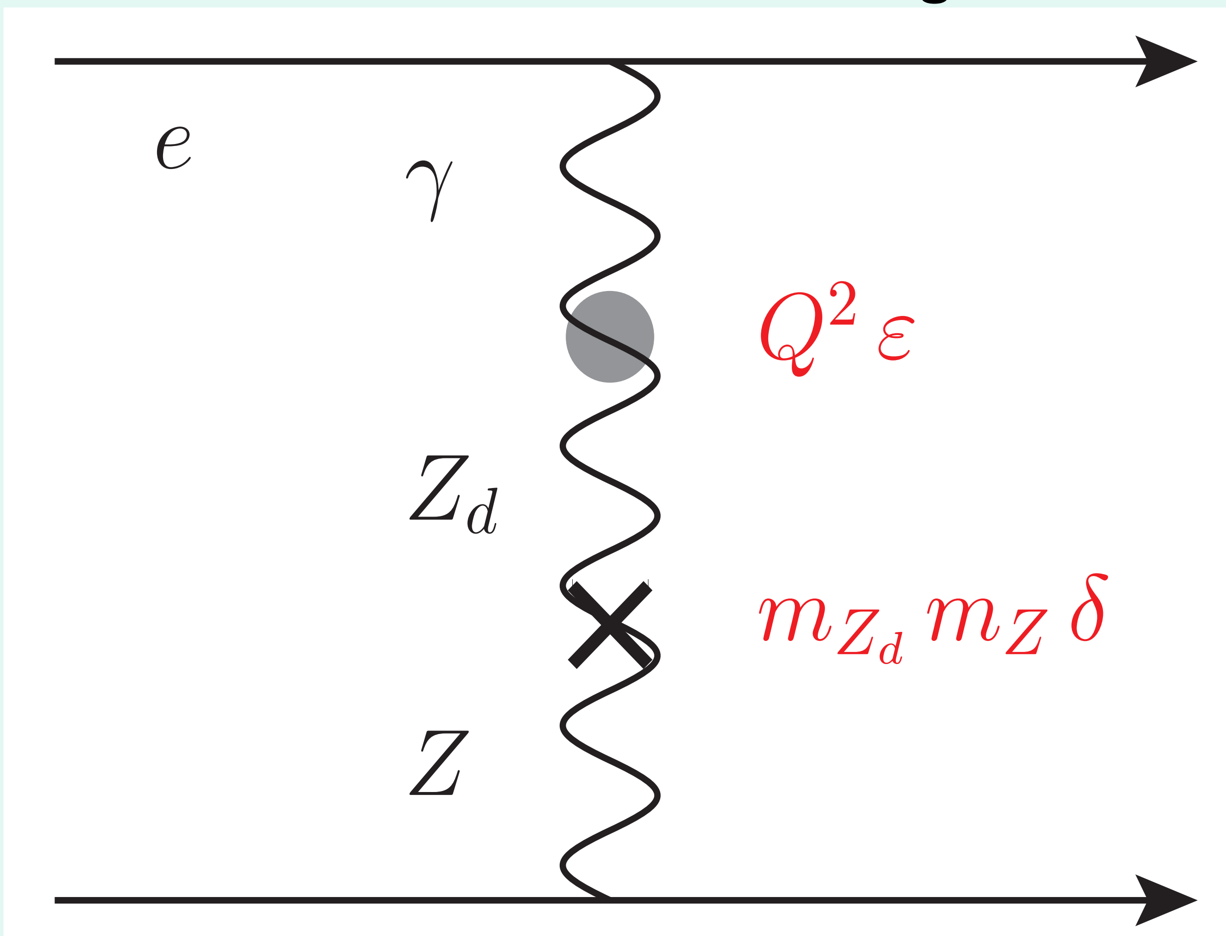


Fig. 2. $\gamma - Z$ mixing diagrams and W-loop contribution to the anapole moment.

Recent Qweak Result & Future Sensitive Proposals

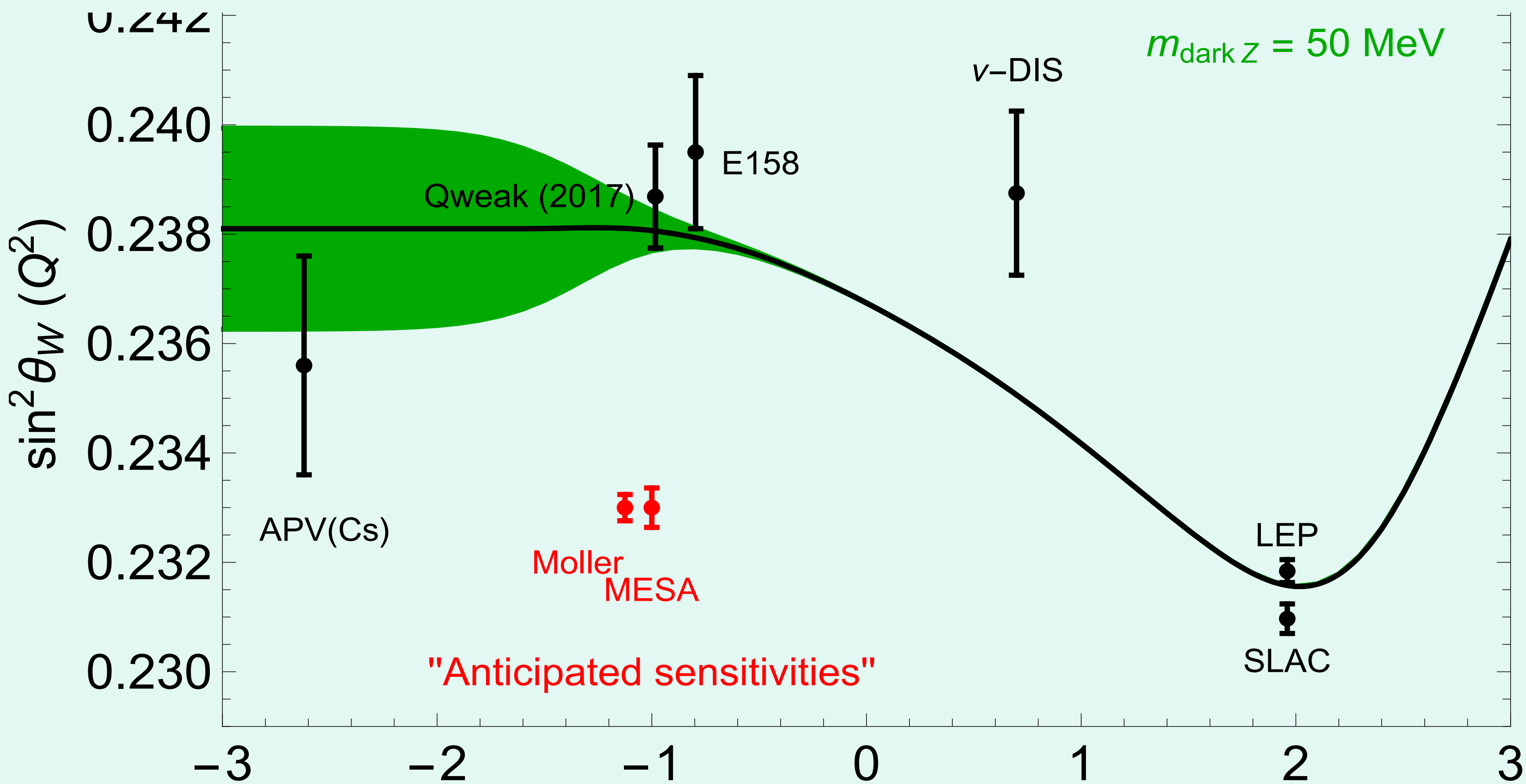


Dark Z Effect on electron scattering
Photon-Z Mixing through Z_d
Kinetic + Mass Mixing



Examples of the effect of "Light" Z_d on Running

H. DAVOUDIASL, H-S LEE, W. MARCIANO



7) $\sin^2\theta_W$ Outlook

Glorious Past

Precision used to unveil: Heavy Top & Higgs Mass Imply New > TeV GUT Threshold

Active Present

P2 at MESA in Mainz & Moller at JLAB

$\Delta\sin^2\theta_W(m_Z)_{MS}=\pm0.00025$!

Comparable to best Z pole studies!

Hopeful Future

Can we do 10X Better? $e^+e^- \rightarrow Z$ factory?

Low Q^2 Precision: APV or Other Method?

New Ideas Welcome