Development of QCD

SLAC Summer Institute: The Standard Model at 50
August 1, 2018
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1. **Strong Interactions in the New High Energy Physics ($\sim$ the 1950s onward)**
2. **Strong Interactions and QFT before QCD**
3. **Enter QCD, and all was light, after a while**
4. **Decades of exploration**
5. **Wrapping it up, with so much unsaid**

Many of these discoveries originated right here at SLAC.

Much, particularly regarding actual calculations, often of crucial importance, is left aside.

Just one perspective on some of the currents that led to today's vibrant activity.

1. Strong Interactions in the New High Energy Physics

Pre-QCD ideas that guided its development and remain in its foundations

A. QFT lessons from QED: The joys of a renormalizable perturbation theory. IR divergences and the inclusive-elastic connection. The KLN generalization.

B. Yang-Mills theories, waiting in the wings.

C. Hadron spectroscopy and the strangeness-inspired quark model.

D. What would a theory of the strong interactions be like? What would one want?

A. QFT lessons from QED: The joys of a renormalizable perturbation theory. IR divergences and the inclusive-elastic connection. The KLN generalization.

- **Already by 1950, Quantum Electrodynamics, based on a Dirac Lagrangian was a well-established success.**

- **And because it was Lagrangian, it could respect locality, causality, classical limits, manifest Lorentz invariance and unitarity in a well-defined renormalizable perturbation theory – one in which incalculable sums over energetic virtual states are replaced by a limited number of measured quantities: electron charge and mass.**

- **But maintaining unitarity and manifest Lorentz invariance is complicated – for a gauge theory, demanding Lorentz invariance** $(-g^{\mu\nu})$ **in the photon propagator unavoidably introduces unphysical degrees of freedom for physical photons** $(-g_{00},\ -g_{ii}$ **for** $\hat{n}_i \propto \vec{k}$**).**

- **The key to unitarity was "Ward identities", which decouple unphysical polarizations from the S-matrix.**

$$\langle \mathrm{phys}\,|\partial^\mu A_\mu(0)\,|\,\mathrm{phys}\rangle \;=\; 0\,,$$

ensures that the photon is massless at all orders.

- **IR divergences in QED: Although virtual corrections are IR divergent, finite predictions for cross sections result from introducing an "energy resolution" – an energy below which soft photons are unobservable. Allowing soft photon emission means that the <u>lowest order, elastic</u> electron scattering cross section can be a good approximation. The inclusive-elastic correspondence will recur below.**

- **The "KLN" theorem formalizes this result in its most general form: Summing over all of states initial and final states within an energy range**

$$E_0 - \delta E \ \le E \ \le E_0$$

always gives an IR finite result, for any theory. Not immediately useful in practical situations, but shows the power of unitarity in QFT.

- **What it means to <u>really</u> work:**

$$\frac{g_e - 2}{2} = 1159.6521869 \pm 0.0000041 \times 10^{-6}$$

B. Yang-Mills theory (1954):

promotes the unobservable phase of quantum mechanical wave functions and the electron field to group transformations, SU(2), U(2), SU(3) . . . N **charges in SU(3) (for example).**

$$\mathcal{L}_{QCD} = \sum_q \bar{q}\,(i\not\partial - g_s\not A + m_q\,)\,q - \frac{1}{4}\,F^2_{\mu\nu}[A]$$

- **with the vector field a matrix (in the Lie algebra):**

$$F_{\mu\nu} \;=\; \partial_\mu\,A_\nu - \,\partial_\nu\,A_\mu \;-\; [A_\mu,,\,A_\nu\,]$$

The immediate problem: in SU(2), three massless analogs of the photon that carry charges. The necessary Ward identities remained undiscovered for over a decade.

C. Hadron spectroscopy and the quark model

– **From** $p,\ n \Rightarrow$

p n N(1440) N(1520) N(1535) N(1650) N(1675) N(1680) N(1700) N(1710) N(1720) N(1900) N(1990) N(2000) N(2080) N(2090) N(2100) N(2190) N(2200) N(2220) N(2250) N(2600) N(2700) N(3000 Region) Δ(1232) Δ(1600) Δ(1620) Δ(1700) Δ(1750) Δ(1900) Δ(1905) Δ(1910) Δ(1920) Δ(1930) Δ(1940) Δ(1950) Δ(2000) Δ(2150) Δ(2200) Δ(2300) Δ(2350) Δ(2390) Δ(2400) Δ(2420) Δ(2750) Δ(2950) Δ(3000 Region)

- **Explanation:** π, N **common substructure: quarks (Gell Mann, Zweig 1964)**
- **spin** $S = 1/2$,
  $I = 1/2$ (u, d) & $I = 0$ (s)
  with approximately equal masses (s **heavier);**

$$\begin{pmatrix} u\ (Q = 2e/3, I_3 = 1/2) \\ d\ (Q = -e/3, I_3 = -1/2) \\ s\ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d})\ , \quad \pi^- = (\bar{u}d)\ , \quad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\ \ldots\ \eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$p = (uud)\ , \quad n = (udd)\ , \quad K^+ = (u\bar{s})\ldots$$

This is the quark model

- **Quark model nucleon has symmetric spin/isospin wave function (return to this later) ⇒ many predictions.**

- **Early success:** $\mu_p/\mu_n =$ **-3/2 ; good to %** **(from** $S = 1/2$**,** $I = 1/2$ (uud)**,** (ddu) **wave functions.)**

- **And now, six: 3 'light': up, down, strange, & 3 'heavy': charm, bottom, top.**

- **Of these all but top form bound states of quark model type:**

$$(q_1\,\bar{q}_2) \qquad (q_1\,q_2\,q_3)$$

- **Special interest: "heavy charmonia":** $(c\,\bar{c}), (b\,\bar{b})$ **"**$J\psi$**" (1974).**

- **But in all this, searches for free quarks came up empty . . . "confinement"**

D. What would a theory of the strong interactions be like? What would one want?

- **A theory of hadron spectroscopy**
- **A dynamical theory for scattering and particle production**
- **An explanation of confinement**
- **A ground for nuclear physics**
- **A Lagrangian description?**

This last criterion was called into question. The candidate was there – Yukawa theory, with the pion as the force carrier – but analysis of nucleon nucleon and pion-nucleon scattering suggested a coupling constant of order 10 in units where the fine-structure constant is 1/137. No obvious perturbative expansion.

2. Strong Interactions and QFT before QCD (the 1960s)

A. Hadronic scattering: Crossing dualities, ladder models, Regge poles, the pomeron and dual resonances, maximal analyticity and the dream of a bootstrap.

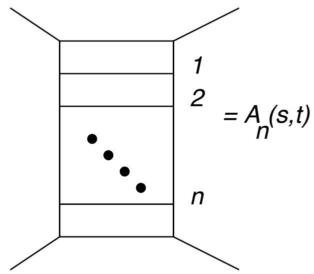
B. Non-Lagrangian-inspired perturbative analysis: Landau equations and Cutkosky rules.

C. Electroweak probes: The axiom of locality, current algebras and Wilson's OPE.

D. Electroweak currents as a doorway to short-distance operator correlations, Bjorken scaling and the parton model.

E. Crossing to 1PI and Drell-Yan cross sections.

A . Hadronic scattering: Crossing dualities, ladder models, Regge poles, the pomeron and dual resonances, maximal analyticity and the dream of a bootstrap. **In lieu of an obvious Lagrangian, abstract general features. Begin with ϕ^3 theory, and it's ladder diagrams, as a function of $s = (p_1 + p_2)^2$ and $t = (p_2 + p_3)^2$ $(\Sigma\, p_i = 0)$:**



- **When time moves "up" (the t-channel) each exchange is like a Yukawa potential, and we expect (and find) bound states.**

$$A(s,t) \;\sim\; \sum_i R_i\, \delta(t \,-\, m_i^2)\,, \quad (t > 0,\, -t \le s \le 0\,.$$

- **When time moves "left to right", with $-s \le t \le 0$, we find $\frac{\alpha(t,m^2)}{n!} \ln^n \frac{s}{m^2}$ from strongly ordered rapidities of particles in the ladder. More generally,**

$$A(s,t) \;\sim\; \sum_i C_i\; (-s)^{\alpha_i(t,m^2)}$$

- **The <u>same</u> diagrams produce bound states <u>and</u> dominant high-energy behavior. The dominant imaginary part is produced from the production of $n+1$ particles at n loops. Regge: each α_i is related to the angular momentum of an "exchanged" particle. Pomeranchuk: the total cross section for the real strong interactions is dominated by an exchange with all quantum numbers vanishing, the "pomeron".**

- **Perhaps no Lagrangian exists. Perhaps the same set of requirements: causality, classical limits, manifest Lorentz invariance and unitarity imposed on the scattering matrix are enough in themselves.**

- **Crossing and analyticity: assume $S_{a+b\to c+d}(s,t)$ is the <u>same</u> function as $S_{a+\bar{c}\to\bar{b}+d}(s',t')$ connected by analytic continuation in variables s,t, and that the same is true for any process with any numbers of particles, not just $2\to 2$, all elements of a unitary S-matrix. This is a lot of conditions! Could it be enough to determine S?**

- **A profound idea, which is currently enjoying a revival in conformal field theory. It was ahead of its time, but led to groundbreaking advances.**

- **Including Veneziano's dual amplitude, built on a linear relation $\alpha(t) = -1\alpha' s)$, provides both the poles and Regge behavior in all three channels**

$$A(s,t) \;=\; \frac{\Gamma(-1-\alpha' s)\,\Gamma(-1-\alpha' t)}{\Gamma(-1-\alpha' u))} \;+\; \text{perms}$$

which led to string theory.

B. **Non-Lagrangian-inspired, yet perturbative analysis: Landau equations and Cutkosky rules.**

- **Invaluable lesson – perturbation theory should be taken seriously. If a "final" theory isn't known, provisional or model Lagrangians can act as a valuable guide. And, if you know the Lagrangian, so much the better. These famous results were derived with no sense the perturbation theory is "the answer".**

  - **Landau Equations – singularities in eternal momenta, p_j are determined by linear equations in loop momenta. They occur when gradients with respect to loops l_c^μ of a set of line momenta $k_i(l_m, p_j)$ become linearly dependent**

$$\sum_{\text{lines } i \text{ in loop } c} \alpha_i \, k^\mu(p_a, l_c) \; = \; 0 \, .$$

  - **Coleman and Norton: the momenta of the on shell lines at Landau equations describe <u>physical</u> processes.**
  - **Cutkosky Rules – once singularities identified, discontinuities are always of the same form**

$$F_m \; = \int \prod_{\text{loops } c} d^4 l_c \, \frac{B \, \delta_+(k_1^2 - M^2) \, \times \, \cdots \, \times \delta_+(k_m^2 - M^2)}{A_1 A_2 \cdots A_N}$$

- **These are the tools for analyzing <u>arbitrary</u> diagrams in <u>arbitrary</u> theories.**

C. Electroweak probes: The axiom of locality, current algebras and Wilson's OPE.

- **An axiom that proved true – the weak and electric interaction couple to hadrons via local operators: the EM current and the currents that appear in the Fermi theory: $j^\mu_{\rm EM}(x)$ and $j^\mu_I(x)$, $j^\mu_{I5}(x)$.**
  - **It wasn't known how to write these operators in terms of fields of the strong interactions, but it was still possible to learn a lot: current algebra:**

$$[\, j^\mu_I(\vec{x})\,,\, j^\mu_J(\vec{y})\,]_{\rm E.T.} \;\sim\; f_{IJK}\,\delta(\vec{x}-\vec{y})\,j_K(\vec{x})$$

  - **Wilson's Operator Product Expansion –**

$$[\, A(x)\,,\, B(y)\,] \;=\; \sum_n E_n(x-y)\,O_n(x)\,.$$

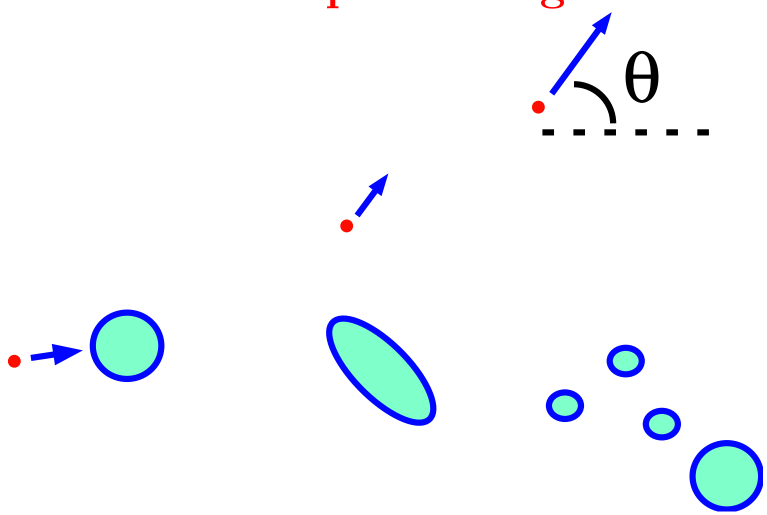
and separation of scales – short distance dynamics manifested by local operators

$$s^{d_A+d_b}\,A(sx)\,B(sy) \;=\; \sum_n C(x-y)s^{d(n)}O_n(sx)$$

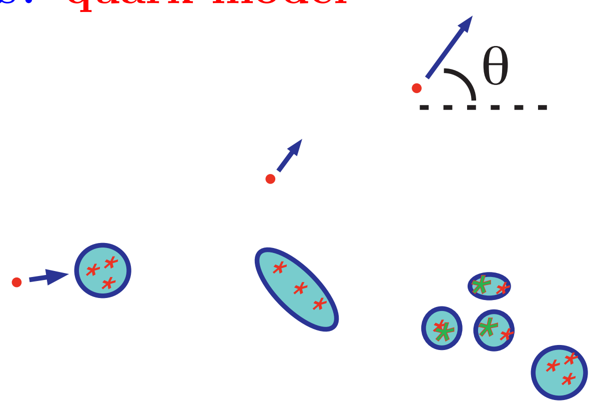
- **Systematics of behavior under scale transformations recurs repeatedly in the systematic treatment of high energy experiments, especially in the following.**

D. **Electroweak currents as a doorway to short-distance operator correlations, Bjorken scaling and the parton model.**

- **Composite yet irreducible? bootstrap $\rightarrow$ strings**



- **Composites of indivisibles? quark model**



- **Yet are the $\star\star$'s "real"? Confinement**

- **Nature makes its choice:**

  - **Prologue: dipole-like elastic (exclusive) form factors:**

$$\frac{d\sigma}{d\Omega_e} = \left[\frac{\alpha_{\rm EM}^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \right] \frac{E'}{E} \left(\frac{|G_E(Q)|^2 \; + \; \tau |G_M(Q)|^2}{1+\tau} \right. \\ \left. + \; 2\tau |G_M(Q)|^2 \tan^2\theta/2 \right)$$

  - **Q: momentum transfer**

  - **Q large: short distance**

  - **Schematically:**

$$\frac{d\sigma_{\rm ep\to ep}(Q)}{dQ^2} \sim \frac{d\sigma_{\rm ee\to ee}(Q)}{dQ^2} \times G(Q)$$

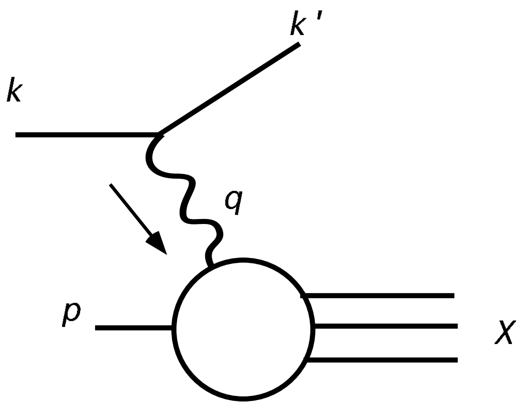
  - **with**

$$G(Q) \sim \frac{1}{\left(1+\frac{Q^2}{\mu_0^2}\right)^2}$$

  - **nonrelativistic analogy:**

  - **power-like form factor $\Rightarrow$ $\star\star$ small & strongly coupled**

- **With higher energy at SLAC: inclusive scattering at then-unprecedented large momentum transfers, $Q^2 > m_p^2$**
- **DIS: Bjorken (1969) remarks on: "the profound state of ignorance on what, even qualitatively, can be expected in this process."**



$$\frac{d\sigma}{dE'\,d\Omega} = \left[\frac{\alpha_{\rm EM}^2}{2SE\sin^4(\theta/2)}\right] \left[2\sin^2(\theta/2)F_1(x,Q^2) + \cos^2(\theta/2)\,\frac{m}{E-E'}\,F_2(x,Q^2)\right]$$

$$\begin{aligned} F_2\left(x=\frac{Q^2}{2p\cdot q}, Q=\sqrt{-q^2}\right) &= \sum_X |\,\langle X|J_\mu(0)|p_N\rangle\,|^2\ \delta(p_X-p_N-q) \\ &= \int \frac{d^4q}{(2\pi)^4}\,\mathrm{e}^{-iq\cdot y}\langle p_N\,|\,[J^\mu(y)\ ,J_\mu(0)\,]\,|p_N\rangle \end{aligned}$$

- **A surprise observaton: IF commutators don't vanish at infinite momentum (i.e., act like FREE FIELD THEORY), then scaling in inclusive deep inelastic scattering,**

$$F_1\left(x = \frac{-q^2}{2P\cdot q}\right) \;\sim\; \frac{F_2(x)}{2x} \;\sim \lim_{P_z\to\infty,\;P_z/q_0\,\mathrm{fixed}} \int d^4y\, e^{-iq\cdot y}\,\langle P\,|\;[J_i(y)\,,\,J_i(0)\,]\;|P\rangle\,,$$

independent of Q.

- **And indeed, when the data came in, in contrast to G's: $F_i(x,Q)'s$ were almost independent of Q!**

OBSERVED BEHAVIOR OF HIGHLY INELASTIC ELECTRON-PROTON SCATTERING

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(Received 22 August 1969)

Results of electron-proton inelastic scattering at 6° and 10° are discussed, and values of the structure function W_2 are estimated. If the interaction is dominated by transverse virtual photons, νW_2 can be expressed as a function of $\omega = 2M\nu/q^2$ within experimental errors for $q^2 > 1$ $(\mathrm{GeV}/c)^2$ and $\omega > 4$, where ν is the invariant energy transfer and q^2 is the invariant momentum transfer of the electron. Various theoretical models and sum rules are briefly discussed.

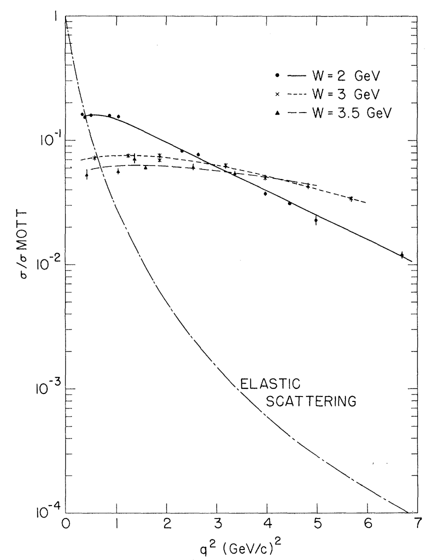
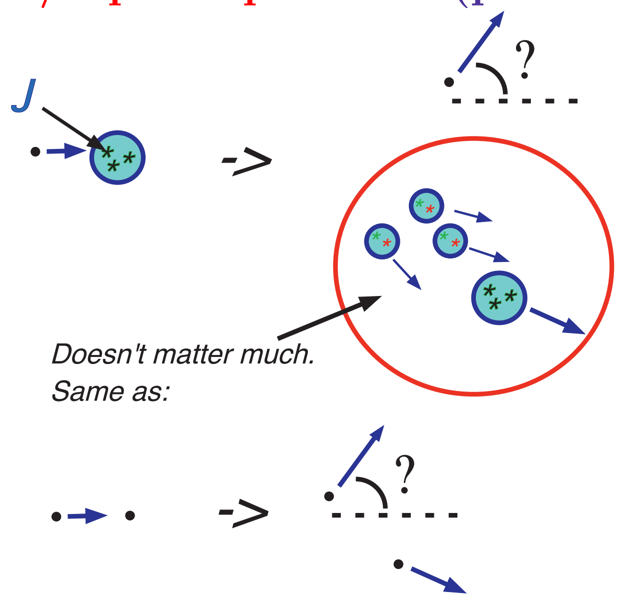


FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\mathrm{Mott}}$, in GeV^{-1}, vs q^2 for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{\mathrm{Mott}}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

Electron sees $\star\star$'s as spin-1/2 point particles (partons)



- **The "strong" force seems weak, almost irrelevant to the electron. But also, reminiscent of QED: an inclusive cross section reverts to "lowest order".**

- **Famously, Feynman's "quark-parton" model: Ignore $\star\star$ interactions**

  – $\frac{d\sigma_{\rm ep\ \ inclusive}(Q)}{dQ^2}$

  $\sim \frac{d\sigma_{\rm e\star\to e\star}(Q)}{dQ^2}$ **x (probability to a find parton)**

$$\frac{d\sigma_{\rm ep\ \ inclusive}(Q)}{dQ^2} = \sum_{\rm partons\ \ i} \frac{d\sigma_{\rm e+i\to e+i}(Q)}{dQ^2}\ f_i(x)$$

  with $f_i(x)$ the distribution for parton i in the target hadron.

- **But what happened to confinement?**

- **And could this make sense in quantum field theory? Not so easy . . .**

– **Wilson $\rightarrow$ Symanzik $\rightarrow$ Christ, Hasslacher, Mueller (1972):**

- **Expand the matrix element "along the light cone":**

$$\int d^4y\, e^{iq\cdot y} \langle p_N \,|J^\mu(y)\, J_\mu(0)\,|p_N\rangle \;=\; \sum_n \int d^4y\, e^{iq\cdot y}\, a_n(y^2)\,(p_N\cdot y)^n$$

- **In the Fourier transform:** $p_N \cdot y \;\leftrightarrow\; 2p_N\cdot q/Q^2 \;=\; 1/x$**,** $y^2 \leftrightarrow q^2$

- **If** $a_n(y^2 = 0) \neq 0 \Rightarrow$ **scaling. Paradox: in theories known to that time,** a_n **always vanished at** $y^2 = 0$ **as a power of** y^2**. (we'll see why)**

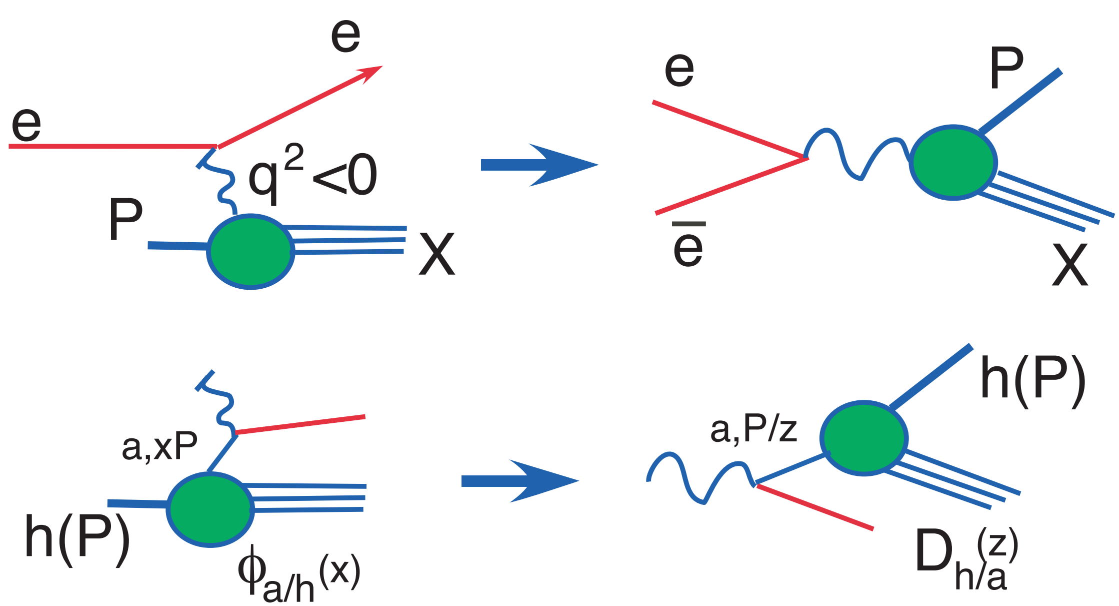
- **Scaling is "radiated away", because as** Q^2 **increases, the system radiated more and more.**

- **Before returning to this issue, a few more profound suggestions from the parton model.**

E. Classic Parton Model Extensions: Crossing to Fragmentation, Jets and Drell Yan

- **Fragmentation functions**
- **Crossing applied to DIS: "Single-particle inclusive" (1PI)**
  From scattering to pair annihilation.

Parton distributions become "fragmentation functions".



- **Parton model relation for 1PI: inclusive hadron from exclusive parton:**

$$\frac{d\sigma_h^{(\mathrm{incl})}(P,q)}{d^3P} = \sum_a \int_0^1 dz\ \frac{d\sigma^{(\mathrm{elas})}_{\mathrm{e^+e^-}\to a}(P/z,q)}{d^3P}\ D_{h/a}(z)$$

- **The direction of the hadron follows the direction of the parton!**

- $D_{h/a}$ **is "universal": could be in DIS (SIDIS), or hadron-hadron scattering.**

- **Heuristic justification from time dilation: Formation of hadron $h(P)$ from parton $a(P/z)$ takes a fixed time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\sigma_a^{(\mathrm{elastic})}$, and is independent of mtm. transf. q (scaling).**

- **For** $\mathrm{e}^+(\mathrm{k}_2)\mathrm{e}^-(\mathrm{k}_1) \to \mathrm{q}(\mathrm{p}_1)\bar{\mathrm{q}}(\mathrm{p}_2)$**.**

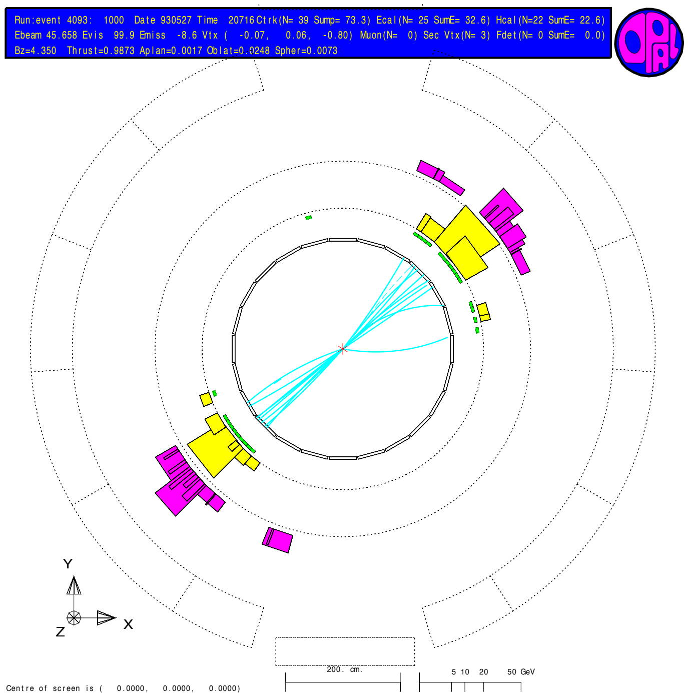
$$\frac{d\sigma^{(\mathrm{elastic})}_{\mathrm{e}^+\mathrm{e}^-\to \mathrm{q}\bar{\mathrm{q}}}(k_1,k_2)}{d\Omega} = \frac{1}{2Q^2}\frac{e_q^2 e^4}{32\pi^2}\, e_q^2 e^4 \left(1+\cos^2\theta\right) ,$$

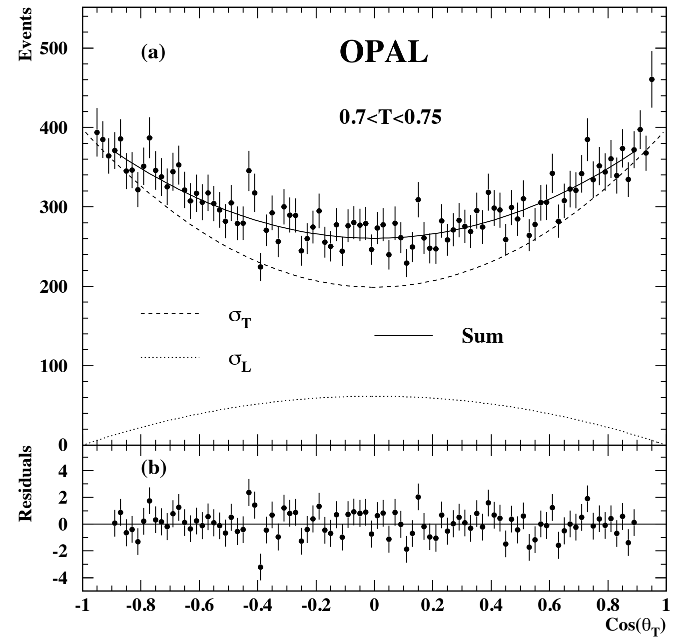
with $Q^2 = (k_1+k_2)^2$**, and** θ **the angle between the electron and the quark.**

- **Jets: the fragmentation picture suggests that almost all hadrons are aligned along parton directions** $\Rightarrow$ **most hadrons come out together as "jets", following the** $1+\cos^2\theta$ **distribution relative to the incoming electron.**

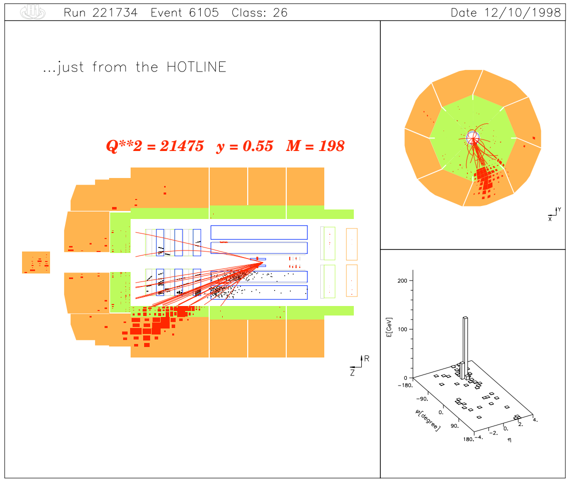
- **"Because of our cutoff** $k_{\mathrm{max}} \ll |q|$ **. . . The distribution of secondaries in the colliding ring frame will look like two jets . . . "**
– S.D. Drell, D.J. Levy and T.-M. Yan, Phys. Rev. D1

- **And this is what happened, at SLAC (Hansen et al., 1975) Petra, SLC and LEP**

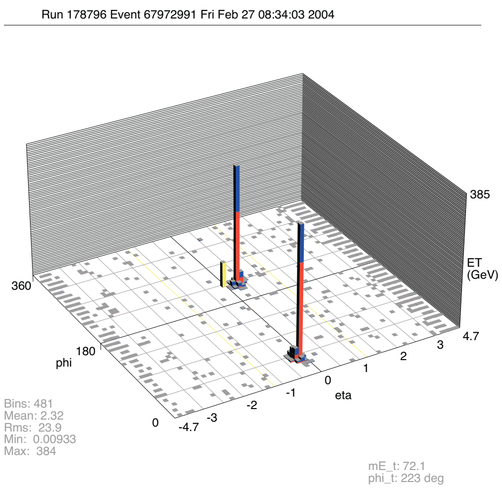


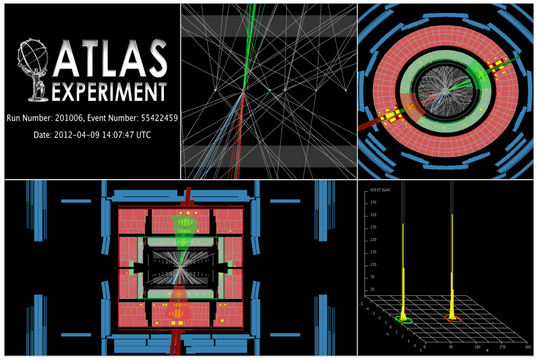


- **For DIS, at HERA, the scattered quark appears as a jet**



- **And in nucleon-nucleon collisions, at Fermilab and the LHC**





- **Finally: the Drell-Yan process**

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q . . . any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim \int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$
$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

  The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS.

- **Template for all hard hadron-hadron scattering**

3. Enter QCD, and all was light, after a while (1973 et. seq.)

A. Gauge theories renormalized, respecting unitarity.

B. Bridging the gap between scaling and confinement.

C. Light cone expansions and anomalous dimensions.

A. Gauge theories renormalized, respecting unitarity.

Meanwhile, a bit of parallel history for the quarks . . .

- **A problem with the quark model: quarks have spin-$1/2$ and share**
- **"isospin":** $u, d \leftrightarrow I = \pm 1/2$
- $p = uud$**(spin 1/2) and** $n = ddu$**(spin 1/2) are symmetric in spin & isospin. And lowest-lying spatial wave function should be symmetric**
- **But spin-$1/2$ particles are all fermions – right?**
- **Greenberg 1964: quarks** *para***fermions of order 3,** *or . . .*

- **Han & Nambu 1965: quarks come in 3 triplets of different Colors**

- **Quarks in baryons are antisymmetric in color quantum number**

- **A new symmetry** $SU(3)$

- **But excited color states not seen (confinement again?)**

- **So what** *is* **this color quantum number? The answer found from**

- **Yang, Mills 1954: Generalization of electric charge to** $N(=3)$ **conserved charges: for color** $3^2 - 1 = 8$**, and as many gluons**

- **Ward identities established to decouple unphysical from physical vectors**
  (Feynman (1963) $\rightarrow$ **Fadeev-Popov (1967))**

- **By 1971 't Hooft,Veltman: Nonabelian Gauge theories consistent & all the rage for the weak interactions (Higgs!) But in contrast to Glashow-Weinberg-Salam . . .**

- **The unbroken nonabelian gauge theory built on color ($q = q_1 q_2 q_3$):**

$$\mathcal{L}_{QCD} = \sum_q \bar{q}\,(i\partial\!\!\!/ - g_s A\!\!\!/ + m_q)\,q - \frac{1}{4}F^2_{\mu\nu}[A]$$

(Pati-Salam 1972, 3; Fritzsch, Gell-Mann, Leutwyler,, Gross and Wilczek and Weinberg 1973)

- **Think of:** $\mathcal{L}_{EM} = K_{\mathrm{e}} + J_{EM} \cdot A + (E^2 - B^2)$

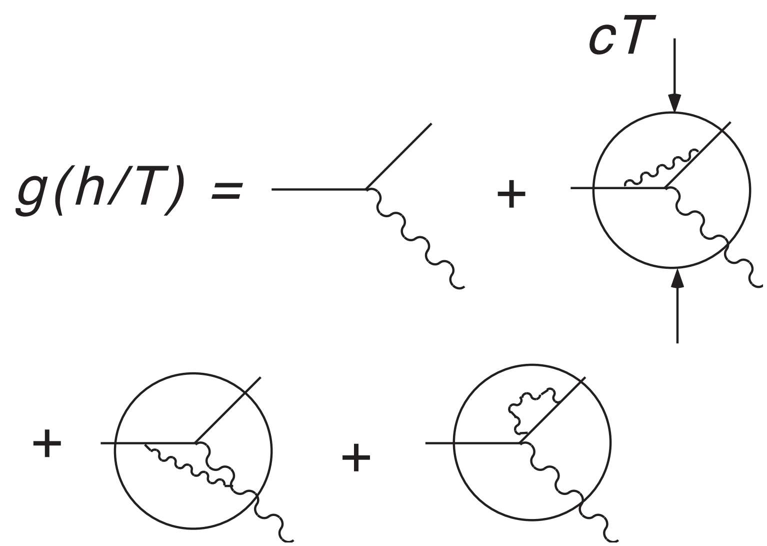
- **The Yang-Mills gauge theory of quarks (q) and gluons (A)**
  Gluons: like "charged photons". The field is a source for itself.

- **Just the right currents to couple to EM and Weak. Its generators commute with those of EW theory, and hence preserve its symmetries and conservation laws. All current algebra results are preserved.**

- **AND . . .**

B. Bridging the gap between scaling and confinement.

- **Just the right kind of forces: QCD charge is "antishielded"**
- **Compute the T (time) -dependence of the coupling:**



and with $\beta_0 = 11 - 2n_{\text{quarks}}/3$ **we get:**

$$\alpha_s(\mu') = \frac{g_s^2}{4\pi} = \frac{\alpha_s(\mu)}{1 + \beta_0 \frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{\mu'}{\mu}\right)^2} = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)}$$

- **This is asymptotic freedom**

$$g_\infty < 0 \Leftrightarrow \alpha_s(\mu = \infty) \to 0!$$

- **Radiation becomes weaker as Q increases.**

- **Gross-Wilczek, Politzer (1973-4), Georgi**
  Near a $\star$ (quark), coupling constant is weak

- **Infrared strong coupling $\to$ quark confinement?**
  Far from a $\star$, coupling constant is strong

- **The template was already there in the light cone analysis . . .**

C. Light cone expansions and anomalous dimensions.

- **Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (1972-77): the entire sum of powers of $p\cdot y$ in the light cone expansion can be organized into a generalization of the parton model form, with only a remaining sum over parton "flavors". So, for example:**

$$F_2^{\gamma N}(x,Q^2) \;=\; \int dy e^{-iq\cdot y} \sum_{\text{powers n}} a_n(y^2\mu^2)\,(p_N\cdot y)^n\, f_n(p_N^2/\mu^2)$$

$$= \sum_{\text{partons}\,q} \int_x^1 d\xi\; C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\alpha_s(\mu)\right) \times\; \phi_{q/N}(\xi,\mu,\alpha_s(\mu))$$

  - C_2: **Expansion in** α_s:
    "perturbative Wilson coefficient" or "hard scattering function"

  - $\phi_{q/N}(x,\mu)$: **fractional momentum distribution for parton** q.

  - μ **called factorization scale: separates large & small** α_s

  - ξ: **fractional momentum of the partons**

  - $\xi\to x$: **radiation that modifies scaling**

– **These parton distributions are operator expectations in hadronic states:**

$$
\begin{aligned}
\phi_{q/h}(\xi,\mu^2) \quad &= \quad \frac{1}{2}\sum_{\text{sigma}}\int_{-\infty}^{\infty}\frac{dy^-}{2\pi}e^{-i\xi p^+ y^-} \ < N(p,\sigma) \mid \bar{q}(0^+,y^-,\mathbf{0}_\perp) \\
&\qquad \times P\exp\left[ig\int_0^{y-} d\lambda\; n\cdot A(\lambda n^\mu)\right]\frac{1}{2}n\cdot\gamma\; q(0)\mid N(p,\sigma)>,
\end{aligned}
$$

Through the path ordered exponential – the parton distribution "sees" the rest of the world as a source of unphysical gluons.

– **Mellin** x **moments give expectations of local (spin-dimension =) "twist-2"local operators.**

$$
\int_0^1 d\xi\,\xi^N\,\phi_{q/h}(\xi,\mu^2) \;=\; \sum_\sigma < h(p,\sigma)\mid \bar{q}(0)\,[n\cdot D(A)]^N\,\frac{1}{2}n\cdot\gamma\; q(0)\mid h(p,\sigma)>
$$

– **The proof of this factorization depends crucially on the Ward identities of QCD – unphysical gluon polarizations organize themselves into an unobservable phase, realizing the concept at the heart of the Yang-Mills construction.**

- **As a clarification – moments** $F_2(N,Q) \equiv \int_0^1 dx x^{N-1} F_2(x,Q)$ **undo the convolution**

$$F_2^{\gamma q}(N,Q) = C_2^{\gamma q}\left(N, Q/\mu, \alpha_s(\mu)\right) \times \phi_q(N,\mu)$$

- **BUT** F_2 **can't depend on** μ

$$\mu \frac{dF_2^{\gamma q}}{d\mu} = 0 \qquad \Rightarrow \mu \frac{d \ln C_2^{\gamma q}}{d\mu} = -\mu \frac{d \ln \phi_q}{d\mu}$$

- **Separate variables:**

$$\Rightarrow \frac{d \ln \phi_q}{d \ln \mu} = -\gamma(N, \alpha_s(\mu)) \equiv \gamma_N^{(1)} \alpha_s(\mu) + \ldots = -\frac{d \ln C_2^{\gamma q}}{d \ln \mu}$$

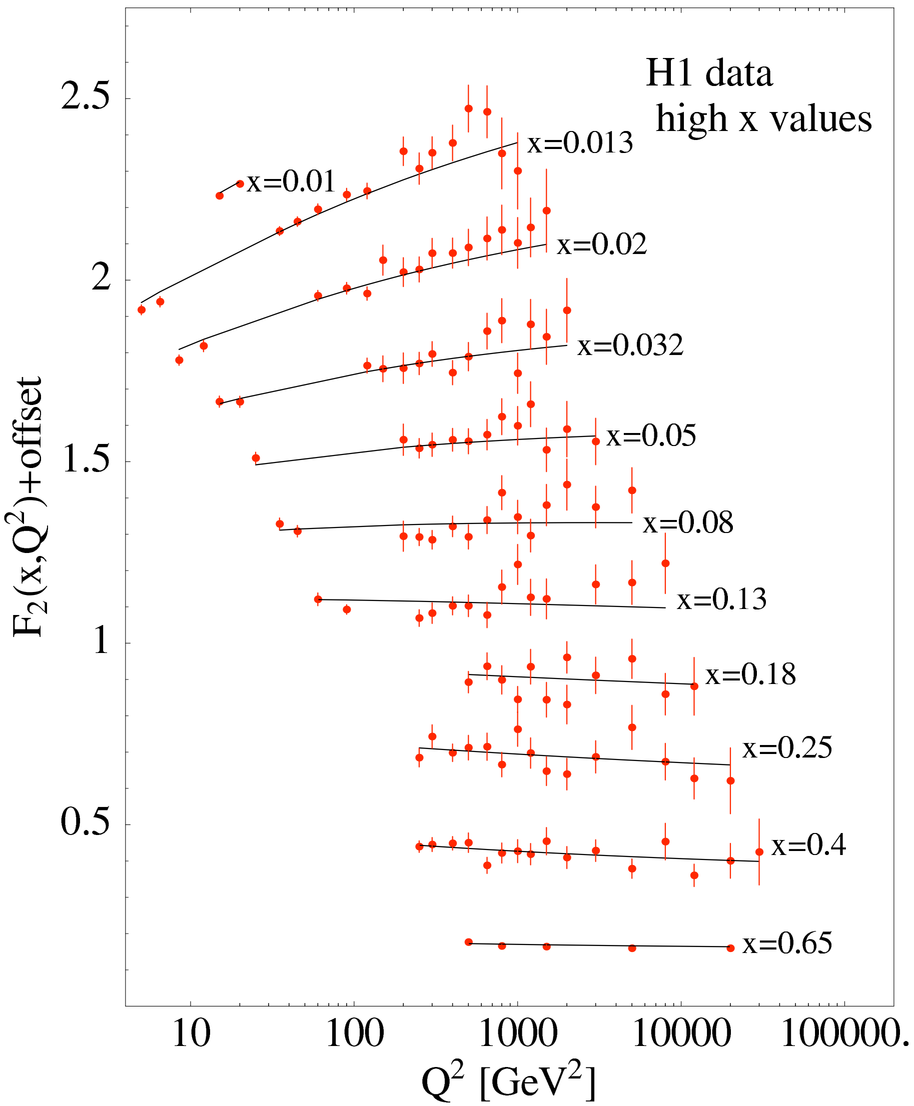
- *Which gives:* **Evolution = scale breaking**

$$\bar\phi(N,\mu^2) \;=\; \bar\phi(N,\mu_0^2)\; \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^{\,2}}{\mu'^{\,2}}\; \gamma\left(N, \alpha_s(\mu')\right) \right]$$

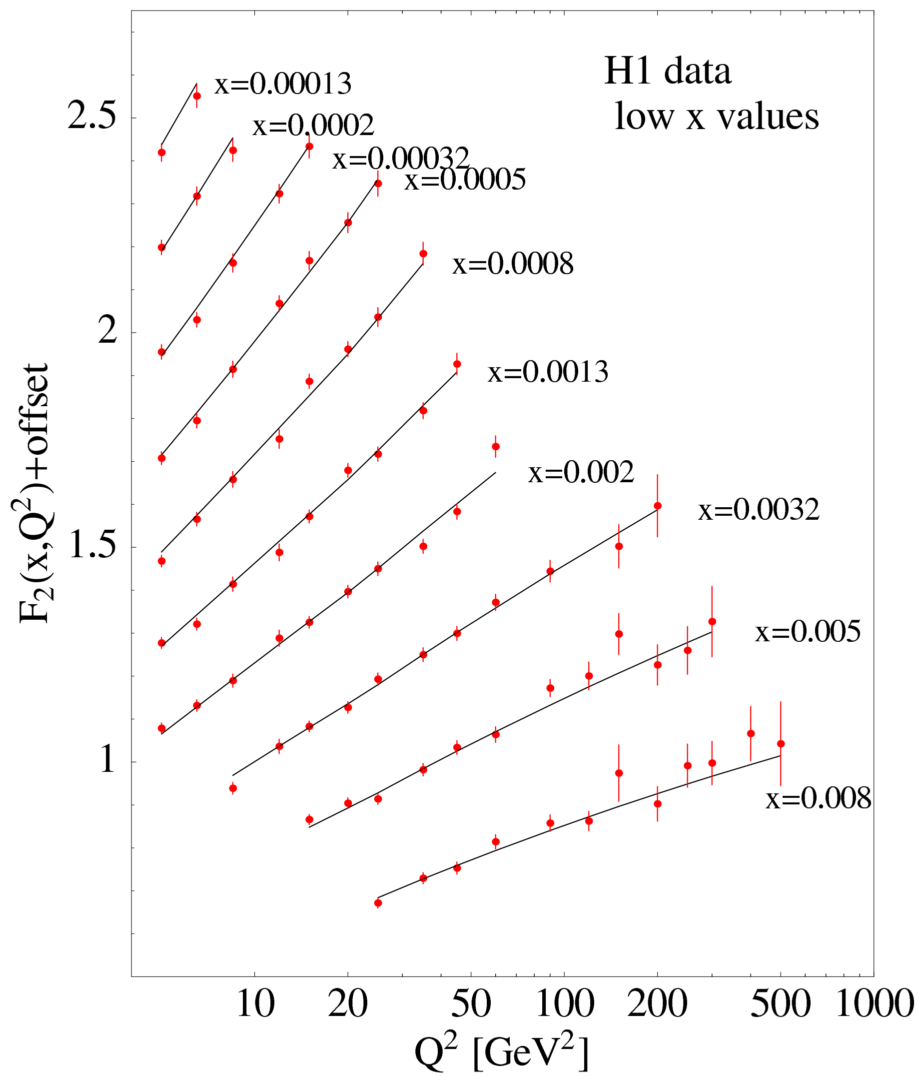
$$\Updownarrow$$

$$\bar\phi(N,Q_0^2) \;=\; \bar\phi(N,Q_0^2) \left(\frac{\ln(Q^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})} \right)^{-2\gamma_N^{(1)}/b_0}$$

- **And it all works. Approximate scaling at moderate x, from HERA:**



- **Predicts pronounced evolution for smaller x:**



- **By the time a struck $\star$ gets far enough to feel a strong force, the electron is long gone. Then, $\star$'s reassemble into hadrons**

- **For most phenomenological applications, reassemble all the moments into the DGLAP evolution equation**

$$\mu \frac{d}{d\mu}\, \phi_{b/A}(x, \mu^2) = \sum_{b=q,\bar{q},G} \int_x^1 \frac{d\xi}{\xi}\, P_{ab}(x/\xi, \alpha_s(\mu))\, \phi_{b/A}(\xi, \mu^2)$$

With P_{ab} computable order by order.

- **The Physical Context of Evolution**

  – **Parton Model: $\phi_{a/A}(x)$ density of parton a with momentum fraction x, assumed independent of Q**

  – **QCD: $\phi_{a/A}(x, \mu)$ is same density, but with transverse momentum $\leq \mu$**

- **If there *were* a maximum transverse momentum Q_0, each $\phi_{a/h}(x, Q_0)$ would freeze for $\mu \geq Q_0$.**

- *Not so* **in renormalized PT.**

- **Scale breaking measures the change in the density as maximum transverse momentum increases.**

- **Cross sections we compute still depend on our choice of "factorization scale" μ through uncommputed higher orders in C and evolution.**

- **But, in summary, we derive approximate scaling and its "breaking".**

- **Experimentally, this took a while, really into the 1980s, with data from SLAC, Fermilab, and eventually, HERA.**

- **Asymptotic freedom is a big deal:**

$$\frac{\text{Scaling}}{\text{QCD}} = \frac{\text{Elliptical Orbits}}{\text{Newtonian Gravity}}$$

- **A beginning, not an end.**

- **For Newtonian gravity, the three-body problem.**

- **For QCD . . . goals like**

$$\frac{\text{Nuclear Physics}}{\text{QCD}} = \frac{\text{Chemistry}}{\text{QED}}$$

- **But can we**
  - **Study the particles that give the currents (quarks)?**
  - **Study the particles that the forces (gluons)?**
  - **Expand in number of gluons?** **Perturbation Theory**

4. Decades of exploration

A. Early windows into nonperturbative QCD

C. Infrared safety: From analyticity and unitarity to jets and event shapes.

D. Factorizations and evolutions in QCD scattering

A. Nonperturbative QCD

Toward longer distance/lower momentum scales, the perturbative coupling increases, and an unimproved perturbative expansion loses all predictive power. It had to be this way for a credible theory of the strong interactions. The exploration of this limit will go on for a long, long time, but within a few years of its discovery, several aspects of nonperturbative QCD contributed to its acceptance. Here is brief homage to a few.

Instantons, anomalies and the eta-prime.

The instanton, "centered" at x_0 of "size" λ
(A.A. Belavin, A.M. Polyakov, A.S. Schwartz, Yu.S. Tyupkin, 1975):

$$A^a_\mu(x) \;= \frac{2}{g}\,, \frac{\eta_{a\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\lambda^2}$$

where η rotates the gauge field on the "sphere at infinity" relative to x_0^μ. Instantons provide nonperturbative contributions to the opeator $F_{\mu\nu}\widetilde{F}^{\mu\nu}$, which appears in the SU(3)-neutral axial vector current:

$$\partial^\mu J^5_\mu \;=\; -i(Ng^2/16\pi^2)\,F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

't Hooft (1976): This produces a mass contribution unique to the η' meson out of nine pseudoscalars that can be made out of u, d, s. Recall the η' particle from the quark model: $u\bar{u} + d\bar{d} + s\bar{s}$. Quark model predictions for its mass were stubbornly low. Other very influential contributions to this problem were made around the same time by Veneziano and Witten, based on an expansion in $1/N$.

– The creation of lattice QCD.

Inspired by the early successes of asymptotic freedom, Wilson (1974) created a new Euclidean, lattice-based formalism, for which QCD appears as the continuum limit. A fundamental feature is that while fermion fields take values on lattice sites, gauge degrees of freedom a captured through closed paths around "links" between sites:

$$\exp\left[ig \oint A \cdot ds\right]$$

(See Wilson's recollections in arXiv 0412043). From this starting point, lattice QCD has grown, and driven, epoch-making developments in computing, in and beyond the limits of science.

– Quarkonia and QCD.

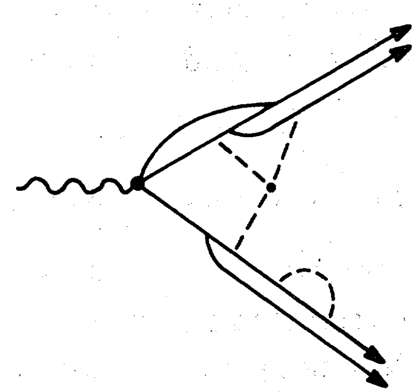
The influence of the discovery of the charmed quark at SLAC and BNL in 1974, followed in 1977 by the bottom at Fermilab cannot be overstated. The description of these states in nonrelativistic models explicitly combined perturbative gluon-exchange, and nonperturbative, confining parameters in the "Cornell potential":

$$V_{Q\bar{Q}}(r) \;=\; \frac{a}{r} \;+\; b\,r$$

upon which many, many improvements can be made.

B. Infrared safety: From analyticity and unitarity to jets and event shapes.

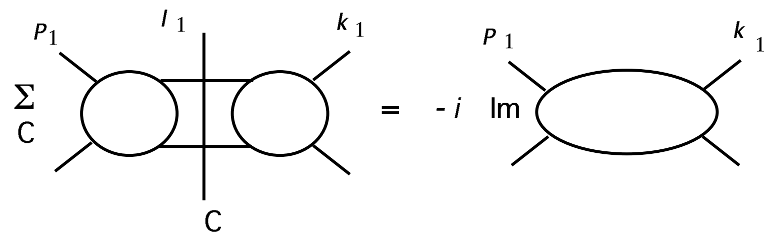
- For an arbitrary diagram, what is the source of long-distance behavior (infrared divergences)? Consult the Coleman-Norton interpretation of Landau equations. For e^+e^- annihilation to hadrons, the only phyiscal pictures are like these (illustrated for two "jets" of collinear particles).



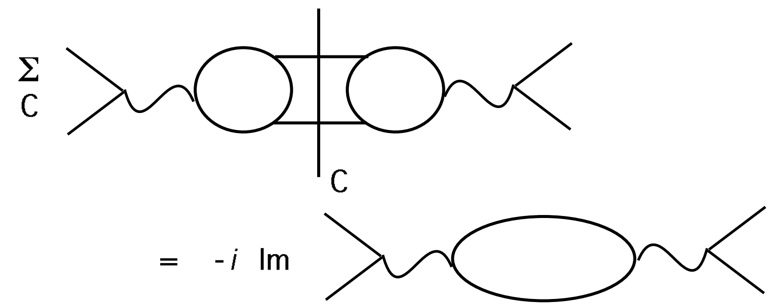
(GS 1978)

- all intermediate states have the same flow of momentum – it's just redistributed between collinear particles, with additional soft radiation. No momentum can flow from one jet to another!

- **For cross sections, cut diagrams and generalized unitarity**
- **Basic expression of unitarity at the level of diagrams:**



- **Or for** e^+e^-,



- **Unitarity for a total cross section or decay rate:**

- **Unitarity in terms of ($S = 1 - iT$)**

$$TT^{\dagger} = -i(T - T^{\dagger})\,.$$

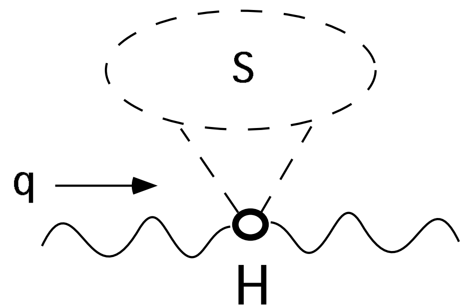
- **Infrared safety for inclusive annihilation and decay**

$$\sigma^{(\mathrm{tot})}_{e^+e^-}(q^2) = \frac{e^2}{q^2}\mathrm{Im}\,\pi(q^2)\,,$$

where the function π is defined in terms of the two-point correlation function of the relevant electroweak currents J_μ (with their couplings included) as

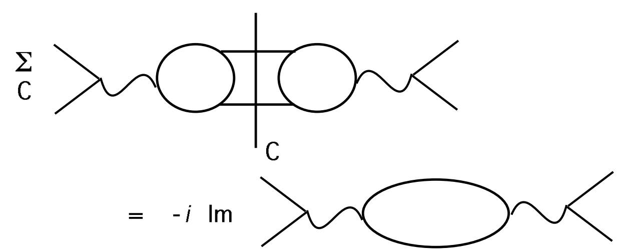
$$\rho(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) = i\int d^4x\, e^{iqx} < 0|\; T\; J_\mu(x)J_\nu(0)\; |0>$$

- **The only physical pictures for $\langle JJ \rangle$ and hence for π:**



- **Power counting confirms finiteness.**

- **But the method is much more general – unitarity holds point-by-point in *spatial* loop momenta $\vec{l}$ of relation (GS 1978)**



$$\sum_{\text{all } C} G_C(p_i, k_j, \vec{l}) = 2 \text{ Im } \left(-iG(p_i, k_j, \vec{l})\right) .$$

- **Proof (and the origin of jet analysis): Do the time integrals for a general amplitude in part I, and get time-ordered perturbation theory. This is equivalent to the sum over Feynman diagrams. The amplitude and its compilex conjugate are given by a sum over virtual states:**

$$\begin{aligned}\sum_m \Gamma^*_m \Gamma_m &= \sum_{m=1}^{A} \prod_{j=m+1}^{A} \frac{1}{E_j - S_j - i\epsilon}\,(2\pi)\delta(E_m - S_m)\prod_{i=1}^{m-1}\frac{1}{E_i - S_i + i\epsilon} \\ &= -i\left[-\prod_{j=1}^{A}\frac{1}{E_j - S_j + i\epsilon} + \prod_{j=1}^{A}\frac{1}{E_j - S_j - i\epsilon}\right]\end{aligned}$$

- **From**

$$i\left(\frac{1}{x+i\epsilon} - \frac{1}{x-i\epsilon}\right) = 2\pi\delta(x)$$

At the level of the loop integrands of TOPT.

$\Rightarrow$ the sum of cut diagrams has the same set of Landau equations as total cross section. Hence the sum at fixed loop momenta is infrared safe.

- **General condition for IR safety: treat states with the same flow of energy the same way.**

- **Introduce an IR safe weight $e_n(\{p_i\})$:**

$$\frac{d\sigma}{de} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(e_n(\{p_1 \dots p_n\}) - w\right)$$

that is, a weight that satisfies

$$\begin{aligned} e_n(\dots p_i \dots p_{j-1}, \alpha p_i + \delta p, p_{j+1} \dots) \; &= \\ & e_{n-1}(\dots (1+\alpha) p_i \dots p_{j-1}, p_{j+1} \dots) + \mathcal{O}\left(\left[\frac{\delta p}{E_{\text{tot}}} \right]^p \right) \end{aligned}$$

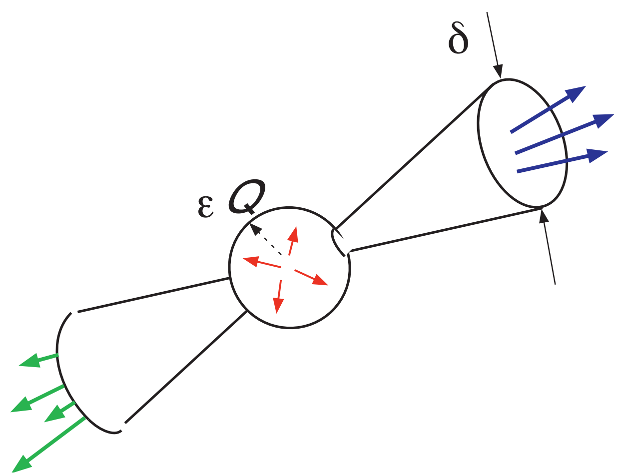
for some $p > 0$.

- **Neglect long times in the initial state for the moment and see how this works in $\mathrm{e}^+\mathrm{e}^-$ annihilation: event shapes and jet cross sections.**

- **Weight function e_n can pick out jets and/or fix their properties.**

- **“Seeing” Quarks and Gluons With Jet Cross Sections. Realizing parton model concepts.**

- **Simplest example: cone jets in $\mathrm{e}^+\mathrm{e}^-$ annihilation. All but fraction ϵ of energy flows into cones of size δ.**



- **Intuition directly from QED: eliminating long-time behavior $\Leftrightarrow$ recognize the impossibility of resolving collinear splitting/recombination of massless particles.**

- **Another example of how a (partially) inclusive cross section “reverts” to lowest order with small corrections.**

How it works at order α_s:

Virtual gluon contribution looks like:

$$\text{virtual}: \quad = \; - \int_0^Q \frac{dk}{2k} \int_{-1}^{1} \; \frac{2\pi \, d\cos\theta}{1 \; - \; \cos^2\theta_{pk}}$$

k = gluon energy, θ = angle to the quark direction.

Real gluon emission has smaller phase space, but still includes all regions where

$$k \; = \; 0 \qquad \textbf{and} \qquad |\cos\theta| \; = \; 1$$

corresponding to soft and collinear configurations (and divergences).

$$\begin{aligned} \text{real}: \quad \sim \; & + \int_0^{\varepsilon Q} \frac{dk}{2k} \int_{-1+\delta^2/2}^{1-\delta^2/2} \; \frac{2\pi \, d\cos\theta}{1 \; - \; \cos^2\theta_{pk}} \\ & + \int_0^Q \frac{dk}{2k} \left(\int_{1-\delta^2/2}^{1} + \int_{-1}^{-1+\delta^2/2} \right) \; \frac{2\pi \, d\cos\theta}{1 \; - \; \cos^2\theta_{pk}} \end{aligned}$$

Singularities cancel (even without IR regularization).

- **No factors Q/m or $\ln(Q/m)$ Infrared Safety.**

- **In this case,**

$$\sigma_{2J}(Q,\delta,\epsilon) \;=\; \frac{3}{8}\sigma_0(1+\cos^2\theta)\left(1-\frac{4\alpha_s}{\pi}\left[4\ln\delta\ln\varepsilon+3\ln\delta+\frac{\pi^2}{3}+\frac{5}{2}\right]\right)$$

- **Perfect for QCD: asymptotic freedom $\rightarrow$ $d\alpha_s(Q)/dQ < 0$.**

- **Some Lessons:**

  – **No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.**

  – **The relation of a jet to quarks and gluons is always approximate but corrections to the approximation computable.**

  – **A single jet may have an enormous amount of information.**

  – **Judicious choices of IR safe event shapes can reveal that information.**

- **The general form of an** $\mathrm{e}^+\mathrm{e}^-$ **annihilation jet cross section:**

$$\sigma_{\rm jet} = \sigma_0 \sum_{n=0}^{\infty} c_n(y_i, N, C_F)\alpha_s^n(Q)$$

- **Dimensionless variables** y_i **include direction and information about the 'size' and 'shape' of the jet:**

- δ**, cone size as above**

- **To specify the jet direction, may use a shape variable, e.g. thrust**

$$T = \frac{1}{s}\max_{\hat{n}}\ \sum_i |\hat{n}\cdot\vec{p}_i| = \frac{1}{s}\max_{\hat{n}}\ \sum_i E_i\,|\cos\theta_i|$$

with θ_i **the angle of particle** i **to the "thust" axis, which we can define as a jet axis.**

- $T = 1$ **for "back-to-back" jets.**

- **Once jet direction is fixed, we can generalize thrust to any smooth weight function:**

$$\tau[f] = \sum_{\text{particles } i \text{ in jets}} E_i\, f(\theta_i)$$

- **For example, the thrust distribution as seen at LEP, compared to experiment** **(Davison & Webber, 0809):**

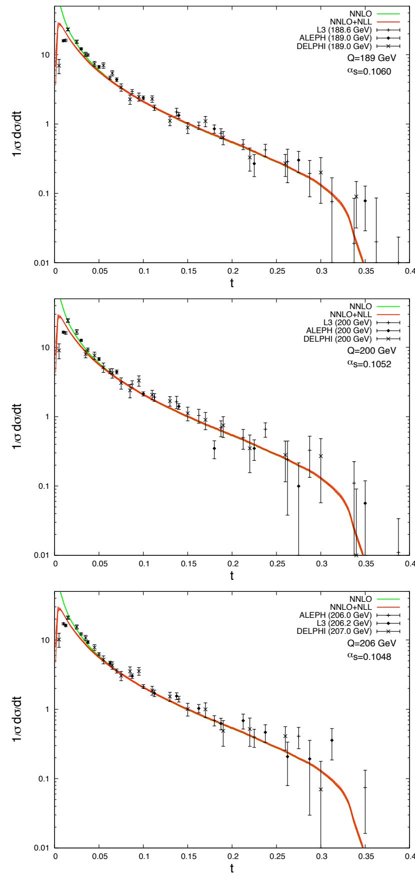


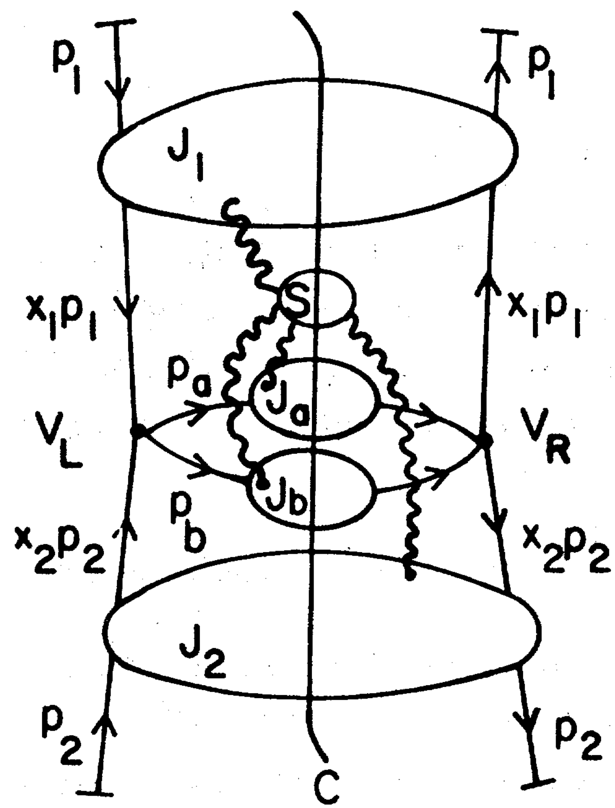
Fig. 3. Fixed-order (NNLO), resummed (NNLO+NLL) and experimental thrust distributions: $Q = 189 - 207$ GeV.

- **Strongly peaked near, but not at, $T = 1$, due to radiation.**

C. Factorizations and Evolutions

- **Essential for cross sections with hadrons in the initial state.**

- **The general Coleman-Norton picture for a large momentum-transfer process in hadron-hadron scattering:**



(GS, S. Libby, 1979)

- **Learning to calculate with a theory that acts differently on different scales . . . In short the key is "factorization". Separate calculable short-distance scattering from long-distance binding of hadrons. Jet cross sections are only one among a larger class.**

- **This theory is still being developed, a development in which new insights into strongly interacting quantum fields is playing a role.**

- **Any factorization leads to an evolution equation.**

- **In effective theories SCET (soft-collinear), these evolution equations typically appears through renormalization group. A very efficient and flexible approach.**
  (Bauer, Fleming, Pirjol, Rothstein, Stewart (2002) Becher, Neubert (2006))

- **Multiscale problems can be dealt with by extended factorization analysis**
  ("kT" and "threshold" resummations, for example: Dokshitzer, Diakonov, Troian; Parisi, Petronzio, Chiapetta, Greco; Catani, Trentedue, Grazzini; Collins, Soper, GS, . . .)

- **As an example, a factorized jet cross sections look like this:**

$$
\begin{aligned}
d\sigma(a+b\to\{p_i\}) \;=\; \int dx_a dx_b\; & f_{a/A}(x_a p_A)\, f_{b'/B}(x_b p_B) \\
\times \quad & C(x_a p_a, x_b p_b, Q)_{ab\to c_1\dots c_{N_{\rm jets}}+X} \\
\times d\; & \left[\prod_{i=1}^{N_{\rm jets}} J_{c_i}(p_i)\right]
\end{aligned}
$$

(Amati, Petronzio, Veneziano; Ellis, Machachek, Efremov, Radyushkin; Politzer, Ross: Libby, GS (1979); Bodwin; Collins Soper, GS (1985,1988), GS & Aybat (2009), Collins (2015))

- **Parton distributions, short distance "coefficients" and functions of the jet momenta tell a story.**

- **In short, the essence of factorization proofs:**

  – **For an IR-safe sum over final states, the effects of final state interactions cancel, including their interference with initial state interactions (so-called "Glauber" or "Coulomb" exchanges).**

  – **Remaining initial state interactions reproduce the same, factorized, parton distributions as in deep-inelastic scattering, as imposed by causality.**

- **Analogous expressions apply in elastic scattering.**
  (Efremov, Radyushkin, Brodsky, Lepage, Farrar (1980) . . . Ji, Raydushkin . . .)
- **The factorized elastic amplitude (mesons)**
  (1979: Brodsky and Lepage, Efremov and Radyushkin)

$$\mathcal{M}(s,t) \;=\; \int \prod_{i=1}^{4}\;[dx_i]\;\phi(x_{m,i})\;M_H\left(\frac{x_{m,i}x_{n,j}p_i\cdot p_j}{\mu^2}\right)$$

with factorized & evolved valence (light-cone) wave functions

$$\phi(x_{m,i},\mu) \;=\; \int \frac{dy^-}{(2\pi)} e^{ix_{1,i}p^+y^-}\,\langle 0 \,|\; T\left(\bar{q}(0)\,\gamma_5\,q(y^-,0^+,\mathbf{0}_\perp)\right)|\,M(p)\rangle$$

and $[dx_i] = dx_{1,i}dx_{2,i}\,\delta\left(1-\Sigma_{n=1}^{2}\,x_{n,i}\right)$**.**

- **BFKL: at very high energies or low x in a class of transverse momentum-dependent amplitudes, from which Balitsky, Fadin, Lipatov, Kuraev solved an eponymous evolution equation for QCD's perturbative pomeron, with asymptotic behavior that grows as a power of s (proportional to α_s).**

- **This rapid growth would violate unitarity, and led to the study of "saturation", in which partons begin to shield each other. These lead to non-linear evolution equations. (Mueller, Qiu (1990) . . . Balitsky, (1995) Kovchegov (1999) . . . JIMWLK (2000+).)**

- **These separations of long- and short-distance made possible (and were confirmed by) explicit calculations, starting with order-α_s (NLO) for electroweak scattering (first corrections to the parton model), then to jet cross sections, then order α_s^2 for EW, and then for QCD cross sections. At each step in this process the splitting kernels increase in power, and extrapolations between energies become more accurate.**

- **Here I'll just remark on innovative uses of analyticity and unitarity as guiding principles, pioneered by Bern, Dixon and Kosower, opening a dialog with string theory and supersymmetry.**

- **Event generators are realizations of factorization at each step in the perturbative shower, concluding with an as-yet model of hadronization.**

4. Wrapping up, with so much unsaid

- A few closing comments, on a few of the many things I couldn't cover in any detail.

- Each of the great Standard Model Machines realized new aspects of the theory.

- Over the decades, pioneering waves of hard-scattering calculations have progressed from LO to NLO to NNLO with real hopes that we can go even beyond (N^3LO for Higgs: Anastasious et al.).

- In many of these developments, a lively dialog has developed with string theory.

- New factorization analyses are probing nucleon structure: transverse-momentum dependent and generalized parton distributions may provide a multi-dimensional image of nucleons.

- Relativistic heavy ion collisions have revealed previously unexpected exotic properties of quarks and gluons at high density.

- **In summary, through asymptotic freedom, QCD has become**

- **A window to the shortest distances/highest energies,**

- **The dog that barks through jets when heavy particles decay.**

- **The dog that doesn't bark through jets yet remains eloquent: Jet quenching.**

- **A testing-ground for string theory ideas.**

- **Retains essential mysteries, only partially understood.**

- **In many ways, the exemplary quantum field theory**