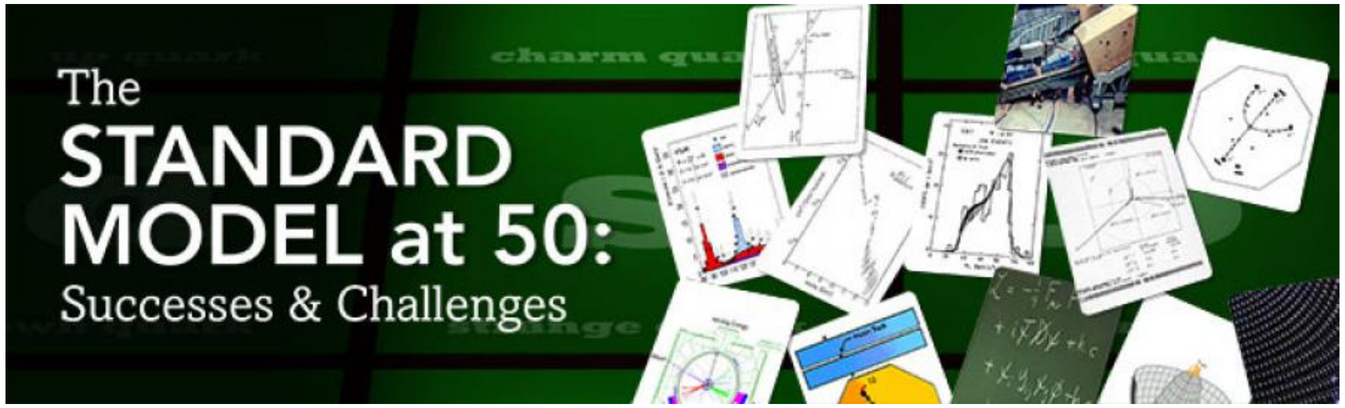

QCD from Lattice Gauge Theory

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Fermilab & IAS TU München

SLAC Summer Institute
August 3, 2018

Outline

- Lattice Gauge Theory for Beginners
  - with a little some history (more in arXiv:1209.3468)
- OMG from Lattice QCD
  - a few remarkable results everyone should know
- Precision QCD in the SM
  - quark masses, flavor physics, and tension headaches; muon g–2
- Lattice Gauge Theory Beyond the Standard Model
- Outlook

Lattice Gauge Theory
for Beginners

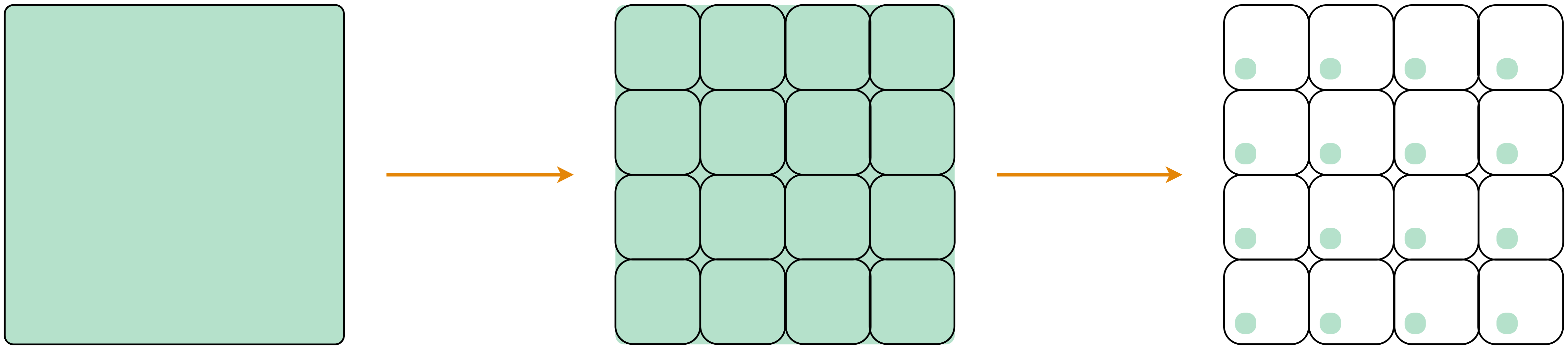
QFT and a Lattice

Heisenberg, Pauli, Z. Phys. 56, 1 (1929)

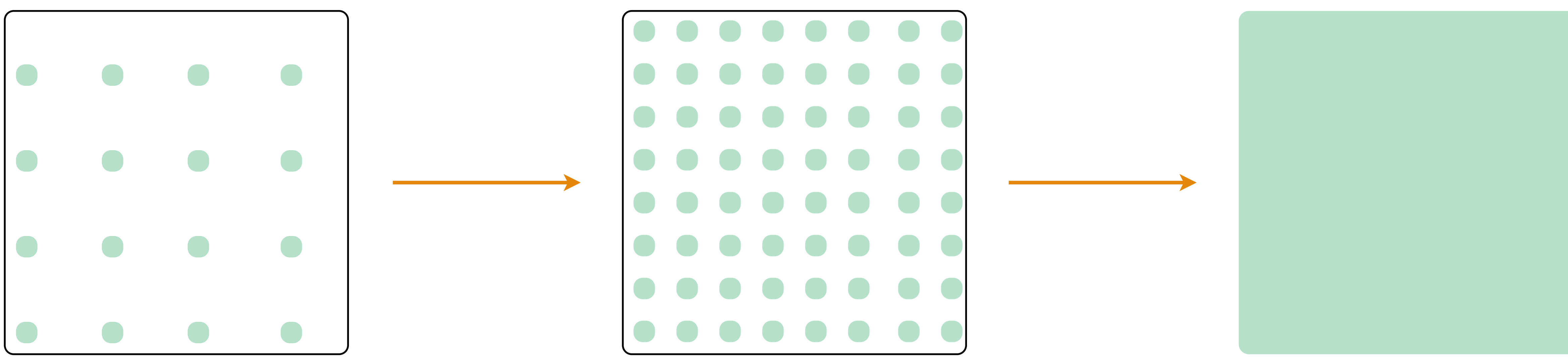
- Quantum field theory contains an uncountably infinite number of degrees of freedom—in order to count straight some sort of regulator is needed:
  - *In der Tat kann man den Fall kontinuierlich vieler Freiheitsgrade, wo die Zustandsgrößen Raumfunktionen sind, stets durch Grenzübergang aus dem Fall endlich vieler Freiheitsgrade gewinnen.*
  - Indeed, one can always obtain the case of continuously many degrees of freedom, where the state variables are functions of space, through a limit of the case of finitely many degrees or freedom. (*My translation*.)


- Heisenberg & Pauli used a spatial lattice.
- Feynman's path integral approach to QM discretizes time:
  - naturally leads to spacetime lattice.

- Break up spacetime continuum into cells; label them with sites:



- Reduce lattice spacing, a, to recover continuum:



- NB: in quantum field theory, bare couplings depend on a, such that physical quantities are held fixed.

Lattice Gauge Theory

K. Wilson, *PRD* **10** (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].

- Gauge symmetry on a spacetime lattice:

$$U(x,y) = \mathbb{P}\exp\left(\int_x^y dz \cdot A(z)\right)$$

$$\mapsto g(x)U(x,y)g^{-1}(y) \quad \text{if} \quad \psi(x) \mapsto g(x)\psi(x)$$

- Naturally associates gauge field with links of lattice: $U_\mu(x) \coloneqq U(x, x + ae_\mu)$.

- Mathematically rigorous definition of QCD functional integrals.

- Enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.

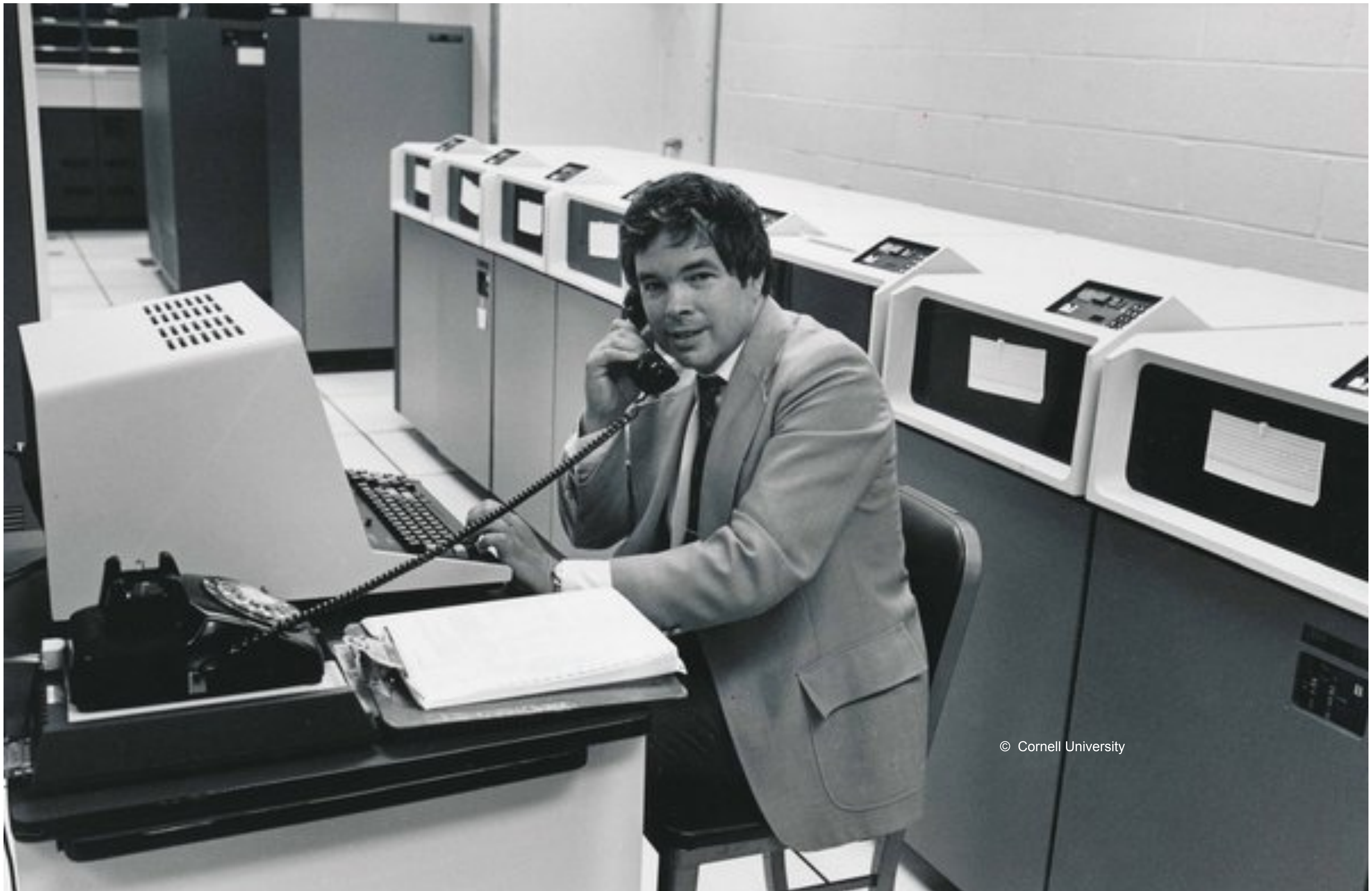
Numerical Lattice QCD

- Nowadays "lattice QCD" usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.

- Big computers:

- Some compromises:
  - finite human lifetime ⇒ Wick rotate to Euclidean time: $x^4 = ix^0$;
  - finite memory ⇒ finite space volume & finite time extent;
  - finite CPU power ⇒ light quarks until recently heavier than up and down.

Computational Science, 1979–



Computers Today

“leadership class”
Mira @ ALCF
49,192 PowerPC A2
(16-core/node)
5D torus

CPU cluster
pi0 @ Fermilab
314 “Ivy Bridge”
16 core/node
Infiniband

GPU cluster
Annie @ BNL
108 2×“Broadwell”
+ 2 Nvidia K80
Infiniband

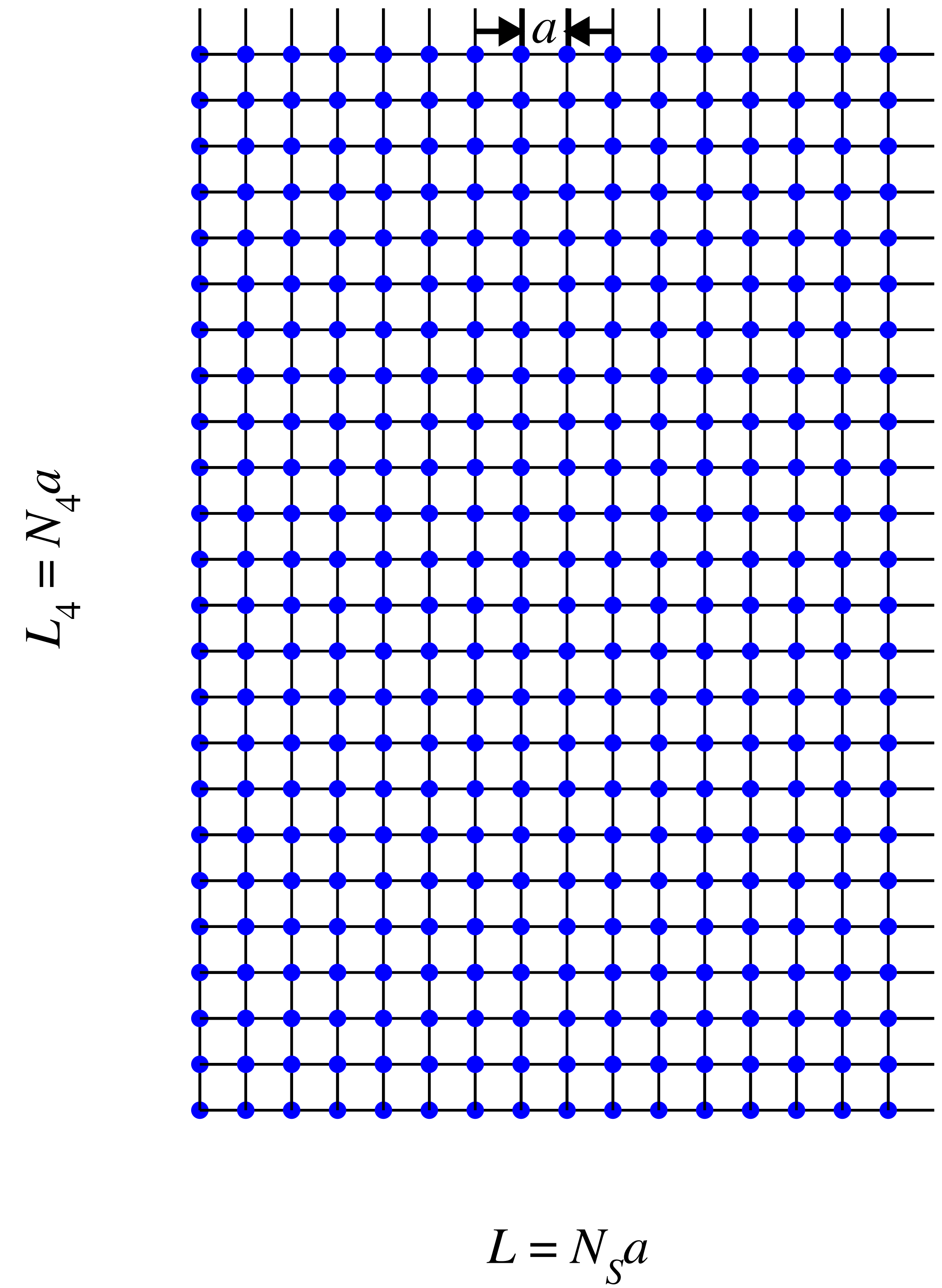
Lattice Gauge Theory

- Infinite continuum: uncountably many d.o.f.
  (⇒ UV divergences);

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

- Infinite lattice: countably many;
  used to define QFT;

- Finite lattice: finite dimension, but ~ 10^8,
  so compute integrals numerically.



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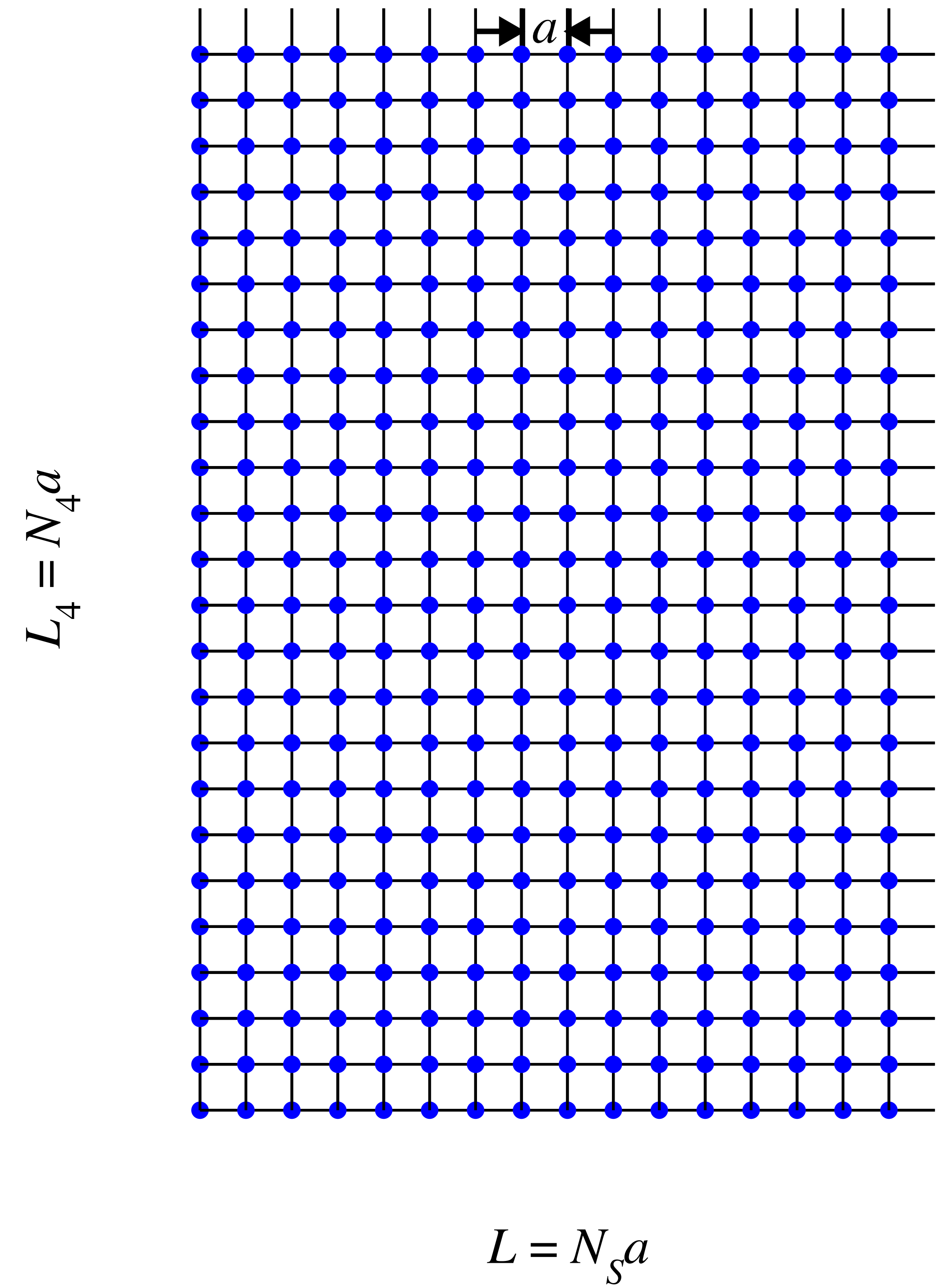

Lattice Gauge Theory

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  (⇒ UV divergences);

$$\langle \bullet \rangle = \frac{1}{Z}\int \underbrace{\boxed{\mathcal{D}U}}_{\text{MC}}\,\underbrace{\boxed{\mathcal{D}\psi\,\mathcal{D}\bar{\psi}}}_{\text{hand}}\,\exp\left(-S\right)\left[\bullet\right]$$

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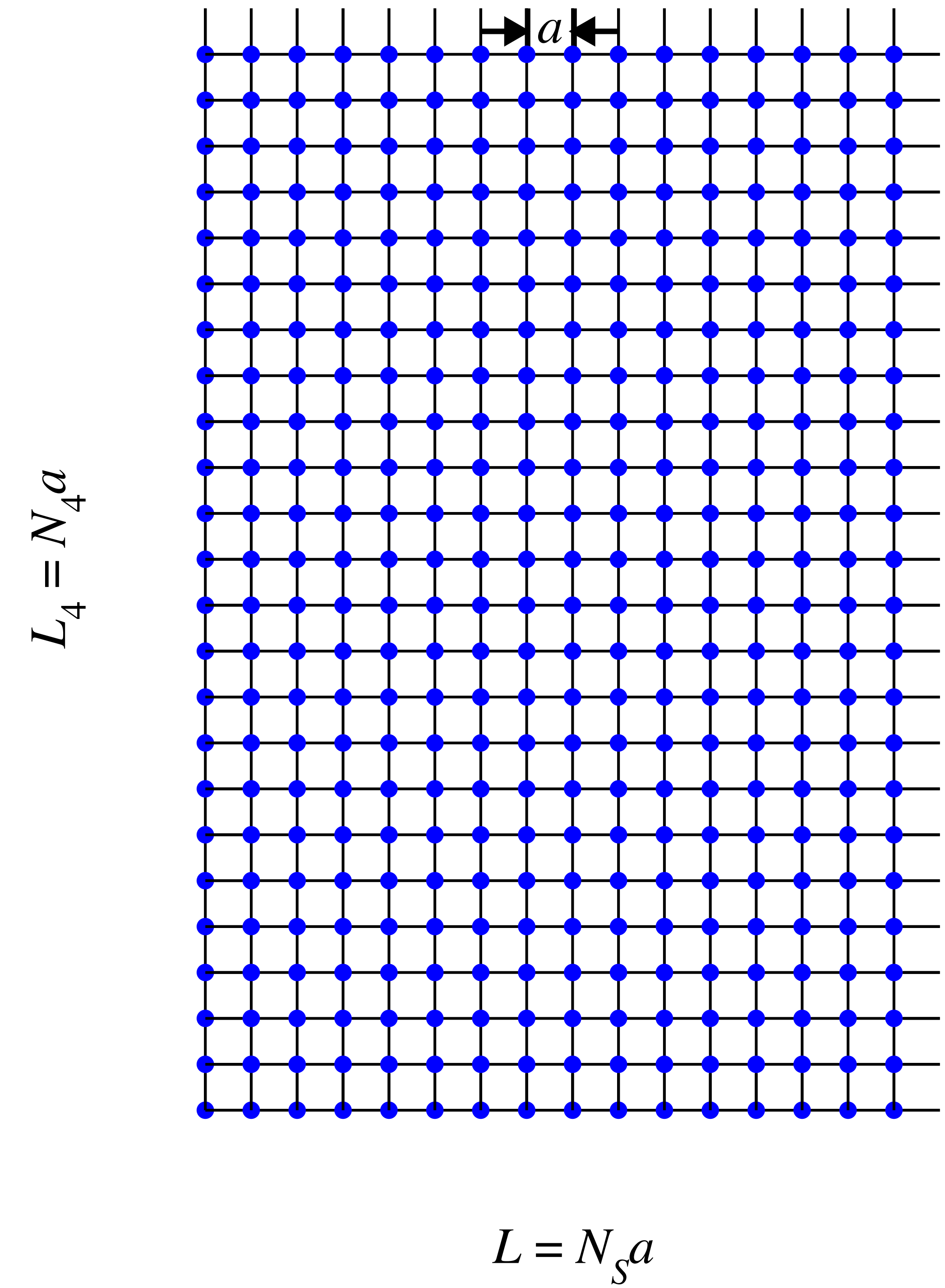


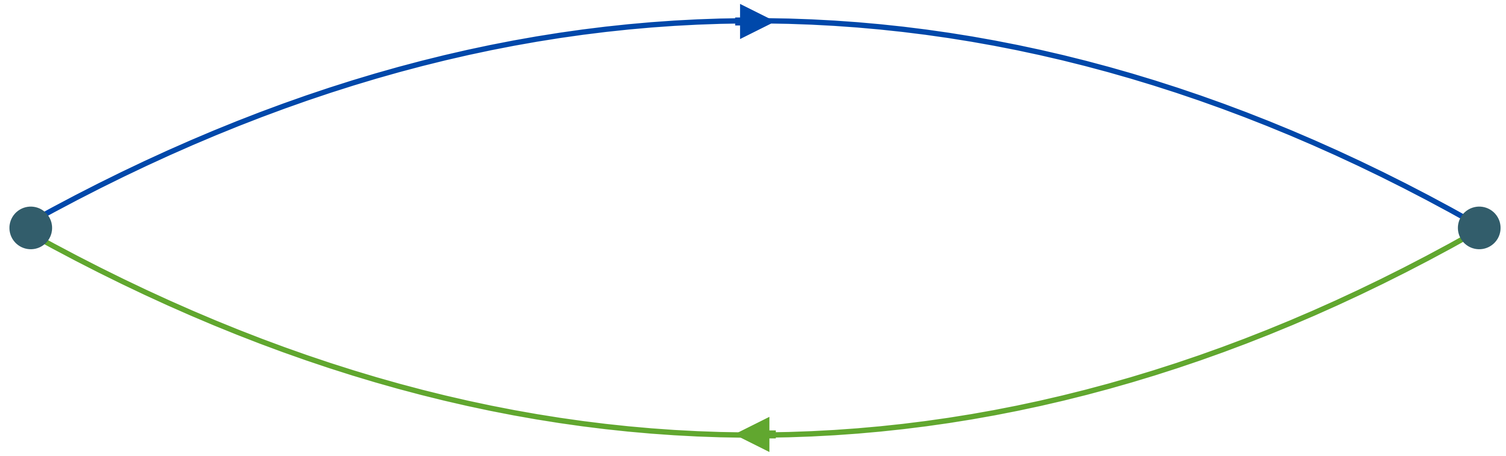
Importance Sampling

- Monte Carlo means to generate a Markov chain of lattice gauge fields.
- The action is extensive ($S \propto L^3T$), so e^{-S} varies over orders of magnitude.
- Importance sampling: generate according to weight $\mathrm{Det}(\not{D} + m)\, e^{-S}$:
    - the determinant (for sea quarks) is the most demanding part;
    - algorithm of choice is HMC ("hybrid Monte Carlo") which introduces fictitious momenta, evolves coordinates and momenta through phase space with Hamilton's equations; accepts end of "trajectory" with Metropolis criterion (Duane *et al*., PLB 195 (1987) 216).
- HMC (dubbed "Hamiltonian Monte Carlo) is now widely used in Bayesian inference and machine learning. Comes from lattice QCD.

Quark Propagators

- Want to compute correlation functions of hadrons but must integrate over quarks “by hand”.

- Wick contractions of $\langle \bar\psi \gamma^5 \psi(x) \bar\psi \gamma^5 \psi(y) \rangle$:



- Solve $(\not{D} + m)_{xz} G_{zy} = b_x$ in background gauge field: $\not{D} + m$ is a sparse matrix:
  - obtain antiquark by noting $\gamma^5 \not{D} \gamma^5 = \not{D}^\dagger$.

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
  - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow$ $\mathrm{O}(a^2)$;
  - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow$ $\mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.

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fast

clean

Correlators Yield Masses & Matrix Elements

- Two-point functions for masses $\pi(t) = \bar{\psi}_u \gamma^5 \psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-m_{\pi_n} t)$$

- Two-point functions for decay constants:

$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \langle 0 | \hat{J} | \pi_n \rangle \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

- Three-point functions for form factors, mixing:

$$\langle \pi(t) J(u) B^\dagger(0) \rangle = \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \langle \pi_n | \hat{J} | B_m \rangle \langle B_m | \hat{B}^\dagger | 0 \rangle \times \exp[-m_{\pi_n}(t-u) - m_{B_m} u]$$

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The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2} \operatorname{tr}[F_{\mu\nu} F^{\mu\nu}] \\ &- \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f \\ &+ \frac{i\theta}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} \operatorname{tr}[F_{\mu\nu} F_{\rho\sigma}] \end{aligned}$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

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$$
\begin{aligned}
\mathcal{L}_{\text{QCD}} &= \frac{1}{g_0^2}\operatorname{tr}[F_{\mu\nu}F^{\mu\nu}] && r_1 \text{ or } m_\Omega \text{ or } \Upsilon(2S\text{-}1S), \ldots. \\
&- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f && m_\pi, m_K, m_{J/\psi}, m_\Upsilon, \ldots. \\
&+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\operatorname{tr}[F_{\mu\nu}F_{\rho\sigma}] && \theta = 0.
\end{aligned}
$$

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Dimensional Transmutation

- Why is the bare coupling traded for a dimensionful quantity?
- To keep physical quantities fixed as a changes, the bare coupling changes:

$$\frac{dg_0^2}{d\ln a^2} = \beta_0 g_0^4, \qquad \beta_0 = \left(11N_c/3 - 2n_f/3\right)/(16\pi^2)$$

$$\frac{dg_0^2}{g_0^4} = \beta_0\, d\ln a^2 = 2\beta_0\, d\ln a$$

$$-\frac{1}{g_0^2} = 2\beta_0 \ln(a\Lambda_{\text{lat}})$$ Λ_{lat} is constant of integration

$$a^{-1} e^{-1/2\beta_0 g_0^2(a)} = \text{constant} =: \Lambda_{\text{lat}} \propto \Lambda_{\overline{\text{MS}}}$$

where the constant of proportionality can be obtained exactly from a one-loop matching calculation. It depends on the details of the lattice action.

The Steps

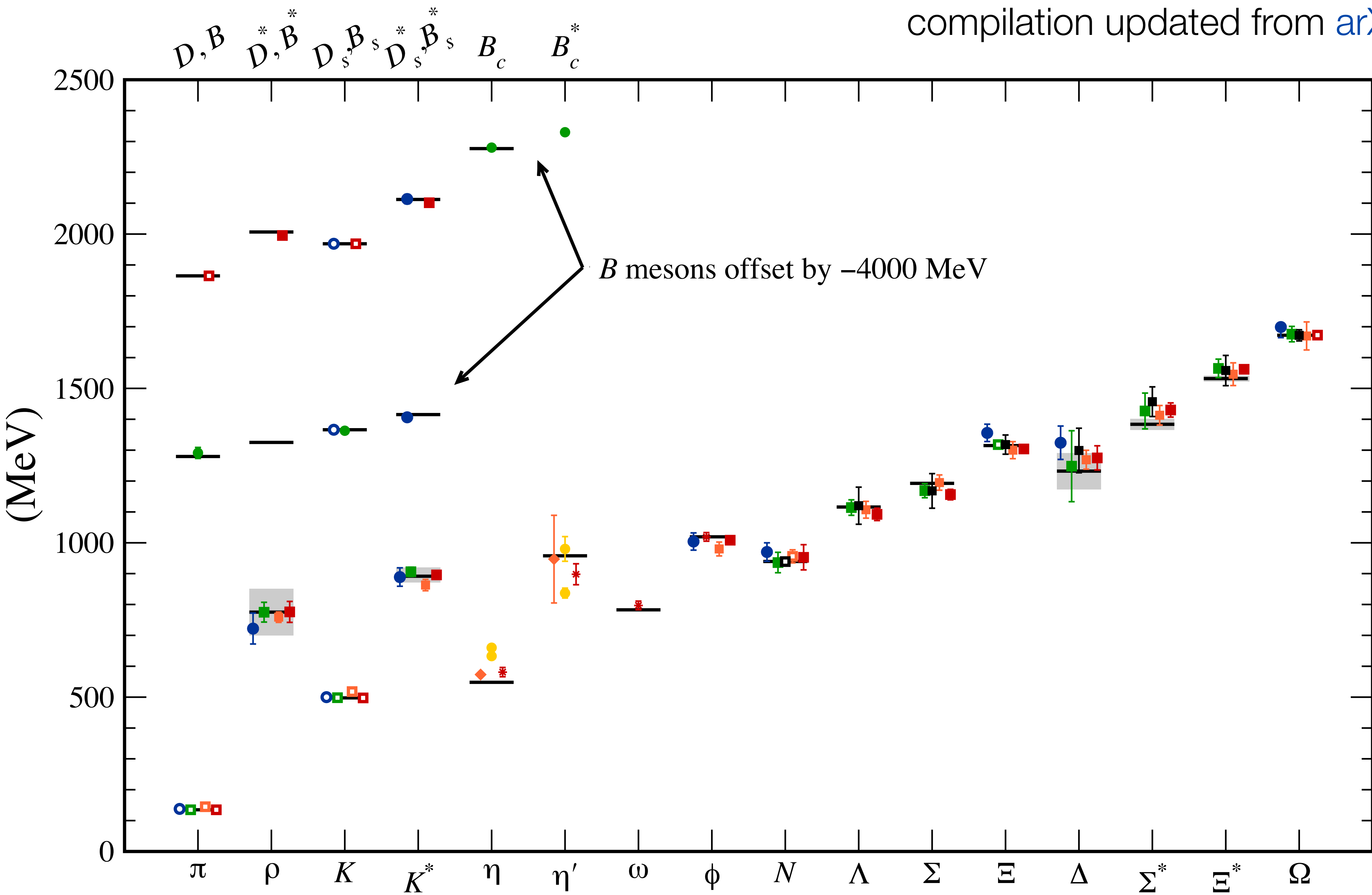
- Use random number generator to create gauge fields distributed $\sim \mathrm{Det}(\not{D}+m)\mathrm{e}^{-S}$.
- Solve $(\not{D}+m)_{xz}G_{zy} = b_x$ for quark propagators in these gauge fields.
- Fit correlation functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.
- Comprehensive analysis of uncertainties!

Lattice QCD for Textbooks

QCD Hadron Spectrum

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler&Woloshyn

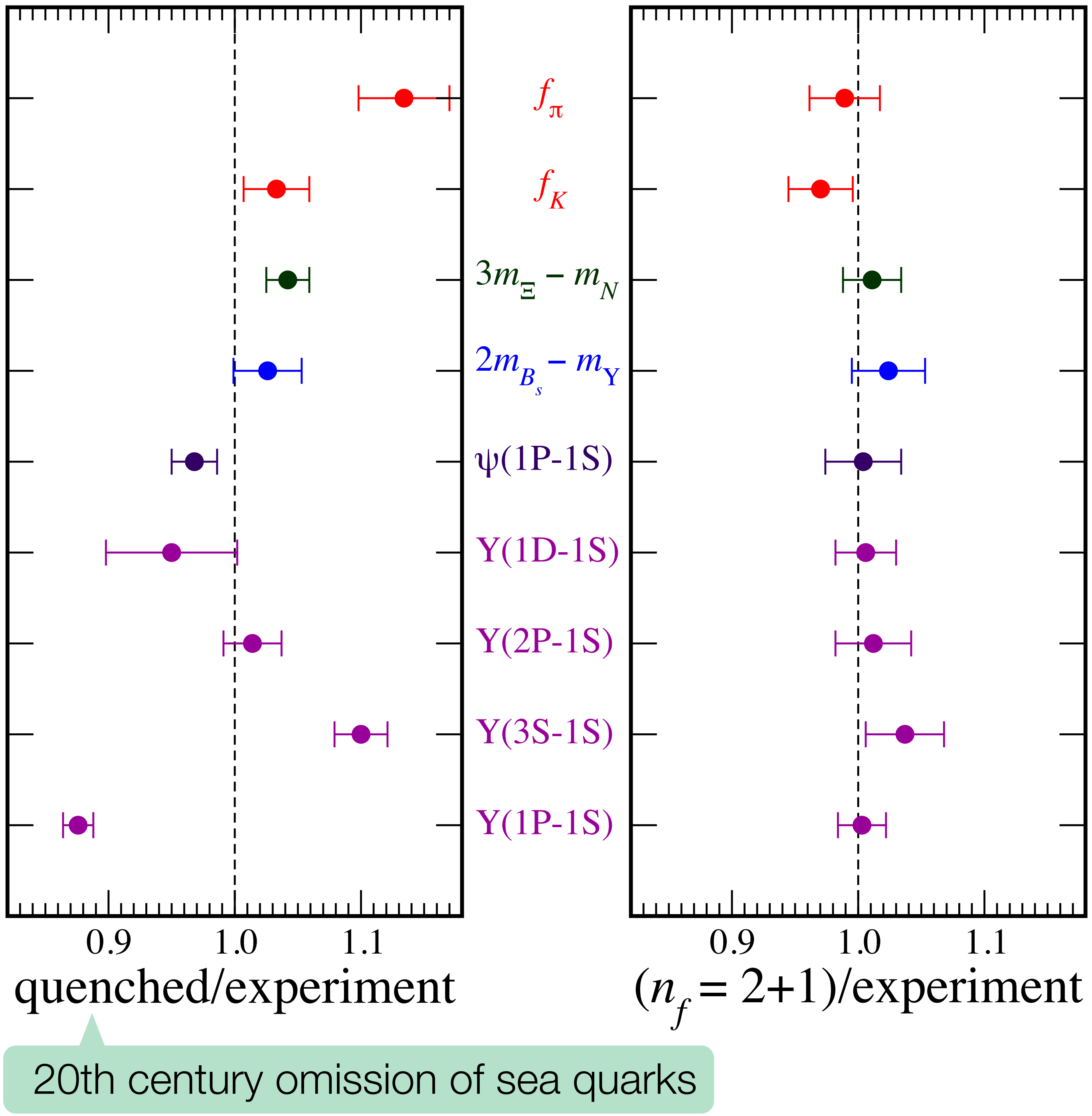
compilation updated from arXiv:1203.1204



numerous quarkonium omitted

Postdictions

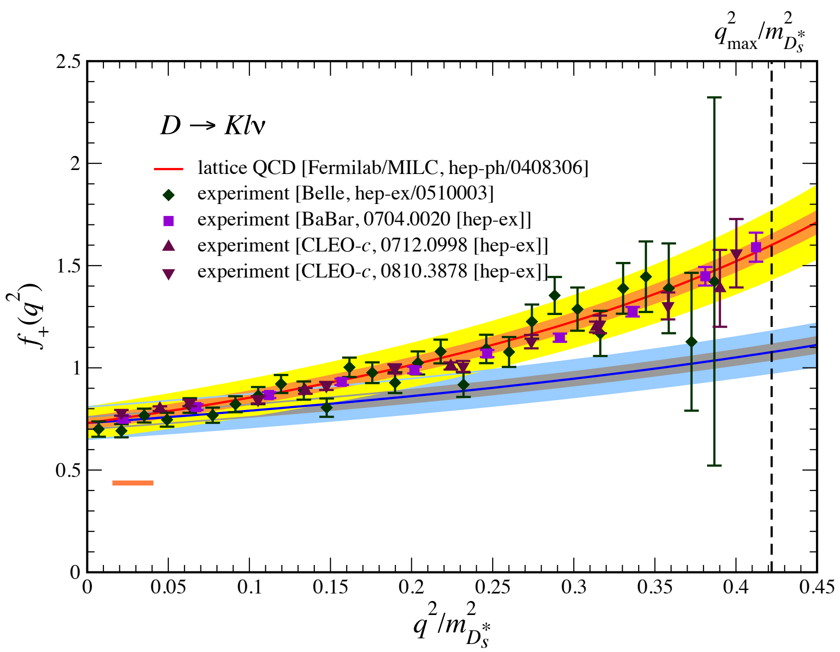
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004

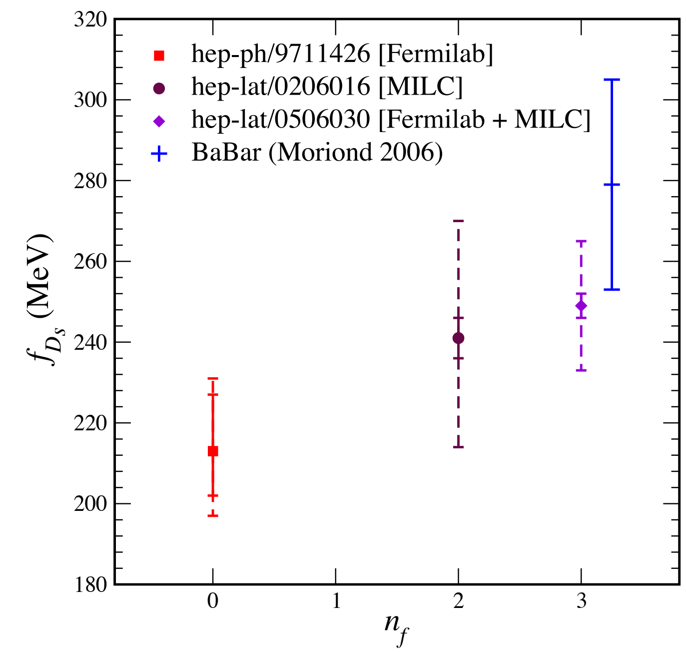


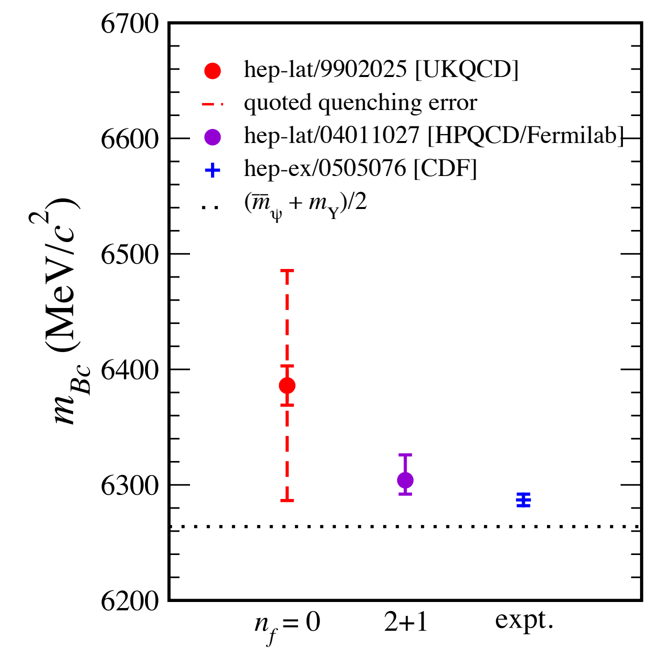
- From 2003!
- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

Predictions

Fermilab Lattice, MILC, HPQCD



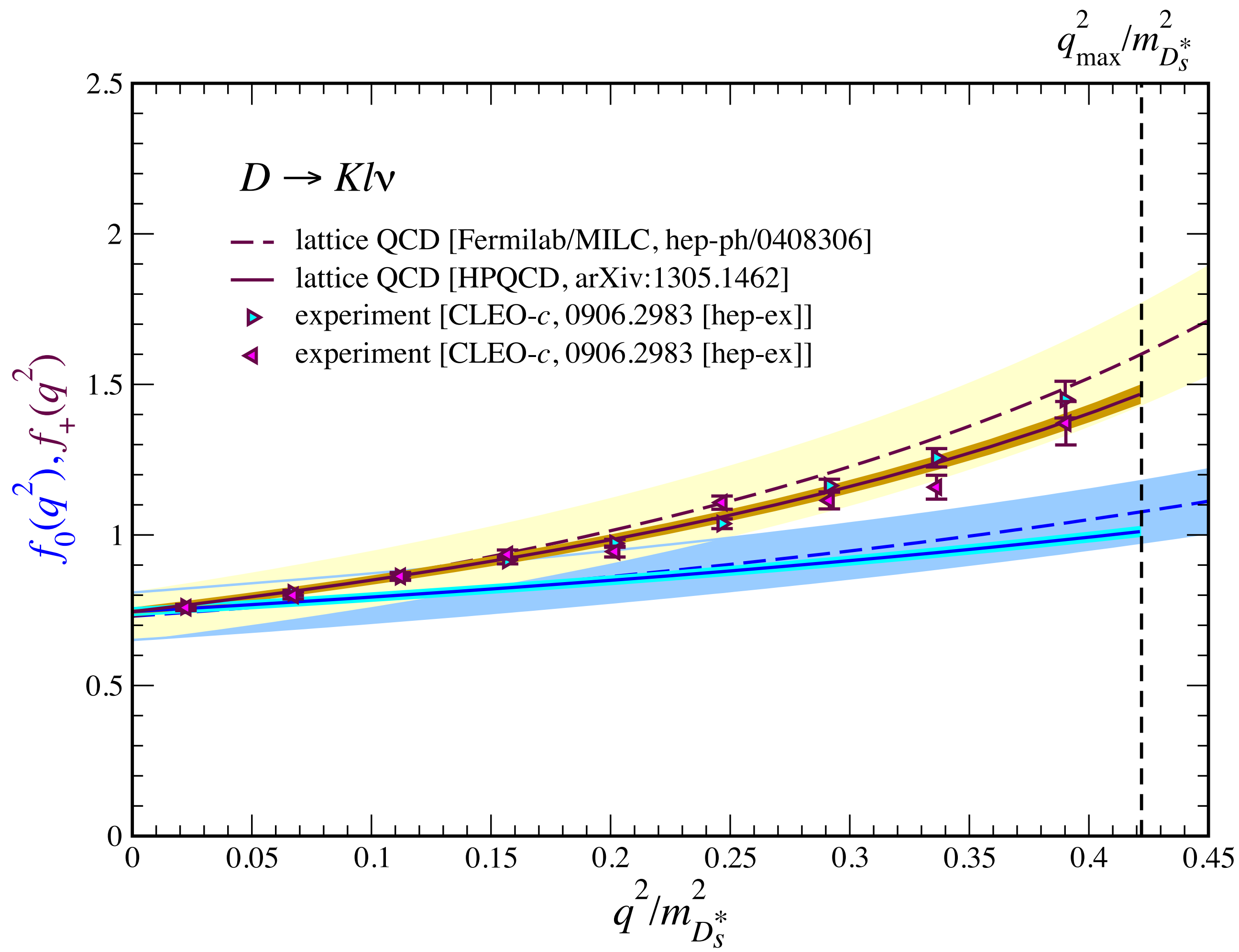




- Semileptonic form factor for $D \to K l \nu$: hep-ph/0408306—updates: arXiv:1008.4562 (normalization), arXiv:1305.1462 (shape);
- Charmed-meson decay constants: hep-lat/0506030—update below;
- Mass of B_c meson: hep-lat/0411027—updates: arXiv:0909.4462, arXiv:1010.3848.

Semileptonic Decays

- Even when the narrative is simple, the best lattice QCD calculation is a moving target.
- FOCUS, Belle, BaBar, & CLEO validated 2004 lattice QCD.
- Lattice QCD is more precise now:
- Instead of check, one can determine $|V_{cs}|$ [arXiv:1305.1462].
- Similarly, determine $|V_{ub}|$, $|V_{cb}|$.



Chiral Condensate

Y. Nambu, *PRL* **4** (1960) 380

- Nambu explained the small pion mass as a consequence of spontaneously broken chiral symmetry; Goldstone: $m_\pi^2\langle\bar{\psi}\psi\rangle = 0$ so either $\langle\bar{\psi}\psi\rangle \neq 0$ or $m_\pi = 0$.

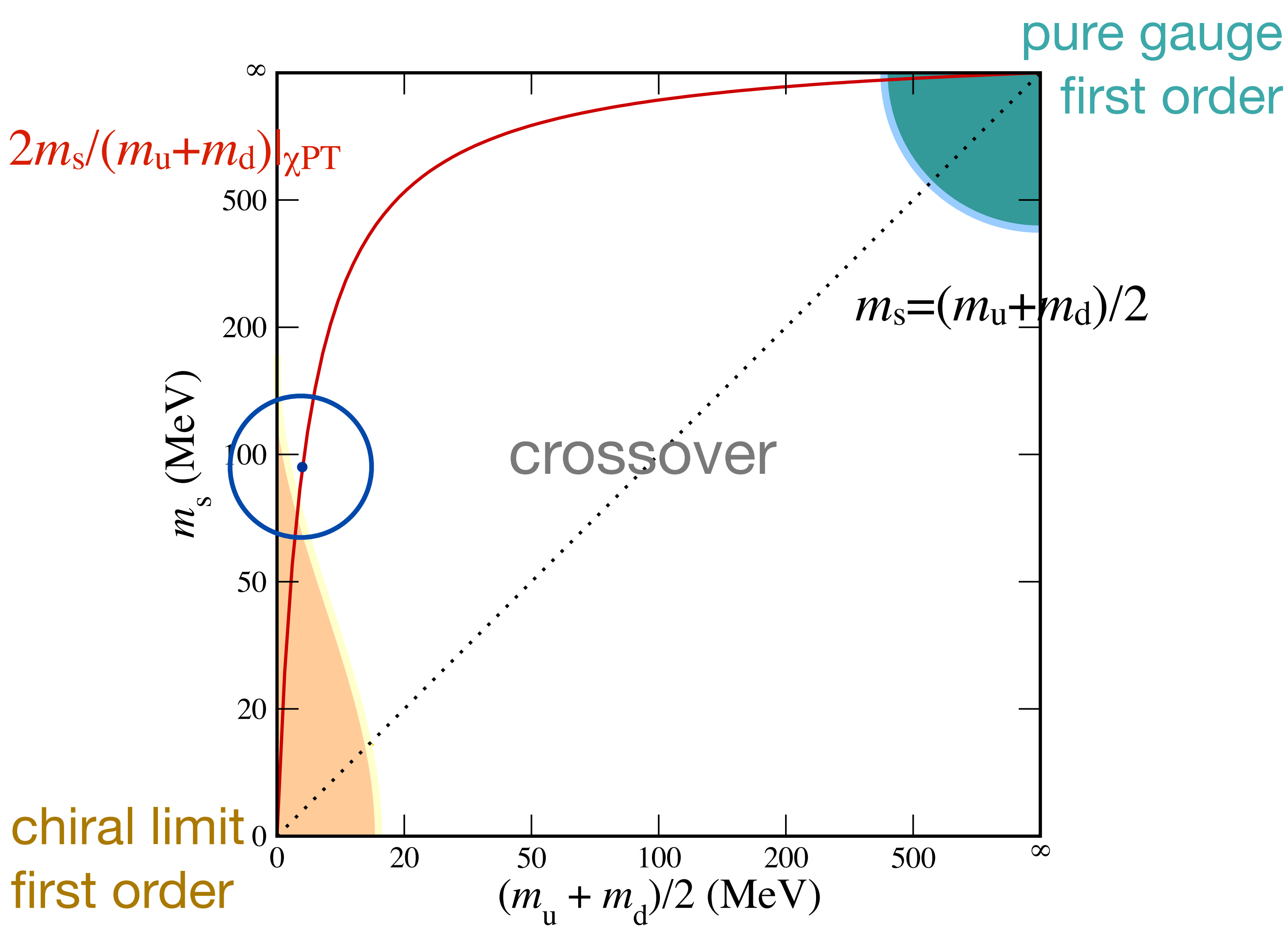
$$\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \left[234 \pm 4_{\mathrm{stat}} \pm 17_{\mathrm{syst}}\ \mathrm{MeV}\right]^3$$

m_{u}, $m_{\mathrm{d}} \to 0$, m_{s} physical [JLQCD, *PRD* **83** (2011) 074501]

- At the hadronic level, the spontaneous breaking of chiral symmetry allows the nucleon mass to be nonzero, even when $m_u = m_d = 0$.

- In nature, m_u & m_d are small, so the physical picture of chiral symmetry is:
  - dominantly spontaneously broken (Nambu's mechanism);
  - small corrections from explicit breaking (chiral perturbation theory).

Quark Masses are Key



after de Forcrand & Philipsen
arXiv:0808.1096

- Explicit χSB softens the transition.
- Quark masses are small, but ...
- ... if even smaller, the $\mu = 0$ transition would be second order, or even first order.
- Implications for the early universe.

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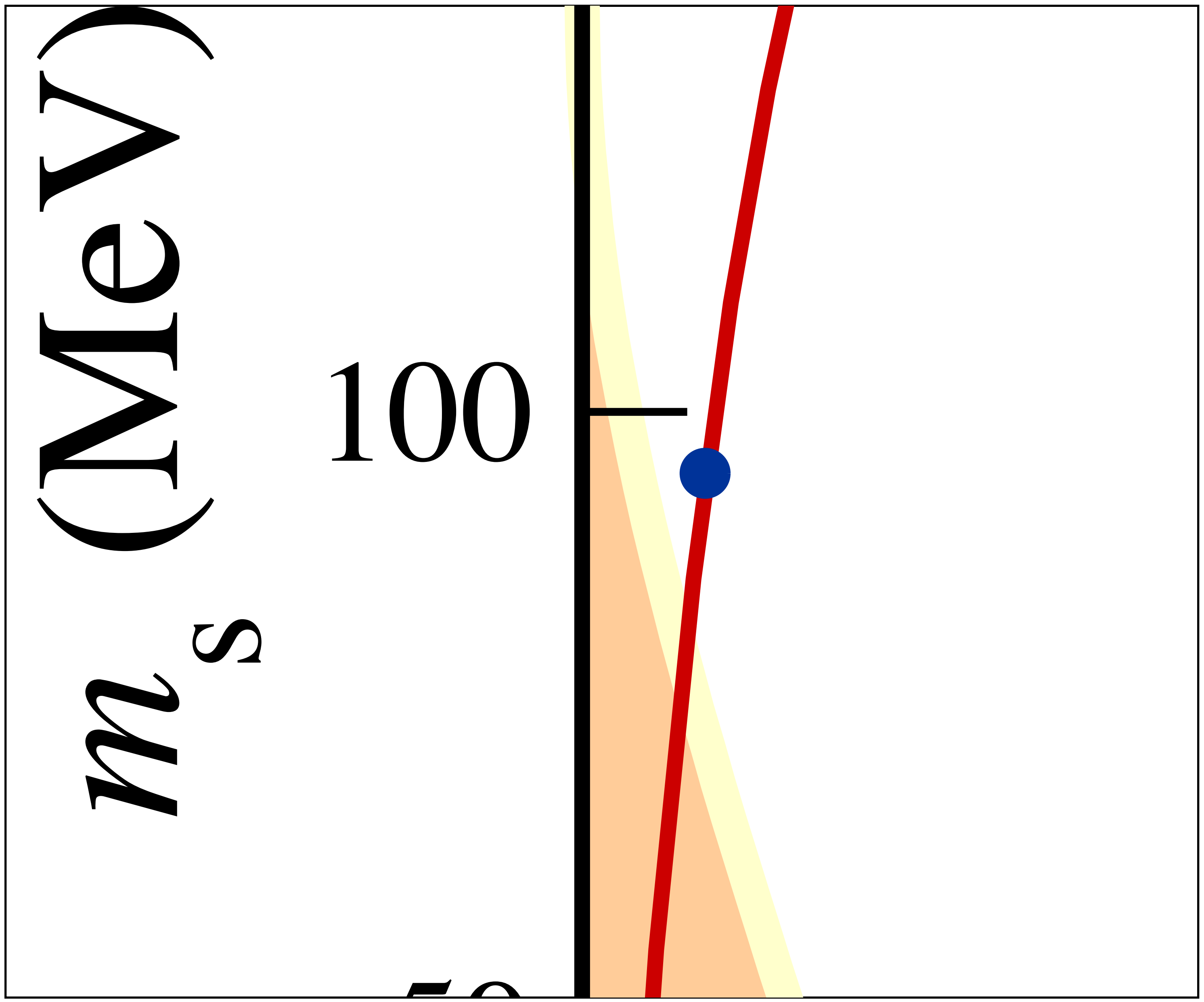

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Lattice QCD and CKM

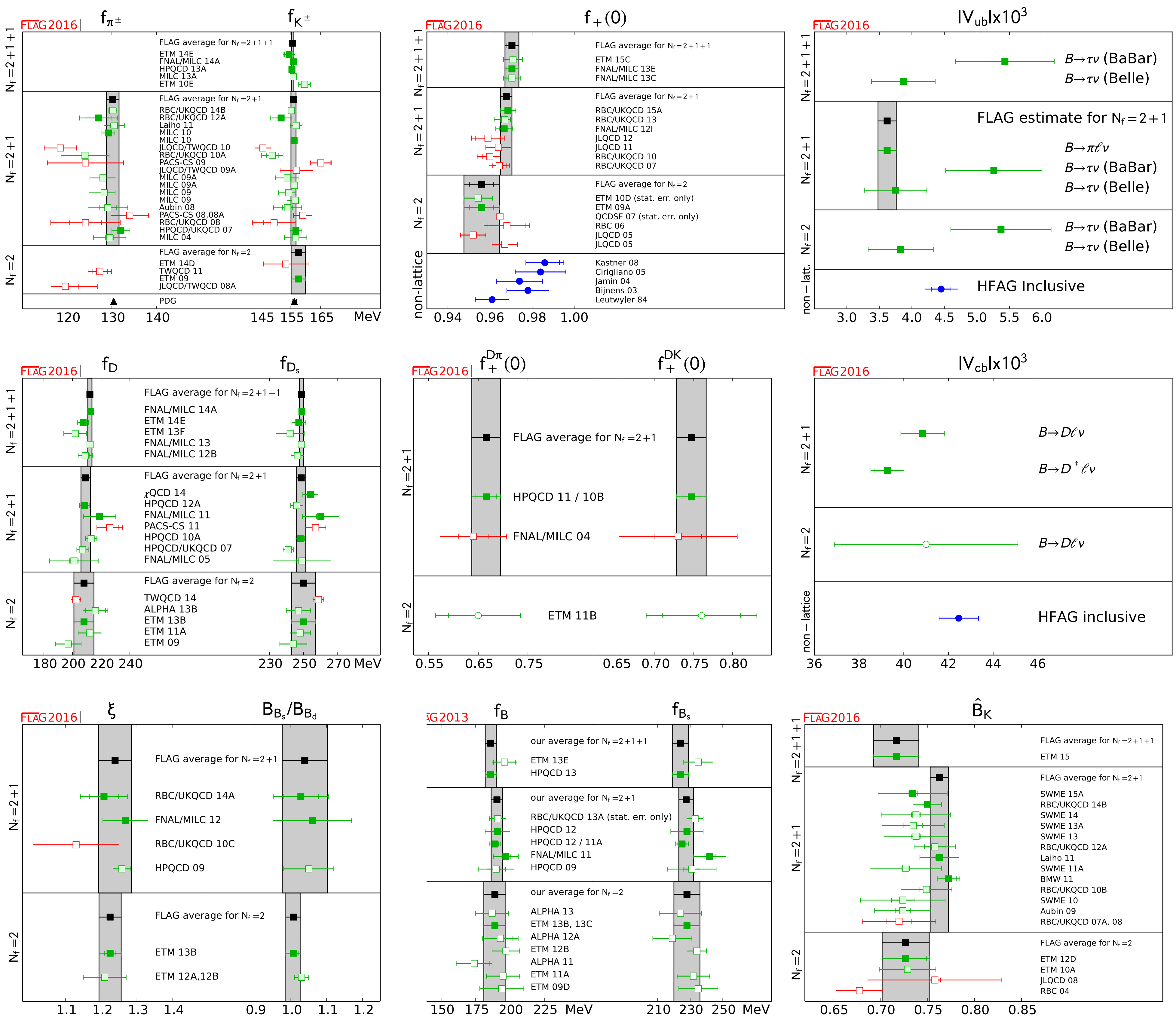
CKM Matrix

Cabibbo, *PRL* **10** (1963) 531; Kobayashi, Maskawa, *Prog. Theor. Phys.* **49** (1973) 652

- Mass couples left to right, breaking $SU_L(2)$ and $U_Y(1)$ (spontaneously).
- Weak and mass eigenbases related by unitary transformation, $D_L = V_{\mathrm{CKM}} D'_L$.
- Global symmetries reduce parameter count of CKM matrix to 4:
  - $|V_{\mathrm{us}}|$, $|V_{\mathrm{cb}}|$, $|V_{\mathrm{ub}}|$, $\delta_{\mathrm{KM}} = \arg V^*_{\mathrm{ub}}$—as fundamental as electron mass.
- Unitarity relations, *e.g.*, $V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb} = 0$: triangles in the complex plane.
- Probed by many measurements + corresponding QCD.

FLAG 2016

arXiv:1607.00299



FLAG2016

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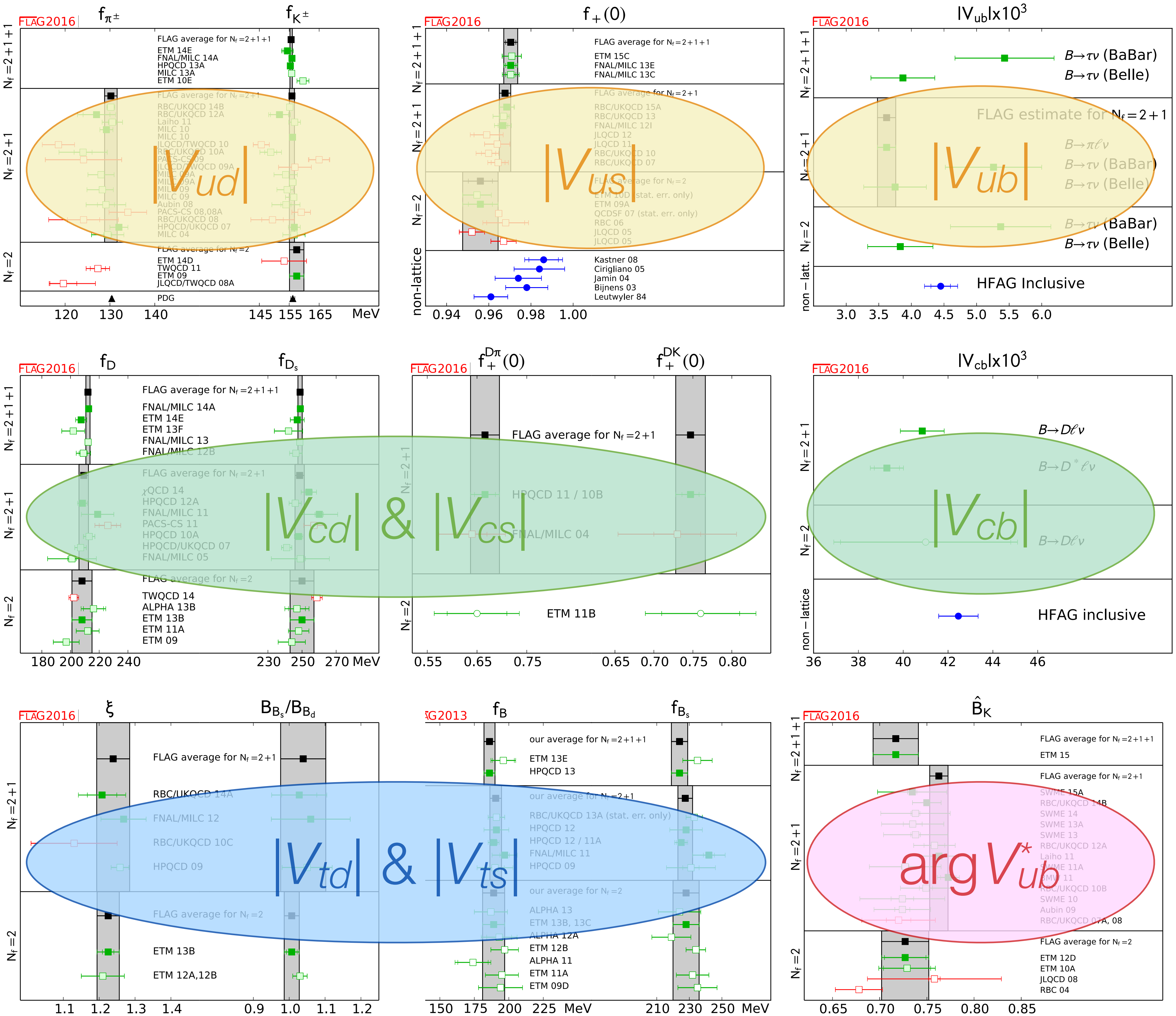
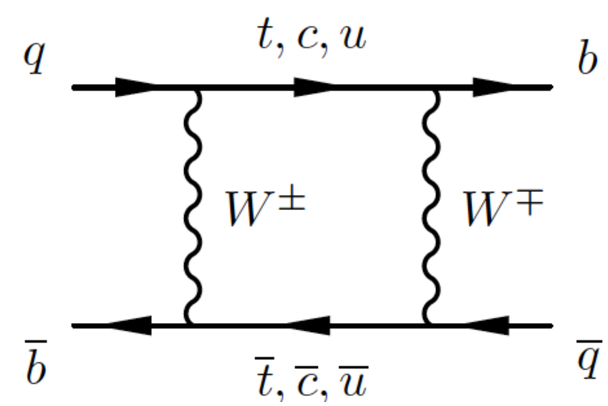


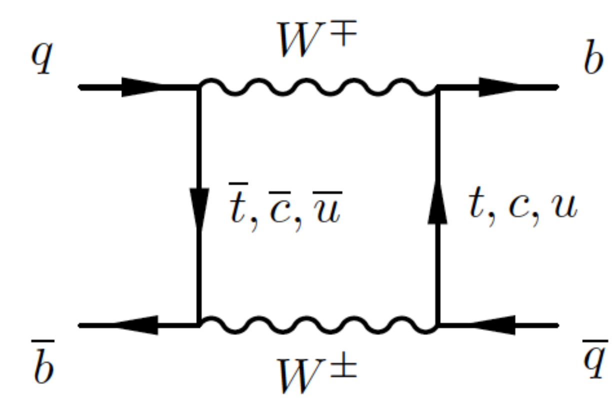
Using FLAG

- FLAG does a useful service: aggregating, evaluating, and averaging results from lattice QCD.
- FLAG asks/insists that users cite the underlying papers:
  - sometimes the “average” contains only one calculation;
  - even if there are several, each one was a lot of work.
- In my view, you need not cite every paper in the FLAG tables, but cite those that lead to the number you use:
  - otherwise you are telling researchers and their funding agencies that the work should stop.

Neutral-Meson Mixing

- In the Standard Model, neutral mesons can oscillate into their antiparticles:





- In extensions of the SM, other particles
  - could appear in the boxes;
  - could appear at the tree level: flavor-changing neutral current.
- Observed for all neutral mesons: K^0, D^0, B^0, B_s.

Basic Observables

- The particle and antiparticle evolve in time via

$$i\frac{d}{dt}\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle = \left(M - i\frac{\Gamma}{2}\right)\left|\begin{array}{c} B^0 \\ \bar{B}^0 \end{array}\right\rangle$$

where M and Γ are 2×2 Hermitian matrices, with $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$.

- Mass difference: $\Delta M \approx 2|M_{12}|$.
- Width difference: $\Delta\Gamma \approx 2|\Gamma_{12}|\cos\phi, \quad \phi = \arg[-M_{12}/\Gamma_{12}]$
- CP asymmetry of flavor-specific decays: $a_{\mathrm{fs}} = \dfrac{|\Gamma_{12}|}{|M_{12}|}\sin\phi$

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP})\mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of all possible operators to 8 = 5 + 3:

$$\mathscr{O}_1 = \bar{b}\gamma^\mu Lq\,\bar{b}\gamma^\mu Lq \qquad \tilde{\mathscr{O}}_1 = \bar{b}\gamma^\mu Rq\,\bar{b}\gamma^\mu Rq$$

$$\mathscr{O}_2 = \bar{b}Lq\,\bar{b}Lq \qquad \tilde{\mathscr{O}}_2 = \bar{b}Rq\,\bar{b}Rq$$

$$\mathscr{O}_3 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Lq^\alpha \qquad \tilde{\mathscr{O}}_3 = \bar{b}^\alpha Rq^\beta\,\bar{b}^\beta Rq^\alpha$$

$$\mathscr{O}_4 = \bar{b}Lq\,\bar{b}Rq$$

$$\mathscr{O}_5 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Rq^\alpha$$

By parity in QCD: $\langle \bar{B}^0|\mathscr{O}_i|B^0\rangle = \langle \bar{B}^0|\tilde{\mathscr{O}}_i|B^0\rangle$

- The off-diagonal terms in the mass and width matrices are related to the $\langle\bar{B}_q|\mathscr{O}_i|B_q\rangle$.

- In the Standard Model, the mass difference (aka oscillation frequency)

$$\Delta M_q = \frac{G_F^2 m_W^2}{4\pi^2 M_{B_q}} \left|V_{tq}^* V_{tb}\right|^2 S_0(m_t^2/m_W^2)\eta_{2B}\langle\bar{B}_q|\mathscr{O}_1|B_q\rangle, \quad q = d, s$$

  where S_0 comes from the box diagrams, η_{2B} from pQCD corrections.

- The product $\eta_{2B}\langle\bar{B}_q|\mathscr{O}_1|B_q\rangle$ is scheme and scale independent.

- In extensions of the Standard Model, any or all of the 5+3 operators could appear.

- In arXiv:1602.03560, Fermilab Lattice + MILC reported the first unquenched lattice QCD calculation of all five matrix elements.

CKM Determination

- If we assume no new physics in $B_{(s)}$-meson mixing, the measured oscillation frequencies can be used to determine the CKM matrix.

- In particular,

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \frac{\Delta M_d}{\Delta M_s} \frac{M_{B_s}}{M_{B_d}}, \qquad \xi^2 = \frac{M_{B_d}^2}{M_{B_s}^2} \frac{\langle \bar{B}_s | \mathscr{O}_1 | B_s \rangle}{\langle \bar{B}_d | \mathscr{O}_1 | B_d \rangle}$$

provides a clean constraint on the CKM unitarity triangle (see below).

- We've reduced the uncertainty on ξ by a factor of 3.

- While we hope mixing is altered by new physics, it is useful to compare different observables by taking the SM to convert them CKM elements.

Chiral-Continuum Extrapolation

nonanalytic terms from NLO HMrSχPT aka “chiral logs”

heavy-quark discretization effects (derived in HQET)

fine tune c-quark hopping parameter

$$F_i = F_i^{\mathrm{logs}} + F_i^{\mathrm{analytic}} + F_i^{\mathrm{HQ\ disc}} + F_i^{\alpha_s a^2\ \mathrm{gen}} + F_i^{\kappa} + F_i^{\mathrm{renorm}}$$

analytic terms in N^{n}LO χPT: base fit $n = 2$

gluon & light-quark cutoff effects a la Symanzik

fit $\alpha_s^2 \rho_{ij}^{[2]}$ (alternatively $\alpha_s^3 \rho_{ij}^{[3]}$)

- Data in r_1 units:

$$r_1^2 F(r_1) = 1$$

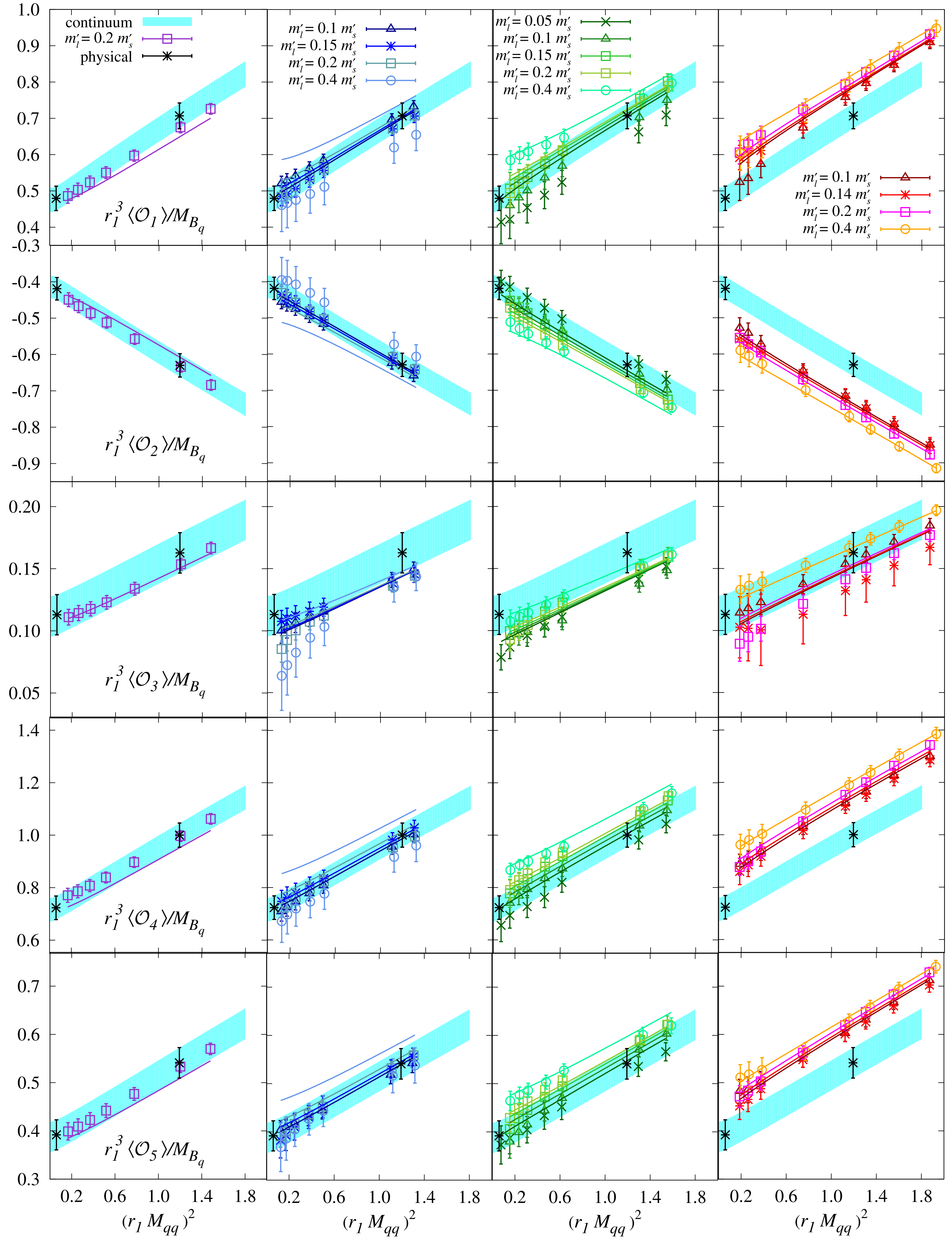
- χPT effects mix the 123 and 45 sets.

- Meson masses are the same for all 5.

- Thus, one fit to all 510 points.

- Reconstitute at $a = 0$ and $m_q = m_d, m_s$.

- $\chi^2_{\text{aug}}/\text{dof} = 137.7/510$



Basic Observables

	B_d–$\bar{B}_d$		B_s–$\bar{B}_s$	
	BMU	BBGLN	BMU	BBGLN
$f_{B_q}^2 B_{B_q}^{(1)}(\overline{m}_b)$	0.0347(27)(7)		0.0503(29)(10)	
$f_{B_q}^2 B_{B_q}^{(2)}(\overline{m}_b)$	0.0290(24)(6)	0.0305(25)(6)	0.0425(26)(9)	0.0451(27)(9)
$f_{B_q}^2 B_{B_q}^{(3)}(\overline{m}_b)$	0.0412(61)(8)	0.0409(60)(8)	0.0585(61)(12)	0.0581(60)(12)
$f_{B_q}^2 B_{B_q}^{(4)}(\overline{m}_b)$	0.0396(28)(8)		0.0540(30)(11)	
$f_{B_q}^2 B_{B_q}^{(5)}(\overline{m}_b)$	0.0366(31)(7)		0.0497(33)(10)	

$$f_{B_d}\sqrt{\hat{B}_{B_d}^{(1)}} = 229.4(9.0)(2.3)\ \text{MeV}$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}^{(1)}} = 276.0(8.0)(2.8)\ \text{MeV}$$

$$\xi = 1.203(17)(6)$$

Correlation Matrix

	$f_{B_d}^2 B_{B_d}^{(1)}$	$f_{B_d}^2 B_{B_d}^{(2)}$	$f_{B_d}^2 B_{B_d}^{(3)}$	$f_{B_d}^2 B_{B_d}^{(4)}$	$f_{B_d}^2 B_{B_d}^{(5)}$	$f_{B_s}^2 B_{B_s}^{(1)}$	$f_{B_s}^2 B_{B_s}^{(2)}$	$f_{B_s}^2 B_{B_s}^{(3)}$	$f_{B_s}^2 B_{B_s}^{(4)}$	$f_{B_s}^2 B_{B_s}^{(5)}$
$f_{B_d}^2 B_{B_d}^{(1)}$	1	0.415	0.124	0.320	0.297	0.845	0.417	0.142	0.323	0.308
$f_{B_d}^2 B_{B_d}^{(2)}$		1	0.332	0.349	0.281	0.416	0.841	0.348	0.360	0.295
$f_{B_d}^2 B_{B_d}^{(3)}$			1	0.204	0.119	0.133	0.316	0.954	0.203	0.125
$f_{B_d}^2 B_{B_d}^{(4)}$				1	0.457	0.343	0.380	0.232	0.848	0.468
$f_{B_d}^2 B_{B_d}^{(5)}$					1	0.312	0.300	0.140	0.449	0.879
$f_{B_s}^2 B_{B_s}^{(1)}$						1	0.464	0.175	0.385	0.357
$f_{B_s}^2 B_{B_s}^{(2)}$							1	0.368	0.437	0.354
$f_{B_s}^2 B_{B_s}^{(3)}$								1	0.257	0.169
$f_{B_s}^2 B_{B_s}^{(4)}$									1	0.508
$f_{B_s}^2 B_{B_s}^{(5)}$										1

Oscillation Frequencies

- Taking CKM from tree-only inputs (from CKMfitter):

$$\Delta M_d^{\mathrm{SM}} = 0.639(50)(36)(5)(13)\ \mathrm{ps}^{-1}$$
$$\Delta M_s^{\mathrm{SM}} = 19.8(1.1)(1.0)(0.2)(0.4)\ \mathrm{ps}^{-1}$$
$$\frac{\Delta M_d^{\mathrm{SM}}}{\Delta M_s^{\mathrm{SM}}} = 0.0323(9)(9)(0)(3)$$

- Contrast with the measured frequencies:

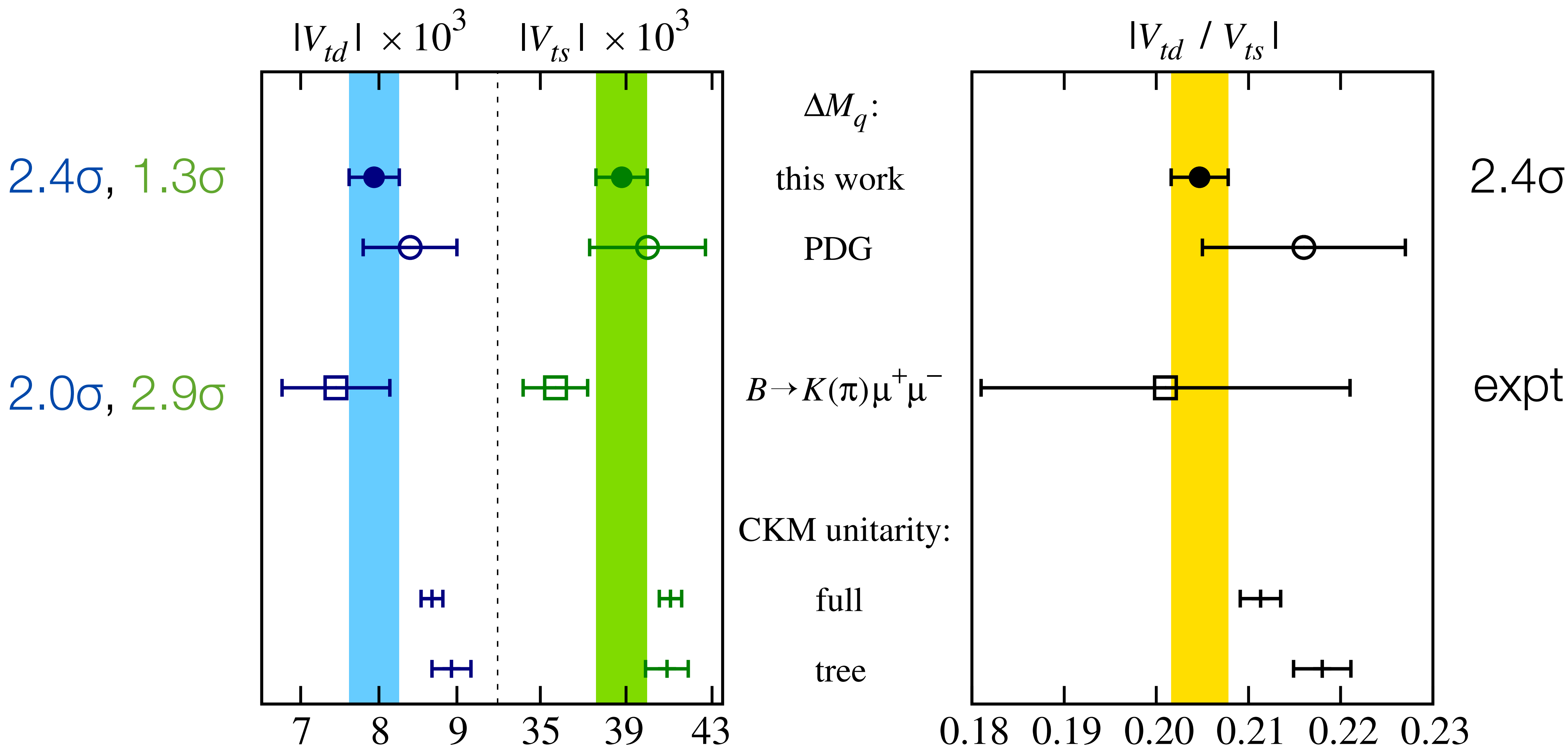
$$\Delta M_d^{\mathrm{expt}} = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1}$$
$$\Delta M_s^{\mathrm{expt}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$

- These amount to discrepancies of 2.1σ, 1.3σ, and 2.9σ, respectively.

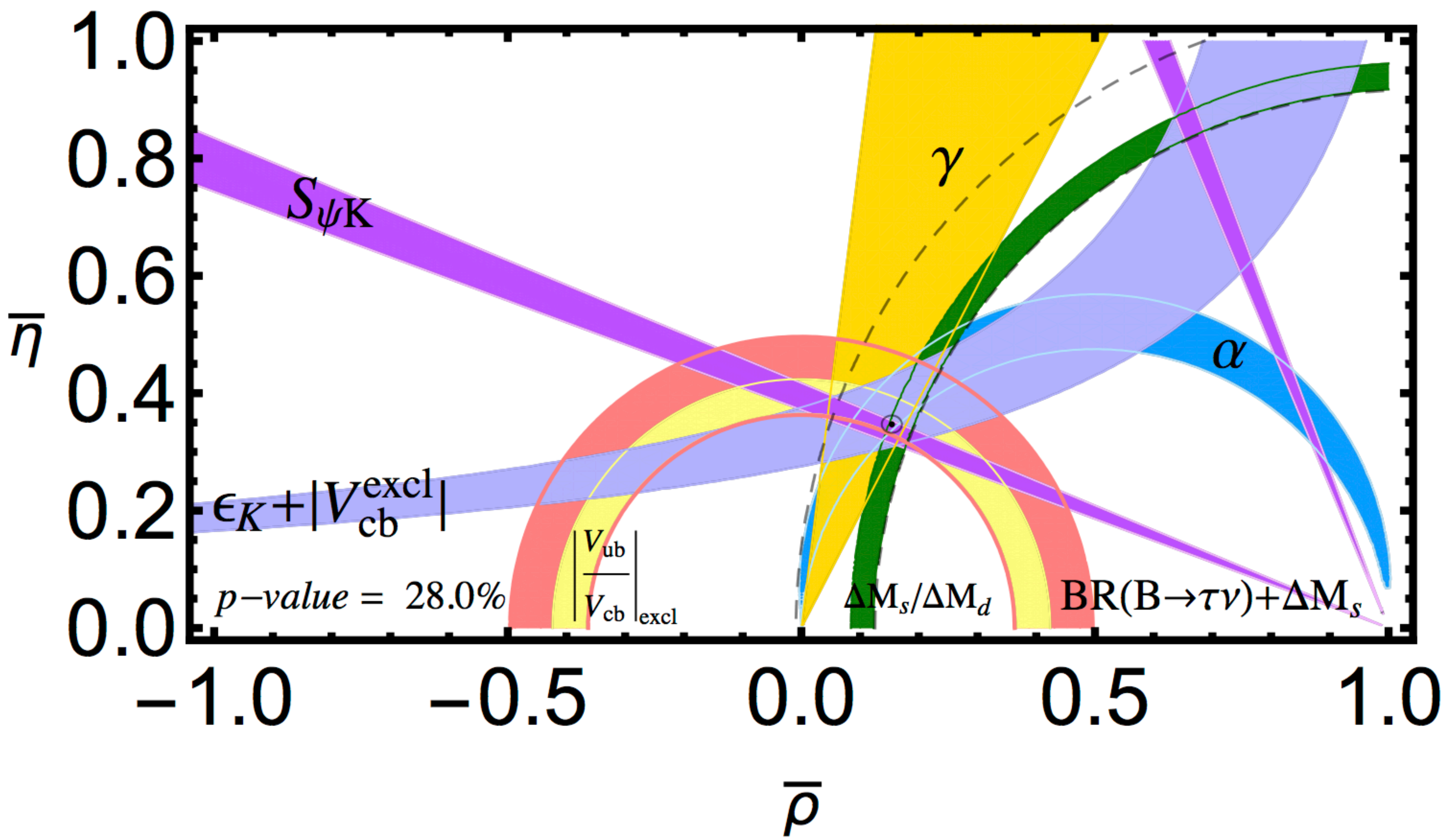
- Examine these tensions with those in other FCNC processes, casting each one as a “CKM determination”.

CKM Comparison

- CKM from FCNC are lower than determinations from trees and unitarity.

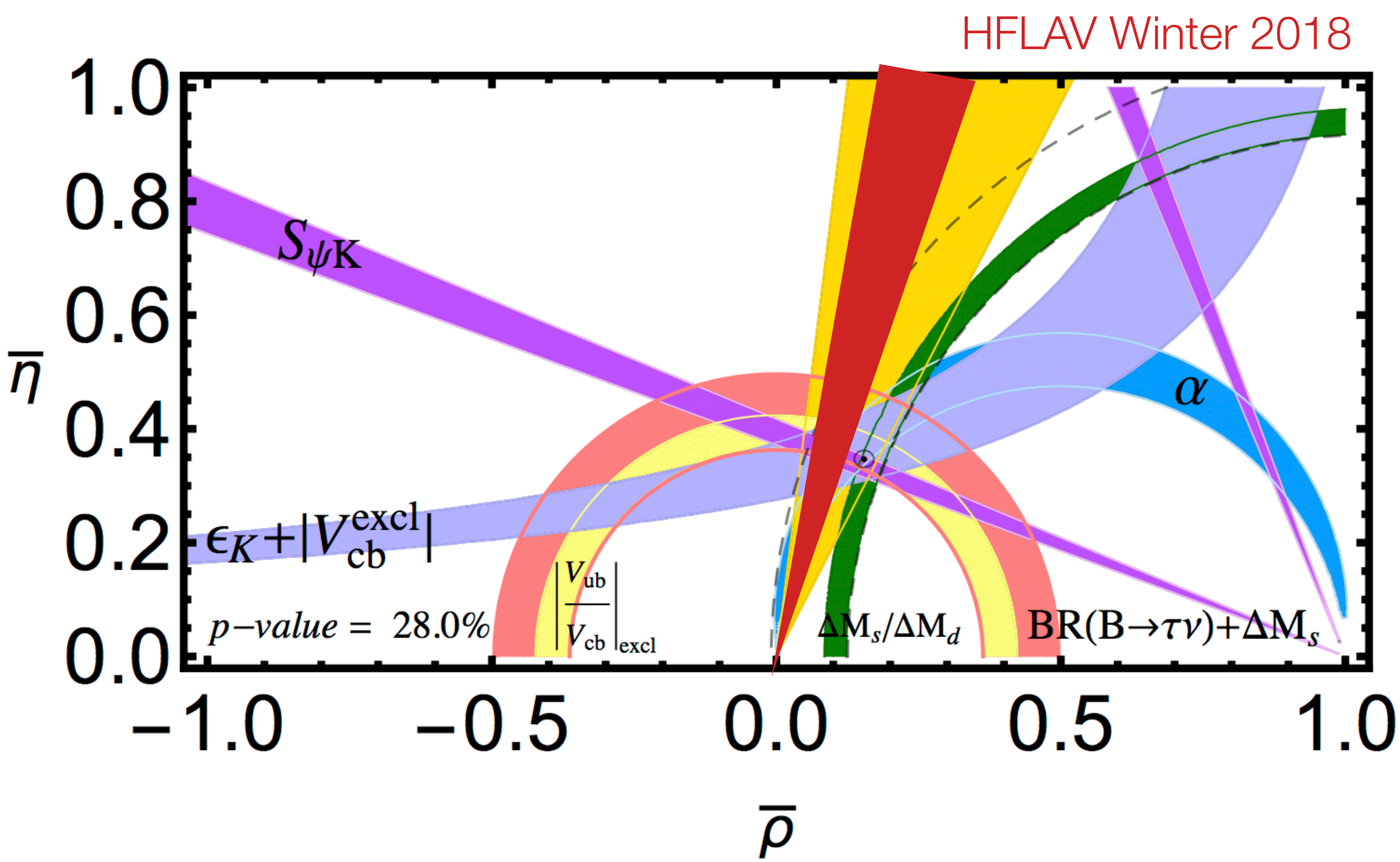


Unitarity Triangle



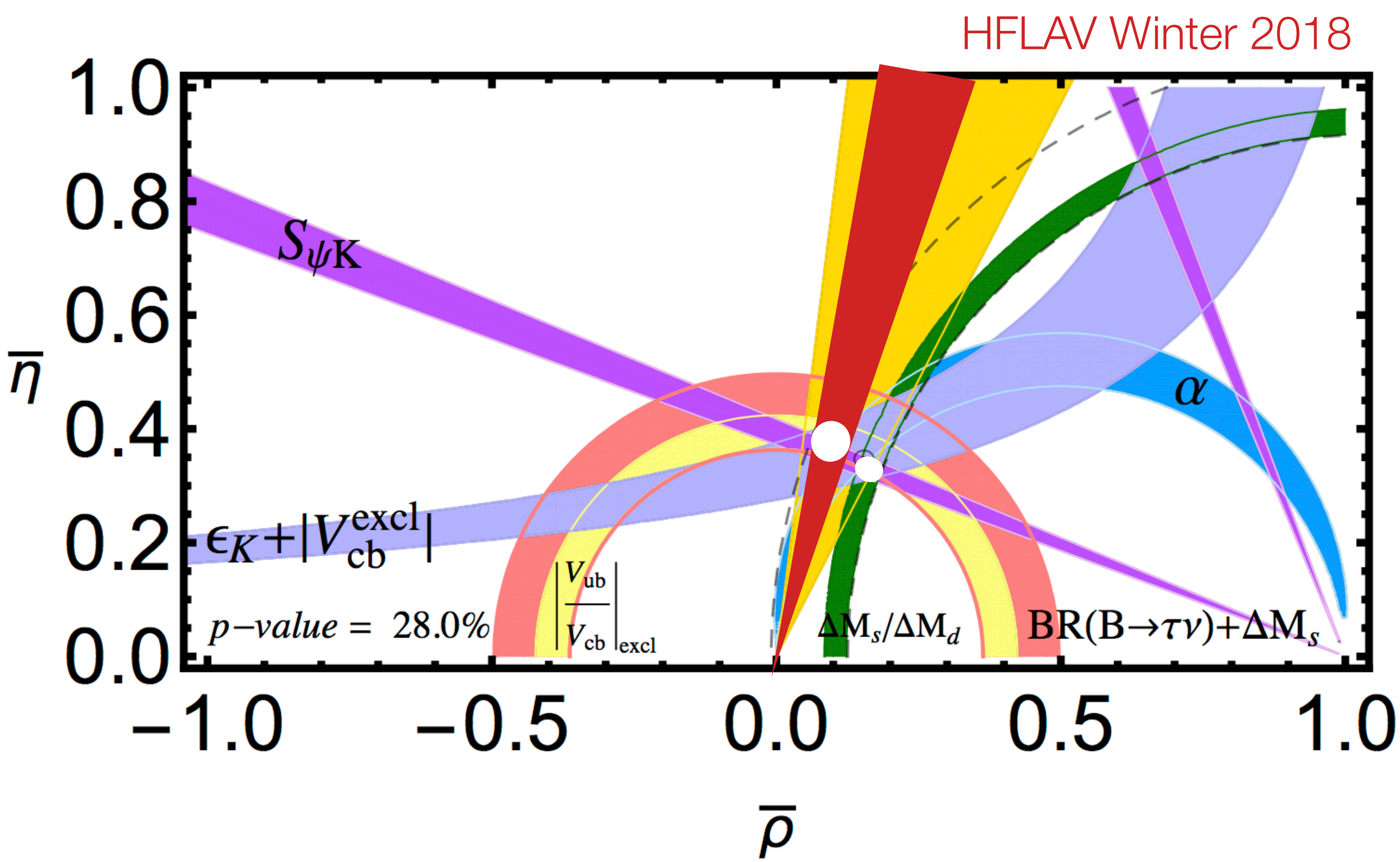
Plot by E.Lunghi

Unitarity Triangle



Plot by E.Lunghi

Unitarity Triangle

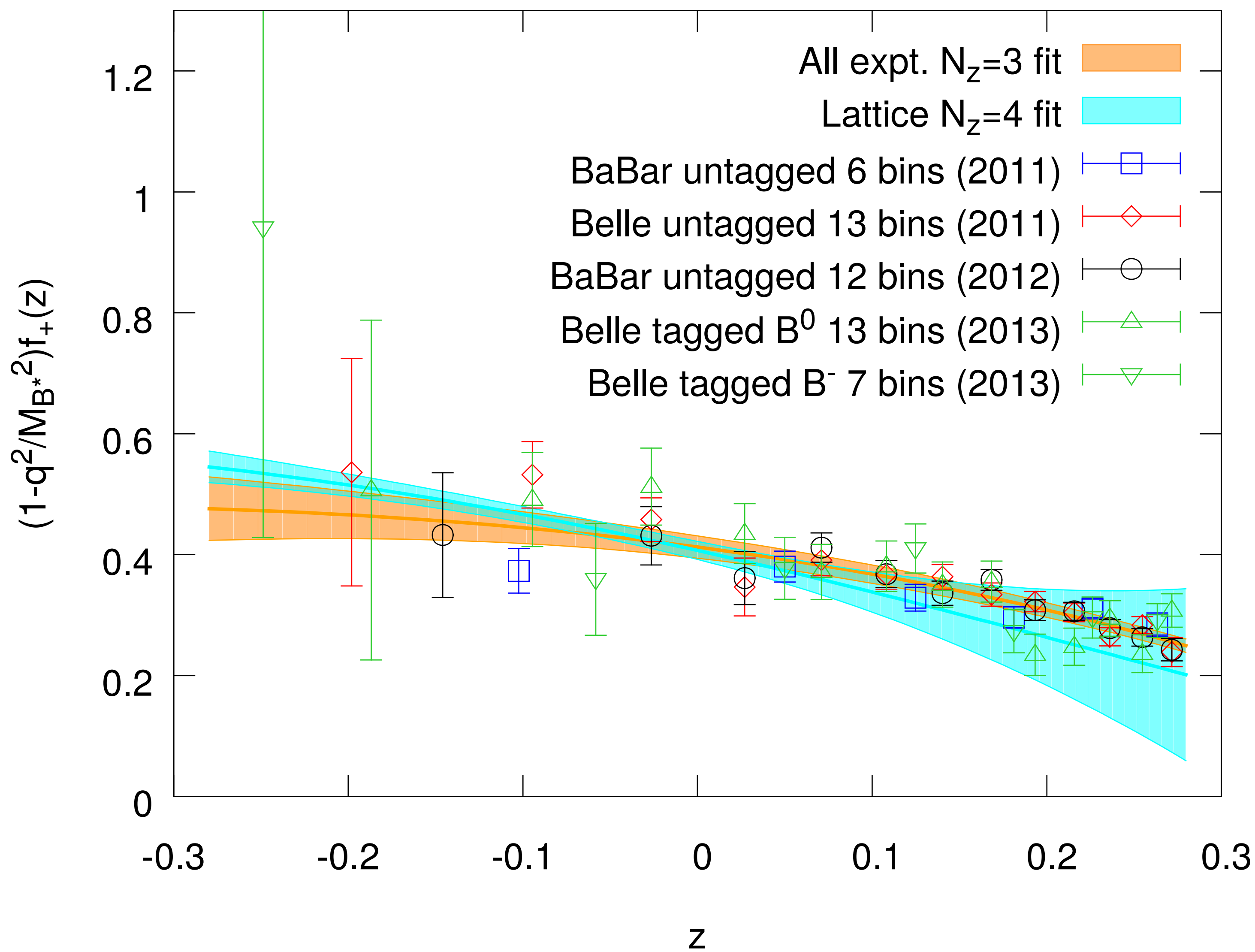


Plot by E.Lunghi

Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

- Much more precise than 2008.
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$ is 4.1%:
  - $10^3|V_{ub}| = 3.72 \pm 0.16$



Analyticity and Unitarity

- The variable z provides a useful expansion: $|z| \leq 0.3$.
- Formulas (Bourrely, Caprini, Lellouch, arXiv:0807.2722):

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z - 1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]$$

$$f_0(z) = \sum_{n=0}^{N_z} b_n^0 z^n$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Subthreshold 1^- pole in f_+; first 0^+ excitation (for f_0) is unstable.

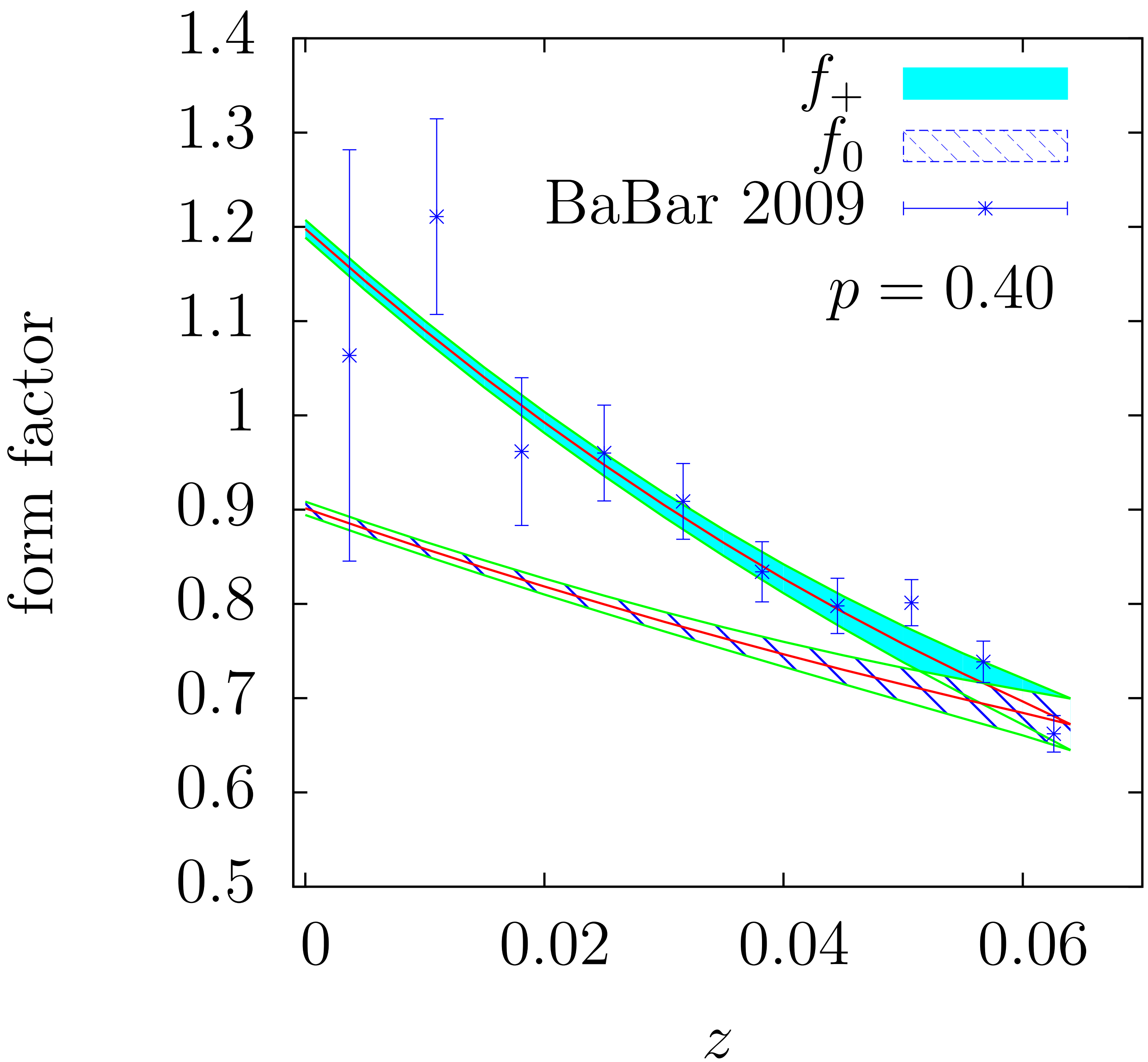
Semileptonic $B \to D l \nu$ for $|V_{cb}|$

arXiv:1503.07237

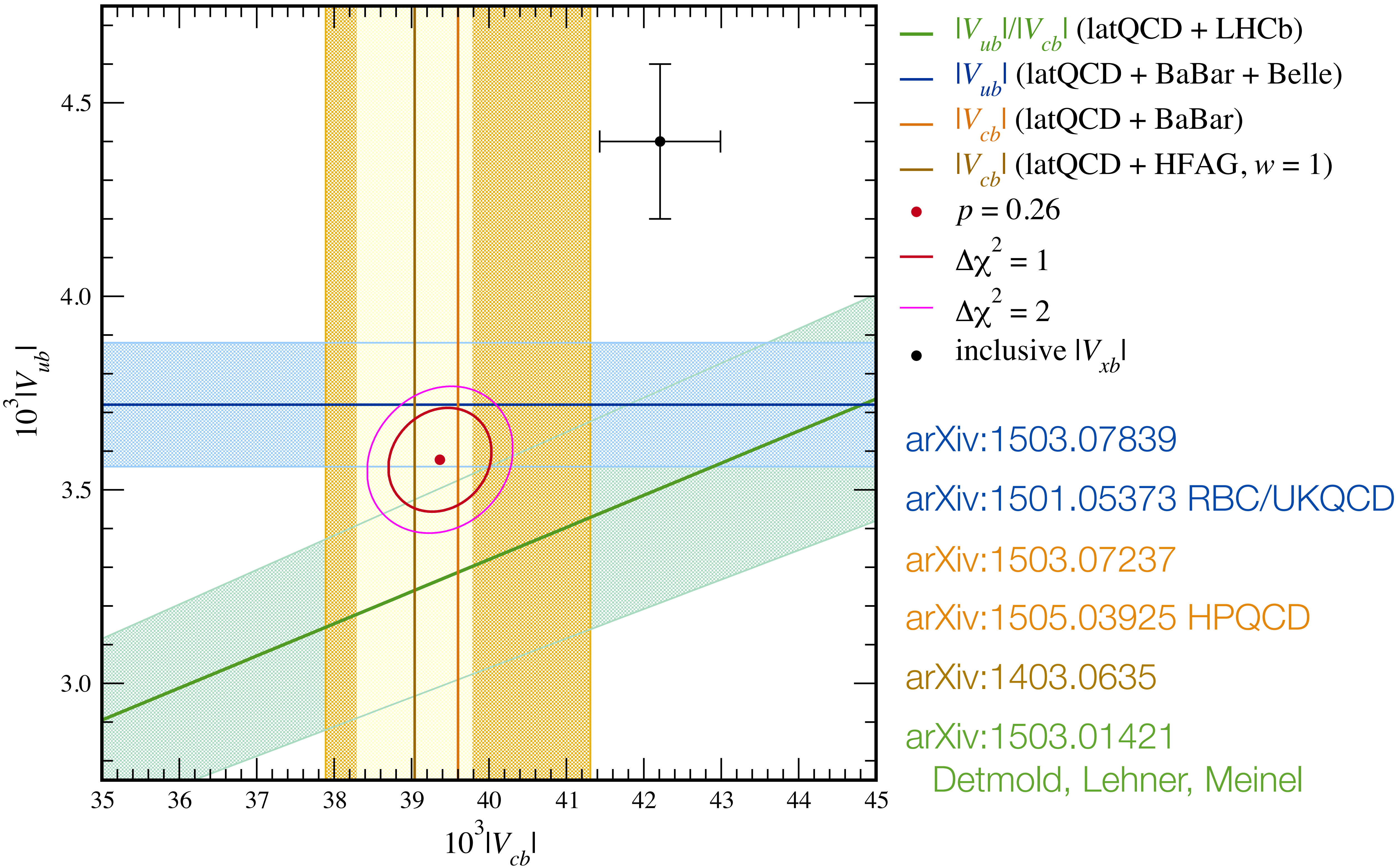
- Similar strategy as above:
  - compute sequence of form-factor values;
  - chiral continuum extrapolation;
  - combined z-expansion fit to obtain $|V_{cb}|$.
- Differences:
  - HQET control of cutoff effects more central [basics in hep-lat/0002008, hep-lat/0112044, hep-lat/0112045];
  - use Boyd, Grinstein, Lebed form of z expansion [hep-ph/9508211].

Combining Lattice QCD with Experiment

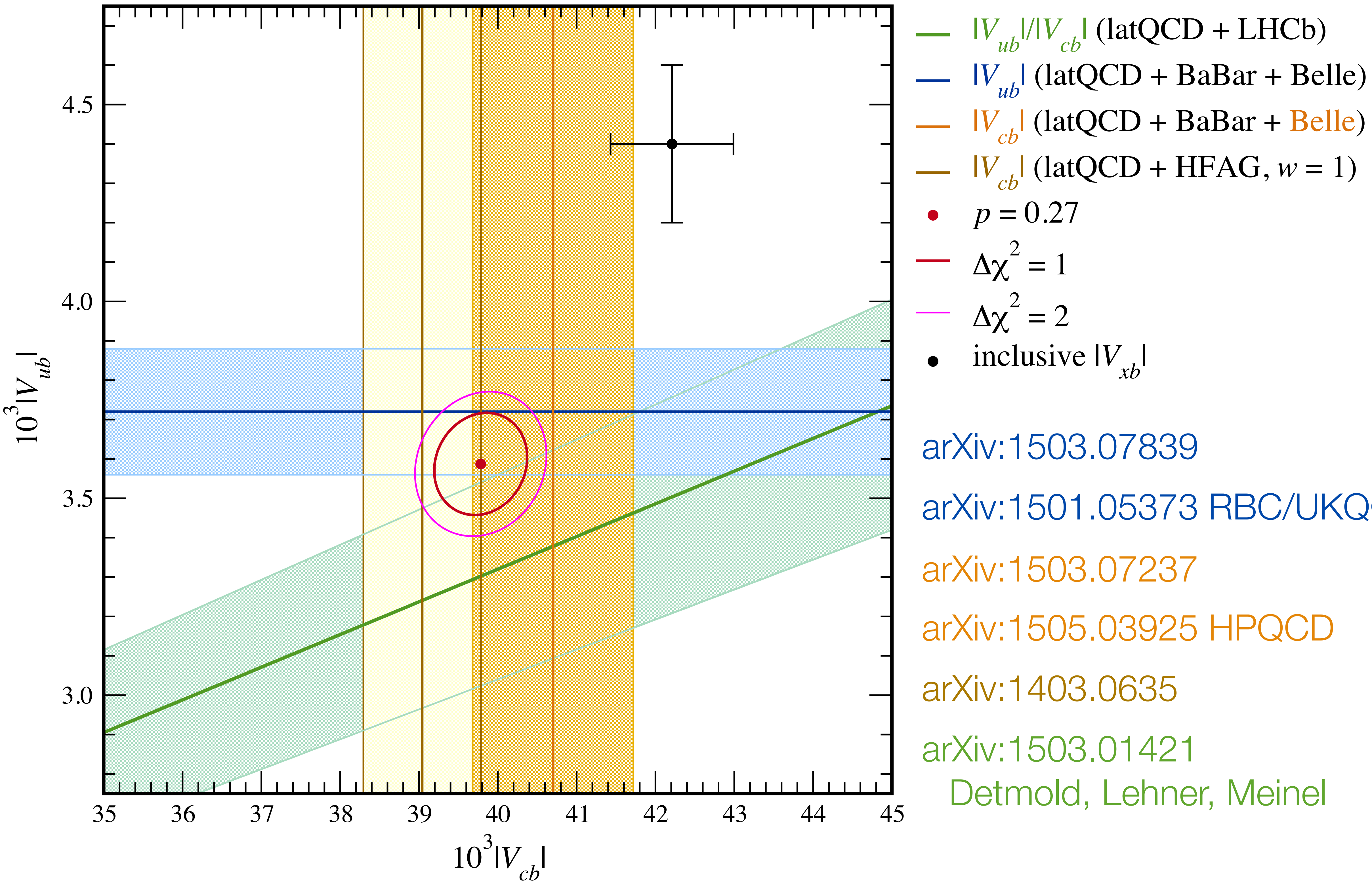
- $B \to Dl\nu$, Fermilab/MILC, arXiv:1503.07237.



$10^3|V_{cb}| = 39.6(1.8)$



4.5
4.0
3.5
3.0
35
36
37
38
39
40
41
42
43
44
45
$10^3|V_{ub}|$
$10^3|V_{cb}|$
$|V_{ub}|/|V_{cb}|$ (latQCD + LHCb)
$|V_{ub}|$ (latQCD + BaBar + Belle)
$|V_{cb}|$ (latQCD + BaBar)
$|V_{cb}|$ (latQCD + HFAG, $w = 1$)
$p = 0.26$
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2$
inclusive $|V_{xb}|$
arXiv:1503.07839
arXiv:1501.05373 RBC/UKQCD
arXiv:1503.07237
arXiv:1505.03925 HPQCD
arXiv:1403.0635
arXiv:1503.01421
Detmold, Lehner, Meinel



arXiv:1503.07839

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arXiv:1503.07237

arXiv:1505.03925 HPQCD

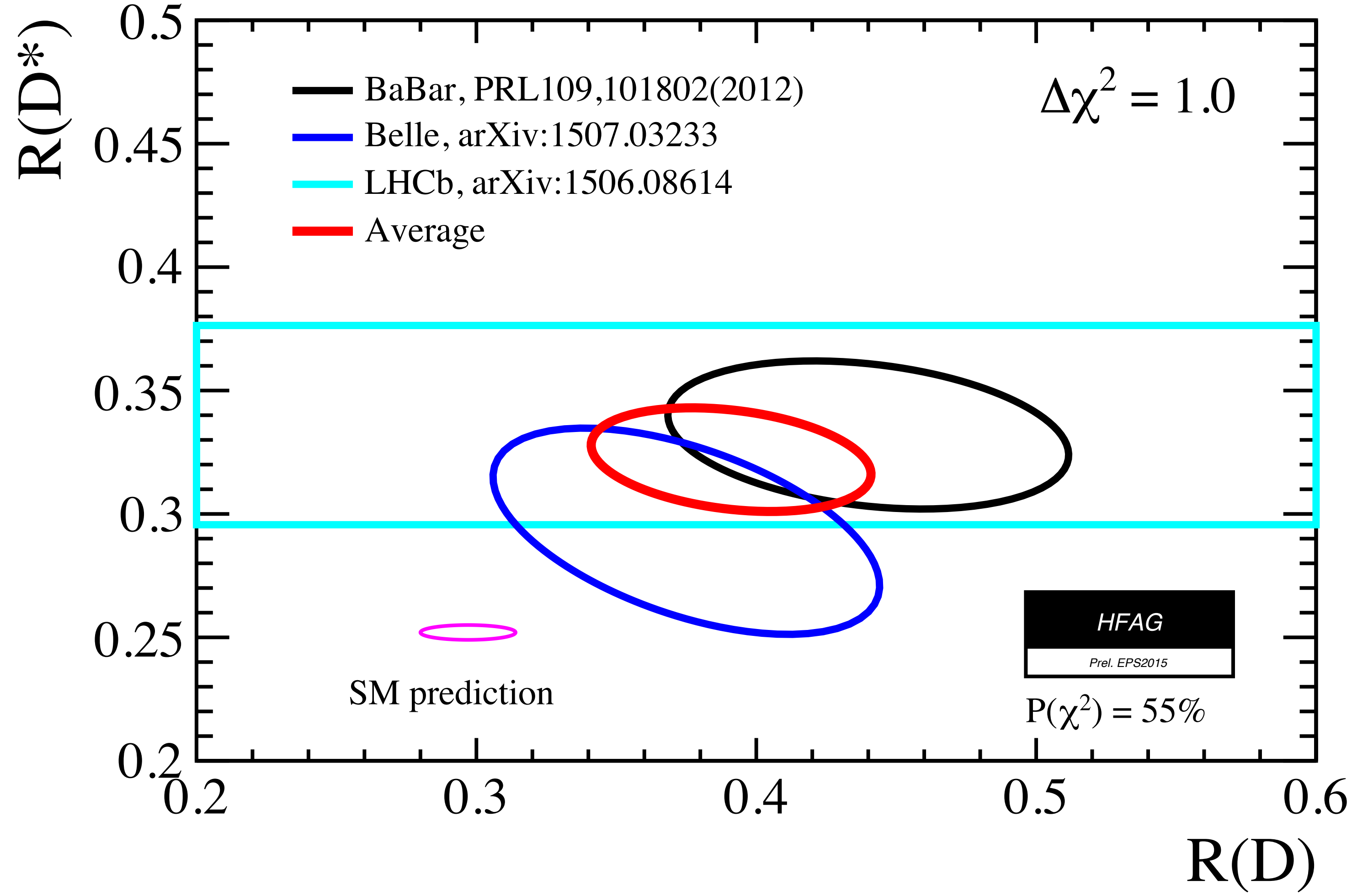
arXiv:1403.0635

arXiv:1503.01421
Detmold, Lehner, Meinel

New Physics in $B \to D^{(*)}\tau\nu$?

BaBar, arXiv:1205.5442; Belle, arXiv:1507.03233; LHCb, arXiv:1506.08614

- BaBar presented evidence for an excess in both channels:
  - 2.0σ for $R(D)$; 2.7σ for $R(D^*)$; 3.4σ combined.
- With Belle & LHCb:
  - 3.9σ combined.
- Estimated form factors w/
  - HQET;
  - quenched QCD.

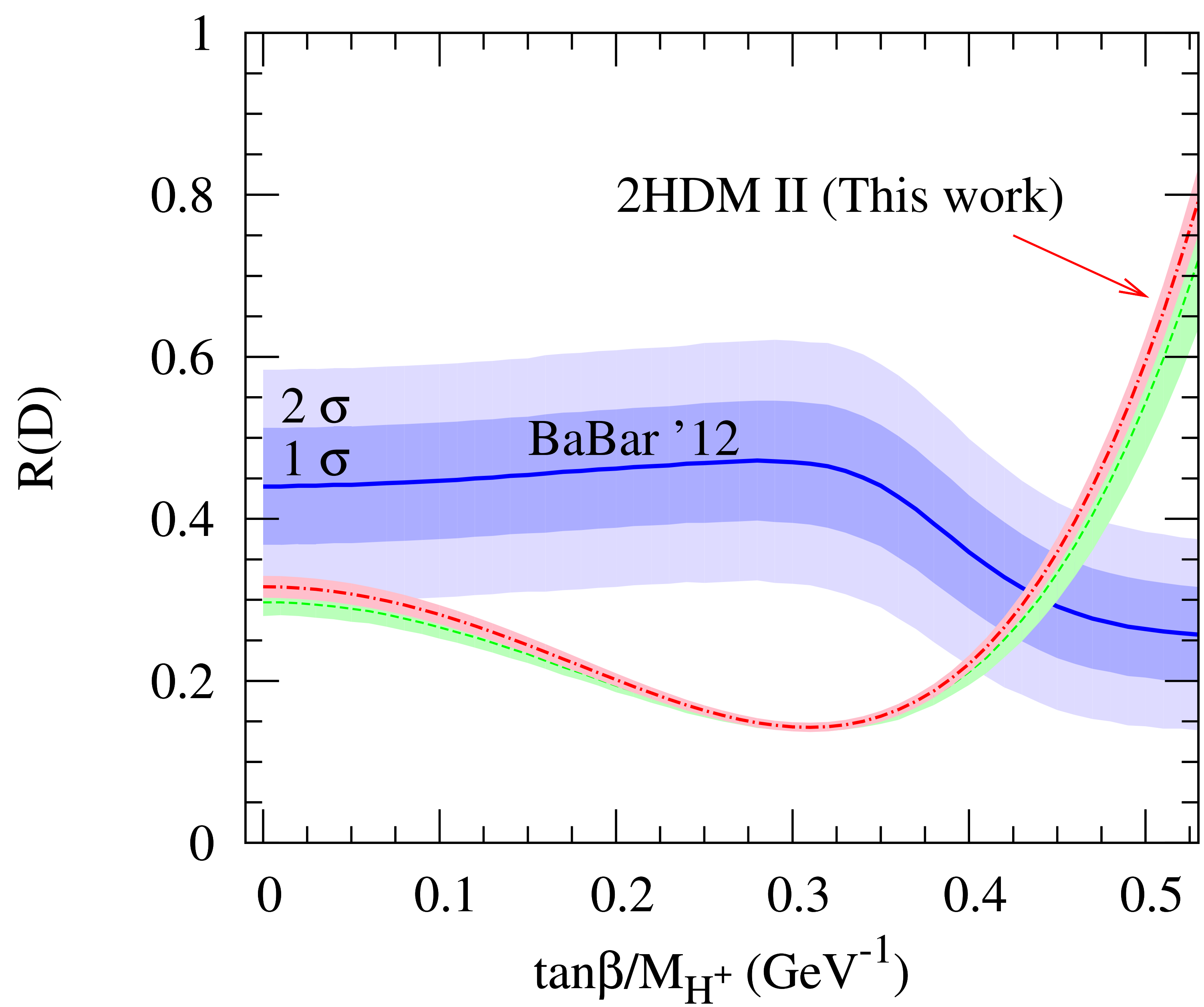


Form Factors for $B \to D^{(*)}\tau\nu$

Fermilab/MILC, arXiv:1206.4992, arXiv:1503.07237; HPQCD, arXiv:1505.03925

see also arXiv:1206.4977.

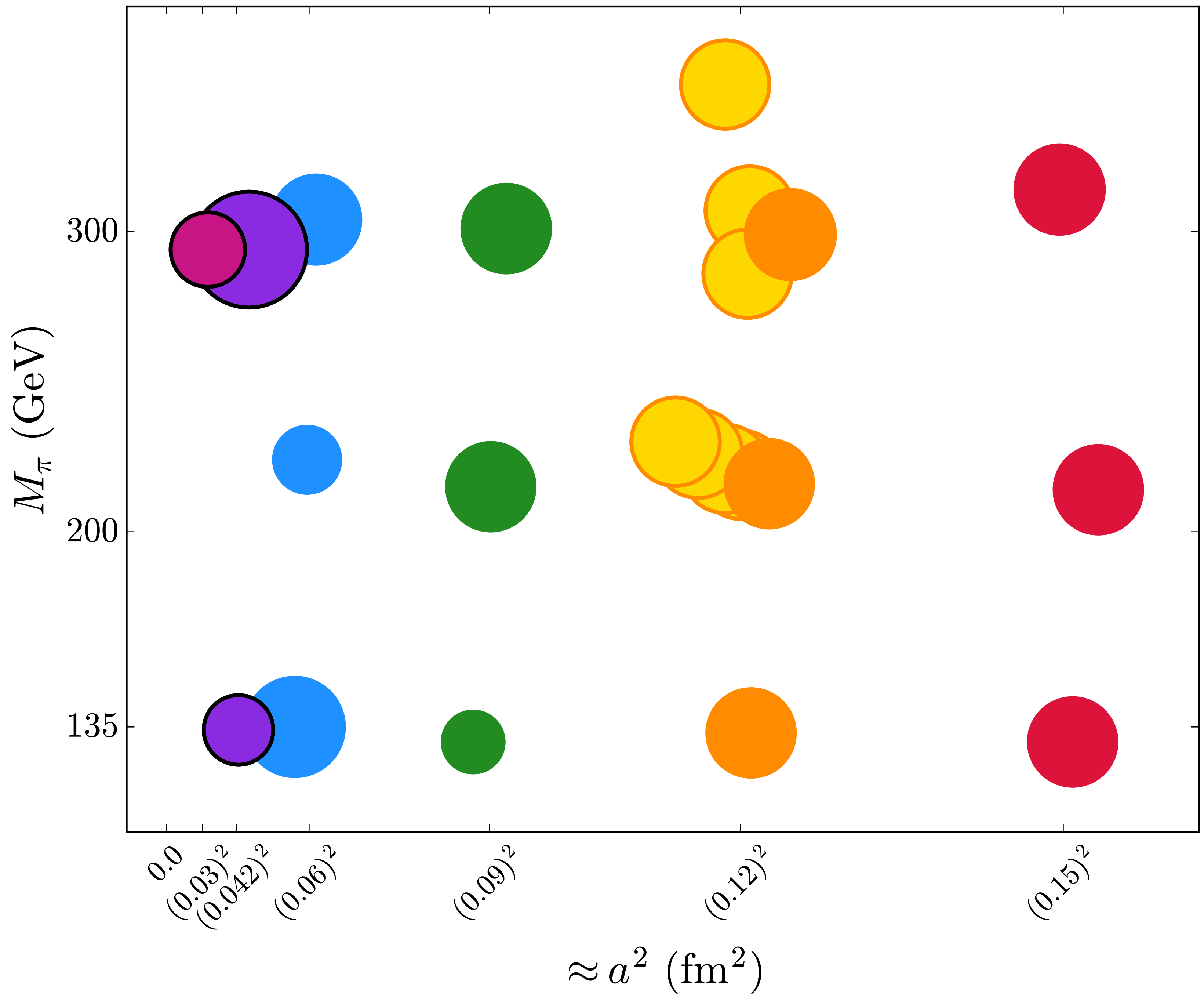
- $R(D)$ values:
  - 0.297 ± 0.017 (est.);
  - 0.316 ± 0.014 (F/M ’12);
  - 0.299 ± 0.011 (F/M ’15);
  - 0.300 ± 0.008 (HPQCD).
- Lattice QCD work for $R(D^*)$ now underway.



MILC HISQ Ensembles

arXiv:1212.4768 + update in arXiv:1712.09262

- 2+1+1 sea quarks;
- 24 ensembles
- 5 w/ $M_\pi = 135$ MeV;
- down to $a = 0.03$ fm;
- typically 1000×4 samples;
- $M_\pi L > 3.2$, often > 5;
- up to $144^3 \times 288$.



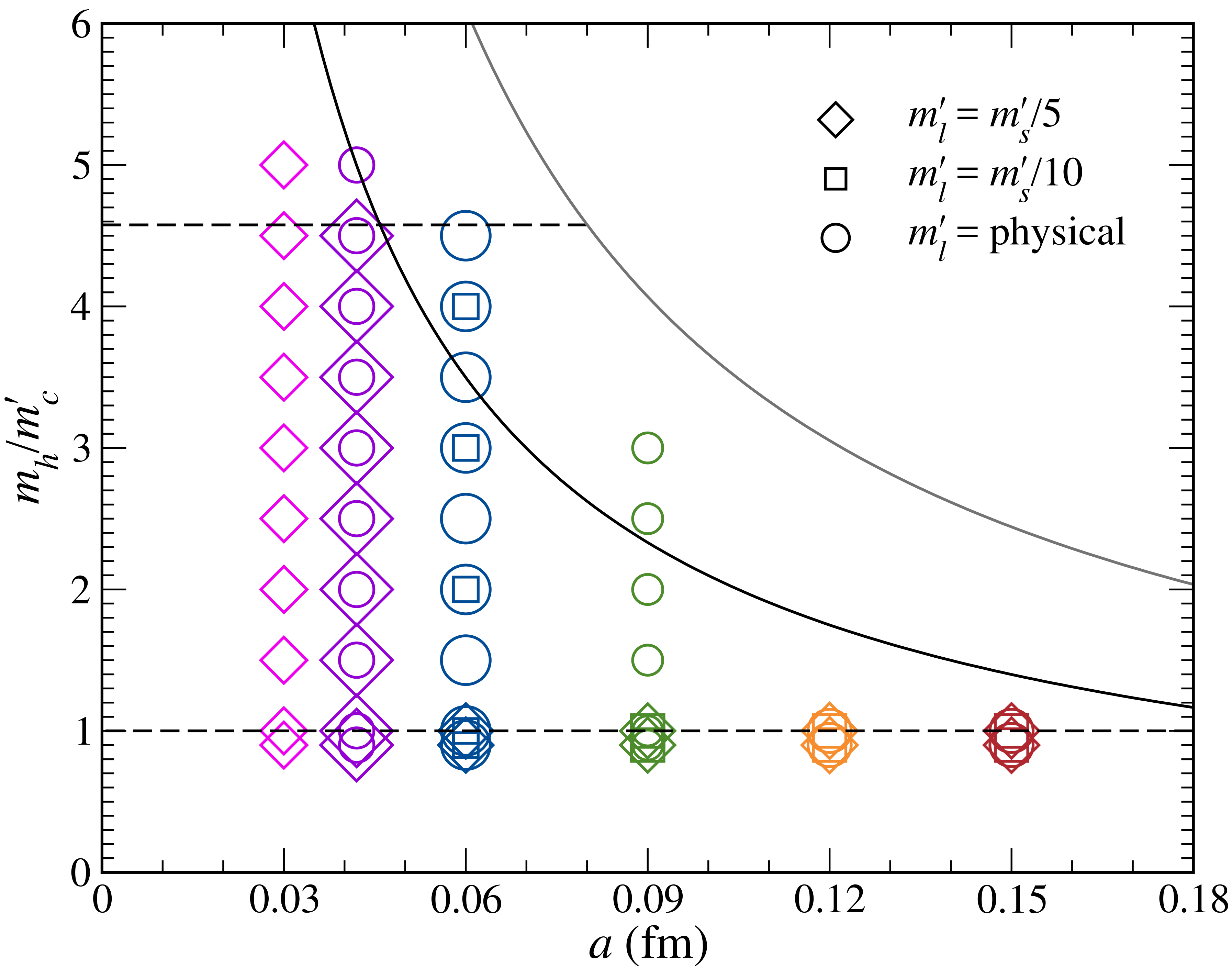
HISQ Ensembles: 2+1+1

MILC, arXiv:1212.4768 + further runs

a (fm)	size	$am_l'/am_s'/am_c'$	# confs	# sources	notes
≈ 0.15	$16^3\times$ 48	0.0130/0.065/0.838	1020	4	
≈ 0.15	$24^3\times$ 48	0.0064/0.064/0.828	1000	4	
≈ 0.15	$32^3\times$ 48	0.00235/0.0647/0.831	1000	4	physical
≈ 0.12	$24^3\times$ 64	0.0102/0.0509/0.635	1040	4	
≈ 0.12	$32^3\times$ 64	0.00507/0.0507/0.628	1020	4	also 24^3, 40^3
≈ 0.12	$48^3\times$ 64	0.00184/0.0507/0.628	999	4	physical
≈ 0.12	$24^3\times$ 64	0.0102/0.03054/0.635	1020	4	$m_s' < m_s$
≈ 0.12	$24^3\times$ 64	0.01275/0.01275/0.640	1020	4	$m_s' = m_l'$
≈ 0.12	$32^3\times$ 64	0.00507/0.0304/0.628	1020	4	$m_s' < m_s$
≈ 0.12	$32^3\times$ 64	0.00507/0.022815/0.628	1020	4	$m_s' < m_s$
≈ 0.12	$32^3\times$ 64	0.00507/0.012675/0.628	1020	4	$m_s' \ll m_s$
≈ 0.12	$32^3\times$ 64	0.00507/0.00507/0.628	1020	4	$m_s' = m_l'$
≈ 0.12	$32^3\times$ 64	0.0088725/0.022815/0.628	1020	4	$m_s' < m_s$
≈ 0.09	$32^3\times$ 96	0.0074/0.037/0.440	1005	4	
≈ 0.09	$48^3\times$ 96	0.00363/0.0363/0.430	999	4	
≈ 0.09	$64^3\times$ 96	0.0012/0.0363/0.432	484	4	physical
≈ 0.06	$48^3\times$144	0.0048/0.024/0.286	1016	4	
≈ 0.06	$64^3\times$144	0.0024/0.024/0.286	572	4	
≈ 0.06	$96^3\times$192	0.0008/0.022/0.260	842	6	physical
≈ 0.042	$64^3\times$192	0.00316/0.0158/0.188	1167	6	
≈ 0.042	$144^3\times$288	0.000569/0.01555/0.1827	429	6	physical
≈ 0.03	$96^3\times$288	0.00223/0.01115/0.1316	724	4	

Heavy-Quark Masses

- always $0.9m_c$, m_c;
- up to $5m_c$;
- omit $am_c \geq 0.9$ from heavy-quark fits (need < $\pi/2$);
- omit 0.15 fm in base fit;
- 492 data points (498 w/ 0.15 fm).



Heavy-Quark Masses

- always $0.9m_c$, m_c;
- up to $5m_c$;
- omit $am_c \geq 0.9$
  f
  f
- c
  base fit;
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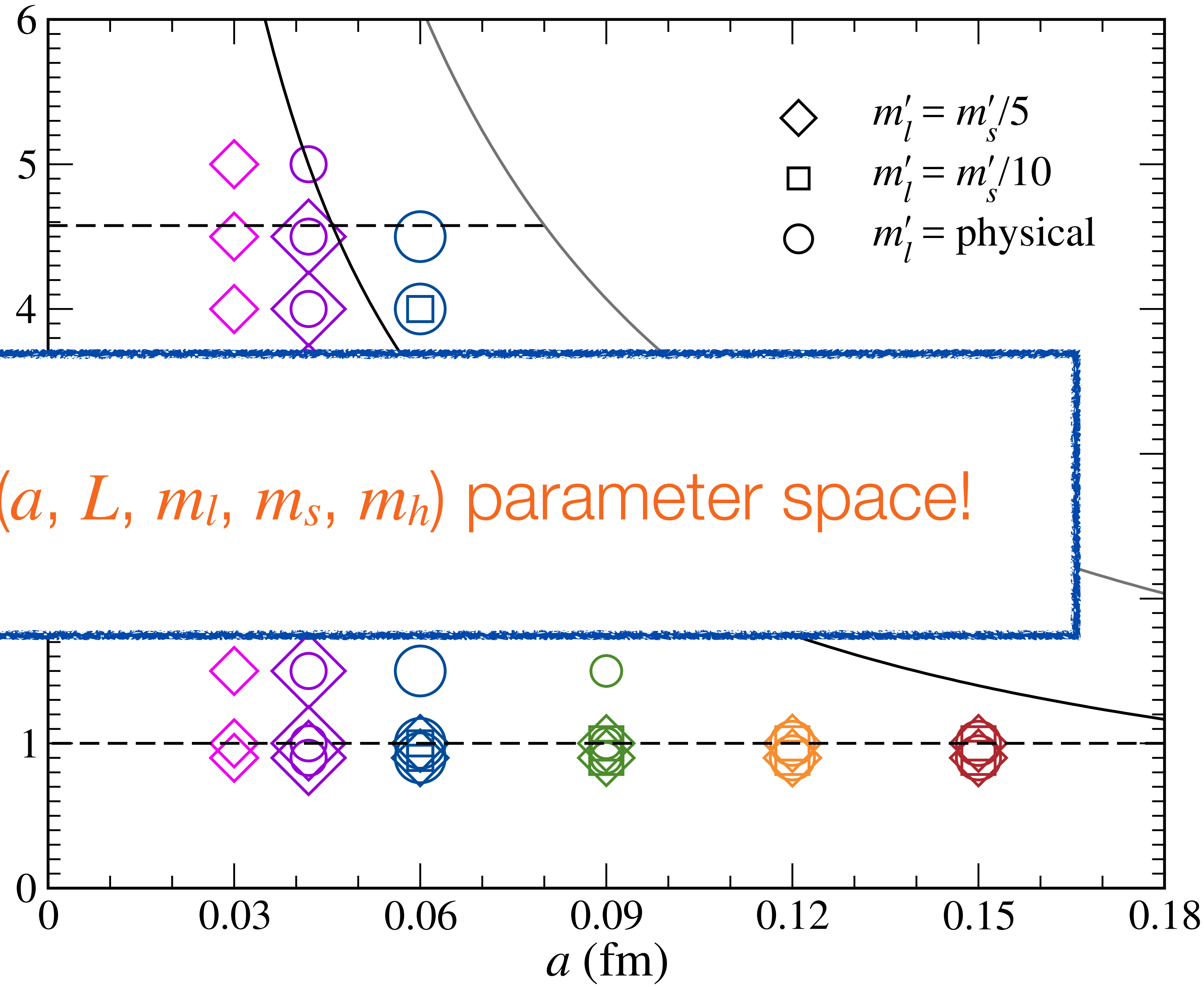
  (498 w/ 0.15 fm).



Huge slab in (a, L, m_l, m_s, m_h) parameter space!

Effective Field Theory
Lattice Gauge

HQET ⊕ Symanzik EFT ⊕ χPT Fits

- As noted, the slab of parameter space (5-dimensional) is huge.
- The raw statistical precision of the simulation data is
  - 0.04–1.4% for heavy-light meson decay constants;
  - 0.005–0.12% for heavy-light meson masses.
- It is insufficient to have a simple function to fit the dependence on (a, m_l, m_s, m_h).
- Functional form follows power-counting and builds in leading chiral logs and HQET anomalous dimension.

Leptonic Decays: $B \to \tau\nu$, $D_s \to l\nu$; $B_s \to \mu^+\mu^-$

- Simplest flavor-physics for lattice QCD.
- Amplitude($B \to \tau\nu$) $\propto |V_{ub}|f_B$ and, so far "yields" $|V_{ub}|$ that is too high.
- Amplitude($B \to \mu^+\mu^-$) $\propto |V_{ts}||V_{tb}|f_B\times$box, so could have BSM loop too.
- "Standard" Fermilab Lattice and MILC Collaboration simulation project;
- Heavy-light pseudoscalar meson two-point functions:

$$\langle P_{qQ}(x_4)P_{qQ}(0)\rangle = \sum_n |\langle 0|\hat{P}_{qQ}|P_{qQ,n}\rangle|^2 e^{-M_{P_{qQ,n}}x_4}$$

with absolutely normalized pseudoscalar density.

Leptonic Decays: $B \to \tau\nu$, $D_s \to l\nu$; $B_s \to \mu^+\mu^-$

- Simplest flavor-physics for lattice QCD.
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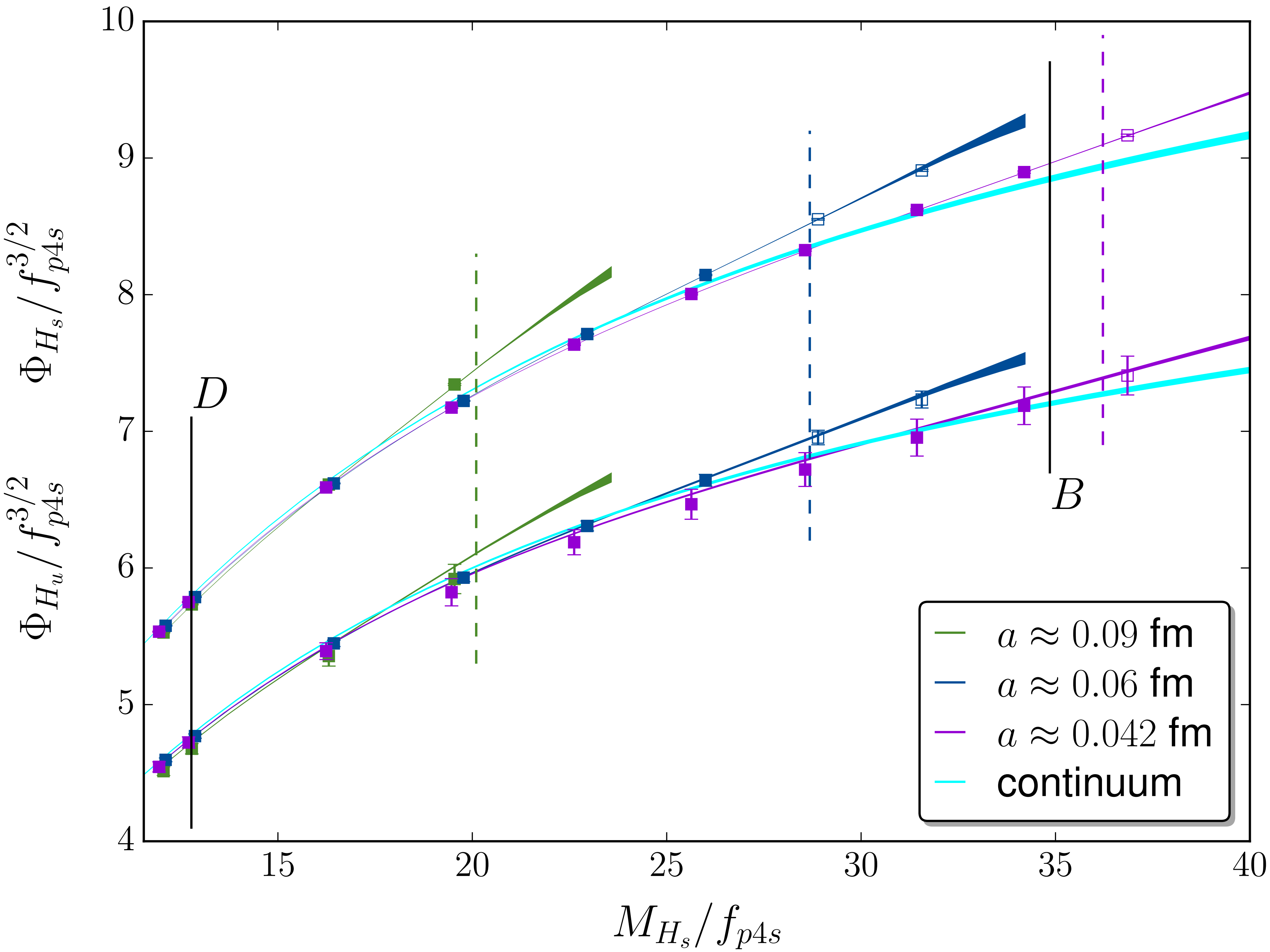
- "Standard" Fermilab Lattice and MILC Collaboration simulation project;
- Heavy-light pseudoscalar meson two-point functions:

$$\langle P_{qQ}(x_4) P_{qQ}(0) \rangle = \sum_n |\langle 0|\hat{P}_{qQ}|P_{qQ,n}\rangle|^2 e^{-M_{P_{qQ,n}} x_4}$$

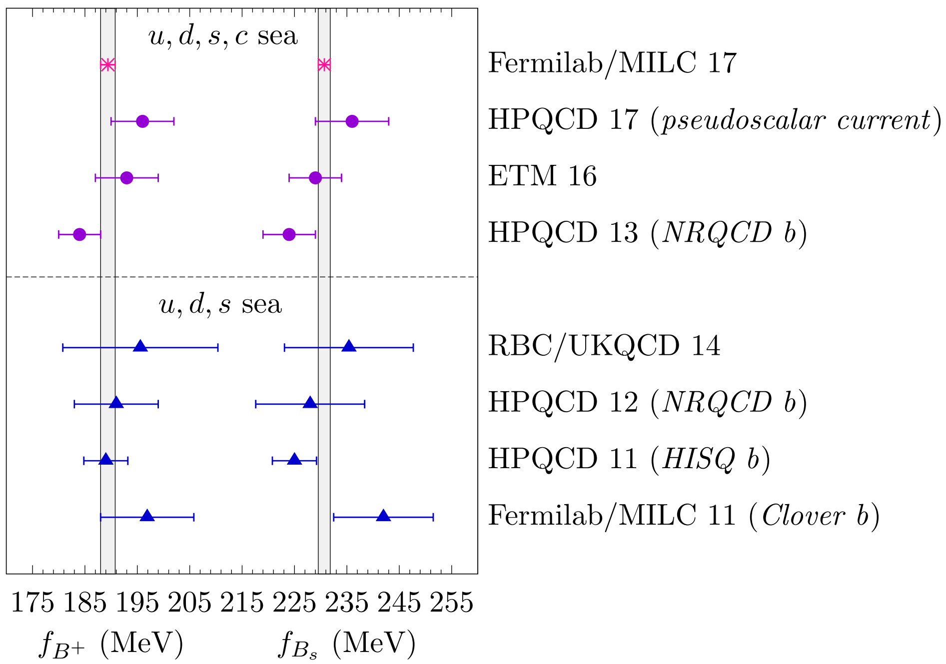
with absolutely normalized pseudoscalar density. $f_{P_{qQ}}$

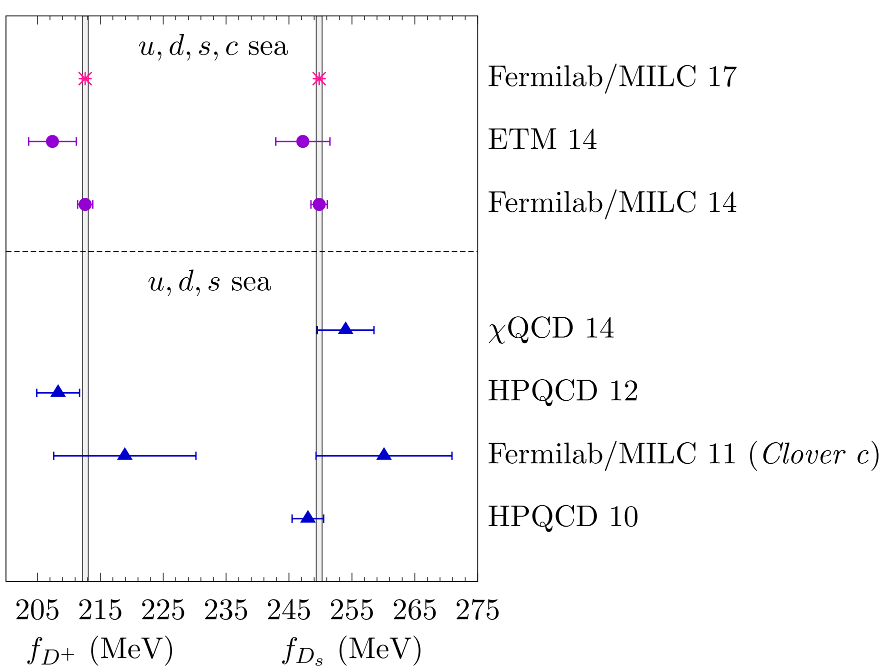
Snapshot of Decay Constants

- 492 data pts;
- 60 parameters;
- $\chi^2/\mathrm{dof} = 466/432$;
- $p = 0.12$;
- stable under fit variations;
- extra errors for FV, topology, EM.



Results for Decay Constants



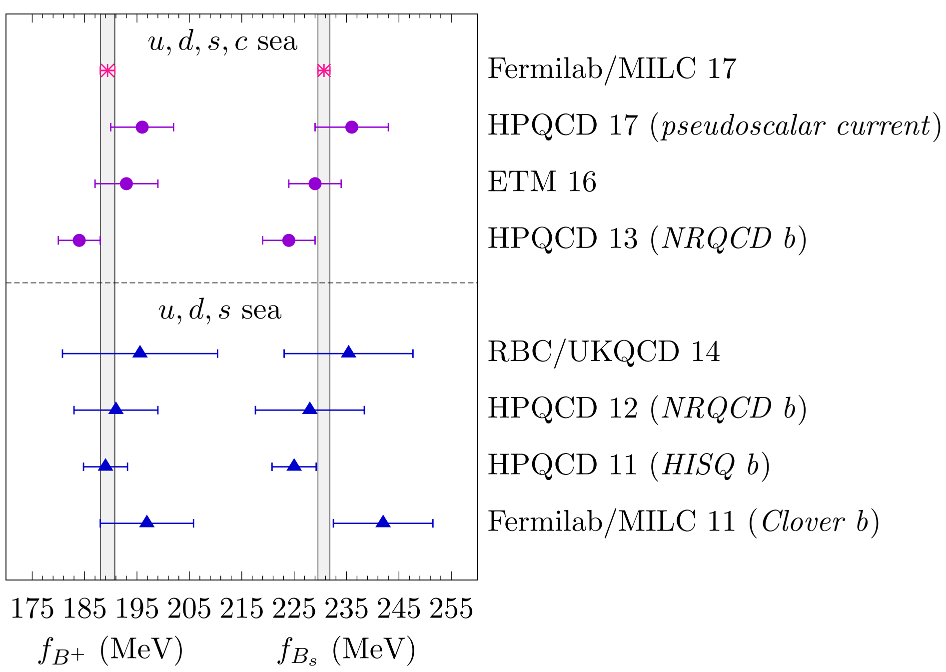


- Fermilab Lattice & MILC [arXiv:1712.09262]:

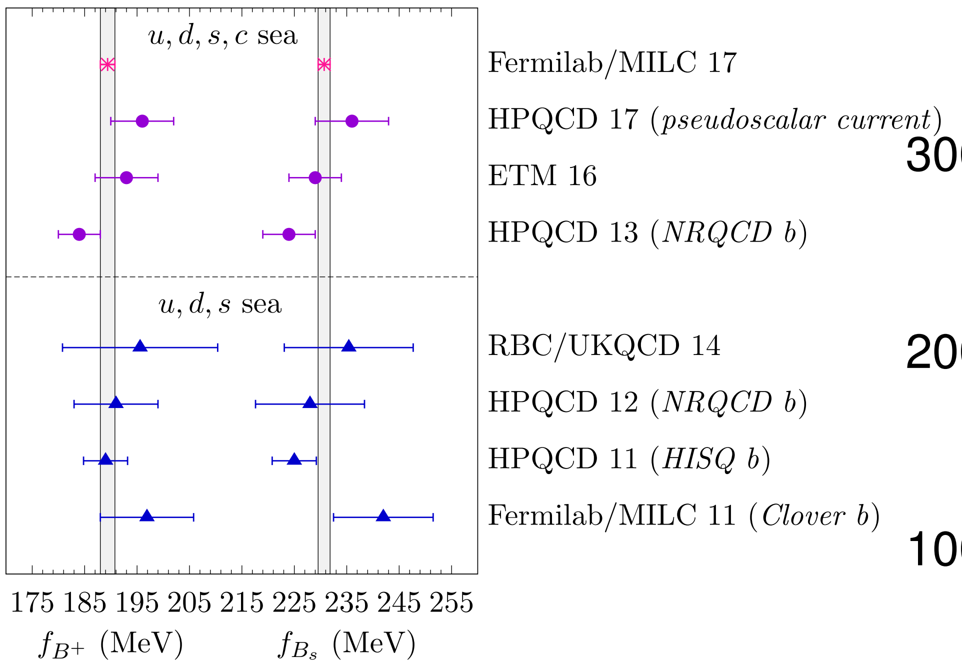
$$f_{D^0} = 211.5(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$
$$f_{D^+} = 212.6(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$
$$f_{D_s} = 249.8(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$
$$f_{B^+} = 189.4(0.8)_{\rm stat}(1.1)_{\rm syst}(0.3)_{f_{\pi,\rm PDG}}\ \text{MeV}$$
$$f_{B^0} = 190.5(0.8)_{\rm stat}(1.0)_{\rm syst}(0.3)_{f_{\pi,\rm PDG}}\ \text{MeV}$$
$$f_{B_s} = 230.7(0.8)_{\rm stat}(0.8)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

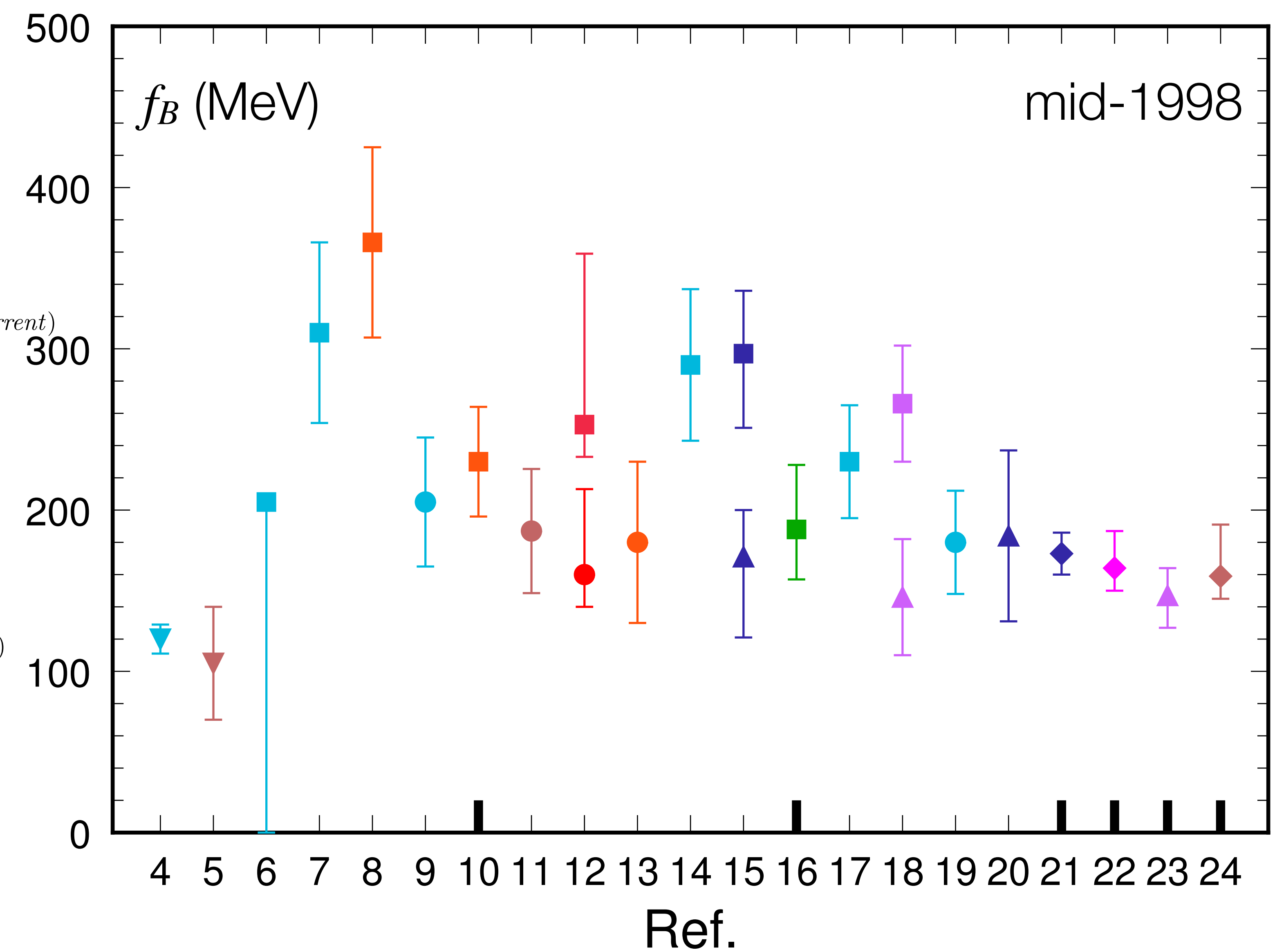
- Overall uncertainty: ~0.2% for D mesons,
  ~0.7% for B mesons.

Archaeology

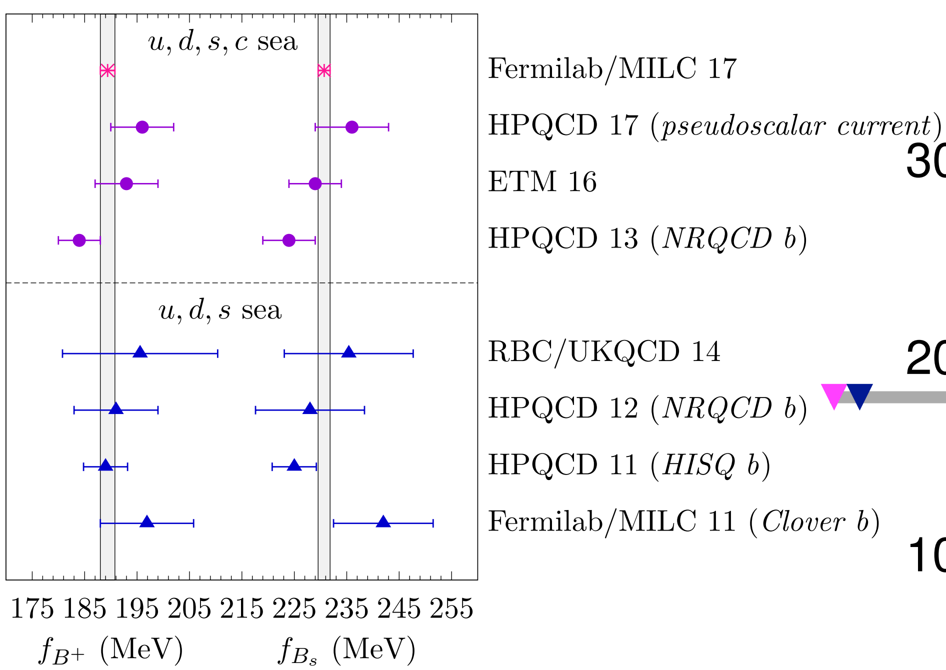


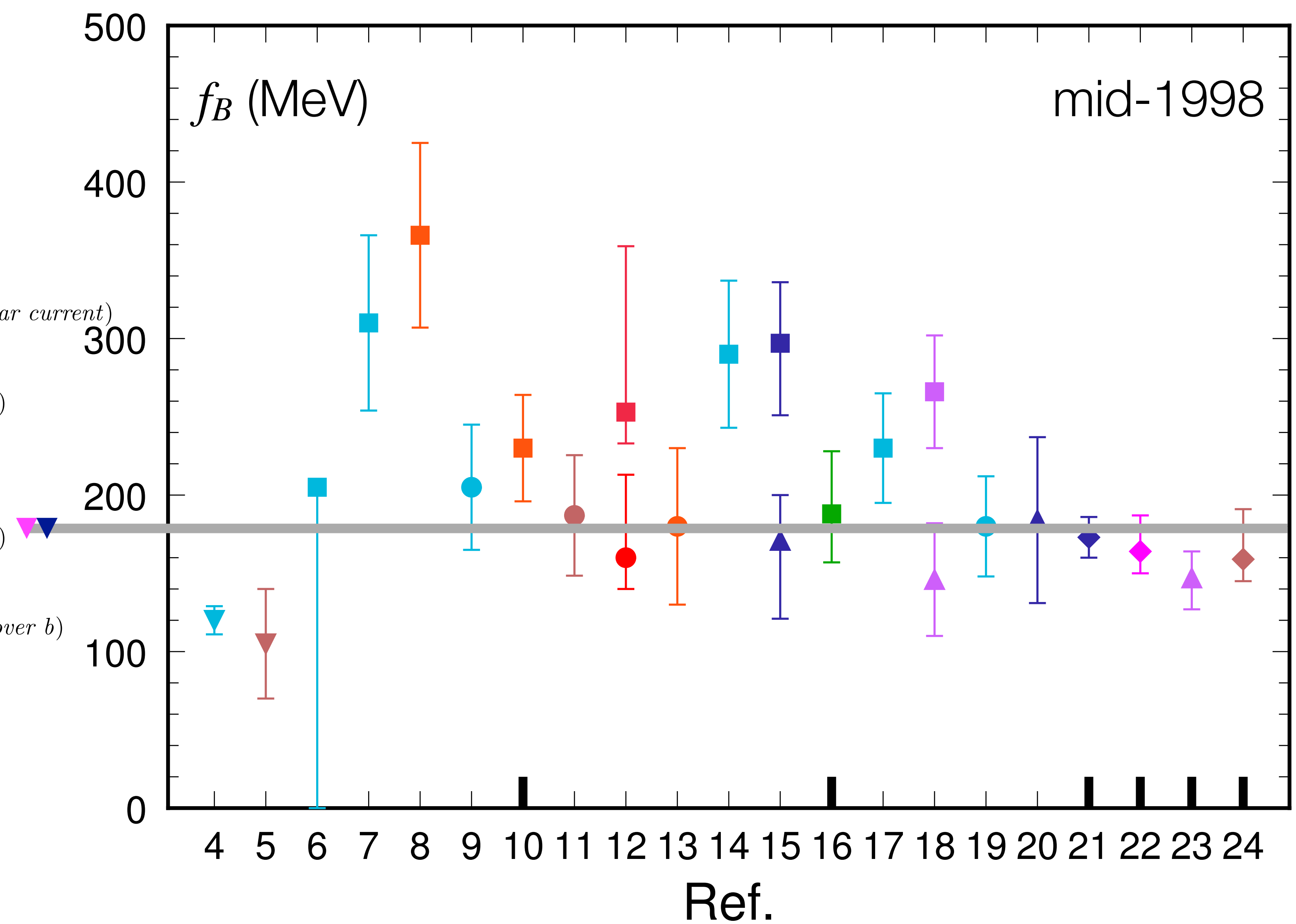
Archaeology





Archaeology





Quark Masses

- From HQET (or other approaches to the $1/m_h$ expansion):

$$M_{H_J} = m_h + \bar{\Lambda} + \frac{\mu_\pi^2}{2m_h} - d_J \frac{\mu_G^2(m_h)}{2m_h}$$

- For ~20 years, I've wanted to vary m_h and use this formula to determine $\bar{\Lambda}$, μ_π^2, and $\mu_G^2(m_b)$ from lattice QCD [arXiv:hep-ph/0006345].

Quark Masses

- From HQET (or other approaches to the $1/m_h$ expansion):

mass of spin-J meson

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- We found a new way to handle the infrared ambiguity of the quark mass: “minimal renormalon subtraction (MRS) [arXiv:1712.04983].

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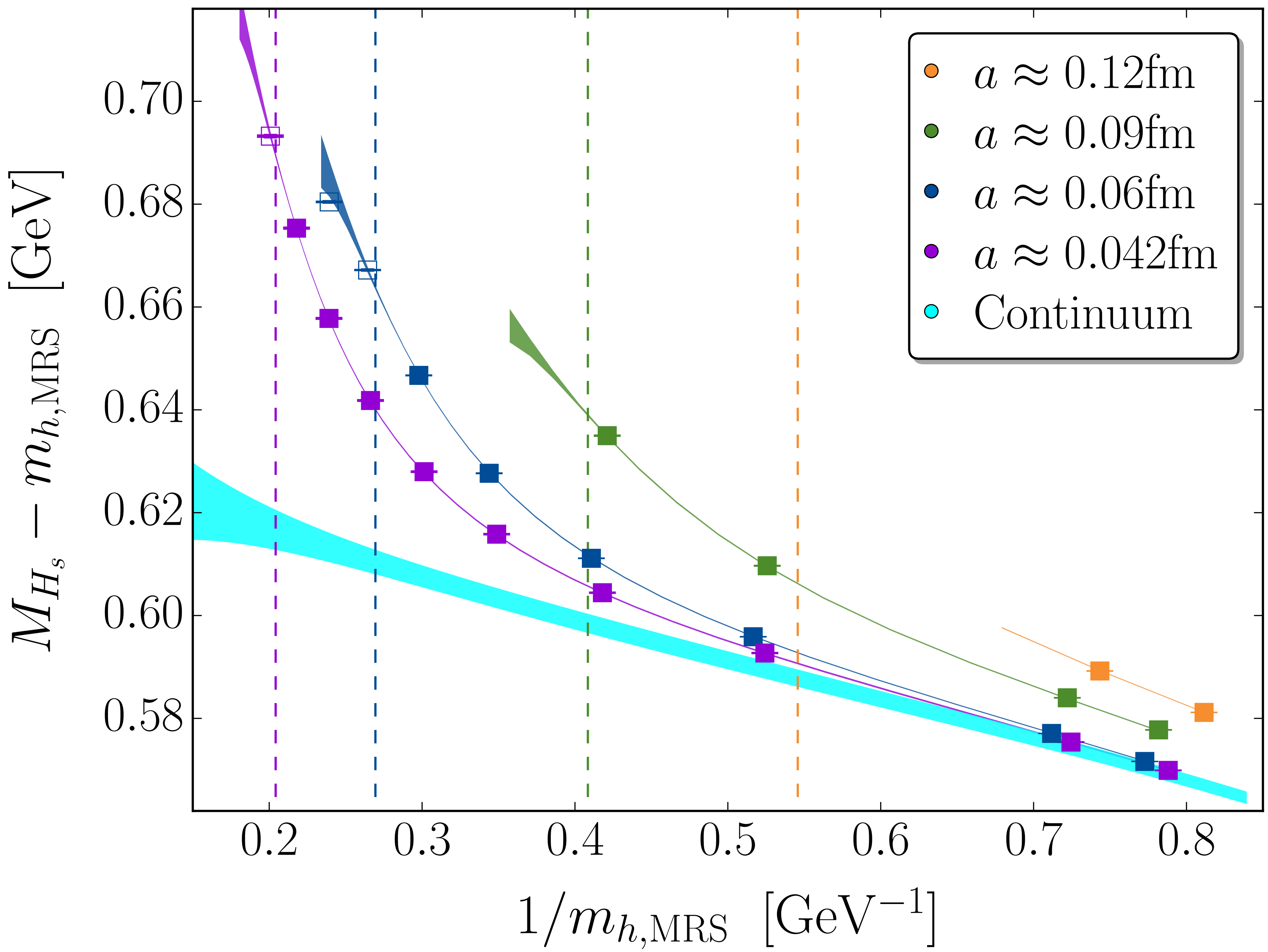
MRS definition

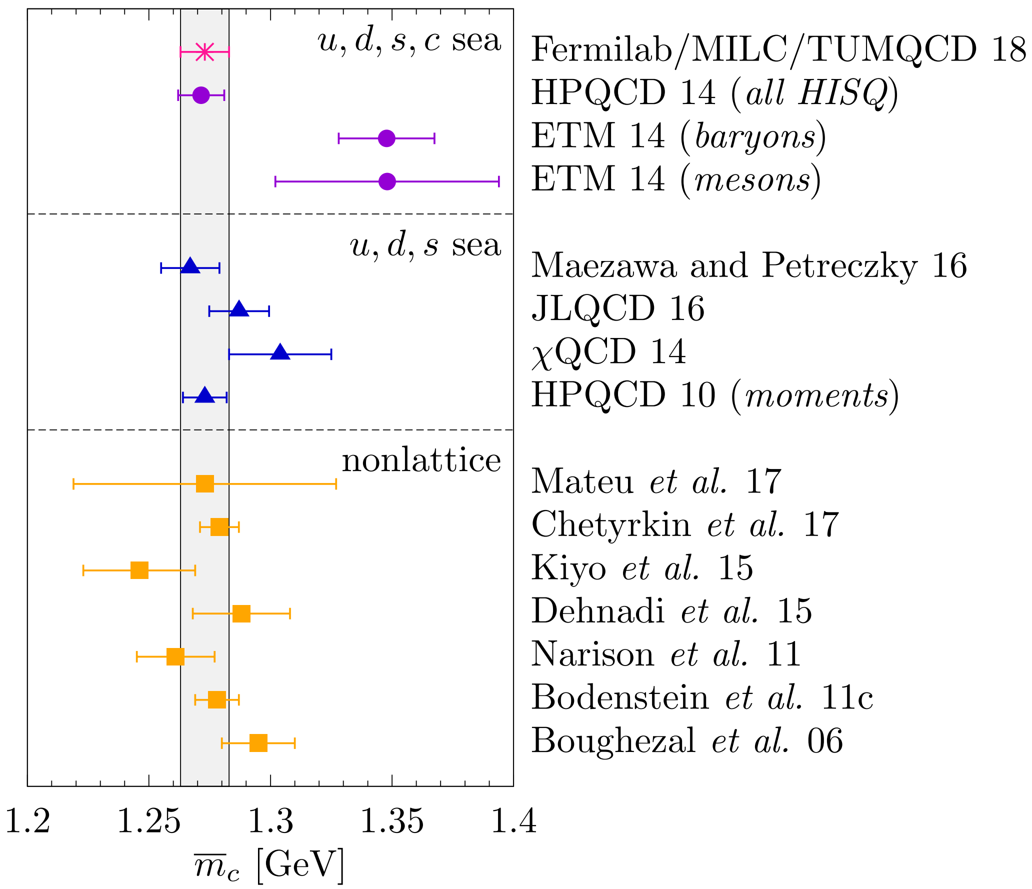
HQET Fit ⊕ Symanzik EFT ⊕ χPT

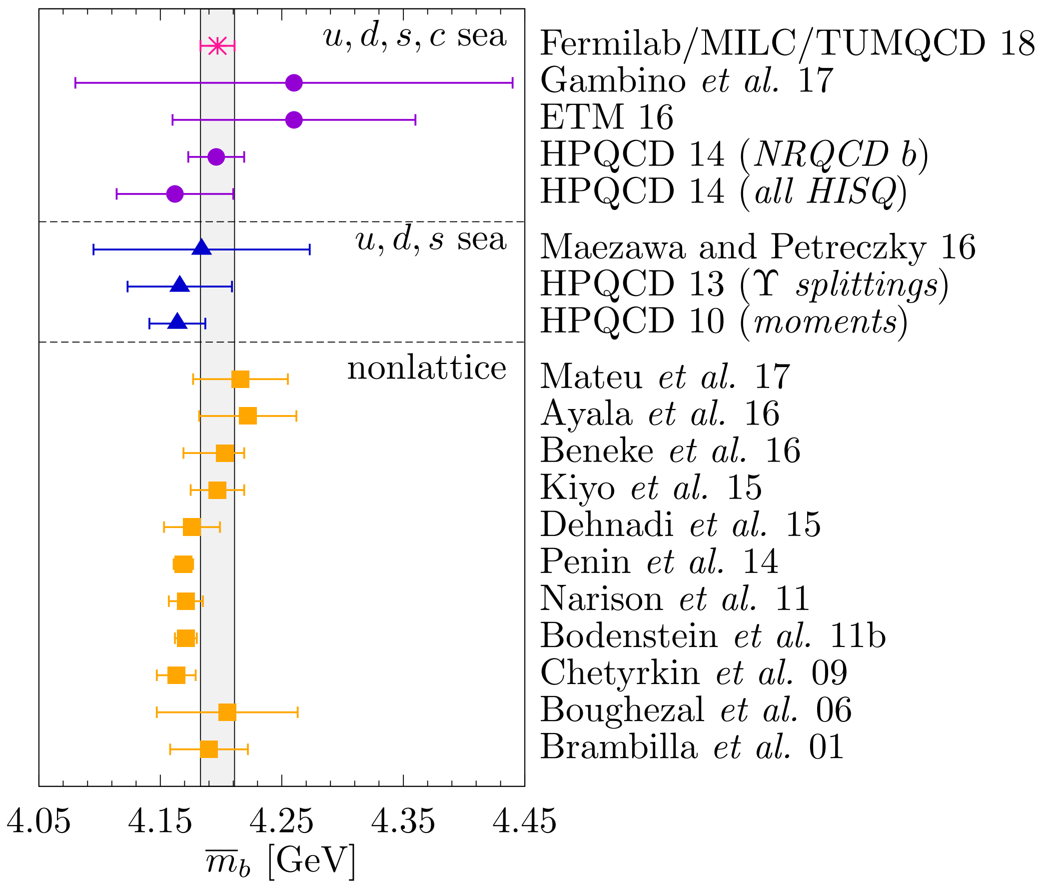
- 384 data pts;
- 77 parameters;
- $\chi^2/\mathrm{dof} = 312/307$;
- $p = 0.3$;
- stable under fit variations;
- extra errors for FV, topology, EM.



Results & Comparisons

- Results form arXiv:1802.04248:





- To our knowledge, first results w/ order-α_s^5 running & order-α_s^4 matching.

- Precision: 0.3% for bottom to 0.5% for charm.

Results & Comparisons 3

- Masses in numerical form:

$$m_{l,\overline{\mathrm{MS}}}(2\text{ GeV}) = 3.404(14)_{\mathrm{stat}}(08)_{\mathrm{syst}}(19)_{\alpha_s}(04)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$
$$m_{u,\overline{\mathrm{MS}}}(2\text{ GeV}) = 2.118(17)_{\mathrm{stat}}(32)_{\mathrm{syst}}(12)_{\alpha_s}(03)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$
$$m_{d,\overline{\mathrm{MS}}}(2\text{ GeV}) = 4.690(30)_{\mathrm{stat}}(36)_{\mathrm{syst}}(26)_{\alpha_s}(06)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$
$$m_{s,\overline{\mathrm{MS}}}(2\text{ GeV}) = 92.52(40)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(12)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$
$$m_{c,\overline{\mathrm{MS}}}(3\text{ GeV}) = 984.3(4.2)_{\mathrm{stat}}(1.6)_{\mathrm{syst}}(3.2)_{\alpha_s}(0.6)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$
$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4203(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}\text{ MeV}$$

- Mass ratios:

$$m_c/m_s = 11.784(11)_{\mathrm{stat}}(17)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_\pi,\mathrm{PDG}}$$
$$m_b/m_s = 53.93(7)_{\mathrm{stat}}(8)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_\pi,\mathrm{PDG}}$$
$$m_b/m_c = 4.577(5)_{\mathrm{stat}}(7)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}$$

Outlook: Nucleon Matrix Elements

Pontecorvo–Maki–Nakagawa–Sakata Matrix

- The PMNS matrix is to neutrino & lepton-flavor physics as the Cabibbo-Kobayashi-Maskawa (CKM) matrix is to quark-flavor physics:

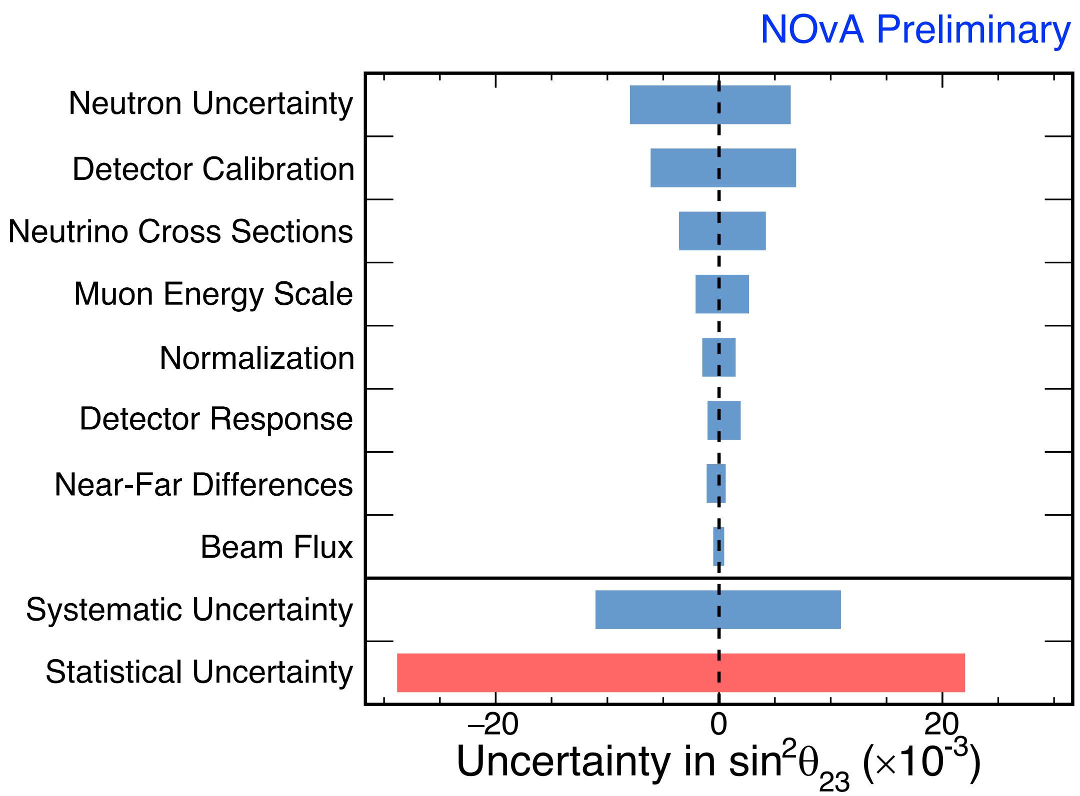
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = [U]_{\ell j}$$

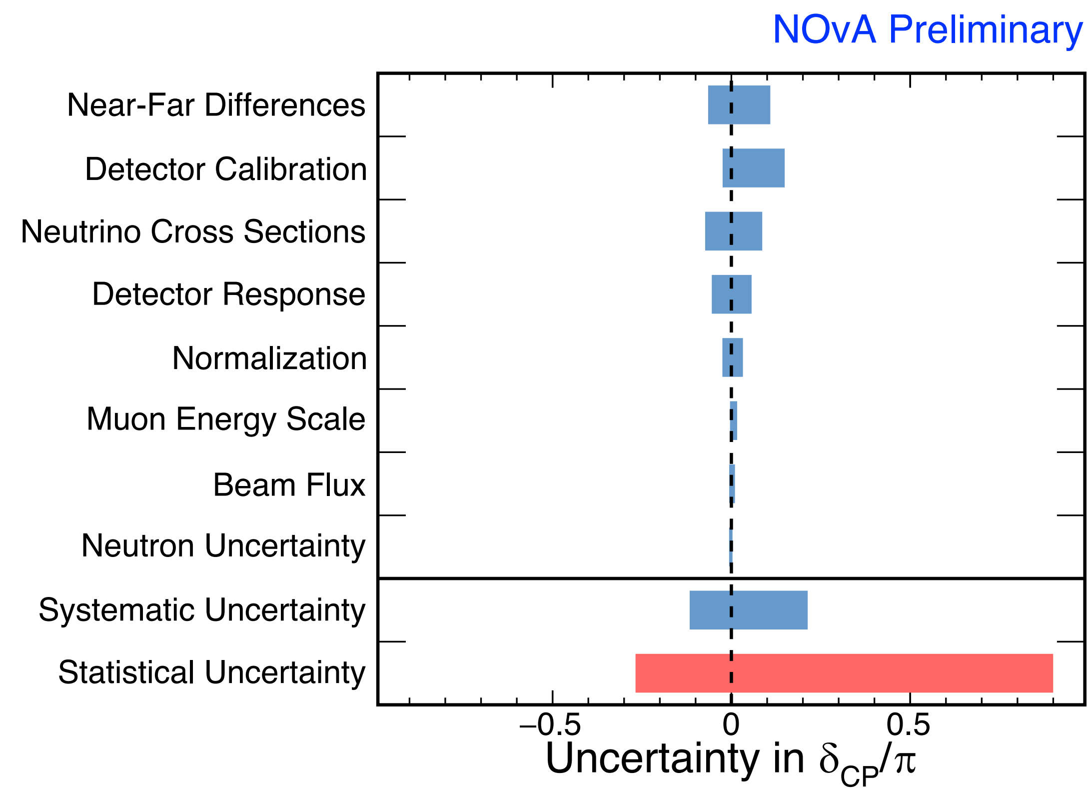
  where $\ell \in \{e, \mu, \tau\}$ labels flavor and $j \in \{1, 2, 3\}$ labels mass eigenstate.

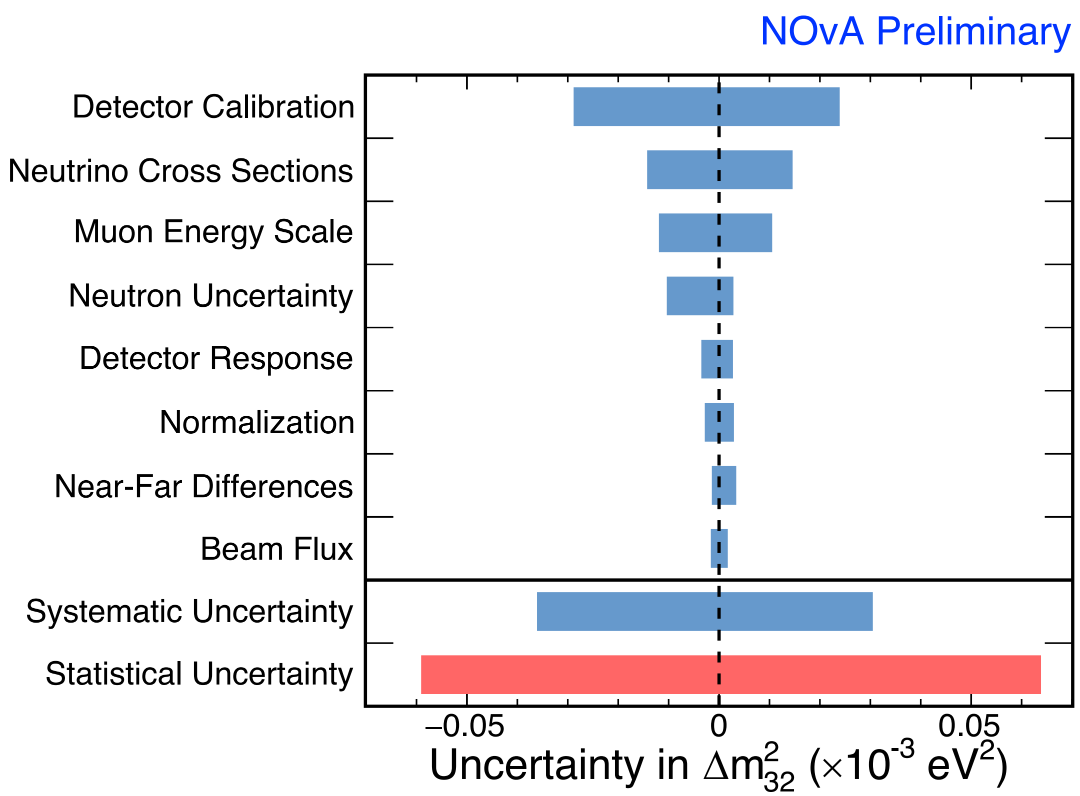
- Like CKM, PMNS has three mixing angles, θ_{lj}, & a CP-violating phase, δ.

- With Majorana mass terms, two fewer field phases absorbed into fields, so two more CP-violating phases, α_i, $i \in \{1, 2\}$, are physical $\leftarrow 0\nu\beta\beta$.

Systematic Uncertainties







Most important systematics:

- Detector Calibration
  - Will be improved by the 2019 test beam program
- Neutrino cross sections
  - Particularly nuclear effects (RPA, MEC)
- Muon energy scale
- Neutron uncertainty – **new** with $\bar{\nu}$'s

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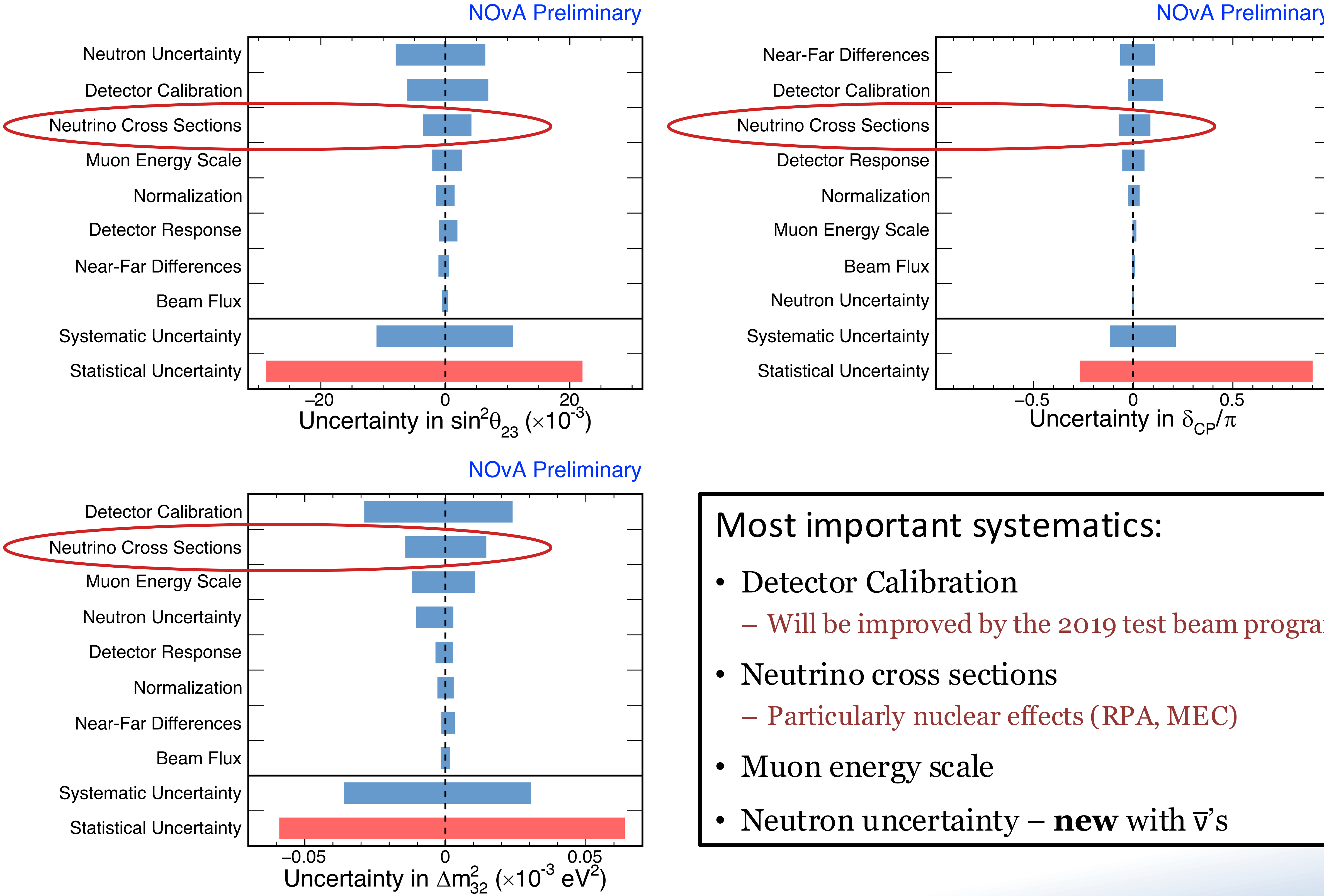

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Looking Forward

- Statistical errors will be reduced via much larger exposure.
- Dominant systematic uncertainty will soon be that from νA cross section:
  - other large ones connected to statistics, test beam, and/or cross section itself.
- Taxpayers are eager to find out about CP and to test the three-family paradigm!

- \$$10^9$ for LBNF/DUNE

Neutrino-Nucleus Scattering

- Neutrinos (and dark matter particles, and charged lepton whose flavor might change) scatter off of nuclei.
- Nuclear theory requires nucleon-level hadronic matrix elements as inputs.
- In neutrino physics, often taken from data, which requires a model of a nucleus and, thus, the potential for a tautology.
- These quantities can, however, be computed from QCD using lattice gauge theory.
- Nucleons are harder (worse signal-to-MCMC-noise), so a bit behind meson calculations for CKM.
- But beginning (with eye on PMNS).

Axial Form Factor from Lattice QCD

- Gupta, Jang, Lin, Yoon, Bhattacharya (PNDME) [arXiv:1705.06834]:

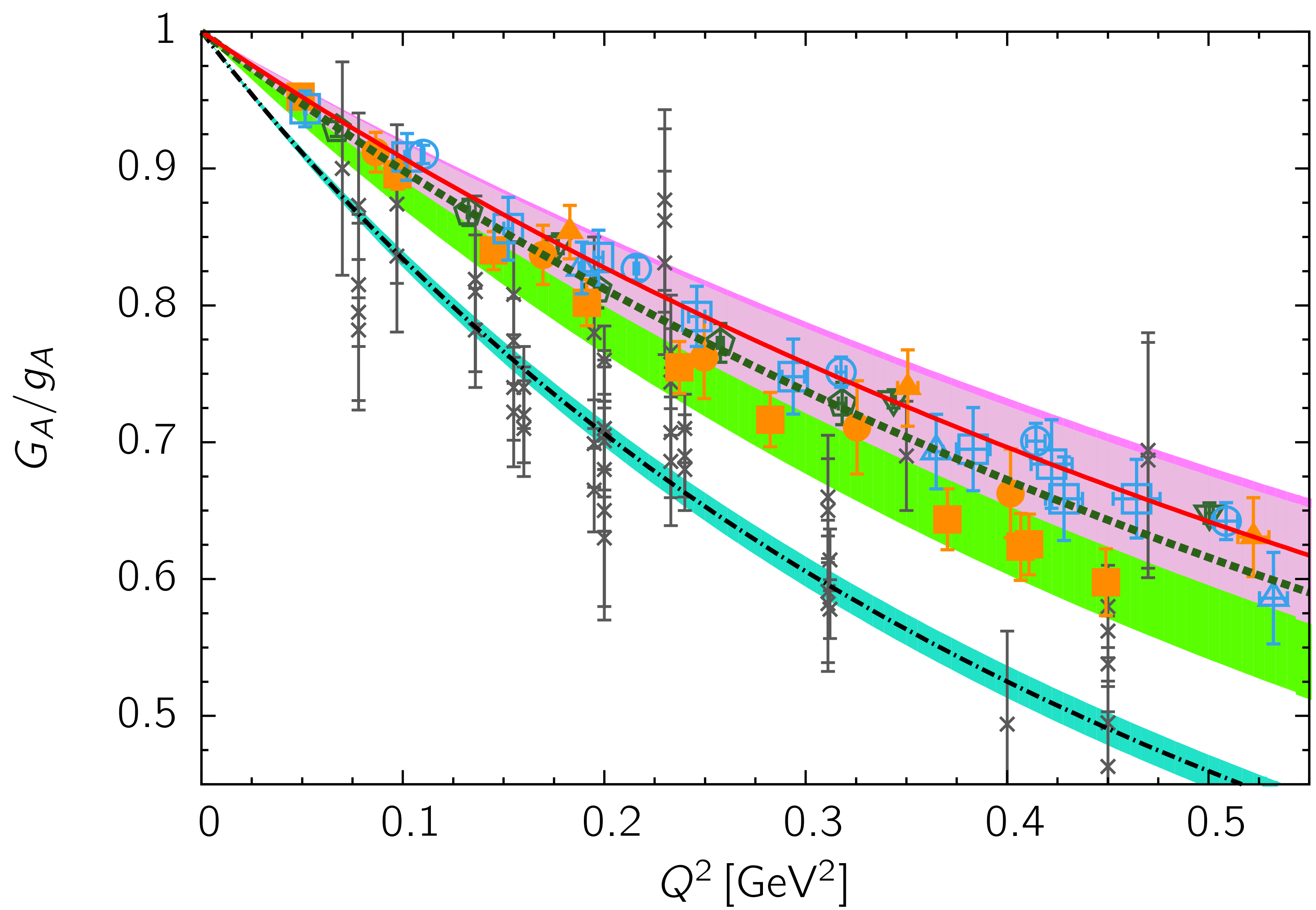
- $a \neq 0$ slope smaller than deuterium & other data.

- Even continuum limit:

  $r_A = 0.46(6)$ fm

  vs Meyer *et al*.:

  $r_A = 0.68(16)$ fm



- Which is more reliable?
  This calculation (says Alex Friedland, SLAC 😉).

Summary and Outlook

- Lattice QCD (for mesons) is now precision QCD.
- Many other vital topics:
    - most important omission is the hadronic contributions to the muon's anomalous magnetic moment:
        - hadronic vacuum polarization is under control and with exascale computing will meet the precision needed for the new experiments at Fermilab and JParc (need 0.2%);
        - hadronic light-by-light is advancing rapidly and may reach the experiments' needs sooner (need 10%).
- Nucleon matrix elements will be essential for all intensity frontier experiments.

QCD is Everywhere

parton distribution functions

hadronic muon $g\text{–}2$

decay & mixing amplitudes

esoteric hadrons:

dark-matter-nucleus cross sections

muon-nucleus cross sections

neutrino-nucleus cross sections

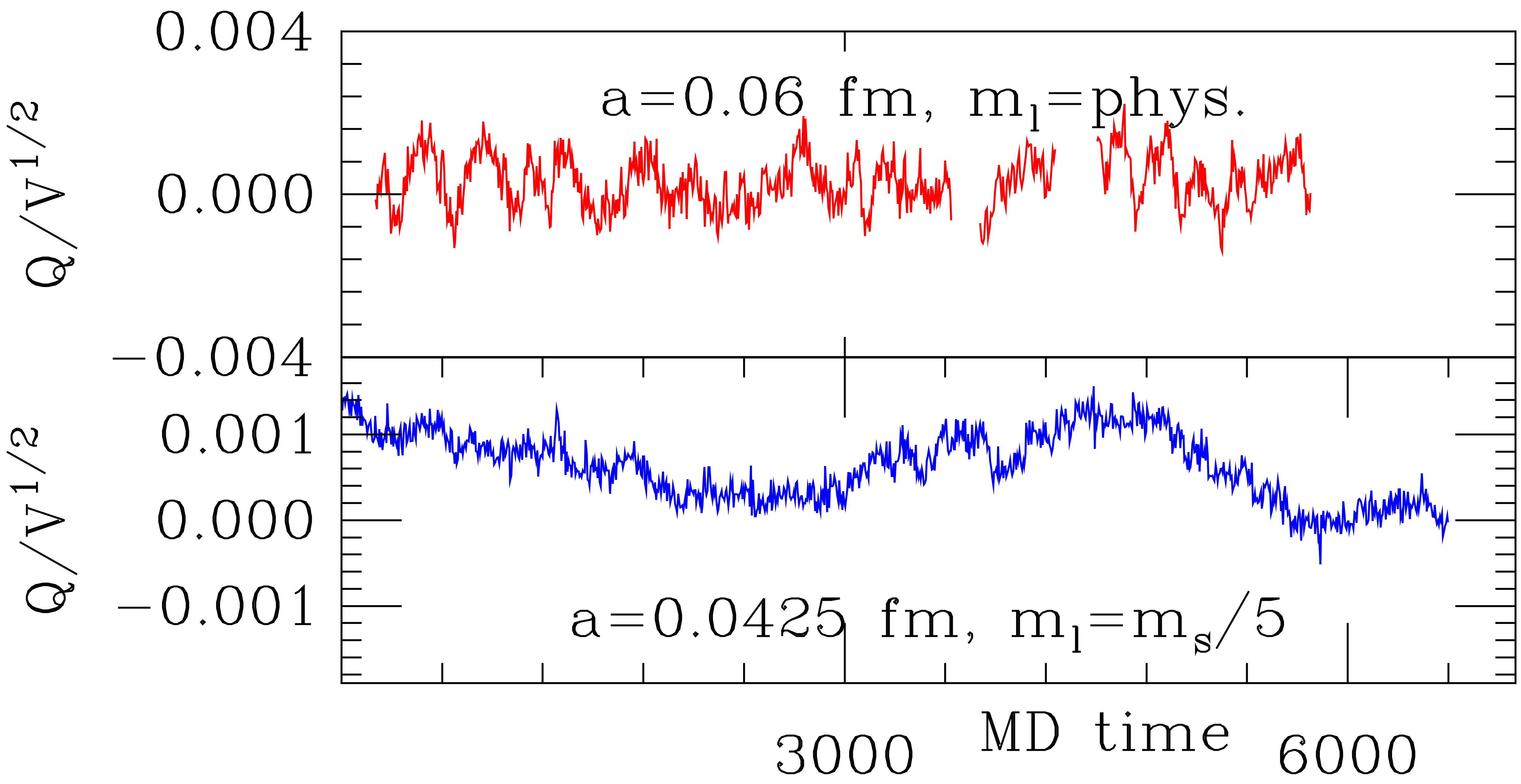
Thank you!

Backup

Frozen Topology

- Continuum gauge fields: topological charge Q cannot change with an infinitesimal change in the gauge field.
- Evolution of lattice gauge fields in CPU time consists of small steps that (in physical units) become smaller and smaller as lattice spacing $a \to 0$.
- Some reactions:
  - "Oh, my! Physics is now impossible!"—anonymous
  - "Physical quantities will suffer a systematic error, and we need to either correct for this error or account for it in our error budgets."
    —Bernard & Toussaint [arXiv:1707.05430]

Good vs. Bad Sampling



- Instead of exponential volume effects, poorly sampled topological charge leads to effects suppressed by

$$\frac{1}{2\chi_T}\frac{1}{V}\left(1-\frac{Q^2}{\langle Q^2\rangle}\right)$$

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$\chi_T = \langle Q^2\rangle / V$
in χPT, $\chi_T \propto f_\pi^2 M_\pi^2$
if $M_\pi L \sim$ const, $\chi_T V \propto f_\pi^2 LT$

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References:
Leutwyler, Smilga [PRD46 (1992) 5607];
Brower *et alia* [hep-lat/0302005];
Aoki *et alia* [arXiv:0707.0396];
Aoki, Fukaya [arXiv:0906.4852].

Typical Corrections

Bernard & Toussaint, arXiv:1707.05430

	$m_l' = m_s'/5$	m_l' = physical
$\langle Q^2 \rangle_{\text{ens}} / \langle Q^2 \rangle_{\chi\text{PT}}$	1.30	0.65
f_K/f_π	1.20508(0.00250) [–0.01271]	1.19680(0.00114) [0.00015]
aM_π	0.031147(0.000172) [–0.000707]	0.028964(0.000020) [0.000008]
aM_D	0.048858(0.000261) [–0.000552]	0.045389(0.000245) [0.000006]
af_D	0.409786(0.000391) [–0.000044]	0.400678(0.000258) [0.000001]
aM_{Ds}	0.054828(0.000068) [–0.000001]	0.053582(0.000025) [0.000000]
af_{Ds}	0.430966(0.000116) [–0.000004]	0.422041(0.000037) [0.000000]

- Must be examined ensemble by ensemble.

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Tiny, and sometimes significant.

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