The woefully incomplete, unabashedly biased history of the Higgs Boson

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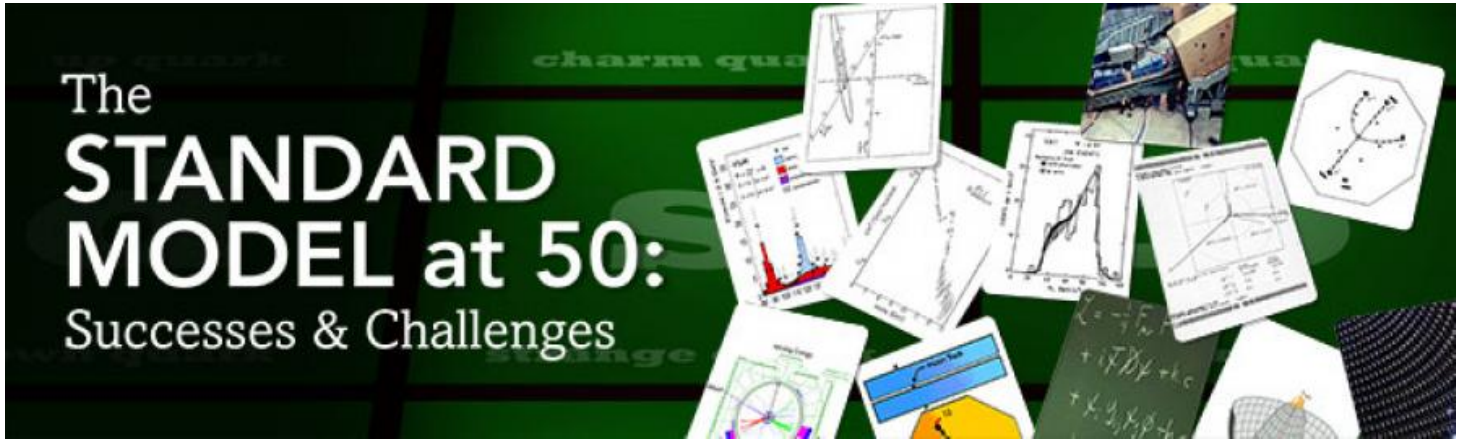
August 6, 2018



SANTA CRUZ

1939: Scalar fields portend an energy scale associated with new phenomena that are close at hand.

JULY 1, 1939 PHYSICAL REVIEW VOLUME 56

On the Self-Energy and the Electromagnetic Field of the Electron

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(Received April 12, 1939)

The charge distribution, the electromagnetic field and the self-energy of an electron are investigated. It is found that, as a result of Dirac's positron theory, the charge and the magnetic dipole of the electron are extended over a finite region; the contributions of the spin and of the fluctuations of the radiation field to the self-energy are analyzed, and the reasons that the self-energy is only logarithmically infinite in positron theory are given. It is proved that the latter result holds to every approximation in an expansion of the self-energy in powers of e^2/hc. The self-energy of charged particles obeying Bose statistics is found to be quadratically divergent. Some evidence is given that the "critical length" of positron theory is as small as $h/(mc)\cdot\exp(-hc/e^2)$.

The situation is, however, entirely different for a particle with Bose statistics. Even the Coulombian part of the self-energy diverges to a first approximation as $W_{st} \sim e^2h/(mca^2)$ and requires a much larger critical length that is $a = (hc/e^2)^{-\frac{1}{2}} \cdot h/(mc)$, to keep it of the order of magnitude of mc^2. This may indicate that a theory of particles obeying Bose statistics must involve new features at this critical length, or at energies corresponding to this length; whereas a theory of particles obeying the exclusion principle is probably consistent down to much smaller lengths or up to much higher energies.

1962: A Spontaneously broken continuous symmetry yields a massless spin-0 (Goldstone) boson

PHYSICAL REVIEW VOLUME 127, NUMBER 3 AUGUST 1, 1962

Broken Symmetries*

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AND

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(Received March 16, 1962)

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

How do Goldstone bosons arise?

Suppose a Lagragian exhibits a continuous global symmetry. If the vacuum state of the theory breaks the global symmetry, then the spectrum contains a massless scalar state—the Goldstone boson. This is a rigorous result of quantum field theory.

Goldstone's theorem can be exhibited in a model of elementary scalar dynamics. Suppose I have a multiplet of real scalar fields ϕ_i with Lagrangian

$$\mathscr{L} = \tfrac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i - V(\phi_i)\,,$$

which is invariant under $\phi_i \longrightarrow \phi_i + \delta\phi_i$, where

$$\delta\phi_i = -i\theta^a T^a_{ij}\phi_j\,.$$

The generators iT^a are real antisymmetric matrices and the θ^a are real parameters. By assumption, $\delta\mathscr{L} = 0$ which yields

$$\delta V = \frac{\partial V}{\partial\phi_i}\delta\phi_i = \frac{\partial V}{\partial\phi_i}T^a_{ij}\phi_j = 0\,.$$

The global symmetry is spontaneously broken if the vacuum state does not respect the symmetry. That is, the potential minimum occurs at $\phi_i = v_i$ where $\exp(-i\theta^a T^a)v \neq v$ [or equivalently, $T^a v \neq 0$]. Define new fields $\widetilde{\phi}_i \equiv \phi_i - v_i$, in which case

$$\mathscr{L} = \tfrac{1}{2}\partial_\mu \widetilde{\phi}_i \partial^\mu \widetilde{\phi}^i - \tfrac{1}{2} M^2_{ij} \widetilde{\phi}_i \widetilde{\phi}_j + \text{interactions}\,,$$

where M^2 is a non-negative symmetric matrix,

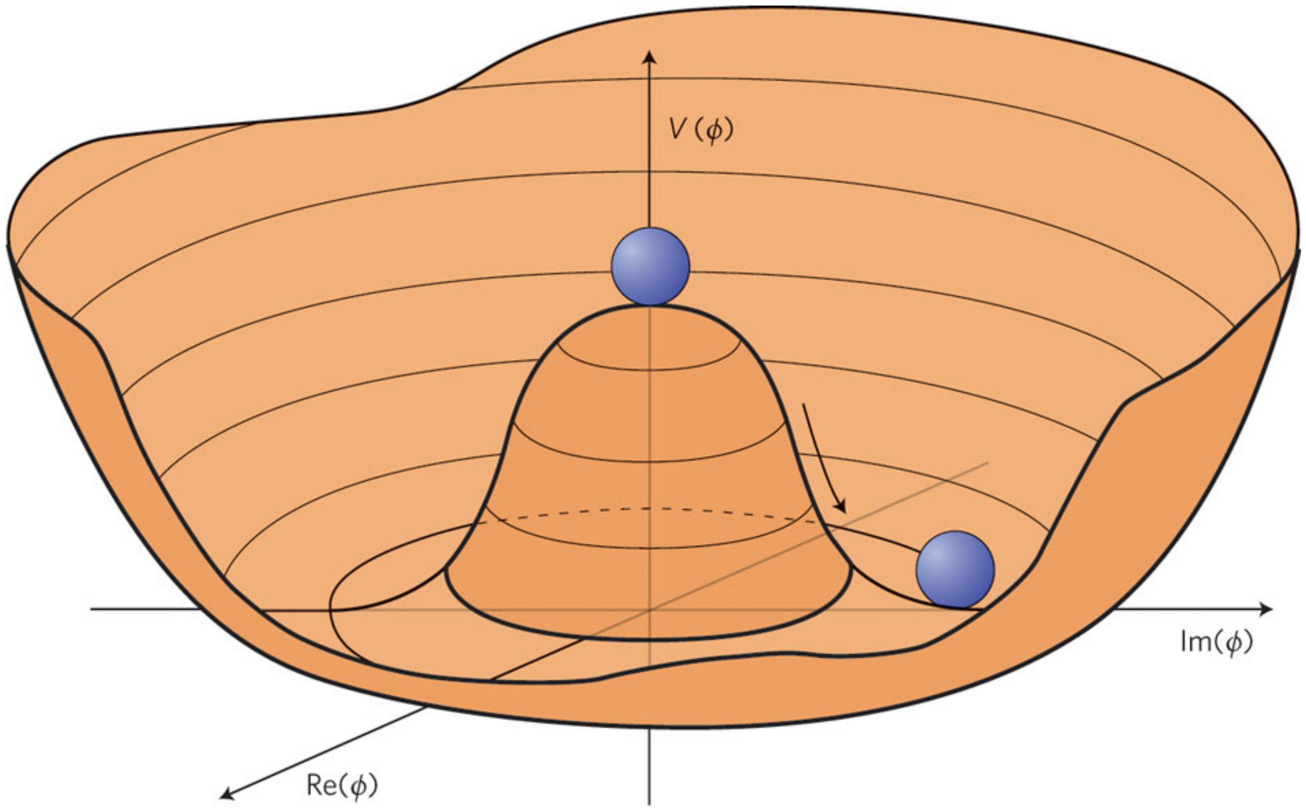
$$M^2_{ij} \equiv \left.\frac{\partial V}{\partial \phi_i \partial \phi_j}\right|_{\phi_i = v_i}.$$

Recall the condition for the global symmetry, $(\partial V/\partial \phi_i) T^a_{ij} \phi_j = 0$. Differentiating this equation with respect to ϕ_j and setting $\phi_i = v_i$ and $(\partial V/\partial \phi_i)_{\phi_i = v_i} = 0$ then yields

$$M^2_{ki} T^a_{ij} v_j = 0\,.$$

The T^a (which may be linear combinations of the original symmetry generators) are re-organized to identify the maximal number of unbroken linearly independent generators (i.e. $T^a v = 0$), which determine the residual unbroken symmetry. As for the remaining broken generators (i.e. $T^a v \neq 0$), we see that $(T^a v)_i$ is an eigenvector of M^2 with zero eigenvalue. In particular, there is one Goldstone boson, $G^a \sim i\phi_i T^a_{ij} v_j$ for each broken generator.

If the potential energy density V(ϕ) of the scalar fields is such that the lowest energy state corresponds to a non-zero value of the field, then the vacuum will possess a non-zero “charge” (condensation), and the global continuous symmetry is broken.



But excitations around the bottom of the “Mexican hat” do not cost energy, and correspond to the excitation of a massless spin 0 particle---the Goldstone boson.

PHYSICAL REVIEW VOLUME 127, NUMBER 3 AUGUST 1, 1962

Broken Symmetries*

If this is so, then there seem only three roads open to an understanding of broken symmetries based on the noninvariance of the vacuum:

(A) The particle interpretation of such theories might be revised (as in the Gupta-Bleuler method) so that the massless particles are not physically present in final states if they are absent in initial states. However, all our attempts in this direction have failed.

(B) The massless particles might really exist. The argument against this based on the Eötvös experiment

1963: Massive gauge bosons without violating gauge invariance (in a non-relativistic setting)

PHYSICAL REVIEW VOLUME 130, NUMBER 1 1 APRIL 1963

Plasmons, Gauge Invariance, and Mass

P. W. Anderson
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(Received 8 November 1962)

Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m \neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m=0$, but that the example of superconductivity illustrates that the physical spectrum need not. Some comments on the relationship between these ideas and the zero-mass difficulty in theories with broken symmetries are given.

1964: Massive gauge bosons without violating gauge invariance (in a relativistic setting)

This magic trick was discovered by Peter Higgs, and is called the Higgs phenomenon. (Actually, the terminology is unfair, since the phenomenon was discovered independently by several other investigators, but we will use it anyway, since it is awkward to talk of the Brout–Englert–Guralnik–Hagen–Higgs–Kibble phenomenon.)[6] We can gain further insight into the Higgs phenomenon if we remember the motivation for the minimal-coupling prescription – gauge invariance.

Taken from Sidney Coleman, *Secret symmetry: An Introduction to Spontaneous Symmetry Breakdown and Gauge Fields*, 1973 Erice Lectures, Published in Subnucl. Ser. **11**, 139 (1975).

The Higgs mechanism can be exhibited in our simple model of elementary scalar dynamics by promoting the global symmetry to a local symmetry. This is accomplished by introducing a gauge field A^a_μ corresponding to each symmetry generator T^a. The Lagrangian is now

$$\mathscr{L} = \mathscr{L}_{\rm YM} + \tfrac{1}{2}(D_\mu\phi)^T(D^\mu\phi) - V(\phi)\,,$$

where $\mathscr{L}_{\rm YM}$ is the Yang-Mills Lagrangian and D is the covariant derivative

$$D_\mu \equiv \partial_\mu + igT^aA^a_\mu\,.$$

Assuming that the scalar potential is minimized at $\phi_i = v_i$ as before, we again define shifted fields, $\widetilde{\phi}_i \equiv \phi_i - v_i$. Then,

$$(D_\mu\phi)^T(D^\mu\phi) = M^2_{ab}A^a_\mu A^{\mu b} + \cdots\,,$$

with $M^2_{ab} = g^2v^TT^aT^bv$. For each unbroken generator, the corresponding vector boson remains massless (due to the residual unbroken symmetry). The remaining vector bosons acquire mass. One can show that the corresponding Goldstone bosons are no longer physical states of the theory. Instead, they are "absorbed" by the corresponding gauge bosons and are realized as the longitudinal spin component of the massive gauge bosons.

1964: The Higgs boson makes its first appearance

VOLUME 13, NUMBER 16 | PHYSICAL REVIEW LETTERS | 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
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(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these par-

about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \tag{2a}$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2) = 0, \tag{2b}$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\}. \tag{2c}$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c)

1966: The Higgs potential of the Abelian Higgs model

PHYSICAL REVIEW VOLUME 145, NUMBER 4 27 MAY 1966

Spontaneous Symmetry Breakdown without Massless Bosons*

PETER W. HIGGS†
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(Received 27 December 1965)

We examine a simple relativistic theory of two scalar fields, first discussed by Goldstone, in which as a result of spontaneous breakdown of $U(1)$ symmetry one of the scalar bosons is massless, in conformity with the Goldstone theorem. When the symmetry group of the Lagrangian is extended from global to local $U(1)$ transformations by the introduction of coupling with a vector gauge field, the Goldstone boson becomes the longitudinal state of a massive vector boson whose transverse states are the quanta of the transverse gauge field. A perturbative treatment of the model is developed in which the major features of these phenomena are present in zero order. Transition amplitudes for decay and scattering processes are evaluated in lowest order, and it is shown that they may be obtained more directly from an equivalent Lagrangian in which the original symmetry is no longer manifest. When the system is coupled to other systems in a $U(1)$ invariant Lagrangian, the other systems display an induced symmetry breakdown, associated with a partially conserved current which interacts with itself via the massive vector boson.

II. THE MODEL

The Lagrangian density from which we shall work is given by[29]

$$\mathfrak{L} = -\tfrac{1}{4} g^{\kappa\mu} g^{\lambda\nu} F_{\kappa\lambda} F_{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} \nabla_\mu \Phi_a \nabla_\nu \Phi_a + \tfrac{1}{4} m_0^2 \Phi_a \Phi_a - \tfrac{1}{8} f^2 (\Phi_a \Phi_a)^2 . \quad (1)$$

1967: The birth of the Standard Model of electroweak physics, where the Higgs mechanism is employed

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12] M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters 17, 888 (1966).

[13] The predicted ratio [eq. (12)] from the current algebra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta\to\pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

[14] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

A MODEL OF LEPTONS*

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(Received 17 October 1967)

$$\mathfrak{L} = -\tfrac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu(\partial_\mu - ig'B_\mu)R - L\gamma^\mu(\partial_\mu ig\vec{t}\cdot\vec{A}_\mu - i\tfrac{1}{2}g'B_\mu)L$$

$$-\tfrac{1}{2}|\partial_\mu\varphi - ig\vec{A}_\mu\cdot\vec{t}\varphi + i\tfrac{1}{2}g'B_\mu\varphi|^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L)\boxed{-M_1^{\,2}\varphi^\dagger\varphi + h(\varphi^\dagger\varphi)^2}. \quad (4)$$

1971--1973: The renormalizability of non-abelian gauge theories is proven by ‘t Hooft (following key insights of Veltman). A detailed treatment of the theory of spontaneously broken gauge theories is presented in a series of four papers by B.W. Lee and Jean Zinn-Justin.

The definitive review article introducing a generation of physicists to gauge theories by Abers and Lee appears in 1973.

1973: Deriving the Higgs boson couplings of the Standard Model by applying tree-level unitarity

Volume 46B, number 2 PHYSICS LETTERS 17 September 1973

HIGH ENERGY BEHAVIOUR AND GAUGE SYMMETRY

C.H. LLEWELLYN SMITH
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Received 13 May 1973

The imposition of unitarity bounds is shown to lead to a Yang-Mills structure in a wide class of theories involving vector mesons. Scalar fields are needed and, at least in simple cases, the unique unitary theory is of the Higgs type

Unitarity of scattering amplitudes

Unitarity is equivalent to the conservation of probability in quantum mechanics. A violation of unitarity is tantamount to a violation of the principles of quantum mechanics—this is too sacred a principle to give up!

Consider the helicity amplitude $\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)$ for a $2\to 2$ scattering process with initial [final] helicities λ_1, λ_2 [λ_3, λ_4]. The Jacob-Wick partial wave expansion is:

$$\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2) = \frac{8\pi\sqrt{s}}{(p_ip_f)^{1/2}}e^{i(\lambda_i-\lambda_f)\phi}\sum_{J=J_0}^{\infty}(2J+1)\mathcal{M}^J_\lambda(s)d^J_{\lambda_i\lambda_f}(\theta)\,,$$

where p_i [p_f] is the incoming [outgoing] center-of-mass momentum, $\sqrt{s}$ is the center-of-mass energy, $\lambda\equiv\{\lambda_3\lambda_4\,;\,\lambda_1\lambda_2\}$ and

$$J_0\equiv\max\{\lambda_i\,,\,\lambda_f\}\,,\quad \text{where}\quad \lambda_i\equiv\lambda_1-\lambda_2\,,\quad \text{and}\quad \lambda_f\equiv\lambda_3-\lambda_4\,.$$

Orthogonality of the d-functions allows one to project out a given partial wave amplitude.

For example, if we project out the $J = 0$ partial wave,

$$\mathcal{M}_\lambda^{J=0}(s) = \frac{1}{16\pi s}\int_{-s}^{0} dt\,\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)\,,$$

where $t = -\frac{1}{2}s(1-\cos\theta)$ [and θ is the CM scattering angle] in the limit where s is much larger than all particle squared masses.

Partial wave unitarity implies that:

$$|\mathcal{M}^J|^2 \leq |\mathrm{Im}\,\mathcal{M}^J| \leq 1\,,$$

which yields

$$(\mathrm{Re}\,\mathcal{M}^J)^2 \leq |\mathrm{Im}\,\mathcal{M}^J|\left(1-|\mathrm{Im}\,\mathcal{M}^J|\right) \leq \tfrac{1}{4}\,.$$

In particular, $\mathcal{M}_\lambda^J(s)$ *cannot* grow as $s\to\infty$, as this would constitute *bad high energy behavior*, which would be a clear violation of unitarity.

Consider the scattering process $W_L^+(p_1)W_L^-(p_2) \to W_L^+(p_3)W_L^-(p_4)$ at center-of-mass energies $\sqrt{s} \gg m_W$. Here, L stands for *longitudinal* and corresponds to $\lambda = 0$. The helicity-zero polarization vector at high energies behaves as

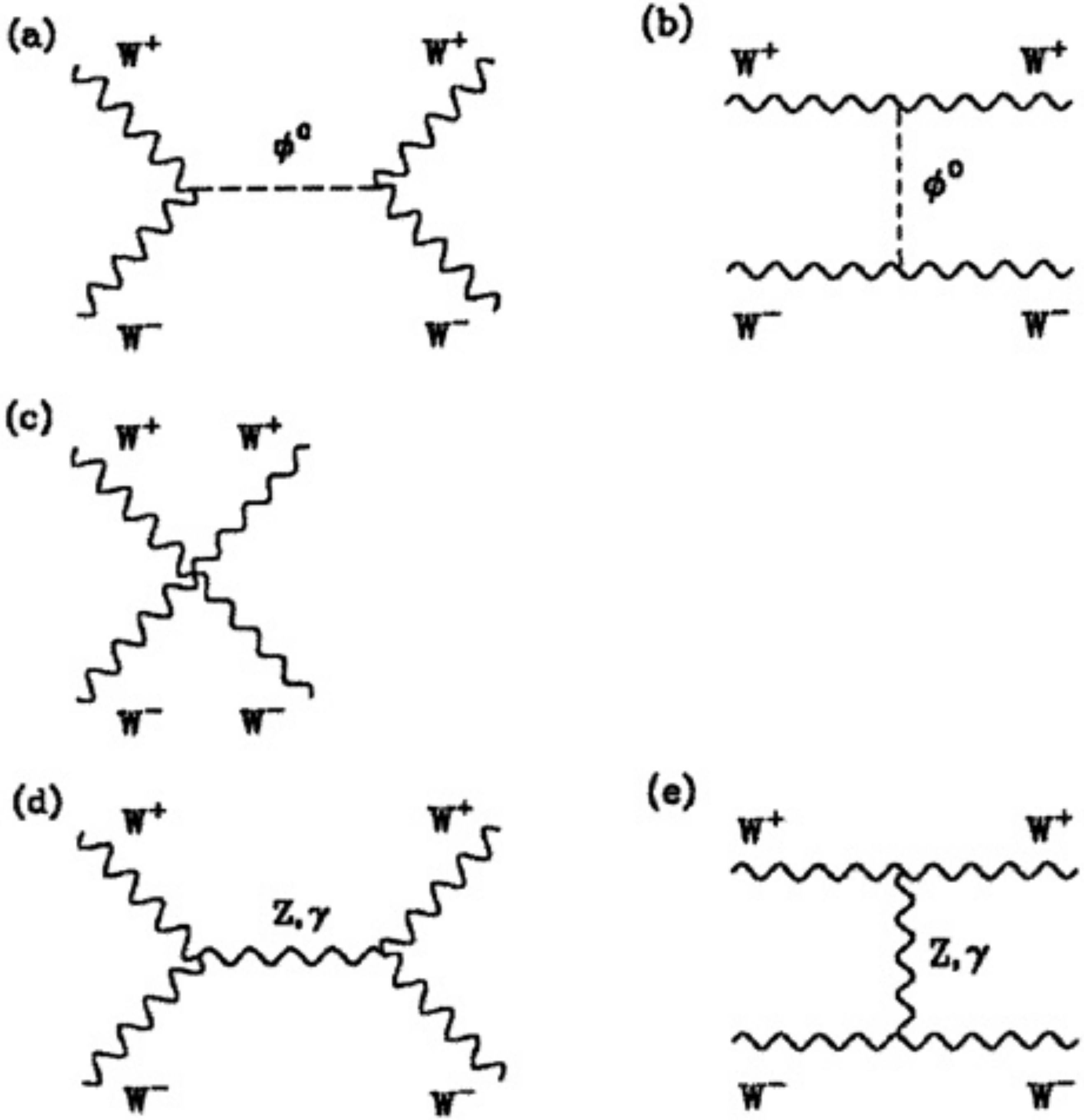
$$\varepsilon_L^\mu(p) \sim p^\mu/m_W \,.$$

Hence, contributions to the tree-level amplitude is proportional to

$$[\varepsilon_L(p_1)\cdot\varepsilon_L(p_2)]\,[\varepsilon_L(p_3)\cdot\varepsilon_L(p_4)] \sim \frac{s^2}{m_W^4}\,,$$

which can potentially lead to bad high energy behavior of the $W_L W_L$ elastic scattering amplitude.

Suppose we compute the tree-level amplitude in the electroweak theory but with the Higgs boson, $H \equiv \phi^0$, removed. (For simplicity, we neglect the fermions.) Instead we put in mass terms for the W and Z bosons by hand.


(a)
W⁺
W⁺
ϕ^0
W⁻
W⁻
(b)
W⁺
W⁺
ϕ^0
W⁻
W⁻
(c)
W⁺
W⁺
W⁻
W⁻
(d)
W⁺
W⁺
Z, γ
W⁻
W⁻
(e)
W⁺
W⁺
Z, γ
W⁻
W⁻

Since the gauge boson self-interactions are of the form specified by the gauge invariant theory with massless gauge bosons, the magic of gauge invariance is responsible for the cancelation of the *leading* bad high energy behavior,

$$\mathcal{M} = \sqrt{2}\, G_F(s+t)\,, \quad \text{for } s \gg m_W^2.$$

where $t \simeq -\frac{1}{2}s(1-\cos\theta)$ and G_F is the Fermi constant of weak interactions. Nevertheless, the amplitude still exhibits bad high energy behavior.

If we repeat the calculation using the electroweak theory with the Higgs boson, then one must include additional contributions to the $W_L W_L$ elastic scattering amplitude. The end result in the limit of $s \gg m_W^2$, m_H^2 is

$$\mathcal{M} = -\sqrt{2} G_F m_H^2 \left(\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2} \right) .$$

Indeed, the bad high energy behavior has been canceled.

1973: Enlarging the Higgs sector

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

A Theory of Spontaneous T Violation*

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(Received 11 April 1973)

A theory of spontaneous T violation is presented. The total Lagrangian is assumed to be invariant under the time reversal T and a gauge transformation (e.g., the hypercharge gauge), but the physical solutions are not. In addition to the spin-1 gauge field and the known matter fields, in its simplest form the theory consists of two complex spin-0 fields. Through the spontaneous symmetry-breaking mechanism of Goldstone and Higgs, the vacuum expectation values of these two spin-0 fields can be characterized by the shape of a triangle and their quantum fluctuations by its vibrational modes, just like a triangular molecule. T violations can be produced among the known particles through virtual excitations of the vibrational modes of the triangle which has a built-in T-violating phase angle. Examples of both Abelian and non-Abelian gauge groups are discussed. For renormalizable theories, all spontaneously T-violating effects are finite. It is found that at low energy, below the threshold of producing these vibrational quanta, T violation is always quite small.

Extended Higgs sectors can provide new sources of CP violation.

This was the first appearance of the two-Higgs doublet extension of the Standard Model (2HDM).

In order to have spontaneous T violation, we assume that there are two SU_2-doublet spin-0 fields

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$$

and (37)

$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix},$$

where the superscript denotes the electric charge. Both ϕ_1 and ϕ_2 are assumed to transform like the product $R^\dagger L$ under the $SU_2 \times U_1$ group; therefore, their coupling to the gauge fields is completely determined by the requirements of gauge invariance. The most general form of a renormalizable, gauge-invariant, and T-invariant potential energy $V(\phi)$ is now given by, instead of (7),

$$\begin{aligned} V(\phi) = & -\lambda_1 \phi_1^\dagger \phi_1 - \lambda_2 \phi_2^\dagger \phi_2 + A(\phi_1^\dagger \phi_1)^2 + B(\phi_2^\dagger \phi_2)^2 \\ & + C(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \overline{C}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + \tfrac{1}{2}[(\phi_1^\dagger \phi_2)(D\phi_1^\dagger \phi_2 + E\phi_1^\dagger \phi_1 + F\phi_2^\dagger \phi_2) + \text{H.c.}], \end{aligned}$$

1975: The ρ-parameter

Nuclear Physics B95 (1975) 135–147

NEUTRAL CURRENTS AND THE HIGGS MECHANISM

D.A. ROSS and M. VELTMAN

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Received 11 April 1975

The consequences of assuming (i) weak and e.m. forces constitute a gauge field theory, and (ii) there are no heavy leptons, are investigated. Relative to the Weinberg model, introduction of a general spontaneous symmetry breaking system leads to a theory with one extra free parameter, namely the neutral vector boson mass. Experimental consequences are indicated. A particular Higgs system containing two multiplets is studied in detail. It is noted that parameters may be chosen such that the cosmological constant is zero before as well as after spontaneous symmetry breakdown.

The ρ-parameter constraint on extended Higgs sectors

Given that the electroweak ρ-parameter is very close to 1, it follows that a Higgs multiplet of weak-isospin T and hypercharge Y must satisfy,[1]

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \qquad \Longleftrightarrow \qquad (2T+1)^2 - 3Y^2 = 1\,,$$

independently of the Higgs vacuum expectation values (vevs). The simplest solutions are Higgs singlets $(T, Y) = (0, 0)$ and hypercharge-one complex Higgs doublets $(T, Y) = (\frac{1}{2}, 1)$. For example, the latter is employed by the two Higgs doublet model (2HDM).

More generally, one can achieve $\rho = 1$ by fine-tuning if

$$\sum_{T,Y}\left[4T(T+1) - 3Y^2\right]|V_{T,Y}|^2 c_{T,Y} = 0\,,$$

where $V_{T,Y} \equiv \langle\Phi(T, Y)\rangle$ is the scalar vev, and $c_{T,Y} = 1$ for complex Higgs representations and $c_{T,Y} = \frac{1}{2}$ for real $Y = 0$ Higgs representations.

[1] Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$.

$$\rho_0 \equiv \frac{M_W^2}{M_Z^2 \hat{c}_Z^2 \hat{\rho}} , \qquad (10.55)$$

which describes new sources of SU(2) breaking that cannot be accounted for by the SM Higgs doublet or m_t effects. $\hat{\rho}$ is calculated as in Eq. (10.12) assuming the validity of the SM. In the presence of $\rho_0 \neq 1$, Eq. (10.55) generalizes the second Eq. (10.12) while the first remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect other radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.15)–(10.18), (10.32), and Γ_Z in Eq. (10.46c). There are enough data to determine ρ_0, M_H, m_t, and α_s, simultaneously. From the global fit,

$$\rho_0 = 1.00039 \pm 0.00019 , \qquad (10.56)$$

Taken from J. Erler and A. Freitas, *Electroweak Model and Constraints on New Physics*, in the 2018 Review of Particle Physics, M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

1976: The gauge hierarchy problem--fine tuning and unnaturalness of elementary scalars

Taken from E. Gildener and S. Weinberg, *Symmetry Breaking and Scalar Bosons*, Phys. Rev. D **13**, 3333 (1976).

The context: in grand unified theories, the unification scale M_U is around 10^{15} GeV, which is significantly larger than 100 GeV, the scale of electroweak physics. So how does one maintain such a large hierarchy of energy scales?

The trouble with this suggestion is that no one has been able to suggest any satisfactory reason why any scalars (aside from Goldstone bosons, which do not count because of their derivative couplings) should escape getting superheavy masses from the superstrong spontaneous symmetry breakdown.[10] One possibility is that the superstrong symmetry breakdown leaves both a chiral symmetry and a supersymmetry[11] unbroken, so that there is a multiplet including massless scalars and fermions. Unfortunately, the subsequent ordinary breakdown which gives masses to the intermediate vector bosons would then produce Goldstone fermions.

GAUGE-SYMMETRY HIERARCHIES REVISITED ★

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Received 30 July 1979

In a previous paper, I showed that in each order of perturbation theory there is an upper bound on the range of validity of a gauge hierarchy. Thus constructing a large hierarchy requires a fine-tuning of the scalar-field parameters. I stated that the possibility of an inherent bound on the hierarchy exists, but the question of the actual existence of such a bound was left completely open. Since then several authors have addressed this problem. Some of what I asserted was misunderstood, and incorrect conclusions have been drawn from recent computations. It has been claimed that the existence of large hierarchies has been demonstrated. It is the purpose of this paper to refute this claim, to help clarify the situation, and to explain why the status of this problem has in fact not really changed in recent years.

1976: The first comprehensive study of how to search for the Higgs boson

Nuclear Physics B106 (1976) 292--340

A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

Received 7 November 1975

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of

See also B.L. Ioffe and V.A. Khoze, Sov. J. Part. Nucl. Phys. **9**, 50 (1978).

The Higgs sector of the Standard Model (SM)

The SM includes a complex hypercharge-one, SU(2) doublet of self-interacting scalar fields, $\Phi \equiv (\Phi^+ \;\; \Phi^0)$ with four real degrees of freedom. The scalar potential is:

$$V(\Phi) = \lambda(\Phi^\dagger\Phi - \tfrac{1}{2}v^2)^2 \,,$$

so that in the ground state, the neutral scalar field takes on a constant non-zero value $\langle\Phi^0\rangle = v/\sqrt{2}$, where $v = 246$ GeV. It is convenient to write:

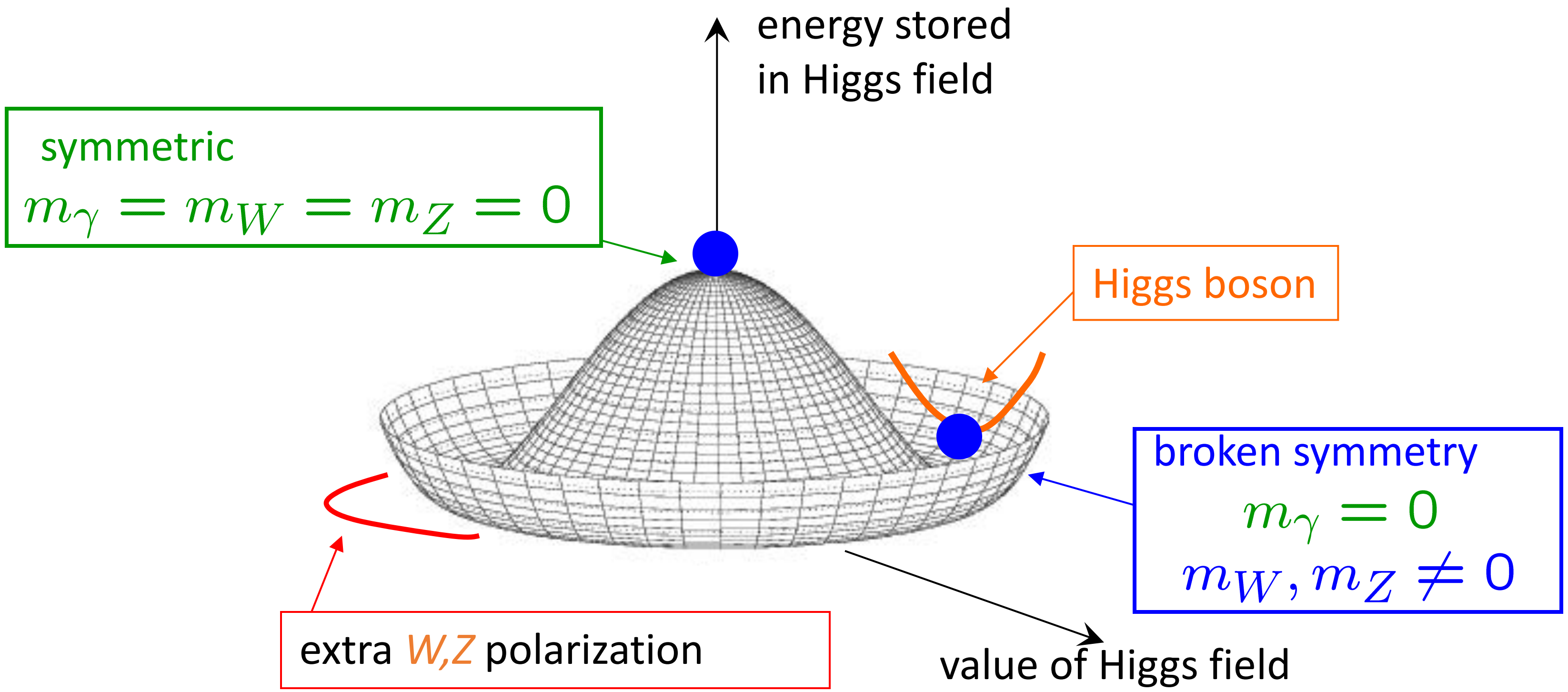
$$\Phi = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}}\left(v + H + i\omega^0\right) \end{pmatrix} \,,$$

where $\omega^\pm \equiv (\omega^1 \mp i\omega^2)/\sqrt{2}$.

The non-zero scalar vacuum expectation value breaks the electroweak symmetry, thereby generating three Goldstone bosons, ω^a $(a = 1, 2, 3)$.

Breaking the Electroweak Symmetry

Higgs imagined a field filling all of space, with a “weak charge”.
Energy forces it to be **nonzero** at bottom of the “Mexican hat”.



After electroweak symmetry breaking, the degrees of freedom represented by the ω^a become the longitudinal modes of the massive W and Z gauge bosons (via the Higgs mechanism), with

$$m_W^2 = \tfrac{1}{4}g^2v^2\,, \qquad\qquad m_Z^2 = \tfrac{1}{4}(g^2 + g'^{\,2})v^2\,,$$

which determines the value of the v.

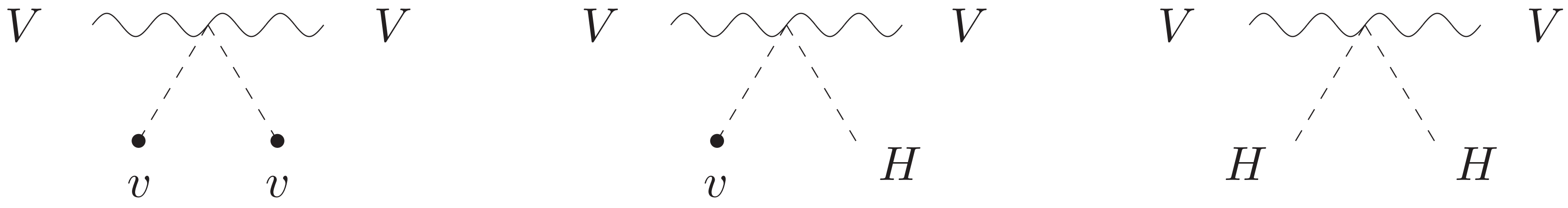
One scalar degree of freedom is left over—the Higgs boson, H, with self-interactions

$$V(H) = \lambda\left[\left(\frac{H+v}{\sqrt{2}}\right)^2 - \frac{v^2}{2}\right]^2 = \tfrac{1}{4}\lambda\left[H^4 + 4H^3v + 4H^2v^2\right]\,.$$

It is a neutral CP-even scalar boson, whose interactions are precisely predicted, but whose squared-mass, $m_H^2 = 2\lambda v^2$, depends on the unknown strength of the scalar self-coupling—the only unknown parameter of the model.

Mass generation and Higgs couplings in the SM

Gauge bosons ($V = W^{\pm}$ or Z) acquire mass via interaction with the Higgs vacuum condensate.



Thus,

$$g_{HVV} = 2m_V^2/v\,, \qquad \text{and} \qquad g_{HHVV} = 2m_V^2/v^2\,,$$

i.e., the Higgs couplings to vector bosons are proportional to the corresponding boson squared-mass.

Likewise, by replacing V with the Higgs field H in the above diagrams, the Higgs self-couplings are also proportional to the square of the Higgs mass:

$$g_{HHH} = 6\lambda v = \frac{3m_H^2}{v}\,, \qquad \text{and} \qquad g_{HHHH} = 6\lambda = \frac{3m_H^2}{v^2}\,.$$

Fermions in the Standard Model

Given a four-component fermion f, we can project out the right and left-handed parts:

$$f_R \equiv P_R f\,, \qquad f_L \equiv P_L f\,, \qquad \text{where} \quad P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5)\,.$$

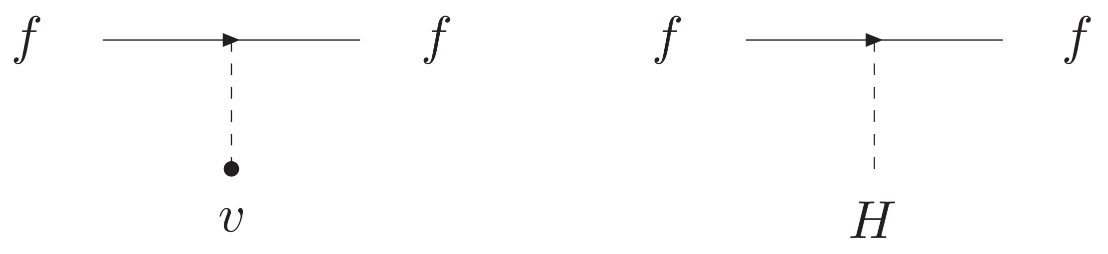
Under the electroweak gauge group, the right and left-handed components of each fermion has different SU(2)×U(1)$_{\rm Y}$ quantum numbers:

fermions	SU(2)	U(1)$_{\rm Y}$
$(\nu\,,\, e^-)_L$	2	-1
e^-_R	1	-2
$(u\,,\, d)_L$	2	$1/3$
u_R	1	$4/3$
d_R	1	$-2/3$

where the electric charge is related to the U(1)$_{\rm Y}$ hypercharge by $Q = T_3 + \frac{1}{2}Y$.

Before electroweak symmetry breaking, Standard Model fermions are massless, since the fermion mass term $\mathcal{L}_m = -m(\bar{f}_R f_L + \bar{f}_L f_R)$ is not gauge invariant.

The generation of masses for quarks and leptons is especially elegant in the SM. The fermions couple to the Higgs field through the gauge invariant Yukawa couplings (see below). The quarks and charged leptons acquire mass when Φ^0 acquires a vacuum expectation value:



Thus, $g_{Hf\bar{f}} = m_f/v$, *i.e.*, Higgs couplings to fermions are proportional to the corresponding fermion mass.

It is remarkable that the neutral Higgs boson coupling to fermions is flavor-diagonal. This is a consequence of the Higgs-fermion Yukawa couplings:

$$\mathcal{L}_{\rm Yukawa} = -h_u^{ij}(\bar{u}_R^i u_L^j \Phi^0 - \bar{u}_R^i d_L^j \Phi^+) - h_d^{ij}(\bar{d}_R^i d_L^j \Phi^{0\,*} + \bar{d}_R^i u_L^j \Phi^-) + \text{h.c.}\,,$$

where i, j are generation labels and h_u and h_d are arbitrary complex 3×3 matrices. Writing $\Phi^0 = (v + H)/\sqrt{2}$, we identify the quark mass matrices,

$$M_u^{ij} \equiv h_u^{ij}\frac{v}{\sqrt{2}}\,, \qquad\qquad M_d^{ij} \equiv h_d^{ij}\frac{v}{\sqrt{2}}\,.$$

One is free to redefine the quark fields:

$$u_L \to V_L^U u_L\,, \qquad u_R \to V_R^U u_R\,, \qquad d_L \to V_L^D d_L\,, \qquad d_R \to V_R^D d_R\,,$$

where V_L^U, V_R^U, V_L^D, and V_R^D are unitary matrices chosen such that

$$V_R^{U\,\dagger} M_u V_L^U = \mathrm{diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad V_R^{D\,\dagger} M_d V_L^D = \mathrm{diag}(m_d\,,\, m_s\,,\, m_b)\,,$$

such that the m_i are the positive quark masses (this is the *singular value decomposition* of linear algebra).

Having diagonalized the quark mass matrices, the neutral Higgs Yukawa couplings are automatically flavor-diagonal.* Hence the SM possesses no flavor-changing neutral currents (FCNCs) mediated by neutral Higgs boson (or gauge boson) exchange at tree-level.

*Independently of the Higgs sector, the quark couplings to Z and γ are automatically flavor diagonal. Flavor dependence only enters the quark couplings to the $W^\pm$ via the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $K \equiv V_L^{U\,\dagger} V_L^D$.

We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm [3,4] and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.

1976: The Bjorken process

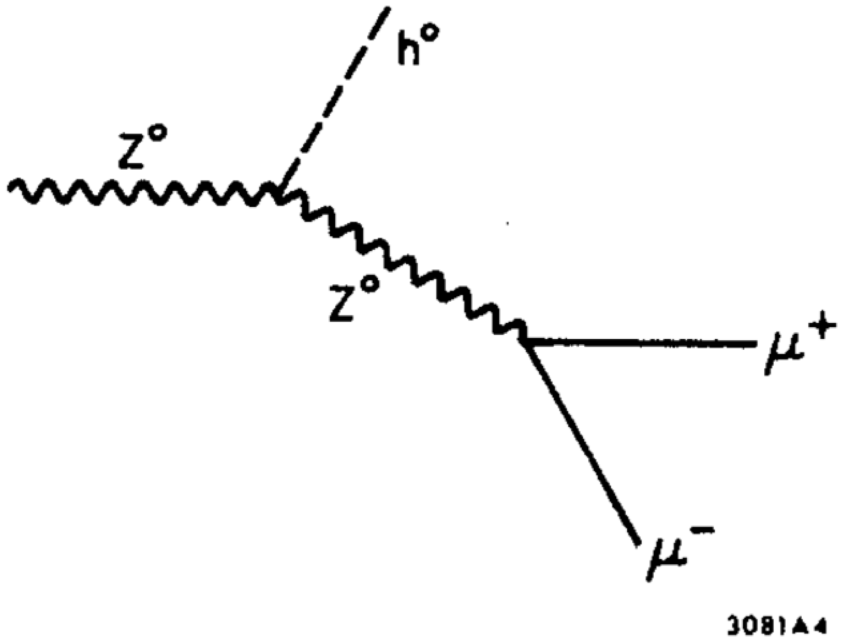


Fig. 4. Diagram for the decay $Z^0 \to h^0\mu^+\mu^-$, where h^0 is a neutral J=0 Higgs-boson.

From J.D. Bjorken, Proceedings of the 1976 SLAC Summer Institute, SLAC-R-198 (1977) pp. 1—42.

($B(We\bar{\nu}_e) \lesssim 10^{-6}$). More interesting is the decay $Z^0 \to h^0\mu^+\mu^-$, with h_0 the Higgs boson in the simple $SU(2) \otimes U(1)$ model. A straightforward calculation (slightly beyond the scope of these lectures), optimistically assuming that a single Higgs boson is responsible for the Z mass, gives

$$\frac{1}{\Gamma(Z \to \mu\mu)}\frac{d\Gamma}{dx} = \frac{\alpha}{4\ \sin^2\theta_W \cos^2\theta_W} \frac{\left[1 - x + \frac{x^2}{12} + \frac{2}{3}\frac{m_h^2}{m_Z^2}\right]\left(x^2 - \frac{4m_h^2}{m_Z^2}\right)^{1/2}}{\left(x - \frac{m_h^2}{m_Z^2}\right)^2} \tag{4.30}$$

where

$$x = \frac{2E_{higgs}}{m_Z} \tag{4.31}$$

and the kinematic limits are

$$\frac{2m_h}{m_Z} \le x \le 1 + \frac{m_h^2}{m_Z^2} \tag{4.32}$$

Rough numerical integration provides the yield shown in Fig. 11. We see that for $m_h \lesssim 40$ GeV, the branching ratio relative to μ pairs $B(Z \to h^0\mu^+\mu^-)/B(Z^0 \to \mu^+\mu^-)$, is $\gtrsim 3 \times 10^{-5}$. Recalling that a 6% $\mu^+\mu^-$ branching ratio still means $\sim$0.6 $Z^0 \to \mu^+\mu^-$ events/second, this leaves a tolerable production of Higgs bosons. The signature evidently is very good; one looks at a peak in the mass recoiling against an energetic acoplanar dilepton pair. We must, however, point out that this estimate, as is any estimate which directly involves the Higgs sector, is very unreliable: the theoretical status is very poorly understood.[42] Indeed there is no certainty that $m_h \lesssim 40$ GeV; Higgs bosons could be ten times more massive.[59] And there could well be several.

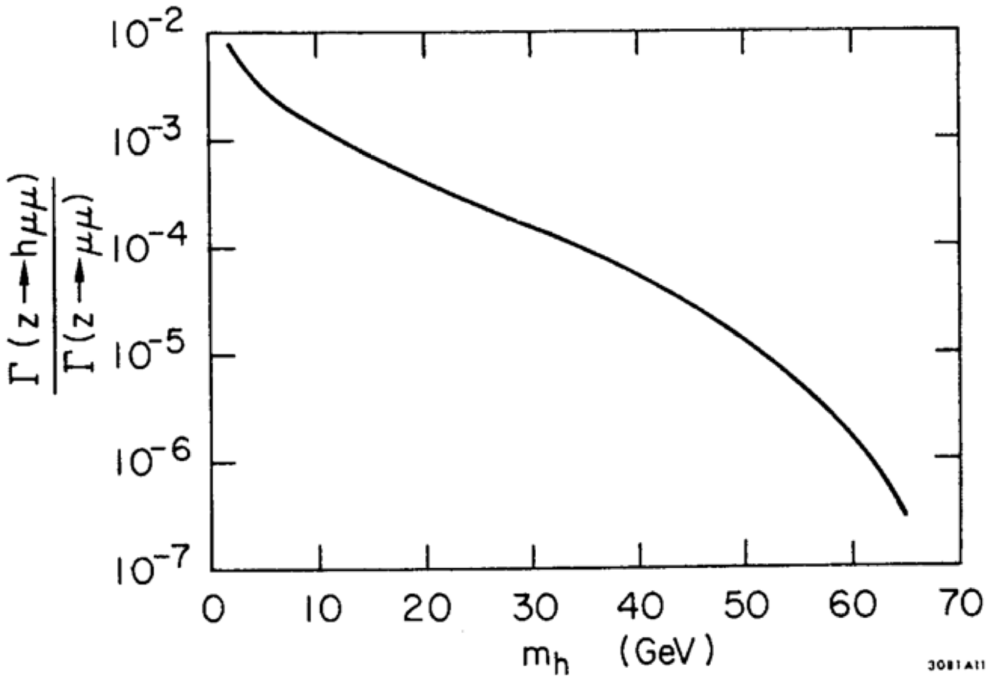


Fig. 11. Estimated branching ratio of $Z \to h^0\mu^+\mu^-$ relative to $Z^0 \to \mu^+\mu^-$. We have taken $\sin^2\theta_W = 1/3$.

1977: Unitarity constraints and an upper bound on the Higgs mass

PHYSICAL REVIEW D VOLUME 16, NUMBER 5 1 SEPTEMBER 1977

Weak interactions at very high energies: The role of the Higgs-boson mass

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(Received 20 April 1977)

We give an S-matrix-theoretic demonstration that if the Higgs-boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$, parital-wave unitarity is not respected by the tree diagrams for two-body scattering of gauge bosons, and the weak interactions must become strong at high energies. We exhibit the relation of this bound to the structure of the Higgs-Goldstone Lagrangian, and speculate on the consequences of strongly coupled Higgs-Goldstone systems. Prospects for the observation of massive Higgs scalars are noted.

Recall that $\mathcal{M} = -\sqrt{2}\, G_F m_H^2 \left(\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2}\right)$ for elastic $W_L W_L$ scattering. Projecting out the $J=0$ partial wave and taking $s \gg m_H^2$,

$$\mathcal{M}^{J=0} = -\frac{G_F m_H^2}{4\pi\sqrt{2}}\,.$$

Imposing $|\mathrm{Re}\ \mathcal{M}^J| \leq \frac{1}{2}$ yields an upper bound on m_H. The most stringent bound is obtained by all considering other possible final states such as $Z_L Z_L$, $Z_L H$ and HH. The end result is:*

$$\boxed{m_H^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700\ \mathrm{GeV})^2\,.}$$

If $m_H \gtrsim 700$ GeV, then the Higgs-self coupling parameter, $\lambda = 2m_H^2/v^2$ is becoming large and our tree-level analysis is no longer reliable. Nevertheless, lattice studies suggest that an upper Higgs mass bound below 1 TeV remains valid even in the strong Higgs self-coupling regime.

*Lee, Quigg and Thacker imposed $|\mathcal{M}^J| \leq 1$, thereby obtaining $M_H^2 \leq 8\pi\sqrt{2}/3G_F$.

1977: Sensitivity to the Higgs mass through radiative corrections (Veltman's screening theorem)

Vol. B8 (1977) *ACTA PHYSICA POLONICA* No 6

SECOND THRESHOLD IN WEAK INTERACTIONS

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and

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(Received January 7, 1977)

The point of view that weak interactions must have a second threshold below 300 — 600 GeV is developed. Above this threshold new physics must come in. This new physics may be the Higgs system, or some other nonperturbative system possibly having some similarities to the Higgs system. The limit of large Higgs mass is thought to be relevant in this context. Radiative corrections proportional to m^2 and $\ln m^2$, m being the Higgs mass, are calculated. Contemplation of the theory in the limit of large Higgs mass suggests that the "new physics" may contain breakdown of μ-e universality and other than V–A neutrino interactions already at relatively low energies.

When radiative corrections are included, one has a number of ways to define the weak mixing angle, θ_W. A scale-dependent ($\overline{\rm MS}$) mixing angle, can be defined,[*] $\hat{s}_Z^2 \equiv \sin^2\theta_W(m_Z)$, and $\hat{c}_Z^2 = 1 - \hat{s}_Z^2$. One possible definition of the ρ parameter in the Standard Model is,

$$\hat{\rho} \equiv \frac{m_W^2}{m_Z^2 \hat{c}_Z^2} = 1 + \delta\hat{\rho}\,,$$

where the leading one-loop radiative corrections, $\delta\hat{\rho}$, are given by

$$\delta\hat{\rho} \simeq -\frac{11 g'^{\,2}}{96\pi^2} \ln\left(\frac{m_H}{m_Z}\right) + \frac{3g^2}{64\pi^2 m_W^2}\left[m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\left(\frac{m_t^2}{m_b^2}\right)\right],$$

and g, g' are the SU(2) and U(1) electroweak gauge couplings, respectively.

Veltman noticed that the contribution of a heavy top quark was quadratic in m_t, whereas the sensitivity of a heavy Higgs boson was only logarithmic.[†]

[*] W.J. Marciano, Phys. Rev. D **20**, 274 (1979).

[†] A related ρ-parameter defined in terms of the ratio of neutral current to charged current neutrino-nucleon scattering cross sections exhibits a similar behavior (with 11/96 above replaced by 3/32).

1977: Implications of flavor-diagonal neutral-Higgs mediated processes for extended Higgs sectors

PHYSICAL REVIEW D VOLUME 15, NUMBER 7 1 APRIL 1977

Natural conservation laws for neutral currents*

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(Received 20 August 1976)

We explore the consequences of the assumption that the direct and induced weak neutral currents in an $SU(2) \otimes U(1)$ gauge theory conserve all quark flavors *naturally*, i.e., for all values of the parameters of the theory. This requires that all quarks of a given charge and helicity must have the same values of weak T_3 and $\vec{T}^2$. If all quarks have charge $+2/3$ or $-1/3$ the only acceptable theories are the "standard" and "pure vector" models, or their generalizations to six or more quarks. In addition, there are severe constraints on the couplings of Higgs bosons, which apparently cannot be satisfied in pure vector models. We also consider the possibility that neutral currents conserve strangeness but not charm. A natural seven-quark model of this sort is described. The experimental consequences of charm nonconservation in direct or induced neutral currents are found to be quite dramatic.

See also E.A. Paschos, *Diagonal Neutral Currents*, Phys. Rev. D **15**, 1966 (1977).

Condition III. We demand that the coupling of each neutral Higgs meson be such as naturally to conserve all quark flavors: strangeness, charm, etc.

Because of our requirement of naturalness, the matrix M_0 must be regarded as an *arbitrary* SU(2)-invariant matrix commuting with Q. Similarly, the matrices Γ_i contain a number of *arbitrary* parameters equal to the number of SU(2)-invariant charge-conserving Yukawa couplings of the Higgs mesons to the quarks. For all such M_0 and for all such Γ_i the couplings of the neutral Higgs mesons must be diagonal in the basis in which M is diagonal.

Suppose that the set of quarks with given charge Q get their mass purely from a single neutral Higgs meson ϕ_Q^0. Then the mass matrix for these quarks of charge Q will be

$$M(Q) = \Gamma_Q \langle \phi_Q^0 \rangle \tag{3.7}$$

and Γ_Q is trivially diagonal in the basis which diagonalizes $M(Q)$. However, if there were more than one neutral Higgs boson contributing to the masses of quarks of a given charge, or if there were both an invariant mass term M_0 and a Higgs contribution, then there would be no reason to expect the couplings of the neutral Higgs bosons to conserve quark flavor. We conclude that Condition

1978: The two-Higgs doublet model (2HDM) takes off…

1978: N.G. Deshpande and E. Ma, *Pattern of Symmetry Breaking with Two Higgs Doublets*

2HDM models satisfying the Glashow-Weinberg-Paschos conditions:

1979: J.F. Donoghue and L.F. Li, *Properties of Charged Higgs Bosons* ("Type-II")
H.E. Haber, G.L. Kane and T. Sterling, *The Fermion Mass Scale and Possible Effects of Higgs Bosons on Experimental Observables* ("Type-I")

1981: L.J. Hall and M.B. Wise, *Flavor Changing Higgs Boson Couplings* (introduced the Type I/II nomenclature)

1990: V.D. Barger, J.L. Hewett and R.J.N. Phillips, *New Constraints on the Charged Higgs Sector in Two Higgs Doublet Models* (introduced Types III and IV, also called Y and X, in which Higgs couplings to quarks and leptons are oppositely treated.

1980: The criterion of naturalness in gauge theories

The Higgs mass parameter of the SM is unnatural, since the symmetry of the theory is not enhanced in the limit in which this parameter is set to zero.

In contrast, light fermions are natural because the limit of $m_f = 0$ corresponds to the presence of a chiral symmetry.

NATURALNESS, CHIRAL SYMMETRY, AND SPONTANEOUS

CHIRAL SYMMETRY BREAKING

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ABSTRACT

A properly called "naturalness" is imposed on gauge theories. It is an order-of-magnitude restriction that must hold at all energy scales μ. To construct models with complete naturalness for elementary particles one needs more types of confining gauge theories besides quantum chromodynamics. We propose a search program for models with improved naturalness and concentrate on the possibility that presently elementary fermions can be considered as composite. Chiral symmetry must then be responsible for the masslessness of these fermions. Thus we search for QCD-like models where chiral symmetry is not or only partly broken spontaneously. They are restricted by index relations that often cannot be satisfied by other than unphysical fractional indices. This difficulty made the author's own search unsuccessful so far.

1981: Attempts to construct natural models of electroweak symmetry breaking (EWSB)

1. Supersymmetry: naturally light elementary bosons are related by supersymmetry to fermions whose small masses are protected by approximate chiral symmetry. However, supersymmetry must be broken at an energy scale not much higher than the scale of EWSB. [Witten, Dimopoulos and Georgi, Sakai,…]

2. Strong EWSB dynamics not based on elementary scalar dynamics. Examples of this approach include technicolor [Weinberg, Susskind,…], and composite Higgs bosons [Kaplan, Georgi, Dimopoulos,…].

PHYSICS REPORTS (Review Section of Physics Letters) 104, Nos. 2–4 (1984) 181–193. North-Holland, Amsterdam

The Gauge Hierarchy Problem, Technicolor, Supersymmetry, and all that

Leonard SUSSKIND*

1. The gauge hierarchy problem

Possible solutions to the GHP

(1) Forget it for now. Some future theory will explain the fine tuning of $\mu(M)$. The boring desert exists just like in the usual SU(5) model. No new discoveries above 250 GeV until 10^{15} GeV.

(2) Technicolor. It is possible for a low energy world to emerge naturally. A familiar example is QCD with massless quarks and gluons. In this case chiral symmetry and gauge invariance insure the absence of any perturbative renormalizations of quark and gluon masses. The coupling constant $g(k)$ is renormalized and therefore depends on the cutoff scale k. The correct dependence is given by the renormalization group

$$\mathrm{d}g(k)/\mathrm{d}\log k = \beta(g(k)) = -\beta_0\, g(k)^3 + \cdots \tag{10}$$

Field content of the MSSM						
Super-multiplets	Super-field	Bosonic fields	Fermionic partners	SU(3)	SU(2)	U(1)
gluon/gluino	$\widehat{V}_8$	g	$\widetilde{g}$	8	1	0
gauge/	$\widehat{V}$	$W^{\pm}\,,\ W^0$	$\widetilde{W}^{\pm}\,,\widetilde{W}^0$	1	3	0
gaugino	$\widehat{V}'$	B	$\widetilde{B}$	1	1	0
slepton/	$\widehat{L}$	$(\widetilde{\nu}_L, \widetilde{e}^-_L)$	$(\nu, e^-)_L$	1	2	-1
lepton	$\widehat{E}^c$	$\tilde{e}^+_R$	e^c_L	1	1	2
squark/	$\widehat{Q}$	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$	3	2	$1/3$
quark	$\widehat{U}^c$	$\widetilde{u}^*_R$	u^c_L	$\bar{3}$	1	$-4/3$
	$\widehat{D}^c$	$\widetilde{d}^*_R$	d^c_L	$\bar{3}$	1	$2/3$
Higgs/	$\widehat{H}_d$	$(H^0_d\,,\ H^-_d)$	$(\widetilde{H}^0_d, \widetilde{H}^-_d)$	1	2	-1
higgsino	$\widehat{H}_u$	$(H^+_u\,,\ H^0_u)$	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$	1	2	1

The fields of the minimal supersymmetric extension of the SM (MSSM) and their SU(3)×SU(2)×U(1) quantum numbers are listed. The electric charge is given in terms of the third component of the weak isospin T_3 and U(1) hypercharge Y by $Q = T_3 + \frac{1}{2}Y$. Generation labels of quarks and leptons are suppressed. For each lepton, quark, and Higgs super-multiplet, there is a corresponding anti-particle multiplet of charge-conjugated fermions and their associated scalar partners. The L and R subscripts of the squark and slepton fields indicate the helicity of the corresponding fermionic superpartners.

The Higgs sector of the MSSM is a 2HDM (whose interactions are constrained by supersymmetry). The second Higgs doublet is needed to cancel gauge anomalies in one-loop triangle diagrams with three external gauge bosons. A theory that possesses gauge anomalies is inconsistent as a quantum theory.

To cancel the gauge anomalies, we must satisfy certain group theoretical constraints.

$$\begin{aligned} &W^i W^j B \text{ triangle} \qquad &\Longleftrightarrow \qquad &\mathrm{Tr}(T_3^2 Y) = 0\,, \\ &BBB \text{ triangle} \qquad &\Longleftrightarrow \qquad &\mathrm{Tr}(Y^3) = 0\,. \end{aligned}$$

Example: contributions of the fermions to $\mathrm{Tr}(Y^3)$

$$\mathrm{Tr}(Y^3)_{\rm SM} = 3\left(\tfrac{1}{27} + \tfrac{1}{27} - \tfrac{64}{27} + \tfrac{8}{27}\right) - 1 - 1 + 8 = 0\,.$$

Suppose we only add the higgsinos $(\widetilde H_u^+\,,\ \widetilde H_u^0)$. The resulting anomaly factor is $\mathrm{Tr}(Y^3) = \mathrm{Tr}(Y^3)_{\rm SM} + 2$, leading to a gauge anomaly. This anomaly is canceled by adding a second higgsino doublet with opposite hypercharge.

1983: Supersymmetry constraints on Higgs masses

ANNALS OF PHYSICS **148**, 95–134 (1983)

Higgs Masses in the Standard, Multi-Higgs and Supersymmetric Models

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Received November 12, 1982

Theoretical constraints and limits on the masses of Higgs scalars in the standard electroweak model, in electroweak models with additional Higgs doublets and in various supersymmetric models are presented. In the standard model, the lower limit on the Higgs mass, based on vacuum stability arguments, is reviewed in detail, as are "upper limits" based on perturbative constraints. In most grand unified and all supersymmetric models, however, at least two doublets are needed. The masses of the various Higgs scalars in the two-doublet model are discussed and constraints on their masses are found, including the generalization of the above limits. The results are then generalized to models with more than two doublets. Finally, recent attempts at constructing models with low-energy supersymmetry are reviewed and it is shown that in many models, fairly stringent tree-level mass relations among the Higgs scalars can be found. These relations are interesting in that they do not refer to the supersymmetric partners of ordinary particles, and they are most restrictive in models in which the supersymmetry is explicitly broken, i.e., via *arbitrary* mass terms.

The eigenvalues are

$$m^2_{\phi,\eta} = \frac{1}{2}\left[(m^2_{\chi^0} + m^2_z) \pm \sqrt{(m^2_{\chi^0} + m^2_z)^2 - 4m^2_z m^2_{\chi^0}\left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2}\,\right] \tag{5.14}$$

which implies

$$m^2_\phi + m^2_\eta = m^2_{\chi^0} + M^2_Z. \tag{5.15}$$

This is our second constraint. More importantly, one can see that the lighter scalar *must* be lighter than the Z (and also lighter than the χ_0 which in turn is lighter than

That is, at tree level the MSSM yields $m_H \leq m_Z$.

1978--1990: The study of the phenomenology of Higgs bosons becomes mature

- **1978:** Higgs production via gluon-gluon fusion (via a top quark loop) at hadron colliders

VOLUME 40, NUMBER 11 PHYSICAL REVIEW LETTERS 13 MARCH 1978

Higgs Bosons from Two-Gluon Annihilation in Proton-Proton Collisions

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(Received 27 December 1977)

We estimate the cross section for Higgs-boson production in proton-proton collisions. We find that most of the cross section comes from a two-gluon annihilation process, in which the gluons couple to Higgs bosons via heavy-quark loops.

➢ 1984: Higgs production via vector boson at hadron colliders

Volume 136B, number 3 PHYSICS LETTERS 1 March 1984

PRODUCTION OF VERY MASSIVE HIGGS BOSONS ☆

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Received 5 December 1983

We compare Higgs boson production mechanisms at multi-TeV hadronic colliders. In addition to the previously investigated processes gluon + gluon → H and $q\bar{q} \to V^* \to VH$ ($V = W, Z$), we consider Higgs boson formation by pairs of virtual W's or Z's, a process analogous to two-photon collisions in e^+e^- scattering. The Higgs production process $W^*W^* \to H$ is dominated by longitudinal W's and is the most important mechanism for $M_H > 6\,M_W$, if the top quark mass is about 30 GeV.

➢ 1984: Higgs decay to WW*

PHYSICAL REVIEW D VOLUME 30, NUMBER 1 1 JULY 1984

Higgs-scalar decays: $H \to W^{\pm} + X$

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(Received 28 March 1984)

Decays of a Higgs scalar in the mass range $m_W \lesssim m_H \lesssim 2m_W$ ($m_W = W^{\pm}$ mass $\simeq 83$ GeV) are examined. For $m_H \gtrsim 125$ GeV, the branching ratio for $H \to W^{\pm} + X$ is found to be substantial, provided the top quark is heavy, $m_t > m_H/2$. Implications of our results for hadron-hadron-collider experiments are briefly discussed.

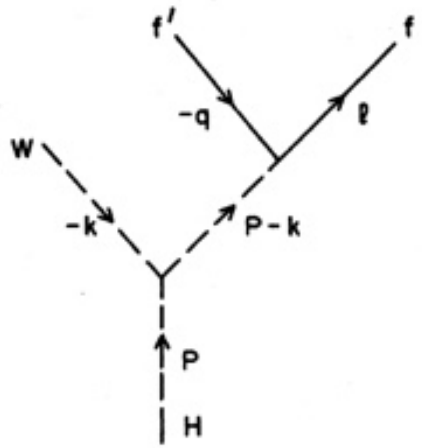

FIG. 2. Feynman diagram for $H \to W f \bar{f}'$.

➢ 1986: MSSM Higgs boson phenomenology begins in earnest

Nuclear Physics B272 (1986) 1–76
North-Holland, Amsterdam

HIGGS BOSONS IN SUPERSYMMETRIC MODELS (I)*

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Received 3 June 1985
(Revised 6 August 1985)

We describe the properties of Higgs bosons in a class of supersymmetric theories. We consider models in which the low-energy sector contains two weak complex doublets and perhaps one complex gauge-singlet Higgs field. Supersymmetry is assumed to be either softly or spontaneously broken, thereby imposing a number of restrictions on the Higgs boson parameters. We elucidate the Higgs boson masses and present Feynman rules for their couplings to the gauge bosons, fermions and scalars of the theory. We also present Feynman rules for vertices which are related by supersymmetry to the above couplings. Exact analytic expressions are given in two useful limits – one corresponding to the absence of the gauge-singlet Higgs field and the other corresponding to the absence of a supersymmetric Higgs mass term.

➢ 1988: The importance of Higgs decay to $\gamma\gamma$ and ZZ* at a hadron collider.

Nuclear Physics B299 (1988) 231–278
North-Holland, Amsterdam

SEARCH TECHNIQUES FOR CHARGED AND NEUTRAL INTERMEDIATE-MASS HIGGS BOSONS*

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Received 12 October 1987

If fundamental scalar bosons exist, a natural implication of theories based on supersymmetry or superstrings, Higgs boson masses are likely to be of order m_Z within roughly a factor of two. We term this the intermediate mass range. We study how such Higgs bosons could be detected at e^+e^- and, particularly, at hadron colliders. At a hadron collider it appears to be necessary to use rare decay modes of the Higgs boson, and we examine $H^0 \to \gamma\gamma$, $\tau^+\tau^-$, $Z\gamma$, $Z\ell^+\ell^-$, $\Theta\gamma$, and $b\bar{b}$.

➢ 1990: The decoupling limit of an extended Higgs sector

Nuclear Physics B335 (1990) 363–394
North-Holland

MULTI-SCALAR MODELS WITH A HIGH-ENERGY SCALE*

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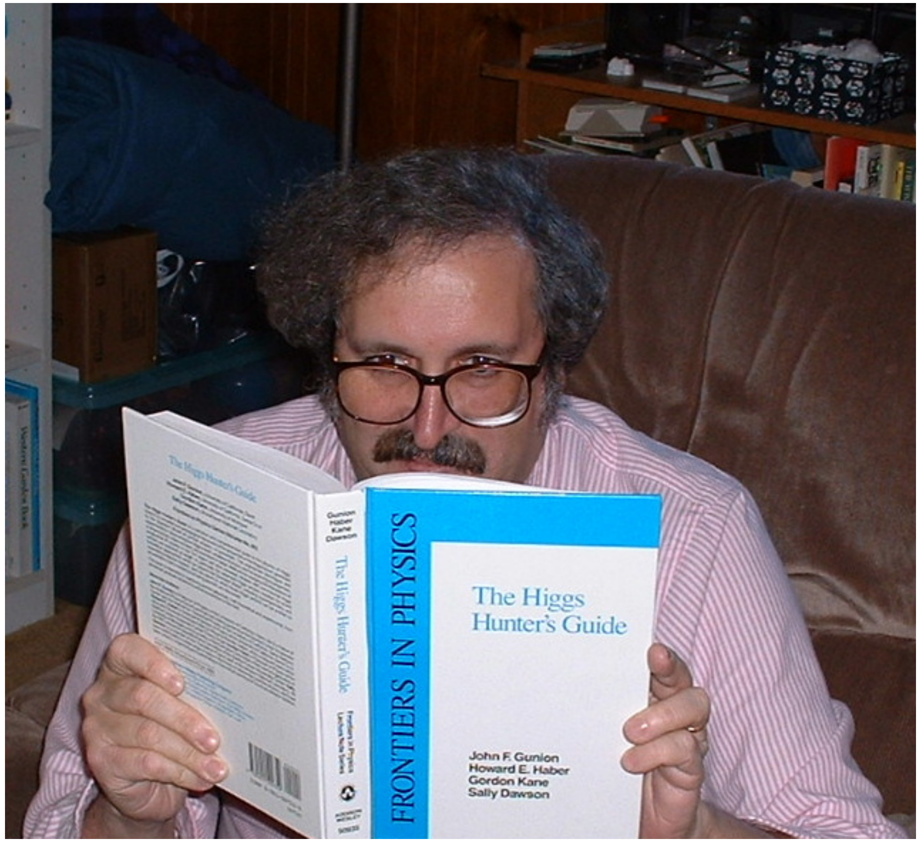
We study multi-Higgs models under the assumption that new physics exists at some high-energy scale (Λ_{NP}). If we perform the minimally required fine-tuning in order to set the electroweak scale (Λ_{EW}), we find that the low-energy scalar spectrum is identical to that of the Standard Model with minimal Higgs content, up to corrections of order $\Lambda^2_{EW}/\Lambda^2_{NP}$. If, in

1990: The status of the Higgs boson is summarized, as LEP and SLC embark on the first dedicated searches for the Higgs boson.



Michael Peskin peruses *The Higgs Hunter's Guide*

HHG authors anticipate the discovery of the Higgs boson

1991: Discovery of the Higgs boson of the MSSM at LEP is no longer guaranteed

VOLUME 66, NUMBER 14 PHYSICAL REVIEW LETTERS 8 APRIL 1991

Can the Mass of the Lightest Higgs Boson of the Minimal Supersymmetric Model be Larger than m_Z?

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(Received 3 January 1991)

In the minimal supersymmetric model (MSSM), the *tree-level* mass of the lightest Higgs scalar h^0 cannot be larger than the mass of the Z boson. We have computed the one-loop radiative correction to the upper bound on m_{h^0} as a function of the free parameters of the MSSM. We find that the dominant correction to $m_{h^0} - m_Z$ is large and positive and grows like m_t^4, where m_t is the top-quark mass. As a result, the MSSM cannot be ruled out if the CERN e^+e^- collider LEP-200 fails to discover the Higgs boson.

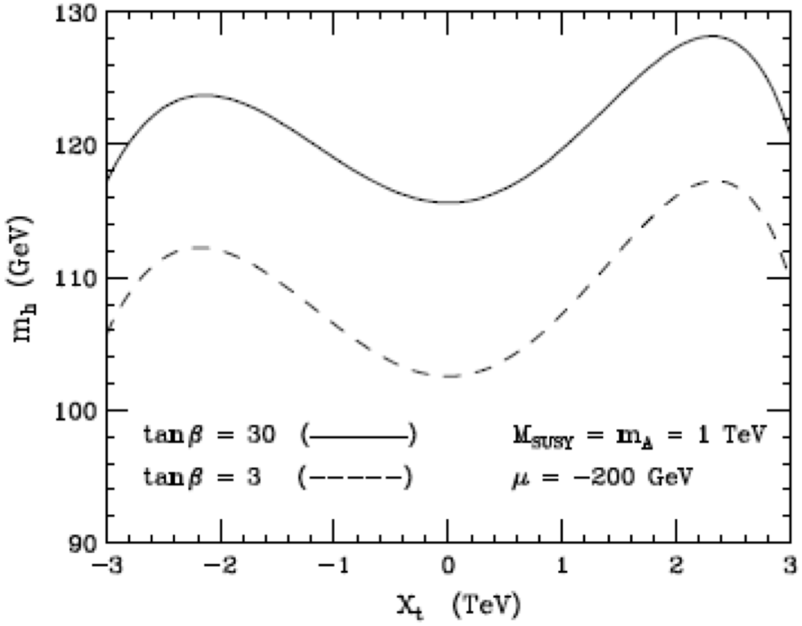
See also, Y. Okada, M. Yamaguchi and T. Yanagida, *Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model*, Prog. Theor. Phys **85**, 1 (1991); J.R. Ellis, G. Ridolfi and F. Zwirner, *Radiative corrections to the masses of supersymmetric Higgs bosons*, Phys. Lett. B **257**, 83 (1991).

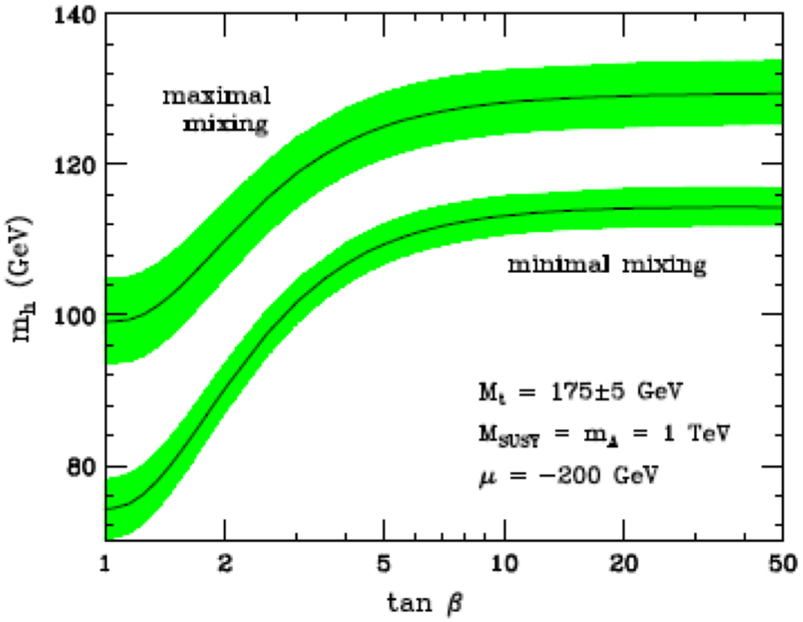
The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancelation, which would have been exact if supersymmetry were unbroken):

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

where $X_t \equiv A_t - \mu\cot\beta$ governs stop mixing and M_S^2 is the average squared-mass of the top-squarks $\widetilde{t}_1$ and $\widetilde{t}_2$ (which are the mass-eigenstate combinations of the interaction eigenstates, $\widetilde{t}_L$ and $\widetilde{t}_R$).

The state-of-the-art computation includes the full one-loop result, all the significant two-loop contributions, some of the leading three-loop terms, and renormalization-group improvements. The final conclusion is that $m_h \lesssim 130$ GeV [assuming that the top-squark mass is no heavier than about 2 TeV].





Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that m_h is maximized (for a fixed $\tan\beta$). This occurs for $X_t/M_S \sim 2$. As $\tan\beta$ varies, m_h reaches is maximal value, $(m_h)_{\max} \simeq 130$ GeV, for $\tan\beta \gg 1$ and $m_A \gg m_Z$.

2000: At the end of SLC/LEP, the data imply that the Higgs boson mass must lie between 114 GeV and 285 GeV (95% CL limits)

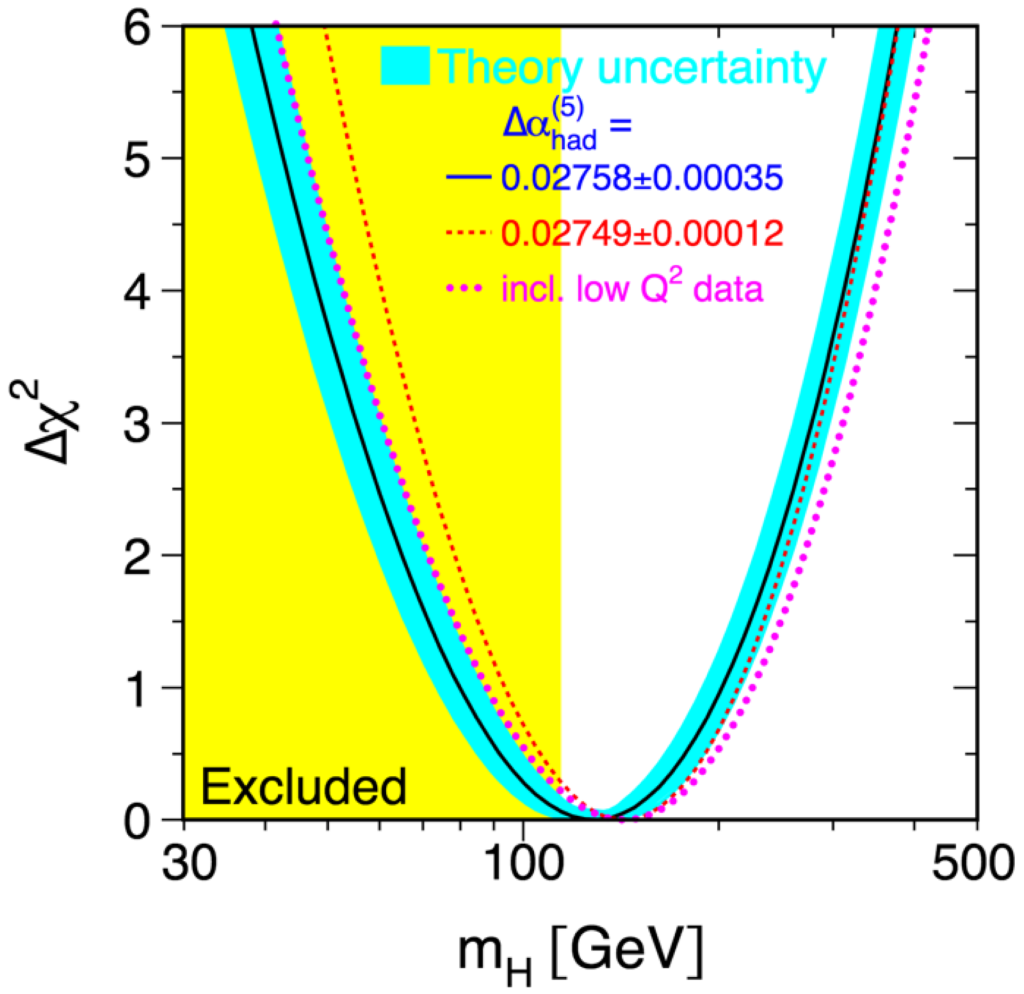
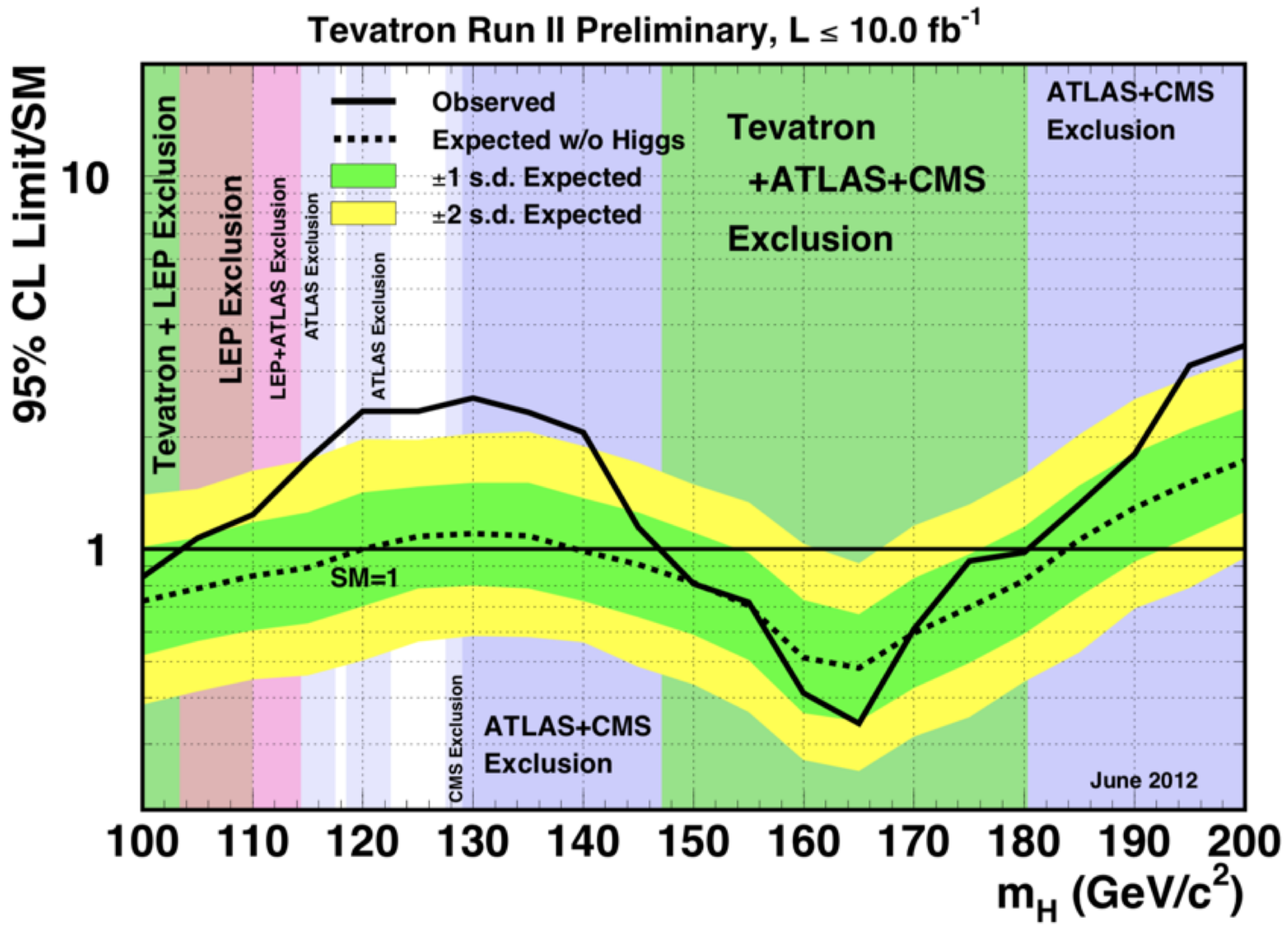


Figure 8.13: $\Delta\chi^2(m_H) = \chi^2_{min}(m_H) - \chi^2_{min}$ as a function of m_H. The line is the result of the fit using all 18 results. The associated band represents the estimate of the theoretical uncertainty due to missing higher-order corrections as discussed in Section 8.4. The vertical band shows the 95% confidence level exclusion limit on m_H of 114.4 GeV derived from the direct search at LEP-II [39]. The dashed curve is the result obtained using the theory-driven $\Delta\alpha^{(5)}_{had}(m_Z^2)$ determination of Equation 8.4. The direct measurements of m_W and Γ_W used here are preliminary.

Taken from the ALEPH, DELPHI, L3 and OPAL Collaborations, the SLD Collaboration, the LEP Electroweak Working Group, the SLD electroweak, heavy flavour groups, Physics Reports **427**, 257 (2006).

2011: Closing in on the Higgs boson. The Tevatron completes a decade of running, as the LHC turns on and begins to take data.



The LHC discovery of 4 July 2012

The CERN update of the search for the Higgs boson, simulcast at ICHEP-2012 in Melbourne, Australia

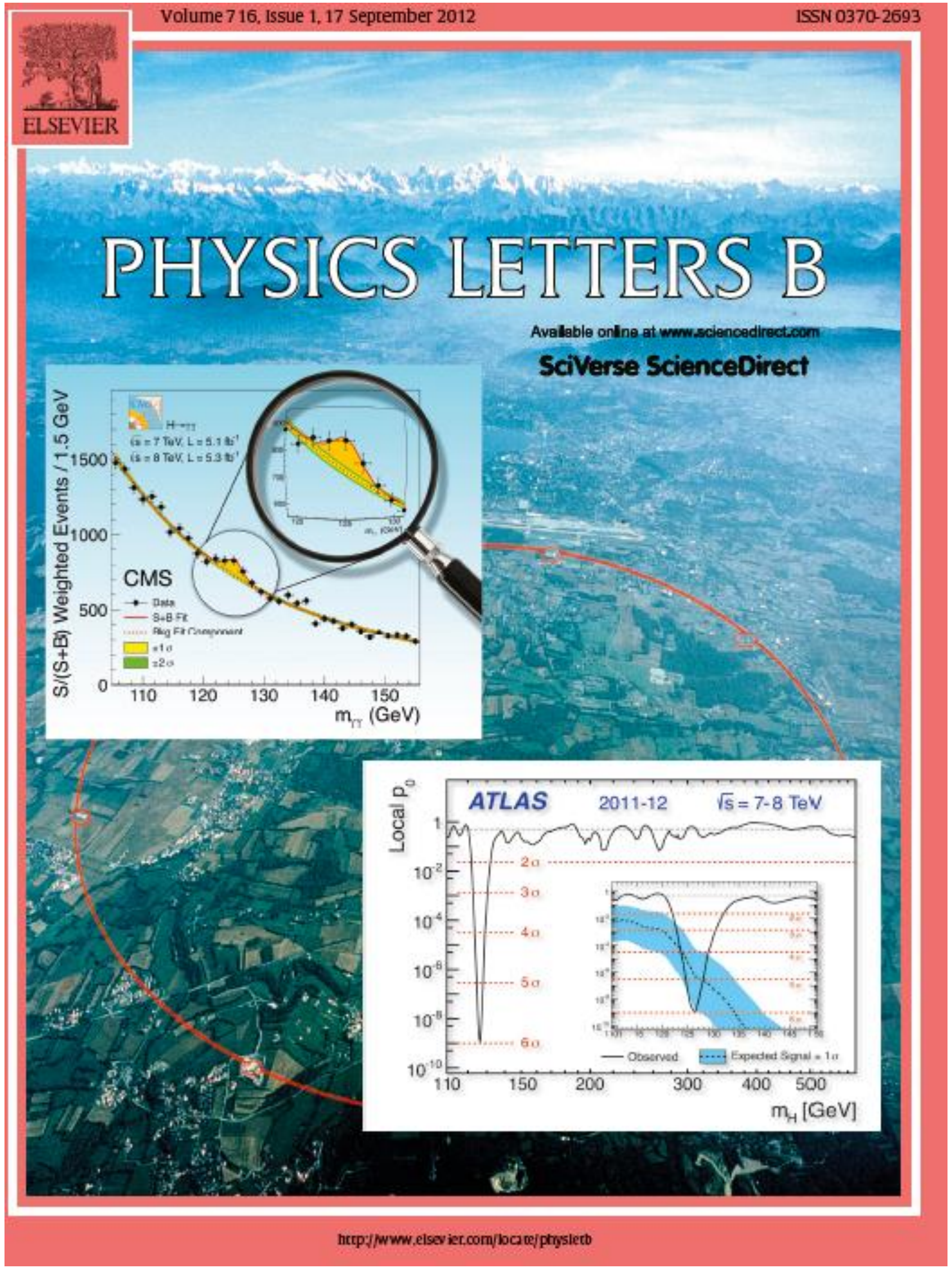
The discovery of a new boson, which may be the long sought after Higgs boson, is reported In two papers pubished in Physics Letters B.

ATLAS Collaboration:

Physics Letters B716 (2012) 1—29

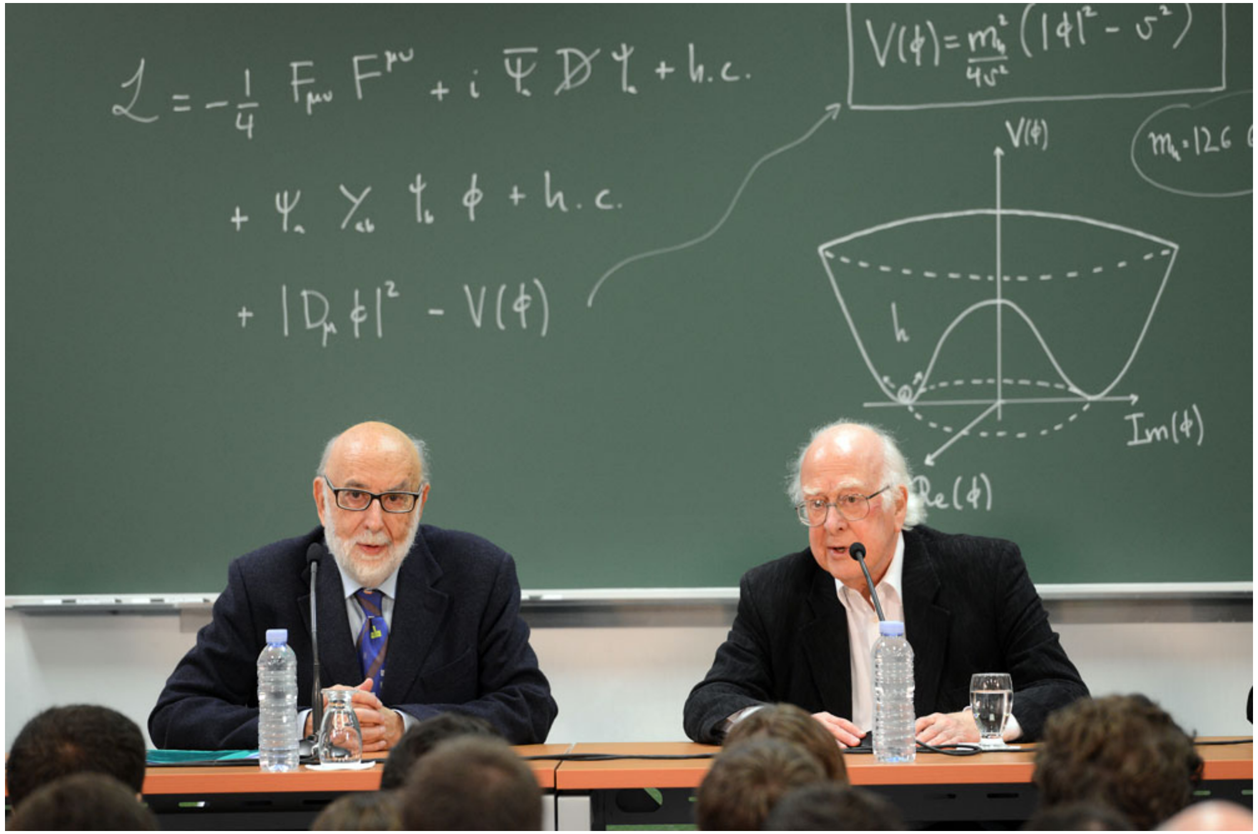
CMS Collaboration:

Physics Letters B716 (2012) 30—61




We found it! We found the Higgs boson!
SCIENCE

Winners of the 2013 Nobel Prize in Physics



François Englert

and

Peter Higgs

Higgs production at hadron colliders

At hadron colliders, the relevant processes are

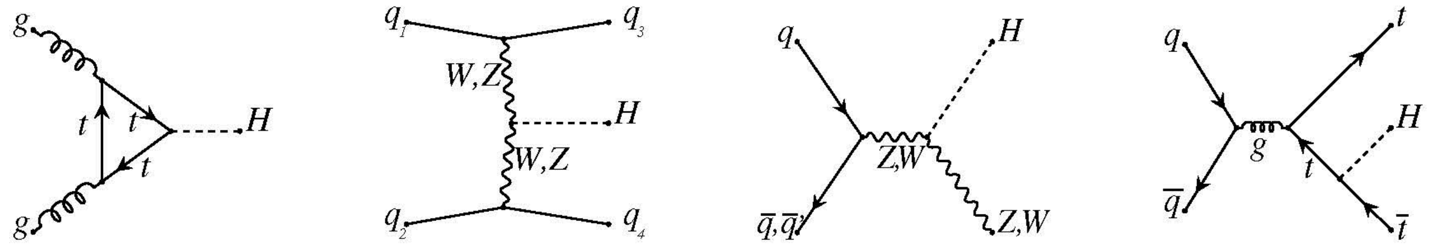
$$gg \to h^0\,, \quad h^0 \to \gamma\gamma\,, VV^{(*)}\,,$$

$$qq \to qqV^{(*)}V^{(*)} \to qqh^0, \quad h^0 \to \gamma\gamma,\, \tau^+\tau^-,\, VV^{(*)}\,,$$

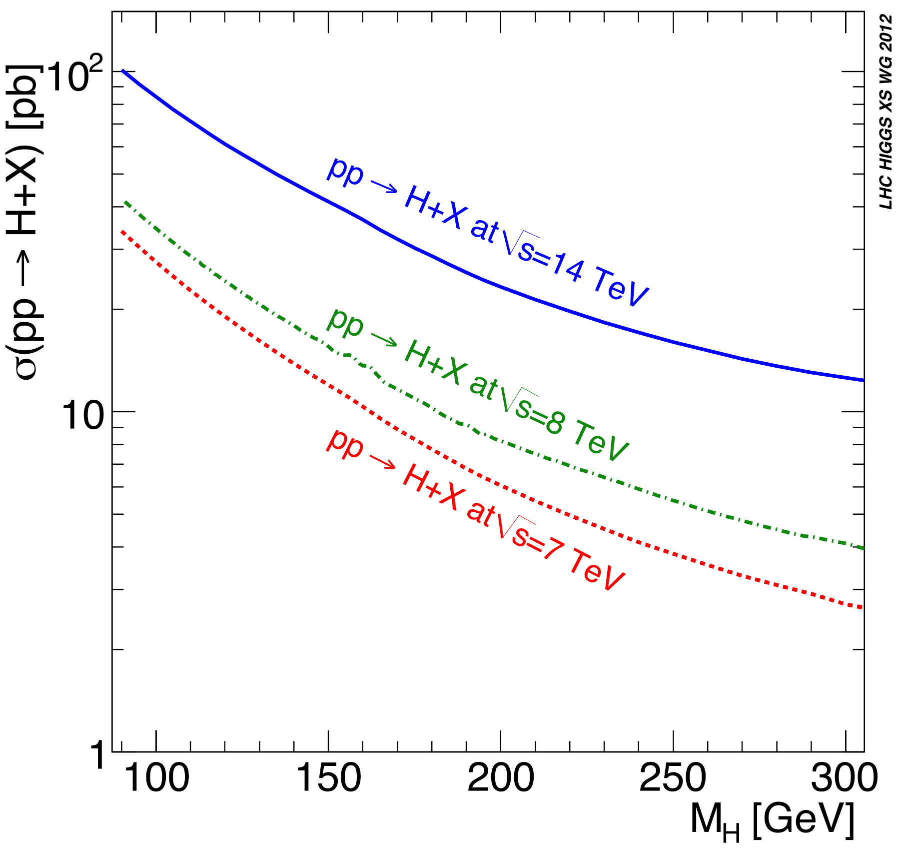
$$q\bar{q}^{(\prime)} \to V^{(*)} \to Vh^0\,, \quad h^0 \to b\bar{b}\,, WW^{(*)}\,,$$

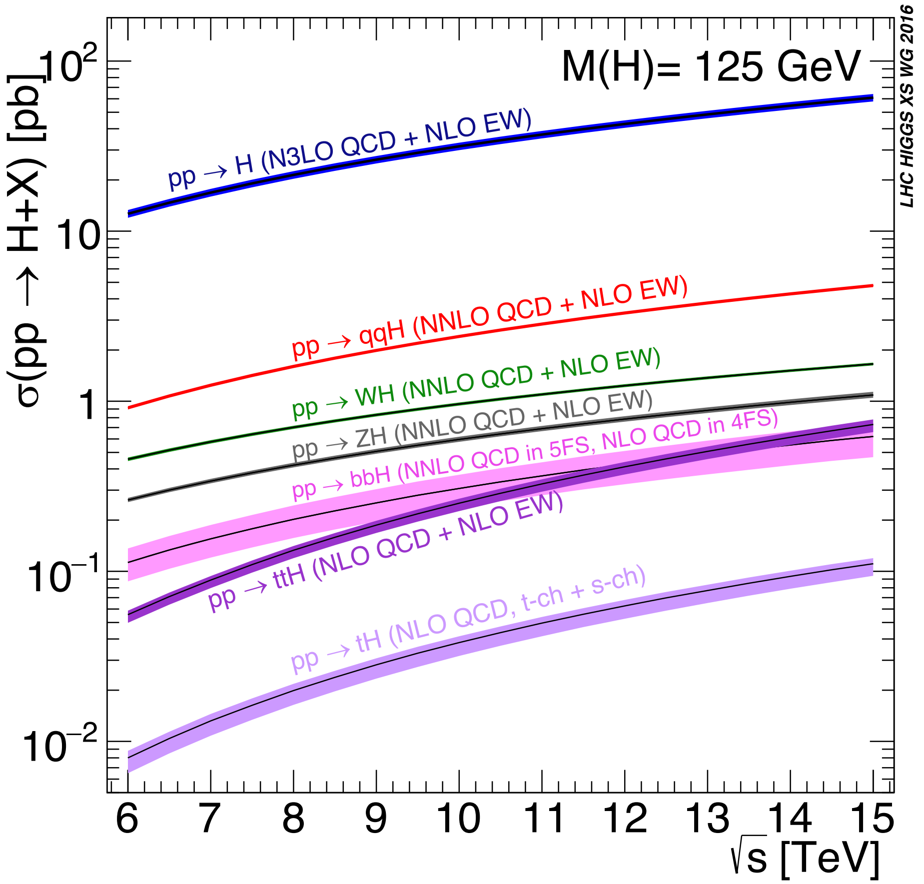
$$gg, q\bar{q} \to t\bar{t}h^0, \quad h^0 \to b\bar{b},\, \gamma\gamma,\, WW^{(*)}\,.$$

where $V = W$ or Z.



Higgs boson production cross sections at a pp collider

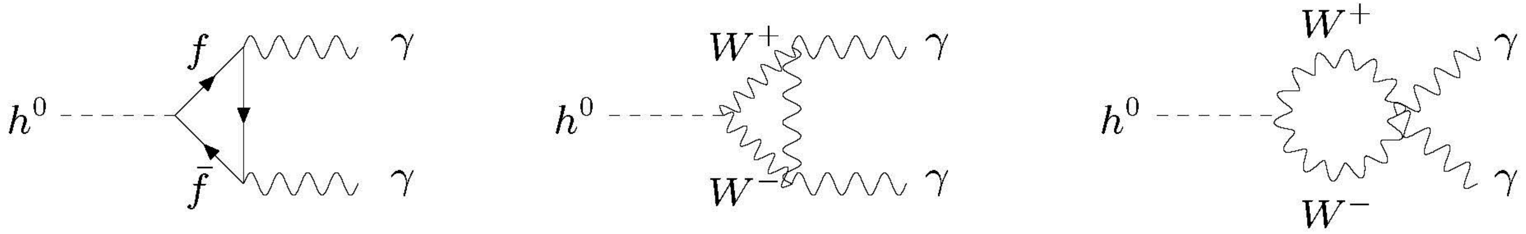




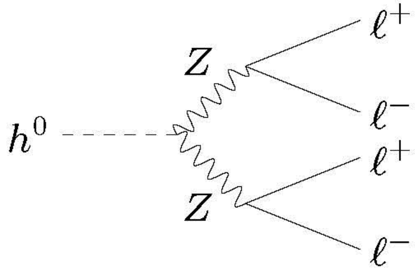
With 36 fb^{-1} of data delivered by the LHC to both ATLAS and CMS in 2015—2016 at a center of mass energy of 13 TeV, roughly 1.8 x 10^6 Higgs bosons per experiment were produced, assuming the Higgs mass is 125 GeV. Still to be analyzed: 50 fb^{-1} of 2017 data and at least another 50 fb^{-1} of data in 2018.

SM Higgs decays at the LHC for $m_h \sim 125$ GeV

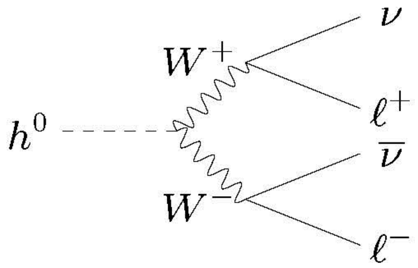
1. The rare decay $h^0 \to \gamma\gamma$ is the most promising signal.



2. The so-called golden channel, $h^0 \to ZZ \to \ell^+\ell^-\ell^+\ell^-$ (where one or both Z bosons are off-shell) is a rare decay for $m_h \sim 125$ GeV, but is nevertheless visible.



3. The channel, $h \to WW^* \to \ell^+\nu\ell^-\overline{\nu}$ is also useful, although it does not provide a good Higgs mass determination.



Higgs boson decay channels observed at the LHC

Higgs boson decay mode	Branching ratio (for m_h = 125 GeV)
$H^0 \rightarrow \mathbf{bb}$	0.582
$H^0 \rightarrow \boldsymbol{\tau}^+ \boldsymbol{\tau}^-$	6.27×10^{-2}
$h^0 \rightarrow \boldsymbol{\ell}^+ \boldsymbol{\ell}^- \boldsymbol{\nu\nu}$ ($\boldsymbol{\ell}$ = $\boldsymbol{e}$ or $\boldsymbol{\mu}$)	1.06×10^{-2}
$h^0 \rightarrow \boldsymbol{\gamma\gamma}$	2.27×10^{-3}
$h^0 \rightarrow \boldsymbol{\ell}^+ \boldsymbol{\ell}^- \boldsymbol{\ell}^+ \boldsymbol{\ell}^-$ ($\boldsymbol{\ell}$ = $\boldsymbol{e}$ or $\boldsymbol{\mu}$)	1.24×10^{-4}

Taken from https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR#Branching_Ratios

Remarks:

1. $h^0 \rightarrow WW^*$ is observed primarily via the $\boldsymbol{\ell}^+ \boldsymbol{\nu} \boldsymbol{\ell}^- \boldsymbol{\nu}$ ($\boldsymbol{\ell}$ = $\boldsymbol{e}$ or $\boldsymbol{\mu}$) final state.
2. $h^0 \rightarrow ZZ^*$ is observed primarily via the $\boldsymbol{\ell}^+\boldsymbol{\ell}^- \boldsymbol{\ell}^+\boldsymbol{\ell}^-$ ($\boldsymbol{\ell}$ = $\boldsymbol{e}$ or $\boldsymbol{\mu}$) final state.

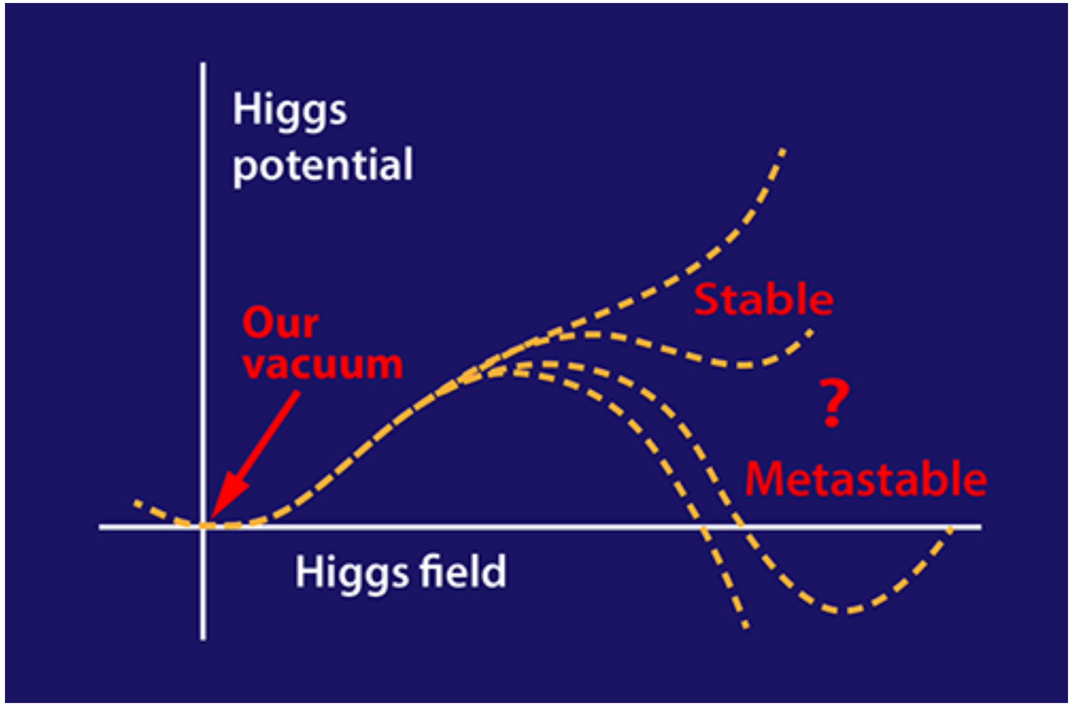
In the decays to the diboson final state, kinematics dictates that one of the vector bosons is off-shell (i.e., "virtual") and is thus indicated by a superscript star.

2012: Is the electroweak vacuum of the SM stable?

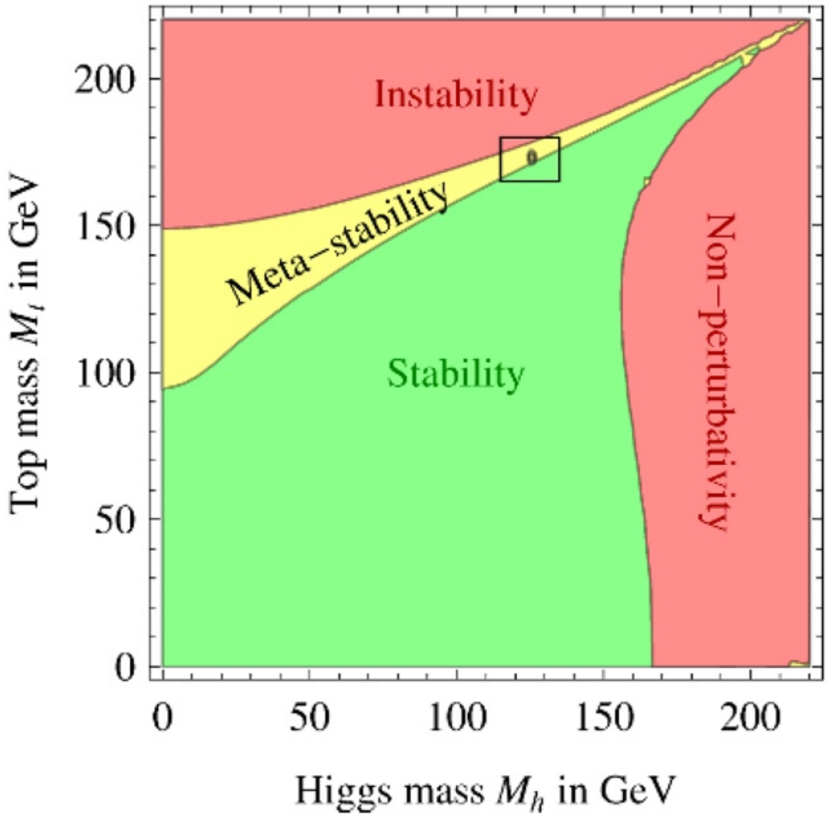
The Higgs field of the SM has a local minimum at $<\Phi>=246$ GeV. However, it is possible that a second minimum develops at very large field values. For field values larger than the Planck scale, $M_{PL} = 10^{19}$ GeV (in units of $c=1$), calculations within the SM are not reliable, as gravitational effects can no longer be neglected.

However, below M_{PL} one can reliably compute the shape of the scalar potential to determine whether our vacuum is stable.

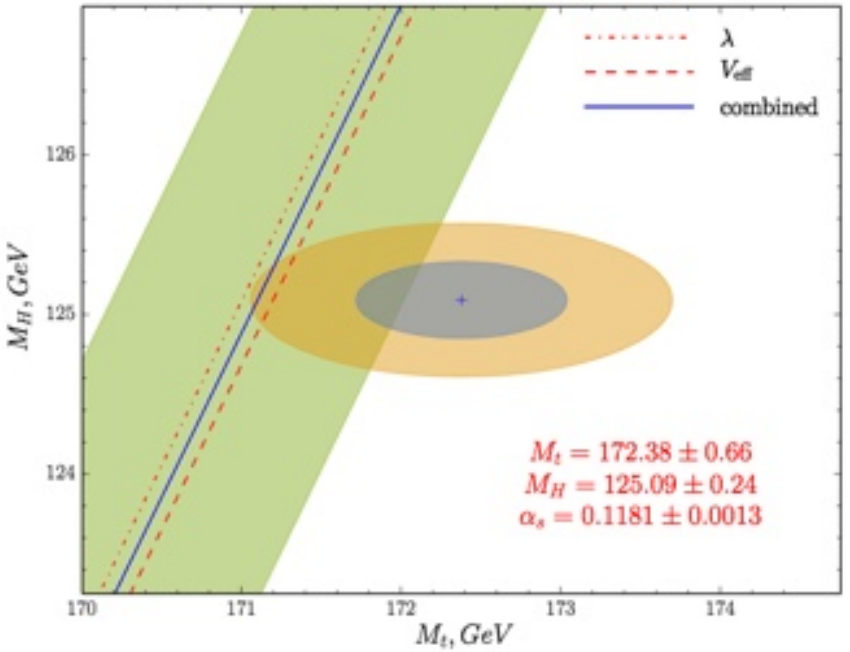
(figure courtesy of A. Kusenko)



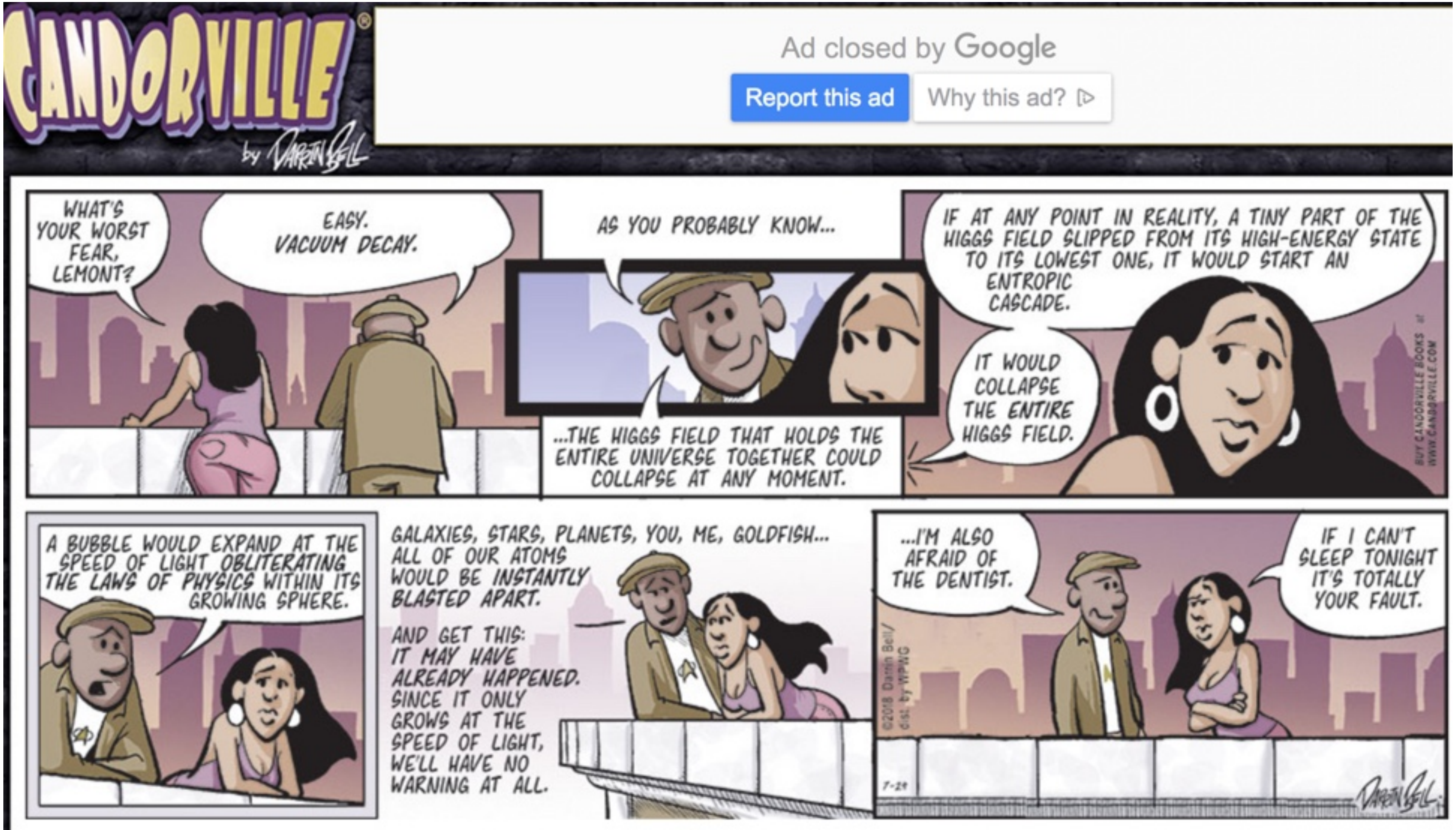
Detailed calculations by G.Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori and A. Strumia (2012)—see figure below on the left, and a more recent treatment by A.V. Bednyakov, B.A. Kniehl, A.F. Pikelner and O.L. Veretin (2015)—see figure below on the right, suggest that the electroweak vacuum is metastable, with a lower secondary minimum below M_{PL} .



However, for a slightly lower value of m_t (compared to the central PDG value), stability up to M_{PL} is recovered.



The popular press has taken notice …



2013: Achieving a SM-like Higgs boson in extended Higgs sectors—the alignment limit

Consider an extended Higgs sector with at n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i. After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vevs (in order to preserve U$(1)_{\rm EM}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$.

We define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^*\Phi_i\,, \qquad\qquad \langle H_1^0\rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations such that $\langle H_i^0\rangle = 0$.

That is H_1^0 is aligned with the direction of the Higgs vev in field space. In the exact alignment limit, $H \equiv \sqrt{2}\,\mathrm{Re}(H_1^0) - v$, whose tree-level properties coincide with the SM Higgs boson, is a mass-eigenstate.

In general, H is not a mass-eigenstate due to mixing with other neutral scalars. In this case, the observed Higgs boson is SM-like if either*†

- the mixing of H with other neutral scalars is suppressed,

and/or

- the diagonal squared masses of the other scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

---

*Although the alignment limit is most naturally achieved in the decoupling regime, it is possible to have a SM-like Higgs boson without decoupling. In the latter case, the masses of the additional scalar states could lie below ~ 500 GeV and be accessible to LHC searches.

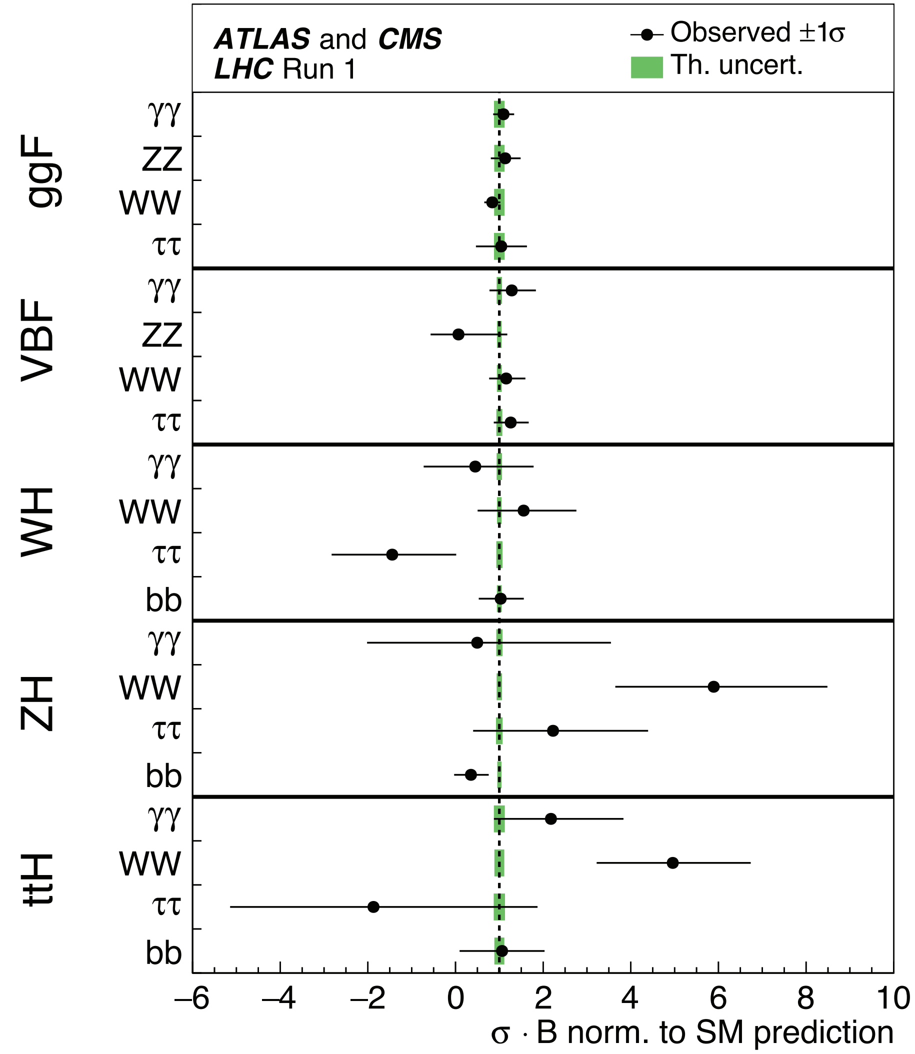
†For further details, see N. Craig, J. Galloway and S. Thomas, *Searching for Signs of the Second Higgs Doublet*, arXiv:1305.2424 [hep-ph]; D.M. Asner *et al.*, *ILC Higgs White Paper*, arXiv:1310.0763 [hep-ph]; J. Bernon, J.F. Gunion, H.E. Haber, Y. Jiang and S. Kraml, *Scrutinizing the alignment limit in two-Higgs-doublet models: $m_h = 125$ GeV*, Phys. Rev. D **92**, 075004 (2015).

2015: Is the observed 125 GeV scalar the SM Higgs boson?

After the end of Run-1 of the LHC (2011—2013), the ATLAS and CMS Collaborations provided a combined analysis of the Higgs boson data.

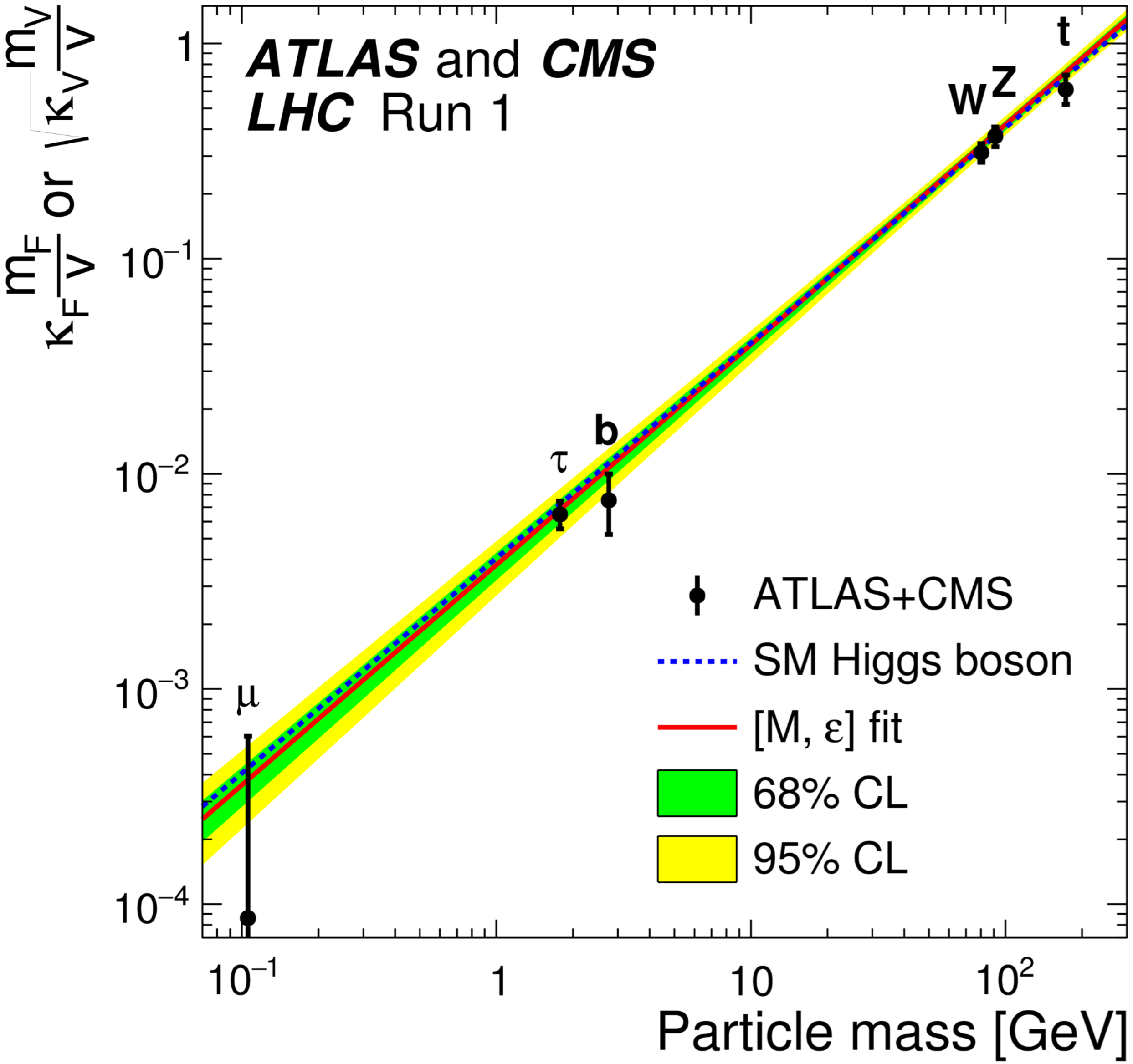
The properties of the Higgs boson are consistent with Standard Model predictions (given the statistical power of the Higgs boson data).

The Higgs data taken at Run-2 of the LHC (2015—2017) have confirmed the Run-1 observations (with potential deviations from the SM further reduced).



Taken from G. Aad et al. [ATLAS, CMS Collaborations], Phys. Rev. Letters **114**, 191803 (2015).

Experimental evidence that Higgs couplings scale with the mass of the particle



2018: Quo Vadis Higgs?

- Do the Higgs properties deviate from those of the SM Higgs boson?

- Are there additional Higgs scalars beyond the SM Higgs boson?
  - Keep in mind that the fermion and gauge boson sectors of the SM are far from being of minimal form. So why shouldn't the the scalar sector be non-minimal as well?

- Are the dynamics of electroweak symmetry breaking natural?
  - Does supersymmetry exist at the TeV scale?
  - Is the any evidence that the Higgs boson is composite?

- The operator $\Phi^\dagger\Phi$ is an electroweak singlet, and thus can be a portal to new physics beyond the SM (BSM). Is such BSM physics accessible at the LHC or at future collider facilities?

Backup slides

Particle content of the Standard Model

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c²	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

Something is missing...

What was missing?

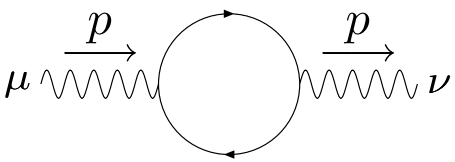
The theory of $W^{\pm}$ and Z gauge bosons must be *gauge invariant*; otherwise the theory is mathematically inconsistent. You may have heard that "gauge invariance implies that the gauge boson mass must be zero," since a mass term of the form $m^2 A^a_\mu A^{\mu a}$ is not gauge invariant.

So, what is the origin of the $W^{\pm}$ and Z boson masses? Gauge bosons are massless at tree-level, but perhaps a mass may be generated when quantum corrections are included. The tree-level gauge boson propagator $G^0_{\mu\nu}$ (in the Landau gauge) is:

$$G^0_{\mu\nu}(p) = \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) .$$

The pole at $p^2 = 0$ indicates that the tree-level gauge boson mass is zero. Let's now include the radiative corrections.

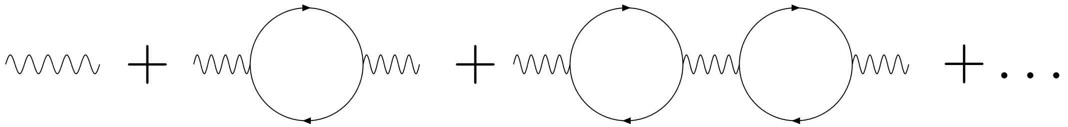
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by covariance with respect to Lorentz transformations, and is constrained by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

The renormalized propagator is the sum of a geometric series



$$= \frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2[1+\Pi(p^2)]}$$

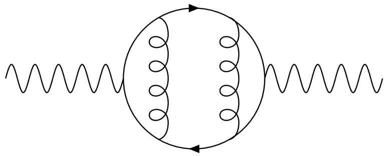
The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2 \to 0}{\simeq} \frac{-g^2 v^2}{p^2}.$$

Then $p^2[1 + \Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Interpretation of the $p^2 = 0$ pole of $\Pi(p^2)$

The pole at $p^2 = 0$ corresponds to a propagating massless scalar. For example, the sum over intermediate states includes a quark-antiquark pair with many gluon exchanges, e.g.,



This is a strongly-interacting system—it is possible that one of the contributing intermediate states is a massless spin-0 state (due to the strong binding of the quark/antiquark pair).

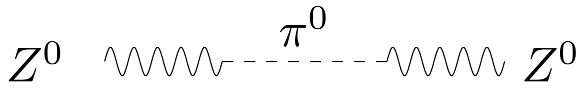
We know that the Z and $W^{\pm}$ couple to neutral and charged weak currents

$$\mathcal{L}_{\rm int} = -g_Z j^Z_\mu Z^\mu - g_W (j^W_\mu W^{+\mu} + {\rm h.c.}) \,,$$

which are known to create neutral and charged pions from the vacuum, e.g.,

$$\langle 0 | j^Z_\mu(0) | \pi^0 \rangle = i f_\pi p_\mu \,.$$

Here, $f_\pi = 93$ MeV is the amplitude for creating a pion from the vacuum. In the absence of quark masses, the pions are massless bound states of $q\bar{q}$ [they are Goldstone bosons of chiral symmetry which is spontaneously broken by the strong interactions]. Thus, the diagram:



yields the leading contribution as $p^2 \to 0$ [shown in red] to the $p_\mu p_\nu$ of $\Pi_{\mu\nu}$,

$$i\Pi_{\mu\nu}(p) = i g_Z^2 f_\pi^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) .$$

Remarkably, the latter is enough to fix the corresponding $g_{\mu\nu}$ part of $\Pi_{\mu\nu}$ [thank you, Lorentz invariance and gauge invariance!]. It immediately follows that

$$\Pi(p^2) = -\frac{g_Z^2 f_\pi^2}{p^2} ,$$

and therefore $m_Z = g_Z f_\pi$. Similarly $m_W = g_W f_\pi$.

Gauge boson mass generation and the Goldstone boson

We have demonstrated a mass generation mechanism for gauge bosons that is both Lorentz-invariant and gauge-invariant! This is the essence of the *Higgs mechanism*. The $p^2 = 0$ pole of $\Pi(p^2)$ corresponds to a propagating massless scalar state called the Goldstone boson. We showed that the W and Z are massive in the Standard Model (without Higgs bosons!!). Moreover, the ratio

$$\frac{m_W}{m_Z} = \frac{g_W}{g_Z} \equiv \cos\theta_W \simeq 0.88$$

is remarkably close to the measured ratio. Unfortunately, since $g_Z \simeq 0.37$ we find $m_Z = g_Z f_\pi = 35$ MeV, which is too small by a factor of 2600.

There must be another source for the gauge boson masses, i.e. new fundamental dynamics that generates the Goldstone bosons that are the main sources of mass for the $W^\pm$ and Z.