Gravitational Radiation from pp Collider

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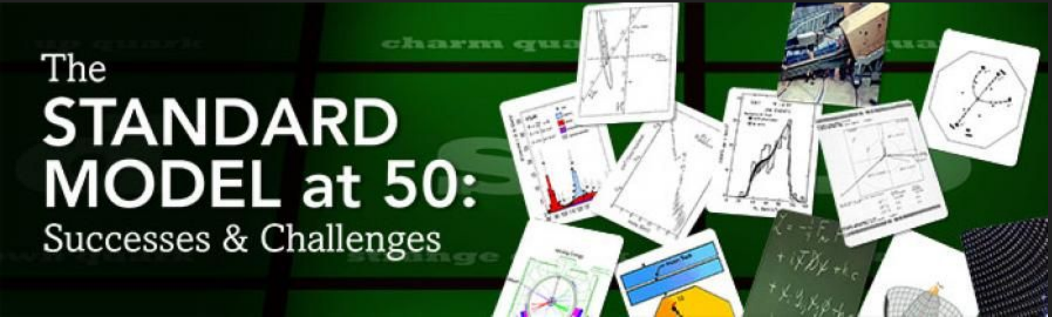


SLAC NATIONAL ACCELERATOR LABORATORY

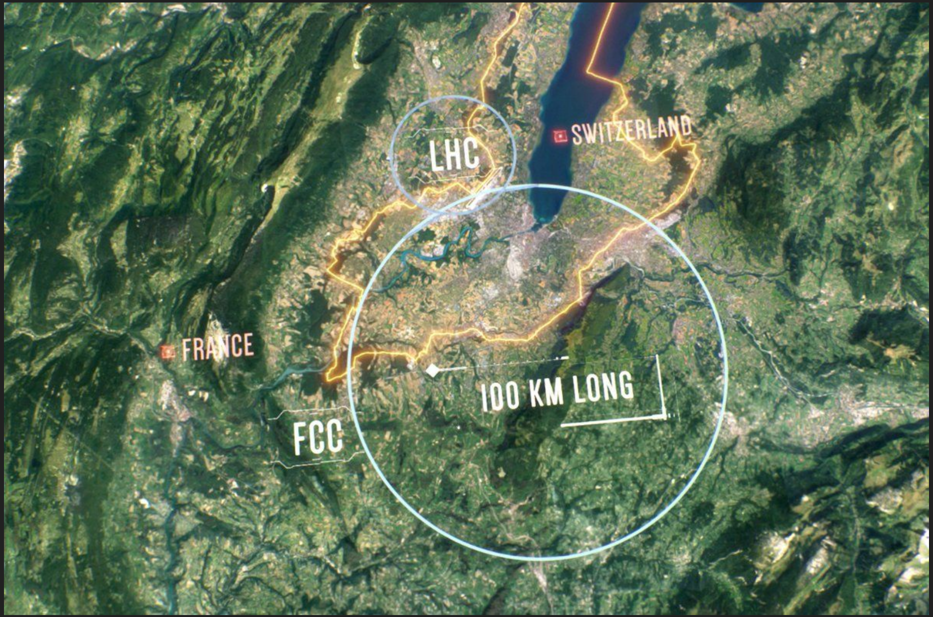
“**LIGO** has observed gravitational radiation from a ~20 solar mass black hole merger at a distance of over ~10^9 light years.

Circulating **proton beams** in an accelerator can also produce gravitational radiation at higher frequency. To be specific, consider a **100 TeV pp collider** with a luminosity of 5×10^{35} cm^2 s^{-1} (and with the other parameters as for the FCC-hh).

Assuming they are coherently produced, determine if a gravitational wave detector with 10x the LIGO sensitivity in the relevant frequency range could detect this radiation at a distance of 10^3 km; make any suitably justifiable approximations.”

What do you think?

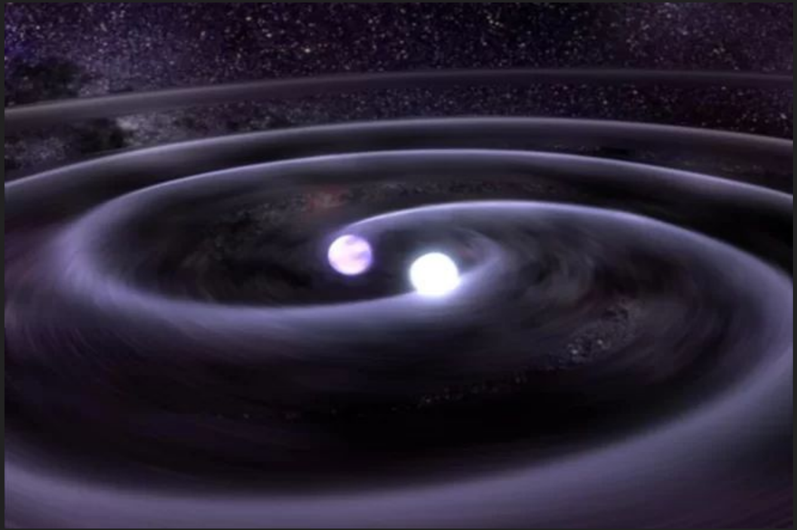
Our scenario: Future Circular Collider (FCC)



- $E = 100$ TeV
- $C = 100$ km
- Masses → proton bunches: $M_{bunch} = N * \gamma * m_p = 1.78 \times 10^{-11}$ kg

Is it straightforward?

From here...



...to here

Electromagnetic-like approach

Analogy between E&M and Newtonian Gravity

$$V_{coulomb} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$V_{gravi} = -G\frac{m}{r}$$

$\hbar, c = 1$

$$\frac{1}{4\pi\epsilon_0} \leftrightarrow -G$$

$$q \leftrightarrow m$$

Vector potential

$$\vec{A}_{EM} = \frac{1}{4\pi\epsilon_0}\frac{q\vec{v}}{r} \rightarrow \vec{A}_g = -G\frac{m\vec{v}}{r}$$

Newtonian Gravitational Wave

$$\vec{A}_G(\vec{R},t) = -\frac{G}{c^2 R}[m_a\vec{v}_a(t_R) + m_b\vec{v}_b(t_R)] + \frac{G\omega^2}{c^3 R}\left[m_a\vec{r}_a(\hat{R}\cdot\vec{r}_a) + m_b\vec{r}_b(\hat{R}\cdot\vec{r}_b)\right]$$

no dipole contribution — quadrapole

$$\vec{g}_{grav} = -\frac{\partial \vec{A}_G}{\partial t} \sim \frac{G\tilde{M}w^3r^2}{R}[angular + oscillation]$$

$$\tilde{M} = \frac{m_a m_b}{m_a + m_b}$$

Coulumb Gauge

let's ignore sines and cosines

$$x(t) \sim \frac{|\vec{g}_{grav}|}{w^2} \sim \frac{G\tilde{M}wr^2}{R}[angular + oscillation]$$

$$\vec{g}_{grav} \sim \ddot{x}(t)$$

$$\Delta L = \Delta x_2(t) - \Delta x_1(t) \sim \frac{G\tilde{M}wr^2}{R}w\Delta t$$

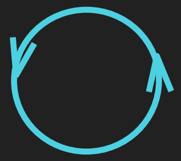
$$\Delta t \sim L$$

Newtonian Gravitational Wave

some math.. then

Strain Signal

$$h = \frac{\Delta L}{L} \sim \frac{G(m_{\odot,p}/2)w^2r^2L}{R}$$



for orbiting two masses

Neutron Star Merger

w ~ 10^3 Hz
r = 10 km
R = 40 Mpc

$$h \sim 10^{-23}$$

FCC-hh two protons

w ~ 10^4 Hz
r = 100 km
R = 1000 km
# of proton = 10^{16}

$$h \sim 10^{-44}$$

Gravitational waves from General Relativity

- take Einstein's equation
- solve for the tensor $h_{\mu\nu}$
- choose the *transverse traceless gauge*

...and you find the **shear strain!**

$$h_+ = \frac{2G}{rc^4}\frac{\partial^2 I^{TT}_{\theta\theta}}{\partial^2 t}$$

$$\frac{\partial^2 \mathcal{I}_{\theta\theta}}{\partial^2 t} = \frac{\partial^2 \mathcal{I}_{xx}}{\partial^2 t}\cos^2\theta$$

$$\mathcal{I}_{xx} = m_a x_a^2 + m_b x_b^2$$

...and how to produce them at FCC - I

There are a few ways that gravitational waves could be produced at the FCC:

1. Two circulating bunches of protons (Analogy with BH Merger scenario):

$$h_+ = \frac{32 m \pi^2 f^2 r_{\mathrm{FCC}}^2 G}{r c^4} \cos^2(4\pi f t)$$

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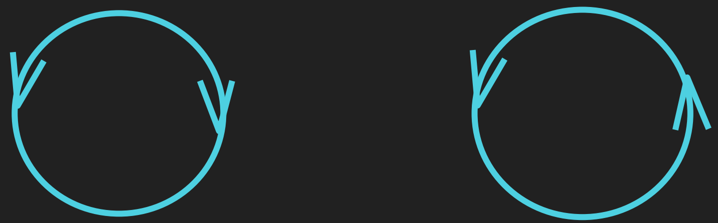

$$h_+ = \frac{32 m \pi^2 f^2 r_{\mathrm{FCC}}^2 G}{r c^4} \cos^2(4\pi f t)$$

$\mathscr{A}(h_+) = 5.295 \times 10^{-44}$

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$\mathscr{A}(h_+) = 5.295 \times 10^{-44}$

$\mathscr{A}(h_+) = 7.857 \times 10^{-20}$
(Black Hole Case)

...and how to produce them at FCC - II

2. pp collisions:

$$h_+ = \frac{8mG}{rc^4}\left(v^2 + vt\frac{dv}{dt}\right)$$

!note:

- ~70 protons out of 10^{11} actually collide at LHC → assuming non-colliding bunches

$$\mathscr{A}(h_+) = 5.287 \times 10^{-44}$$

Detection: LIGO

Largest **gravitational waves observatory**

Twin 4 km-arms **interferometers**

$$\frac{\Delta L}{L} \sim h$$

All you want to know about LIGO already marvellously explained in ***Dr. Jess McIver Lecture 'Gravitational Waves and Multi-messenger Astronomy'!***

LIGO's sensitivity

LIGO sensitivity:

Photon shot noise:

$$h_{\text{min}} = \frac{1}{bL}\left(\frac{\hbar c\tilde{\lambda}}{\tau \mathcal{I}_0}\right)^{\frac{1}{2}}$$

Radiation pressure noise:

$$h_{\text{min}} = \frac{\tau b}{mL}\left(\frac{\tau \hbar \mathcal{I}_0}{c\tilde{\lambda}}\right)^{\frac{1}{2}}$$

Minimize wrt laser power

Optimal limit →

$$h_{\text{min}} = \frac{1}{L}\left(\frac{\tau\hbar}{m}\right)^{\frac{1}{2}} = 10^{-23}$$

L= 4 km

τ = 1 ms

m_{mirror} = 100 kg

Can a **10x** sensitive LIGO detect the GW from pp collider?

$$\mathscr{A}(h_{min}) = 10^{-24}$$

EM-like estimate

$$\mathscr{A}(h_{+}) \sim 10^{-44}$$

GR estimate

$$\mathscr{A}(h_{+}) = 5.295 \times 10^{-44}$$

Our conclusions

Gravitational waves produced by the FCC are, in all 3 scenarios, **20** orders of Magnitude away from being detected by an upgraded LIGO Detector. So… no hope!

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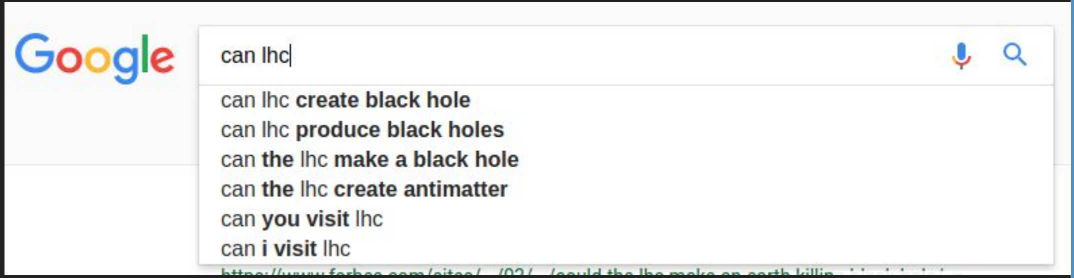
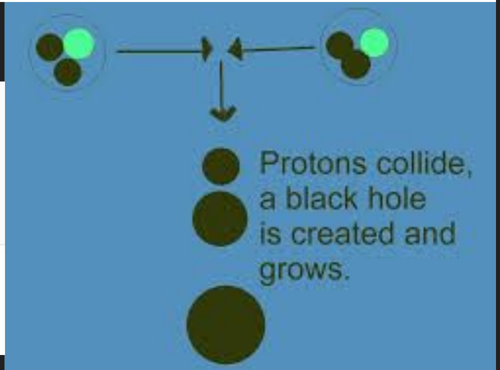
According to some media we might

have missed something, though...





References

- **LIGO Lab - Caltech & MIT**, https://www.ligo.caltech.edu
- **FCC-hh**, http://tlep.web.cern.ch/content/fcc-hh
- **Modern classical physics: optics, fluids, plasmas, elasticity, relativity, and statistical physics**, Kip S. Thorne and Roger D. Blandford
- **Advanced LIGO**, LIGO Scientific Collaboration, LIGO-P1400177-v5

Thanks to Jessica McIver for the fruitful discussions!

Additional material

EM-like approach

For two mass ma, mb - r apart, orbiting at w, R away from detector

$$\vec{A}_G = -\frac{Gm_a\vec{v}_a(t_{Ra})}{c^2 R_a} - \frac{Gm_b\vec{v}_b(t_{Rb})}{c^2 R_b}$$

$$\vec{A}_G(\vec{R},t) = -\frac{G}{c^2 R}[m_a\vec{v}_a(t_R) + m_b\vec{v}_b(t_R)] + \frac{G\omega^2}{c^3 R}\left[m_a\vec{r}_a(\hat{R}\cdot\vec{r}_a) + m_b\vec{r}_b(\hat{R}\cdot\vec{r}_b)\right]$$

$$\vec{g}_{grav} = -\frac{\partial \vec{A}_G}{\partial t} \sim \frac{G\tilde{M}w^3 r^2}{R}[angular + oscillation]$$

$$\tilde{M} = \frac{m_a m_b}{m_a + m_b}$$

arXiv:1710.04635

$$\vec{F} = q\vec{E} \leftrightarrow \vec{F} = m\vec{g}_{grav}$$

$$\vec{g}_{grav} \sim \ddot{x}(t)$$

$$x(t) \sim \frac{|\vec{g}_{grav}|}{w^2} \sim \frac{G\tilde{M}wr^2}{R}[angular + oscillation]$$

$$\Delta L = \Delta x_2(t) - \Delta x_1(t) \sim \frac{G\tilde{M}wr^2}{R}w\Delta t$$

$$\Delta t \sim L$$

GR approach

$$I_{xx} = m_A x_A^2 + m_B x_B^2, I_{yy} = m_A y_A^2 + m_B y_B^2$$

$$\ddot{I}_{\theta\theta} = \ddot{I}_{xx}\cos^2\theta, \ddot{I}_{\phi\phi} = \ddot{I}_{yy}$$

$$\ddot{I}_{\theta\theta}^{TT} = -\ddot{I}_{\phi\phi}^{TT} = \frac{1}{2}(\ddot{I}_{\theta\theta} - \ddot{I}_{\phi\phi})$$

$$h_+ = \frac{2G}{rc^4}\ddot{I}_{\theta\theta,retarded}^{TT}$$

$$\Box h_{\mu\nu} = 0$$

$$h_{\mu\nu}^{TT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\omega(z-t)}$$

Noises

- Photon shot noise:
  - precision → restricted by number of detected photons : prop. beam intensity
  - deltaL can be measured by measuring the number of emerging photons
  - statistical fluctuations proportional to sqrt(N)

$$\delta(\Delta L) \sim \frac{\tilde{\lambda}}{2b\sqrt{N_0}}$$

$$h_{\min} = \frac{\delta(\Delta L)}{L} = \frac{\Delta L}{L} \sim \frac{\tilde{\lambda}}{bLN_0^{1/2}} \sim \frac{1}{bL}\left(\frac{\hbar c\tilde{\lambda}}{\tau I_0}\right)^{1/2}$$

I0 = ~5-10 W

- Radiation pressure:
  - uncertainty on the momentum deposited on the mirrors → unc. in position