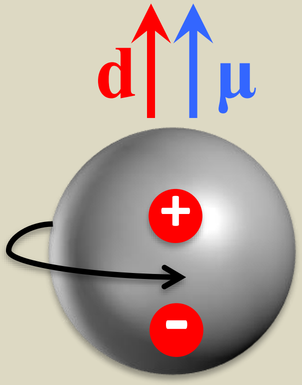
Neutron Electric Dipole Moment



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- Boram Yoon

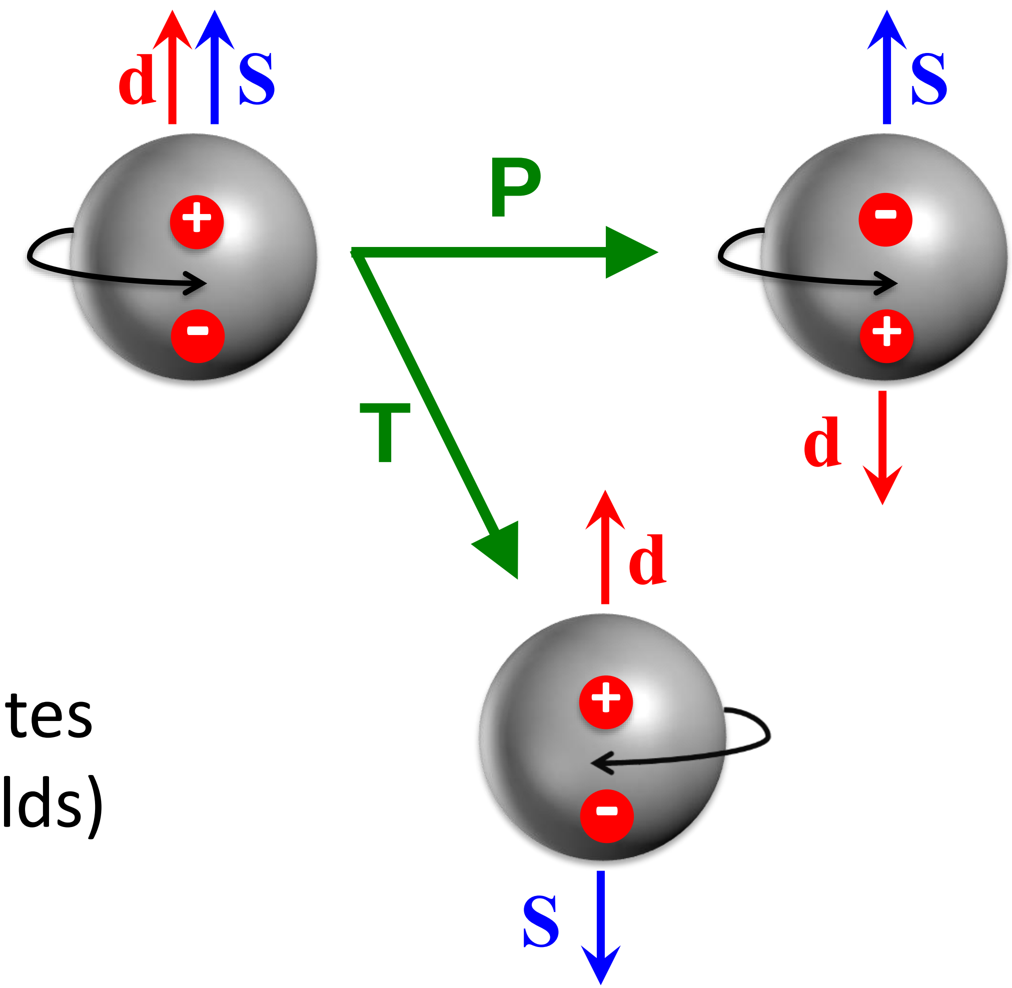
Neutron EDM and CP Violation

- Measures separation between centers of (+) and (-) charges

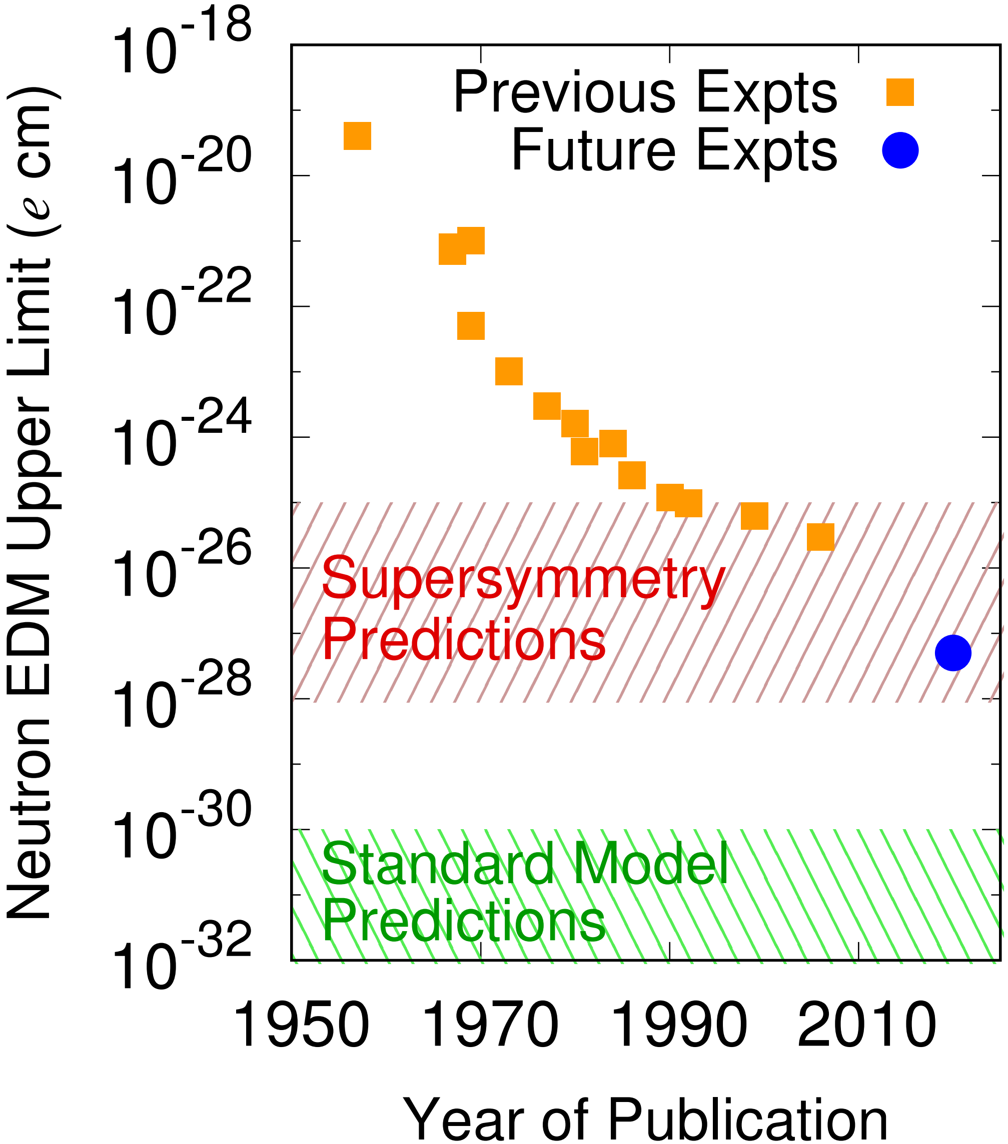
$$\delta H = d_N \hat{S} \cdot \vec{\mathcal{E}}$$

- Current bound:
  $|d_n| < 2.9 \times 10^{-26}\ e \cdot \mathrm{cm}$

- Nonzero nEDM violates P and T (CP if CPT holds)



Neutron EDM Searches



- Predictions
  - Standard Model
    $|d_n| \sim 10^{-31}\ e{\cdot}\mathrm{cm}$
  - Supersymmetry
    $|d_n| \sim 10^{-25} - 10^{-28}\ e{\cdot}\mathrm{cm}$
- Experiments targeting $5 \times 10^{-28}\ e{\cdot}\mathrm{cm}$ precision
  - PSI EDM
  - Munich FRMII
  - RCNP/TRIUMF
  - SNS nEDM
  - JPARC

Impacts

- New source of CP violation
  - CPV in SM is not sufficient to explain observed baryon asymmetry

- Test of Supersymmetry and other BSM models
  - In many BSM theories, nEDM is predicted to be in the range 10^{-26}–10^{-28} $e\cdot$cm

Effective Lagrangian at 2 GeV

$$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\bar{\theta} G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma \cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma \cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+d_w \frac{g_s}{6} G\tilde{G}G$$ dim=6 Weinberg 3g operator

$$+\sum_i C_i^{(4q)} O_i^{(4q)}$$ dim=6 Four-quark operators

- $\bar{q} \pounds O(10^{-9} - 10^{-11})$: Strong CP problem
- Effectively dim=5 suppressed as $d_q \approx v/\Lambda_{\mathrm{BSM}}^2$;
- Dim=6 terms
- Lattice QCD: calculate their ME within nucleon state

Spinor transformation under Parity

	P, CP-even	P, CP-violating
Dirac Eq.	$\left(ip_\mu \gamma_\mu + m\right)u = 0$	$\left(ip_\mu \gamma_\mu + me^{-2i\alpha\gamma_5}\right)\tilde{u} = 0$
Parity Op.	$\boxed{\gamma_4}$ $u_{\vec{p}} \to \gamma_4 u_{-\vec{p}}$	$\boxed{e^{2i\alpha\gamma_5}\gamma_4}$ $\tilde{u}_{\vec{p}} \to e^{2i\alpha\gamma_5}\gamma_4 \tilde{u}_{-\vec{p}}$

- CPV interactions ➔ phase in neutron mass term

  γ_4 no longer parity op of neutron state

- Introduce new parity operator or
- Rotate neutron state so that γ_4 remains the parity op:

$$\boxed{\tilde{u} = e^{i\alpha\gamma_5}u, \qquad \bar{\tilde{u}} = \bar{u}e^{i\alpha\gamma_5}}$$

Abramczyk, et al., PRD96 (2017) 014501

F_3: The CP Violating Form Factor

Expand the matrix element in terms of Lorentz covariant form factors

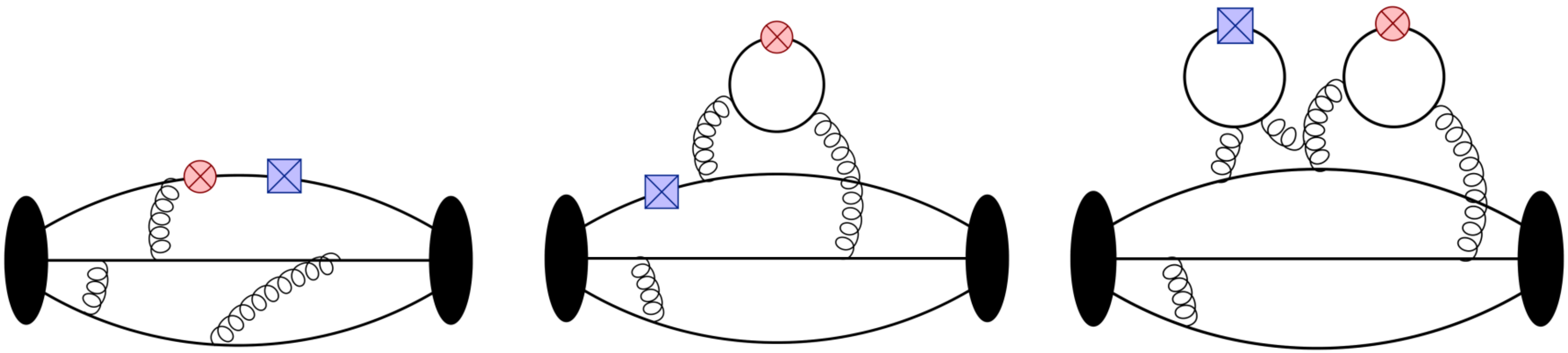
$$\langle N|J_\mu^{EM}|N\rangle_{CPV} = e^{i\alpha(q^2)\gamma_5}\,\bar{u}\,[\gamma_\mu F_1(q^2) + \left(2im_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2\right)\frac{F_A(q^2)}{m_N^2}$$
$$+i\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2m_N} + \sigma_{\mu\nu}\mathrm{q}_\nu\gamma_5\frac{F_3(q^2)}{2m_N}]\,u\,e^{i\alpha(q^2)\gamma_5}$$

With $\sum_s u(p,s)\bar{u}(p,s) = (E\gamma_4 - ip\cdot\gamma + m)/2E$

The contribution of each CPV operator to nEDM is given by

$$d_N = \frac{F_3(q^2=0)}{2m_N}$$

Two equally important challenges for lattice QCD



- **Signal in the CP violating form factor F_3 is small**
  - **Need very high statistics**
- **Renormalization and divergent mixing between operators**
  - **Needs non-perturbative calculation of mixing coefficients in order to obtain results that are finite in the continuum limit**

QCD θ-term

$$-\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}$$

QCD θ-term

- Calculate d_N in presence of CP violating θ-term

$$S = S_{QCD} + S_\theta$$

$$S_\theta = -i\theta \int d^4x\, G\tilde{G} / 32\pi^2 = -i\theta\, Q_{\mathrm{top}}$$

- Lattice calculation strategies
  - Expansion in θ
  - External electric field method
  - Simulations with imaginary θ

Expansion in θ

$$\langle \mathcal{O}(x)\rangle_\theta = \frac{1}{Z_\theta}\int d[U,q,\bar{q}]\,\mathcal{O}(x)\,e^{-S_{QCD}+\boxed{i\theta Q_{\rm top}}}$$

$$= \langle \mathcal{O}(x)\rangle_{\theta=0} + i\theta\langle \mathcal{O}(x) Q_{\rm top}\rangle_{\theta=0} + O\left(\theta^2\right)$$

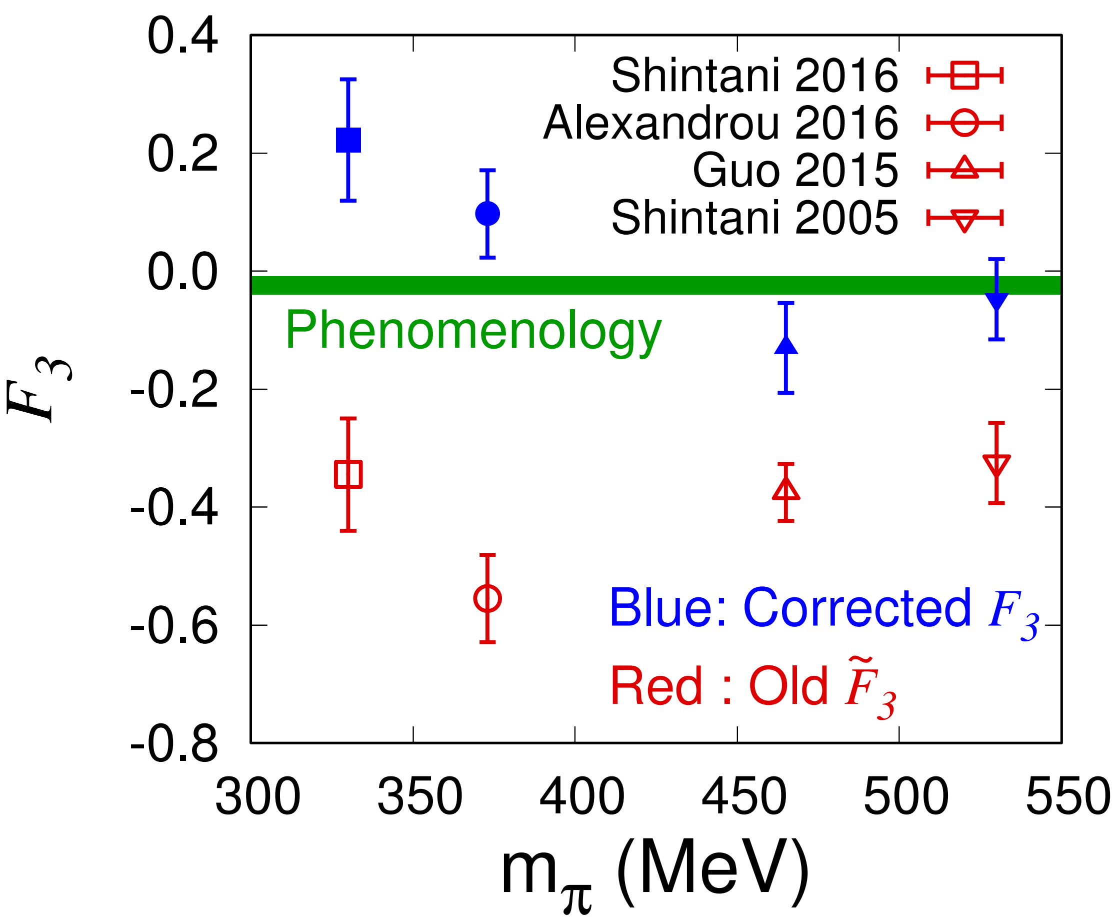
- *O(x)*= $\langle N(\tau)V_\mu(t)N(0)\rangle$ nucleon 3-pt fn with insertion of $V_\mu(t)$
- Nucleon interpolating operator $N = \epsilon^{abc}\left(d^{Ta}C\gamma_5 u^b\right)d^c$
- $\langle O(x)Q_{top}\rangle$ "reweights" the nucleon 3-point fn *O(x)* by Q_{top}
- Measurements performed on regular (θ=0) lattices
- $d_n = \theta F_3(q^2=0)/2M_N$

Form Factors with Parity Mixing

Abramczyk, et al., PRD96 (2017) 014501

- Otherwise Phase $e^{i\alpha\gamma_5}$
  mixes F_2 and F_3

$$\boxed{\begin{aligned} F_2 &= \cos(2\alpha)\tilde{F}_2 - \sin(2\alpha)\tilde{F}_3 \\ F_3 &= \sin(2\alpha)\tilde{F}_2 + \cos(2\alpha)\tilde{F}_3 \end{aligned}}$$



- Corrections calculated with assumptions & approximations
- Corrected lattice data consistent with zero
- May resolve tension between phenomenology and lattice results

External Electric Field Method

- In the presence of uniform electric field $\vec{\mathcal{E}}$, a change of energy for the nucleon state due to the θ-term is

$$\boxed{E^{\theta}_{\vec{S}} - E^{\theta}_{-\vec{S}} \approx 2 d_N \theta\, \vec{S}\cdot\vec{\mathcal{E}}}$$

- Neutron correlator with θ-term via reweighting

$$\left\langle N\bar{N} \right\rangle_{\theta} (\vec{\mathcal{E}}, t) = \left\langle N(t)\bar{N}(0)\, e^{i\theta Q_{\mathrm{top}}} \right\rangle_{\vec{\mathcal{E}}}$$

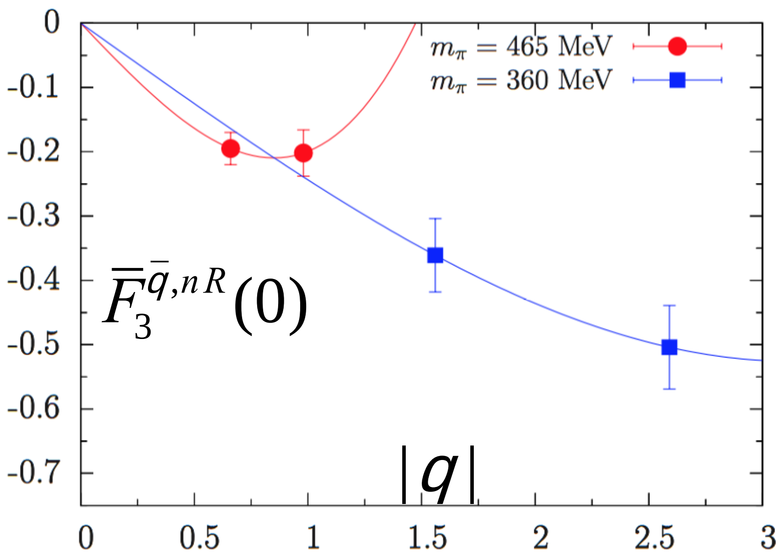
- Electric field applied only to valence quarks
- Does not need form-factor analysis nor extrapolation to q^2=0

Simulation with Imaginary θ

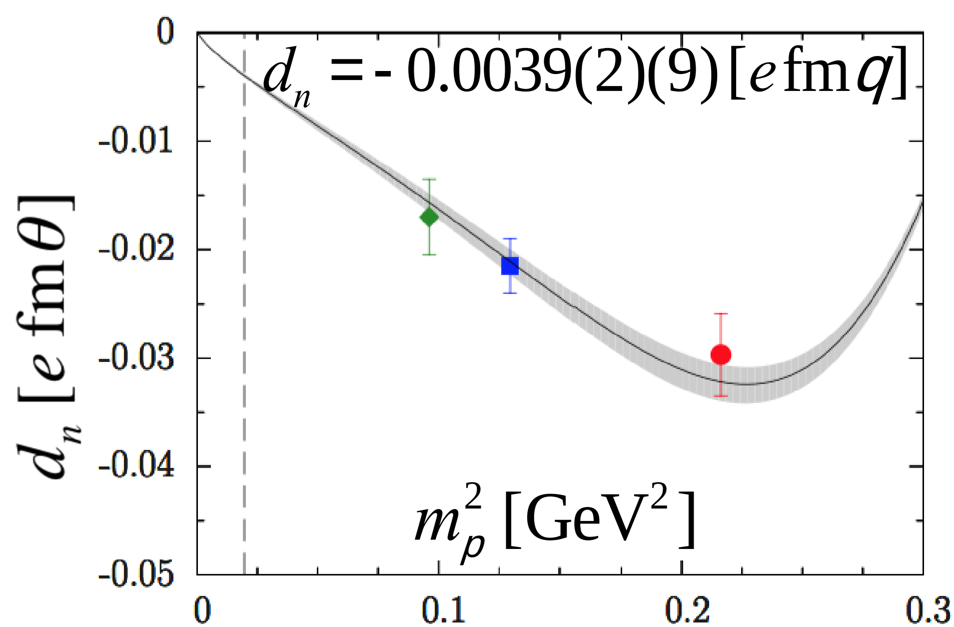
- Avoid imaginary action (sign problem) by

$$\boxed{\theta = i\tilde{\theta}} \qquad S_\theta^q = \tilde{\theta}\frac{m_l m_s}{2m_s + m_l}\sum_x \bar{q}\gamma_5 q$$

- Analytic continuation for small |θ|
- d_n is extracted from F_3



Guo, et al, PRL 115 (2015) 062001



Stout Link Nonperturbative Clover, a = 0.074 fm

Quark EDM

Leading contribution comes from the change in the vector current

$$j_\mu = \partial L/\partial A_\mu$$

Generates an additional piece in the vector current

$$-\frac{i}{2}\sum_{q=u,d,s} d_q\,\bar{q}(\sigma\cdot F)\gamma_5 q$$

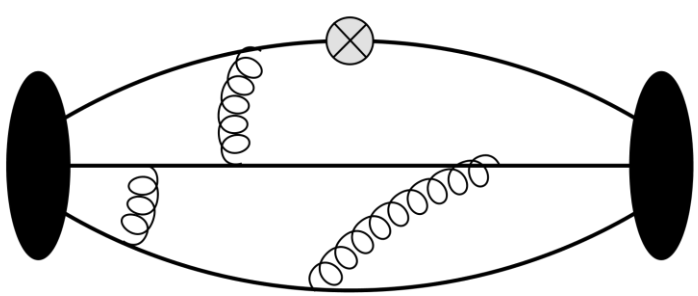
Quark EDM $d_q \bar{q}(\sigma \cdot F)\gamma_5 q$

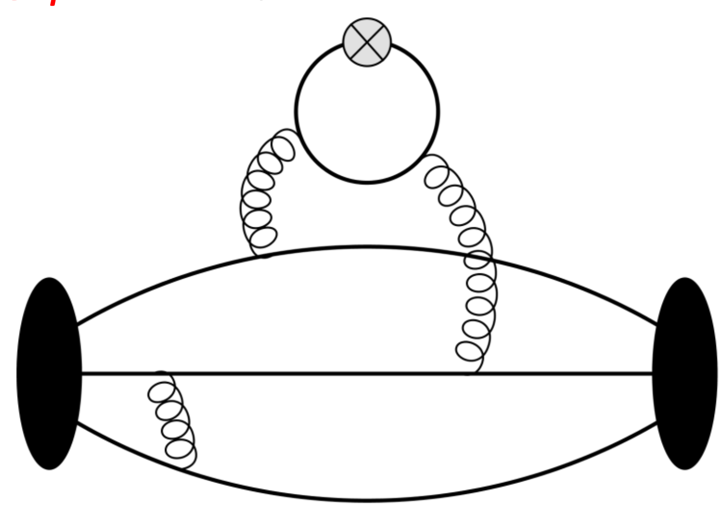
- nEDM from qEDMs given by the tensor charges g_T

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

$$\langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle = g_T^q \, \bar{u}_N \sigma_{\mu\nu} u_N$$

- $d_q \propto m_q$ in many models; $m_u/m_d \approx 1/2$, $m_s/m_d \approx 20$
  Precise determination of g_T^s is important





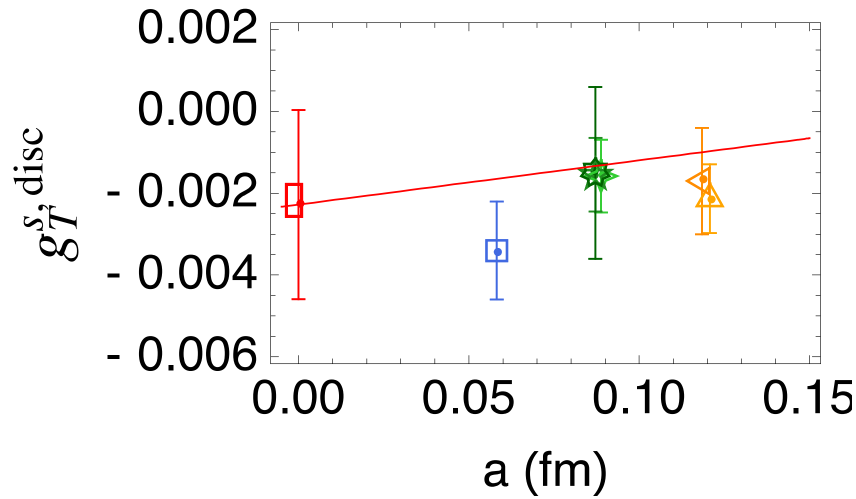
Quark EDM $d_q \bar{q}(\sigma \cdot F)\gamma_5 q$

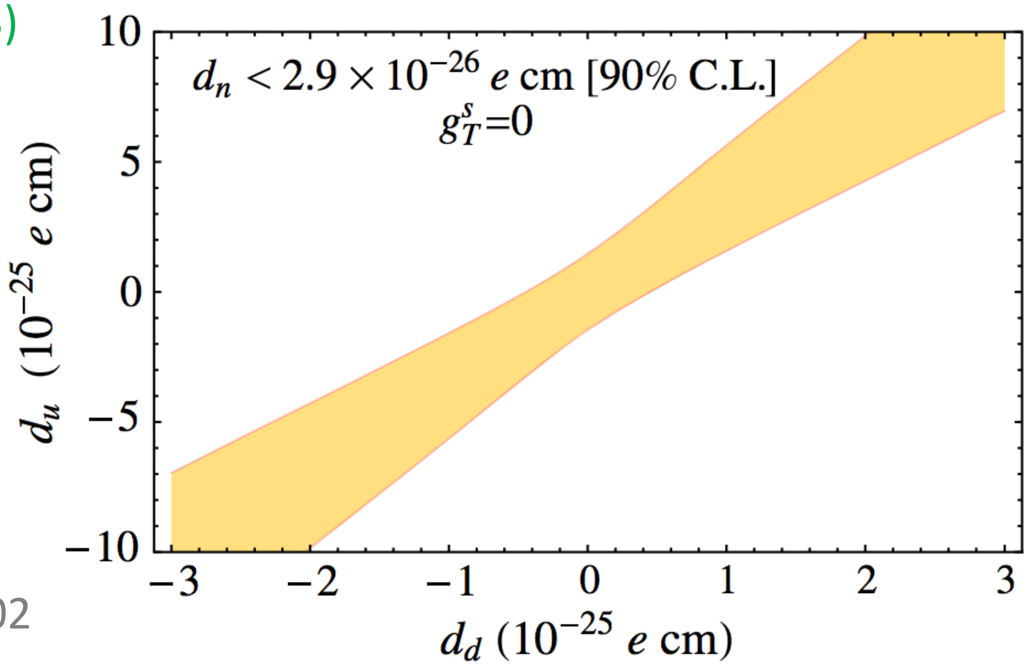
- Tensor charges:
  N$_f$=2+1+1 Clover-on-HISQ
  (a=0, m$_\pi$=135MeV)

- $$g_T^u = -0.23(3) \quad -0.211(16)$$
  $$g_T^d = 0.79(7) \quad 0.811(31)$$
  $$g_T^s = 0.008(9) \quad -0.0023(23)$$

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

- Constraints on d_q using $|d_n| < 2.9 \times 10^{-26}\, e\cdot\text{cm}$





Bhattacharya, et al., PRL 115 (2015) 212002

Quark Chromo EDM (cEDM)

$$-\frac{i}{2}\sum_{q=u,d,s}\tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q$$

Quark Chromo EDM

- Calculate d_N in presence of CP violating cEDM term

$$S = S_{QCD} + S_{cEDM}$$

$$S_{cEDM} = -\frac{i}{2}\int d^4x \;\; \tilde{d}_q g_s \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three methods explored
  - Expansion in $\tilde{d}_q$
  - External electric field method
  - Schwinger source method

Expansion in $\tilde{d}_q$

$$\left\langle N V_\mu \bar{N} \right\rangle_{CPV} = \left\langle N V_\mu \bar{N} \right\rangle + \tilde{d}_q \left\langle N V_\mu \bar{N} \cdot \sum_x O_{\mathrm{cEDM}}(x) \right\rangle + O(\tilde{d}_q^2)$$

$$O_{\mathrm{cEDM}} = \frac{i}{2} g_s \bar{q} (s \times G) g_5 q$$

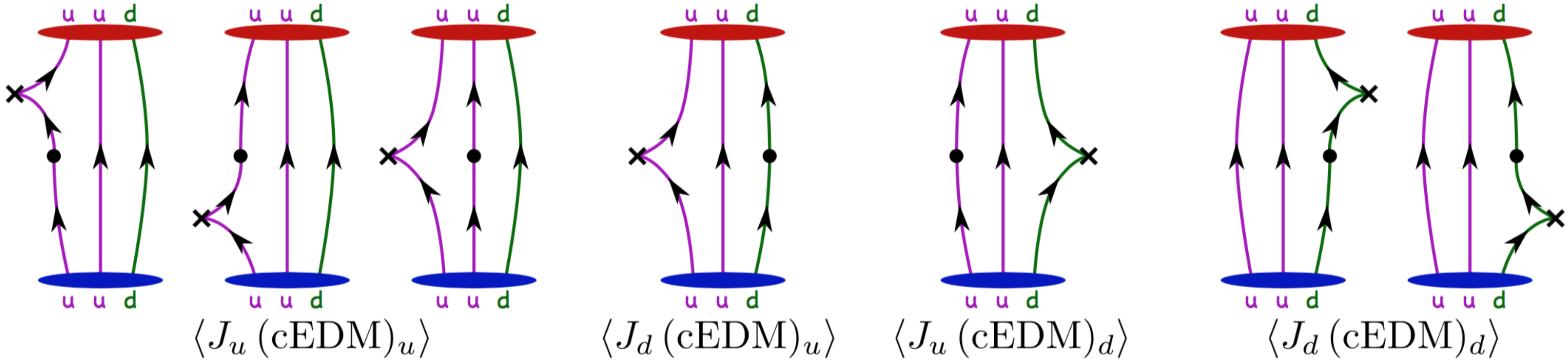
- Calculate the **four-point correlation function**

$$\boxed{\left\langle N V_m \bar{N} \underset{x}{\text{å}} O_{\mathrm{cEDM}}(x) \right\rangle}$$

-

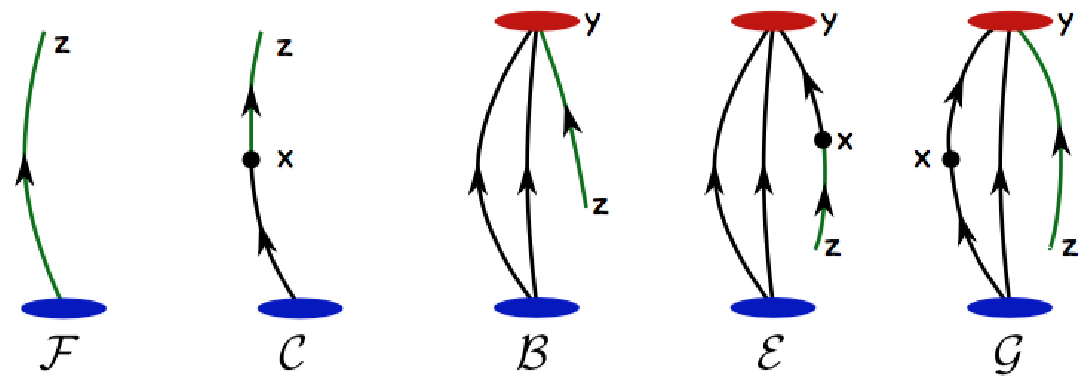
- d_n obtained from the form-factor F_3

Expansion in $\tilde{d}_q$



Connected Diagrams

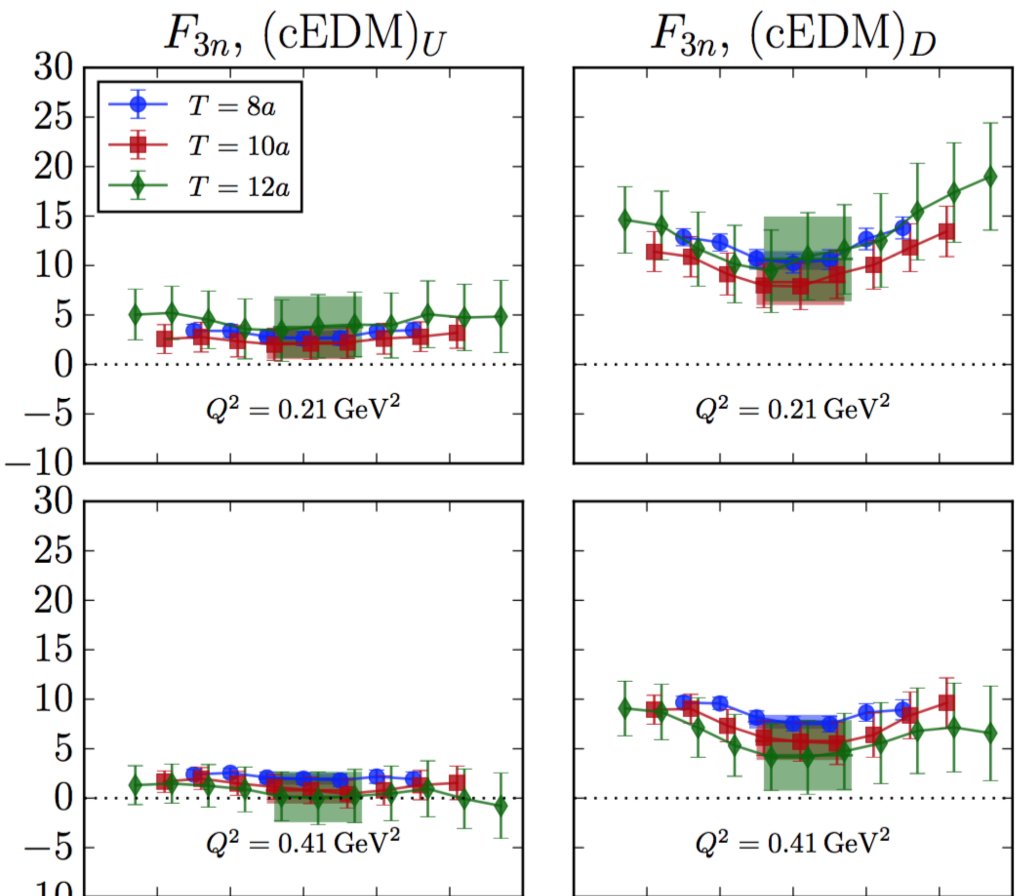
Propagators Needed

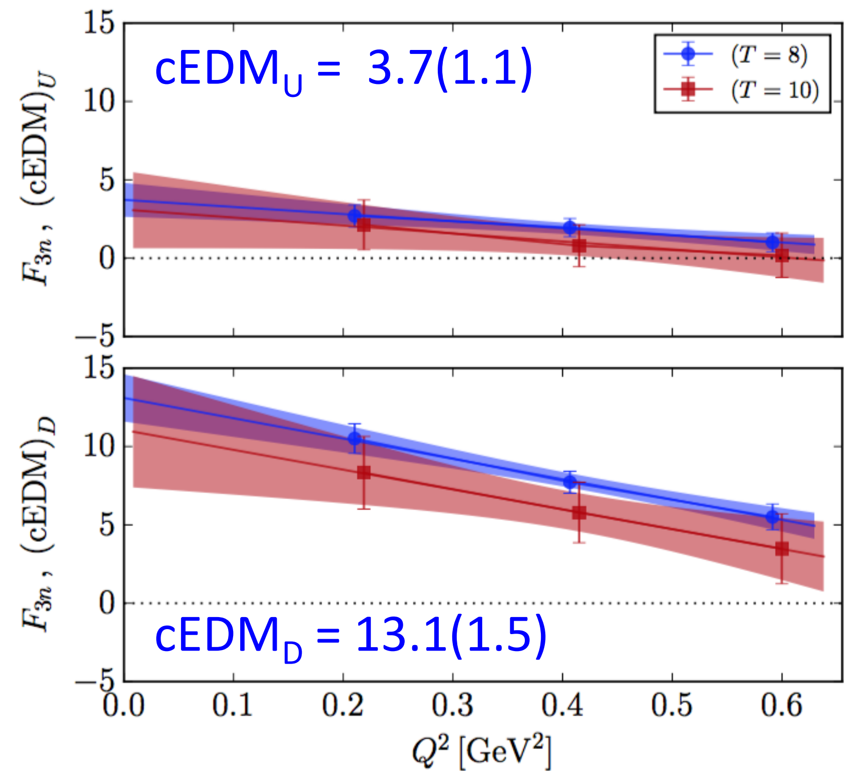


- Four-point correlator is evaluated using Regular and backward props (*F*, *B*), cEDM sequential prop (*C*) and doubly-sequential props (E, G)

Abramczyk, et al., PRD96 (2017) 014501

Expansion in $\tilde{d}_q$





- DWF
- a = 0.11fm
- m_π=340 MeV

Abramczyk, et al., PRD96 (2017) 014501

External Electric Field Method

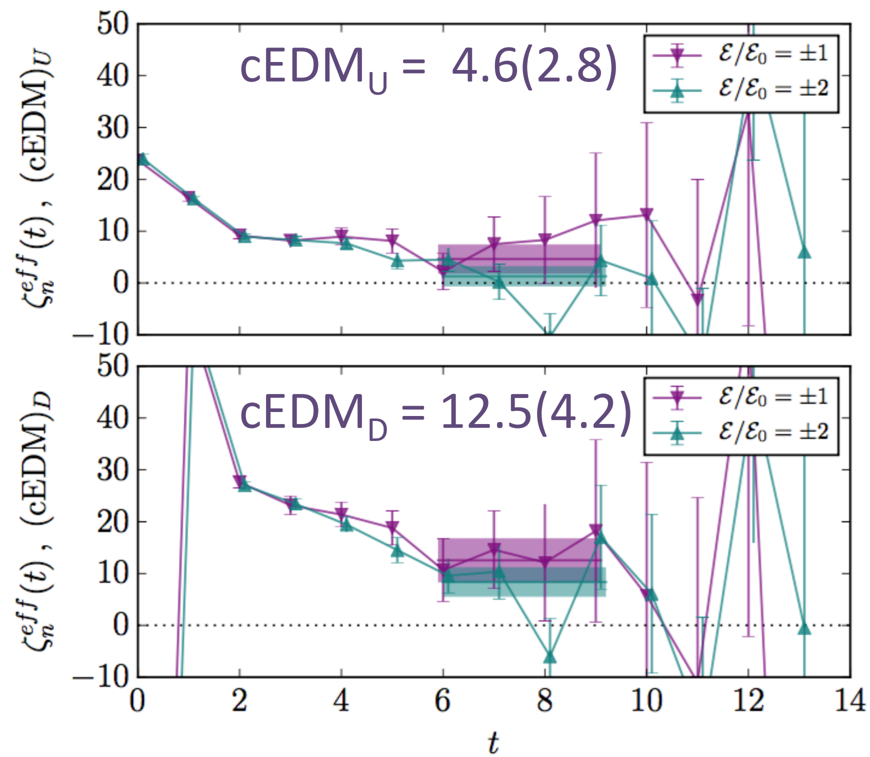
- In the presence of uniform electric field $i\vec{\mathcal{E}}$

$$E_{\vec{S}}^{\mathrm{cEDM}} - E_{-\vec{S}}^{\mathrm{cEDM}} \approx 2d_N \vec{S}\cdot i\vec{\mathcal{E}}$$

- Neutron correlator with CEDM term via reweighting

$$\left\langle N\bar{N}\right\rangle_\theta (\vec{\mathcal{E}},t) = \left\langle N(t)\bar{N}(0)\, O_{\mathrm{cEDM}}\right\rangle_{\vec{\mathcal{E}}}$$

- DWF
- a = 0.11fm
- m_π=340 MeV
- E_0 = 0.039 GeV^2



Abramczyk, et al., PRD96 (2017) 014501

Schwinger Source Method

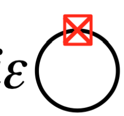
- Quark chromo EDM operator is bilinear in quark fields

$$i\bar{q}(\sigma\cdot G)\gamma_5 q$$

- Modify the Dirac operator M to include the cEDM term.

$$\not{D} + m - \frac{r}{2}D^2 + c_{SW}\Sigma^{\mu\nu}G_{\mu\nu} \rightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{SW}G_{\mu\nu} + i\varepsilon\tilde{G}_{\mu\nu})$$

$$c_{sw}\sigma^{\mu\nu}G_{\mu\nu} \rightarrow \sigma^{\mu\nu}(c_{sw} + i\varepsilon\gamma_5)G_{\mu\nu}$$

- Calculate M^{-1} and M_ϵ^{-1}
- Construct 3-point correlators using M^{-1} and M_ϵ^{-1}
- Change in Fermion determinant = reweighting factor $e^{i\varepsilon}$ 

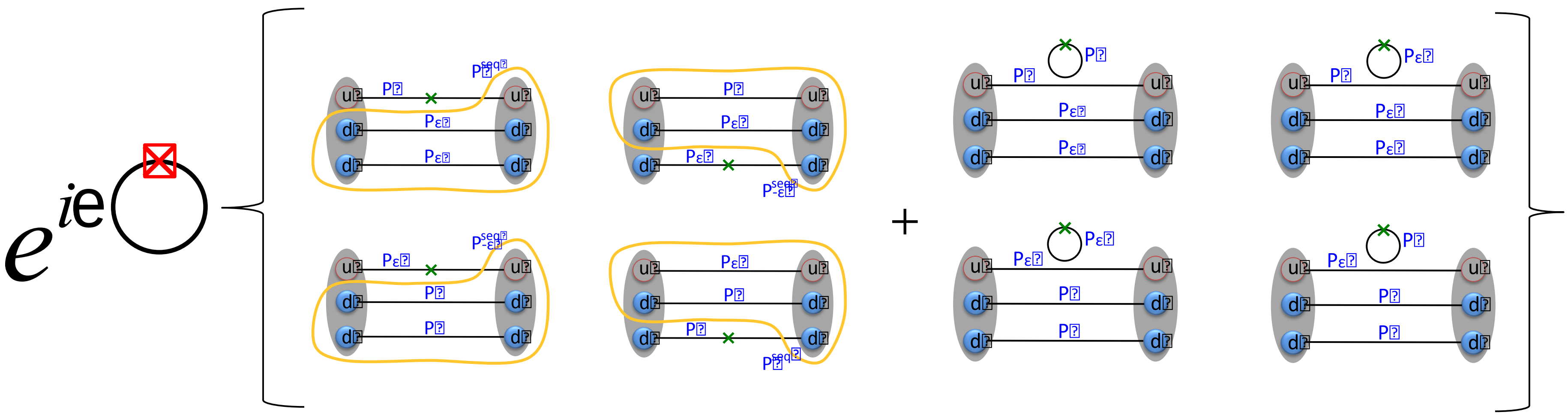
$$\frac{\det\left(D_{\text{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\right)}{\det\left(D_{\text{clov}}\right)} \approx \exp\left[i\varepsilon \mathrm{Tr}\left(\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} D_{\text{clov}}^{-1}\right)\right]$$

Reweight by the ratio of determinants

$$\frac{Det[\not{D}+m-\frac{r}{2}D^2+\Sigma^{\mu\nu}(c_{SW}G_{\mu\nu}+i\varepsilon\tilde{G}_{\mu\nu})]}{Det[\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu}]}$$

$$=\exp\{Tr\ Ln[1+i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}]\}$$

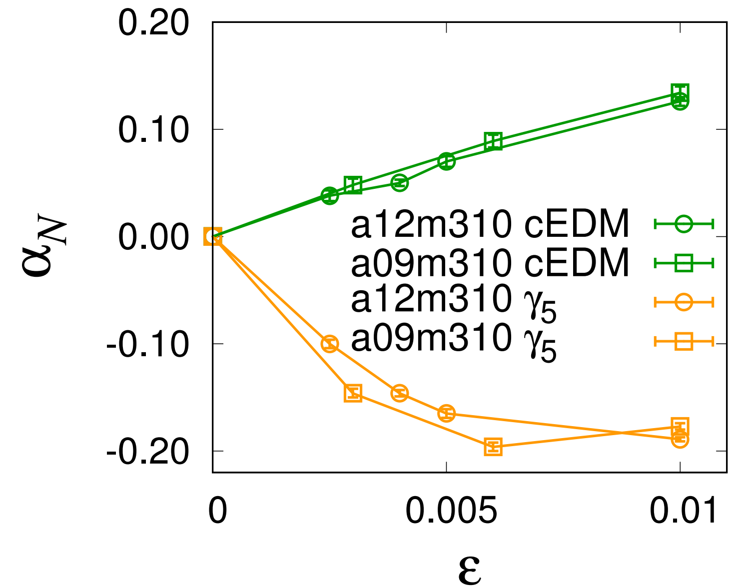
$$\approx\exp\{Tr\ i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D}+m-\frac{r}{2}D^2+c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\}$$



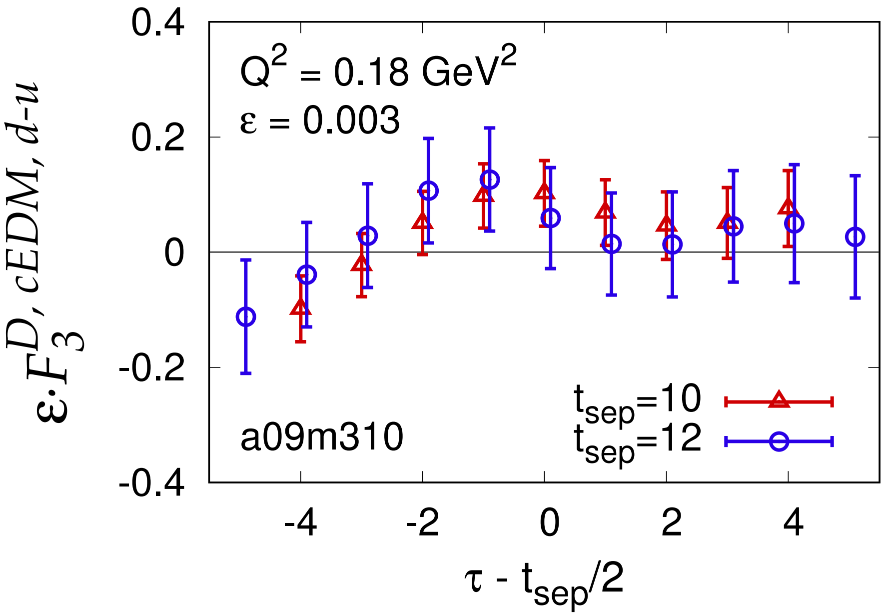
Connected disconnected

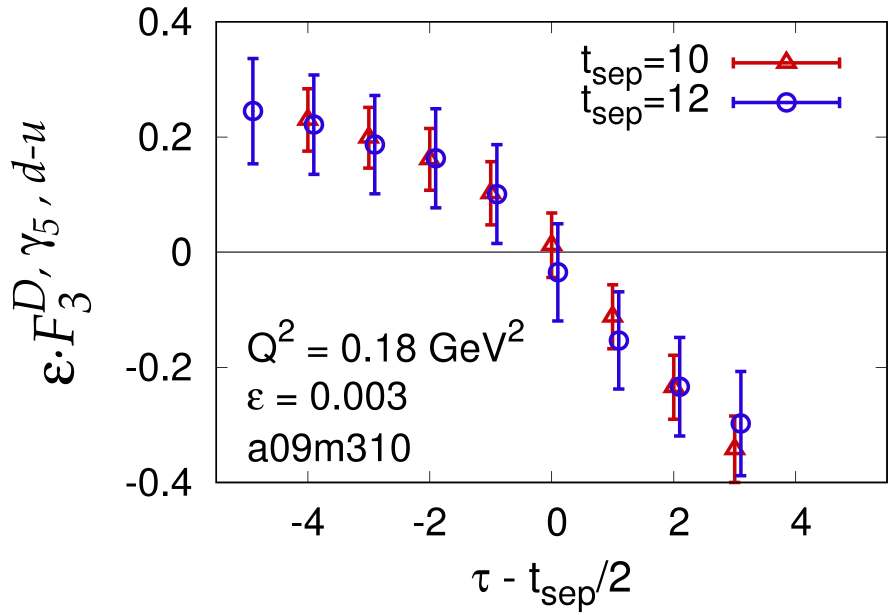
Schwinger Source Method

- Calculation performed at small ε so that results are linear in ε
- cEDM mixes with γ_5, so need calculation with both operators
- Calculate contribution of each to F_3



Tests at a = 0.09 fm, M_π=310 MeV





Renormalization

- Renormalization of cEDM Operators are studied
  - 1-loop perturbation on twisted-mass fermion
    [Constantinou, et al, 2015]
  - 1-loop and Nonperturbative RI-$\widetilde{\mathrm{S}}$MOM
    [Bhattacharya, et al, 2015]

- Mixing with dimension-3 operator: *1/a²* mixing

$$\boxed{\begin{aligned} O_{\mathrm{cEDM}} &= a^2 \bar{q} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu} q \\ O_{\mathrm{P}} &= \bar{q} \gamma_5 q \end{aligned}}$$

Quark Chromoelectric Operator: Mixing

	C	$\partial^2 P$	E	$mF\tilde{F}$	$mG\tilde{G}$	$m\partial \cdot A$	$m^2 P$	P_{EE}	$\partial \cdot A_E$	A_∂	$A_{A(\gamma)}$
C	Z_C	X	X	X	X	X	X	X	X	X	X
$\partial^2 P$	0	Z_P	0	0	0	0	0	0	0	0	0
E	0	0	Z_T	0	0	0	0	0	0	0	0
$mF\tilde{F}$	0	0	0	$Z_m^{-1} Z_{F\tilde{F}}$	0	0	0	0	0	0	0
$m\, G\tilde{G}$	0	0	0	0	$Z_m^{-1} Z_{G\tilde{G}}$	X	0	0	0	0	0
$m\, \partial \cdot A$	0	0	0	0	0	$Z_m^{-1} Z_{\partial A}$	0	0	0	0	0
$m^2 P$	0	0	0	0	0	0	Z_m^{-1}	0	0	0	0
P_{EE}	0	0	0	0	0	0	0	X	X	X	0
$\partial \cdot A_E$	0	0	0	0	0	0	0	0	X	0	0
A_∂	0	0	0	0	0	0	0	X	X	X	0
$\partial \cdot A_E$	0	0	0	0	0	0	0	0	X	0	0
A_∂	0	0	0	0	0	0	0	X	X	X	0
$A_{A(\gamma)}$	0	0	0	0	0	0	0	0	0	0	X

Bhattacharya et al, PRD92 (2015) 114026 (arXiv:1502.07325 [hep-ph])

Ongoing work

- Weinberg Three-gluon Operator $d_w \frac{g_s}{6} G\tilde{G}G$

- Renormalization and mixing
  - Gradient Flow

Summary

- **QCD θ-term**

  Actively being calculated; need better precision
- **Quark EDM** *(Done)*

  Calculated: $g_T^d = 0.811(31)$; $g_T^u = -0.211(16)$; $g_T^s = -0.0023(23)$
- **Quark Chromo EDM**

  Exploratory studies started; need to address disconnected diagrams & renormalization/mixing
- **Weinberg Three-gluon Operator**

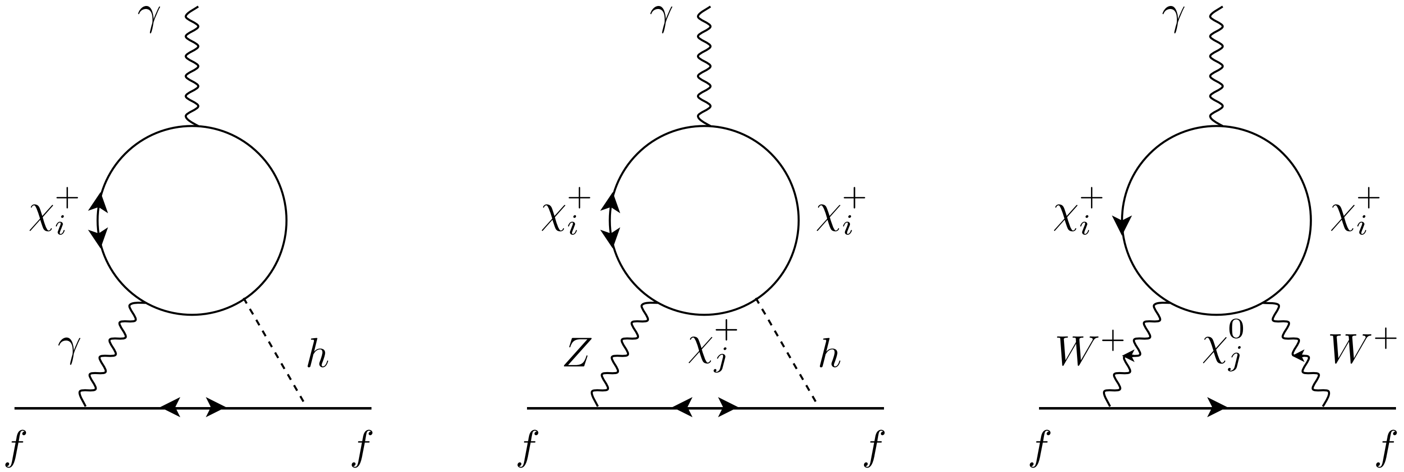
  Exploratory studies just started
- **Four-quark Operators**

  Not yet explored

Should have better estimate of accuracy achievable in 1-2 years

Split Supersymmetry

- All scalars but one Higgs doublet are much heavier than Λ_{EW}
- Has gauge coupling unification, dark matter candidate
- Avoids flavor and CP constraints mediated by 1-loop terms with scalars
- Fermion EDMs arise at 2-loops: phases in gaugino-Higgsino sector communicated to SM fermions through *γh, Zh, WW* exchanges
- chromoEDM, Weinberg, …, operators do not arise at 2-loop

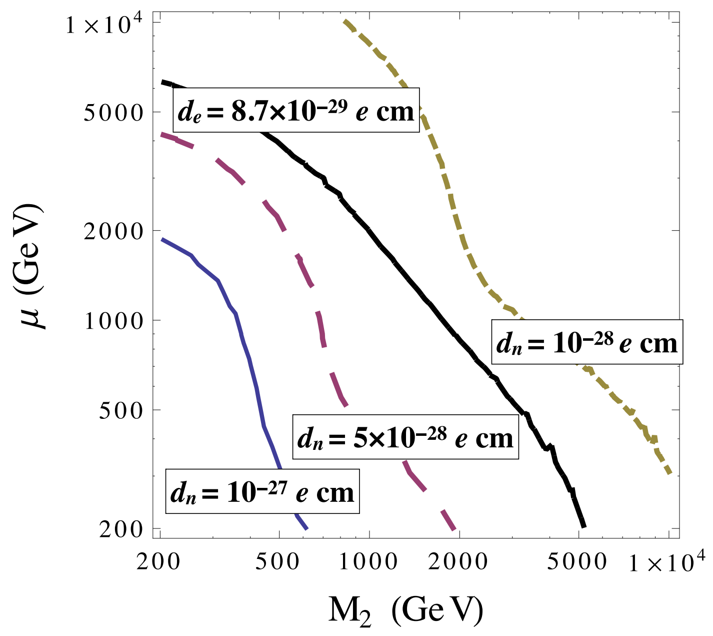


Our analysis followed the work of Giudice & Romanino, PL B634 (2006) 307

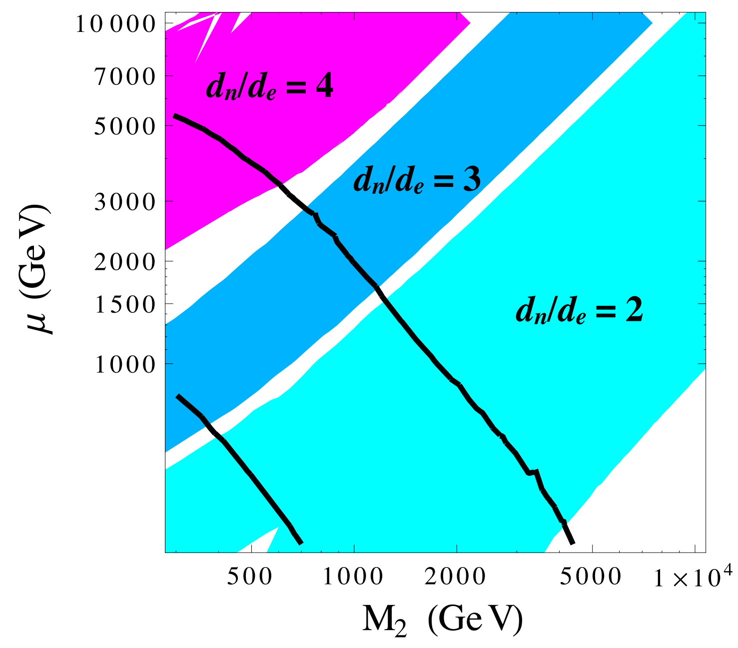
Constraint on Split Supersymmetry

- The correlation between d_n and d_e provides a constraint on Split SUSY.
- Using our estimates of $g_T(u,d,s)$ and $d_e=8.7 \times 10^{-29}$ e cm gives a stringent upper bound:

$$\boxed{d_n < 4 \times 10^{-28} \text{ e cm}}$$



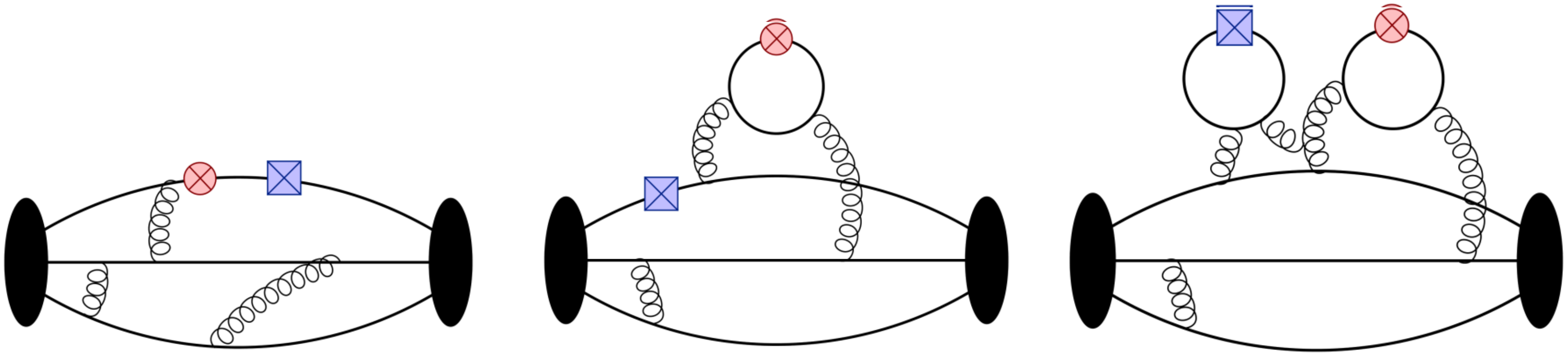
Contours of d_n , d_e versus gaugino (M$_2$) and Higgsino (μ) mass parameters setting tanβ=1 and sinϕ=1



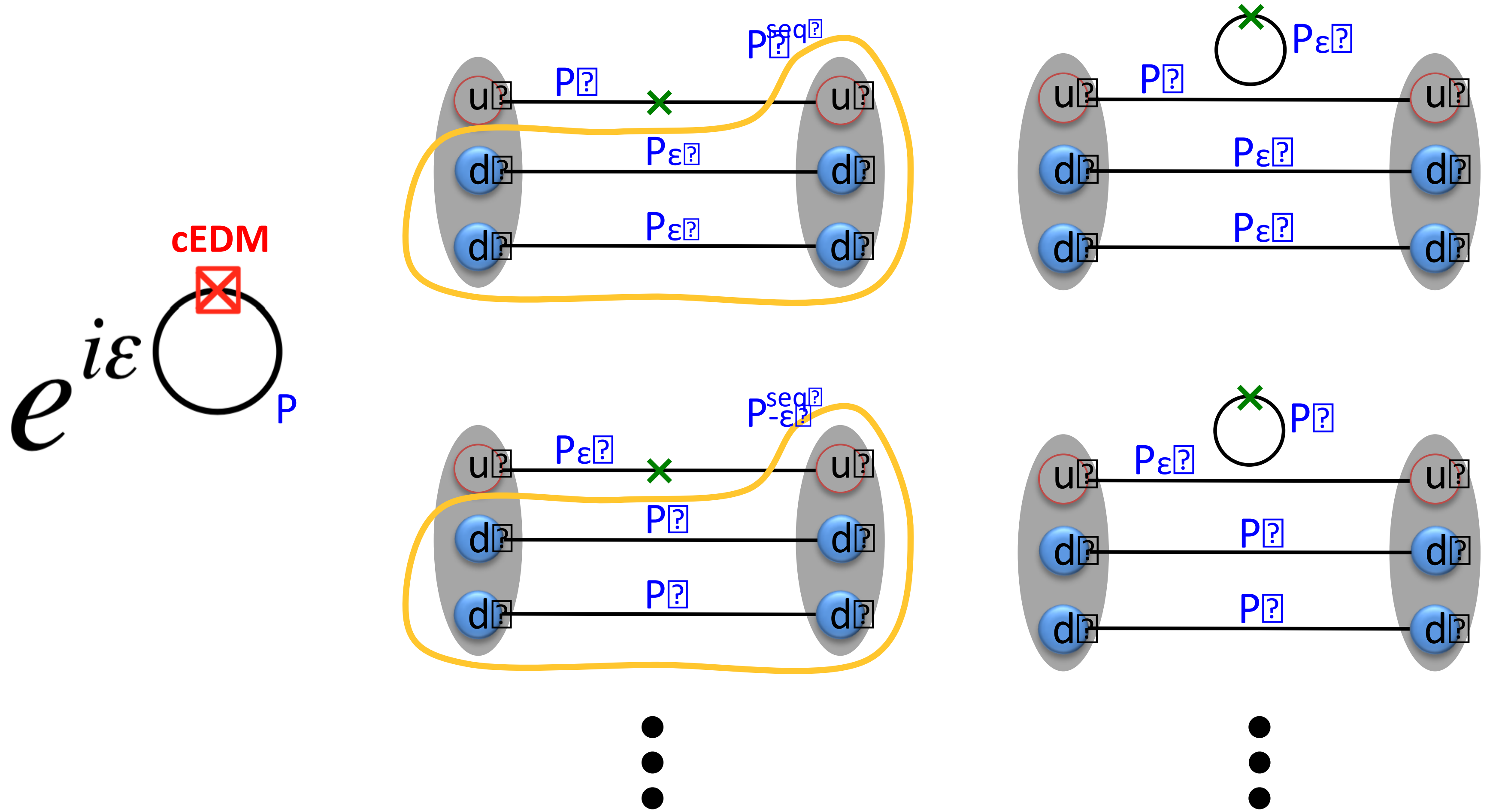
Correlation between d_n , d_e in split SUSY. Bands are for different d_n/d_e (ϕ independent) and solid lines are for $d_e=8.7 \times 10^{-29}$ e cm & sinϕ=0.2 and 1.

Bhattacharya et al, PRL 115 (2015) 212002

4-pt functions: Quark Chromo EDM



Schwinger Source Method



Reweighting **Connected Diagrams** **Disconnected diagrams**

Schwinger Source Method

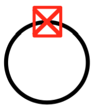
- Quark chromo EDM operator is bilinear in quark fields

$$i\bar{q}(\sigma \cdot G)\gamma_5 q$$

- Modify the Dirac operator to include the cEDM term. Invert this matrix to generate valence quark propagators

Effectively $$\boxed{D_{\mathrm{clov}} \to D_{\mathrm{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}}$$

$$c_{sw}\sigma^{\mu\nu}G_{\mu\nu} \to \sigma^{\mu\nu}(c_{sw} + i\varepsilon\gamma_5)G_{\mu\nu}$$

- 3-point correlators; d_N extracted from F_3
- Change in Fermion determinant = reweighting factor $e^{i\varepsilon}$ 

$$\frac{\det\left(D_{\mathrm{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\right)}{\det\left(D_{\mathrm{clov}}\right)} \approx \exp\left[i\varepsilon \mathrm{Tr}\left(\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} D_{\mathrm{clov}}^{-1}\right)\right]$$