

Physics at the horizon

Mind the Cap!

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with

Nick Warner, Emil Martinec, Jan deBoer, Micha Berkooz, Simon Ross, Gianguido Dall'Agata, Stefano Giusto, Rodolfo Russo, Guillaume Bossard, Masaki Shigemori, Monica Guică, Nikolay Bobev, Bert Verhocke, Andrea Puhm, David Turton, **Stefanos Katmadas**, **Johan Blåbäck**, **Pierre Heidmann**



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ANR

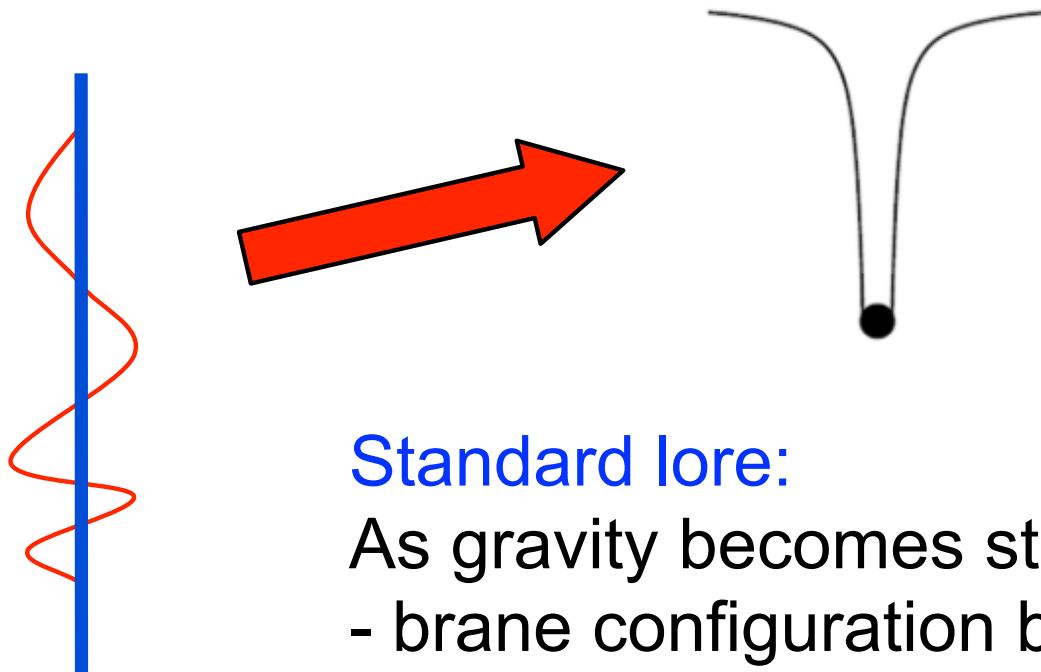


Strominger and Vafa (1996):

*Black Hole Microstates at **Zero Gravity*** (branes + strings)

Correctly match B.H. entropy !!!

One Particular Microstate at **Finite Gravity**:



Standard lore:

As gravity becomes stronger,

- brane configuration becomes smaller
- horizon develops and engulfs it
- recover standard black hole

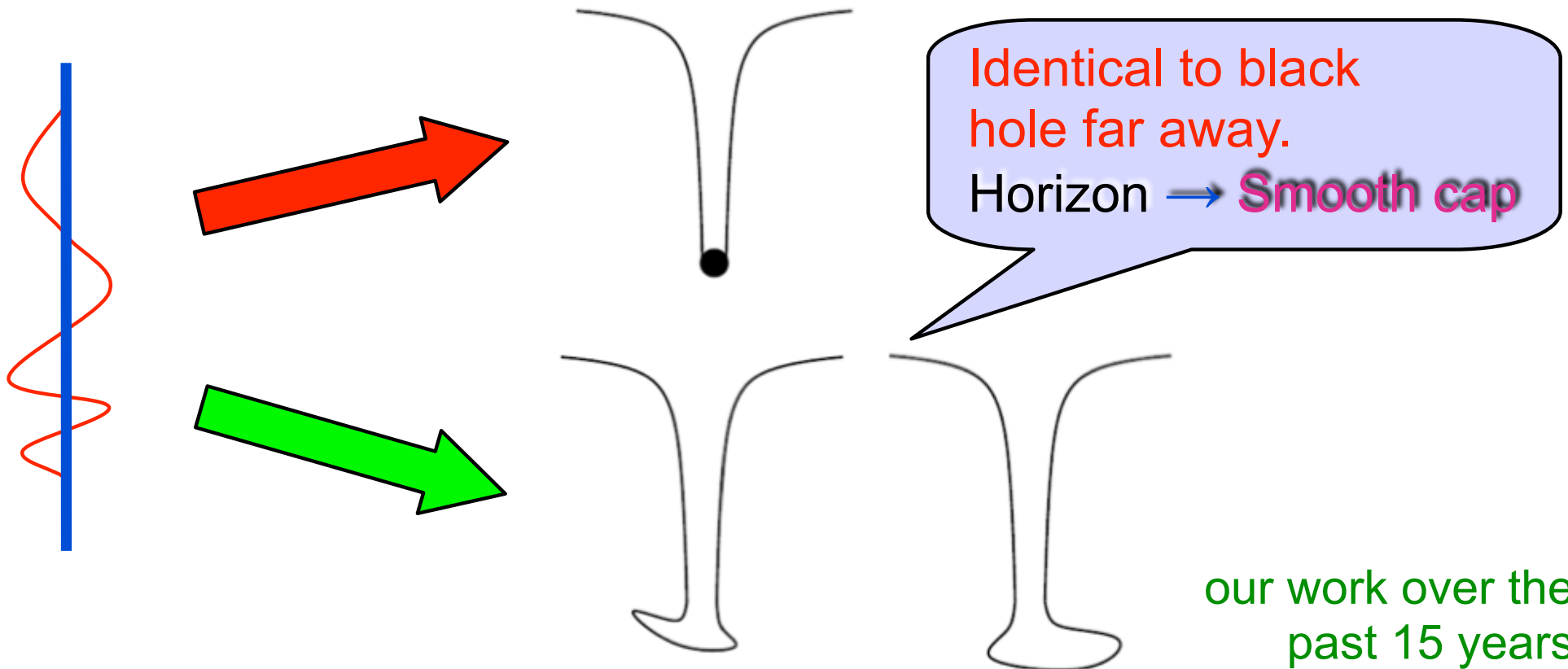
Susskind
Horowitz, Polchinski
Damour, Veneziano

Strominger and Vafa (1996):

*Black Hole Microstates at **Zero Gravity*** (branes + strings)

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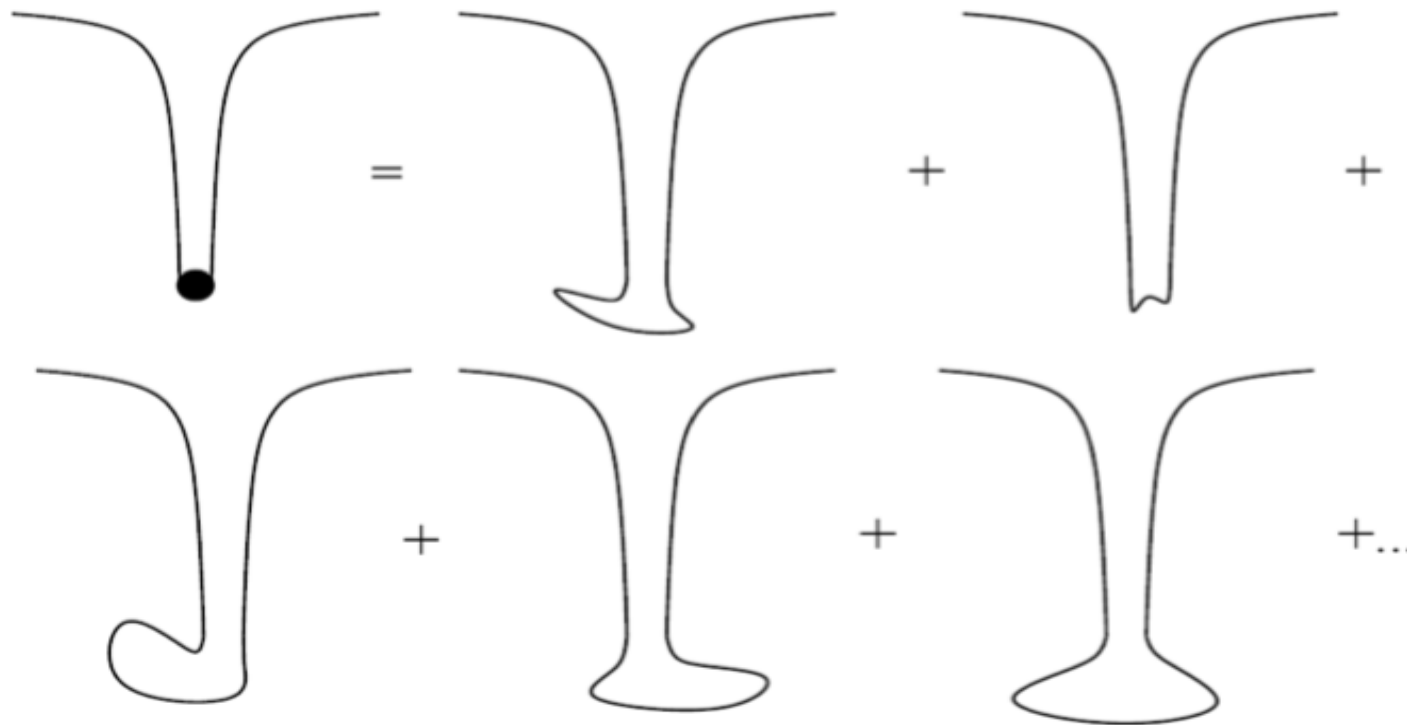
One Particular Microstate at **Finite Gravity**:



BIG QUESTION: Are *all* black hole microstates becoming geometries with no horizon ?

Black hole $\stackrel{?}{=}$ ensemble of horizonless microstate configurations

Mathur 2003



Only way to solve QM-GR conflict

Mathur 2009, Almheiri, Marolf, Polchinski, Sully 2012

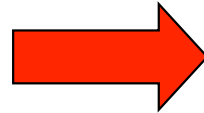
Analogy with ideal gas

Thermodynamics

(Air = ideal fluid)

$$P V = n R T$$

$$dE = T dS + P dV$$



Statistical Physics

(Air -- molecules)

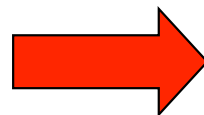
e^S microstates

typical

atypical

Thermodynamics

Black Hole Solution



Statistical Physics

Microstate geometries

Long distance physics

Gravitational lensing

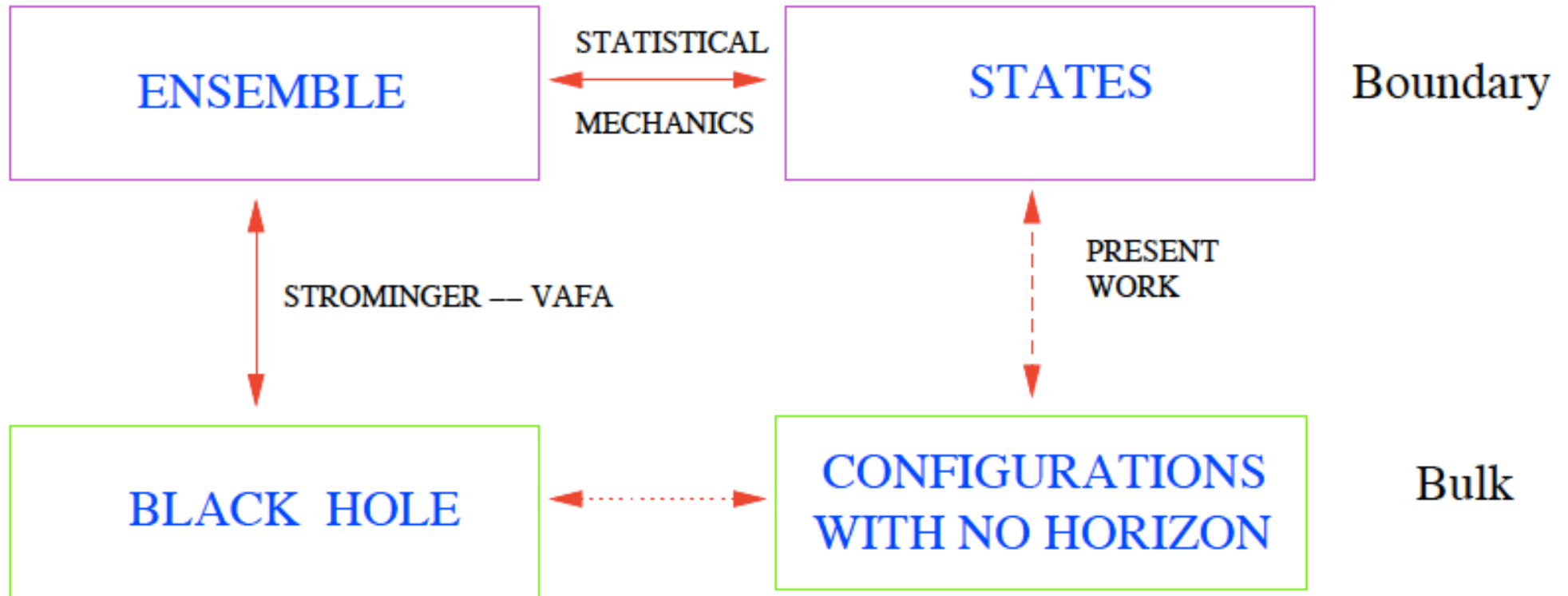
Physics at horizon

Information loss

Gravity waves ?

AdS-CFT formulation:

e.g. Bena, Warner, 2007



Not some **hand-waving** idea - **provable** by rigorous calculations in String Theory

Word of caution

- To replace classical BH by BH-sized object
 - Gravastar, quark-star, boson-star
 - Infinite density firewall hovering just above horizon
 - Gas of wormholes
 - Bose-Einstein condensate of gravitons
 - LQG configuration...

3 very stringent tests:

1. Same growth with G_N !!!

Horowitz

BH size **grows** with G_N ; “normal objects” **shrink**

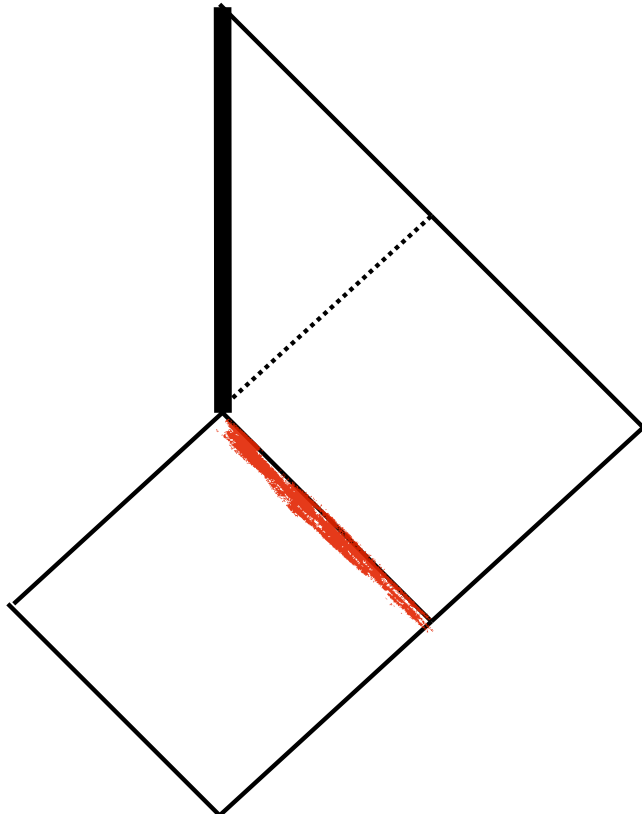
- BH **microstate** geometries **pass this test**
- **Highly nontrivial** mechanism: $G_N = g_s^2$
- D-branes = solitons, **tension** $\sim 1/g_s \rightarrow$ lighter as G_N increases



To build structure@horizon, non-perturbative degrees of freedom you must use !

2. Mechanism not to fall into BH

Very difficult !!!



GR Dogma:

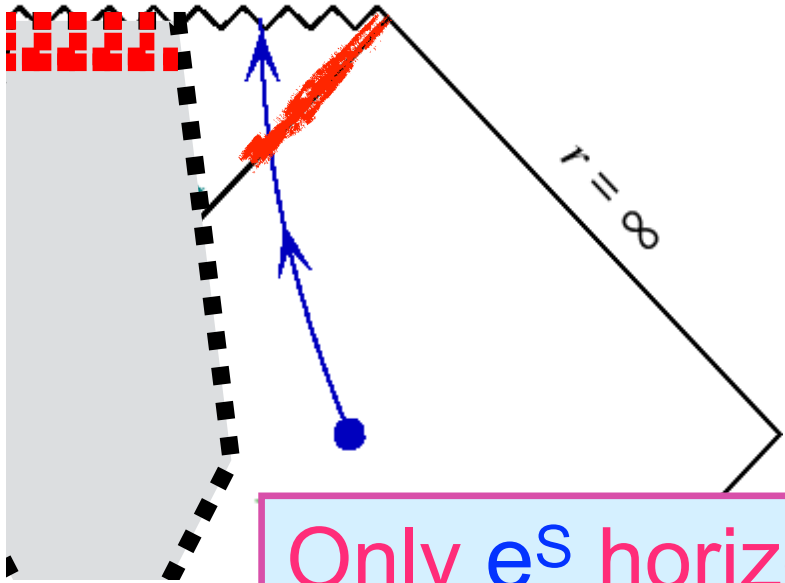
**Thou shalt not put anything
at the horizon !!!**

- Null \rightarrow speed of light.
- If massive: ∞ boost \rightarrow ∞ energy
- If massless: dilutes with time
- Nothing can live there !
(or carry degrees of freedom)
- No membrane, no spins, no “quantum stuff”
- No (fire)wall

*If support mechanism have you not,
go home and find one*

3. Avoid forming a horizon

- Collapsing shell forms horizon Oppenheimer and Snyder (1939)
- If curvature is low, no reason not to trust classical GR
- By the time shell becomes **curved-enough for quantum effects to become important**, horizon in causal past



Backwards in time - **illegal** !

BH has e^S microstates with no horizon

Small tunneling probability = e^{-S}

Will tunnel with probability **ONE** !!!

Kraus, Mathur; Bena, Mayerson, Puhm, Vercnocke

Only e^S horizon-sized microstates can do it !



*If quantum tunneling you are brushing aside,
incorrect physics you are doing*

Microstates geometries

- Where is the **BH charge** ?

$$L = q A_0$$

magnetic

$$L = \dots + A_0 F_{12} F_{34} + \dots$$

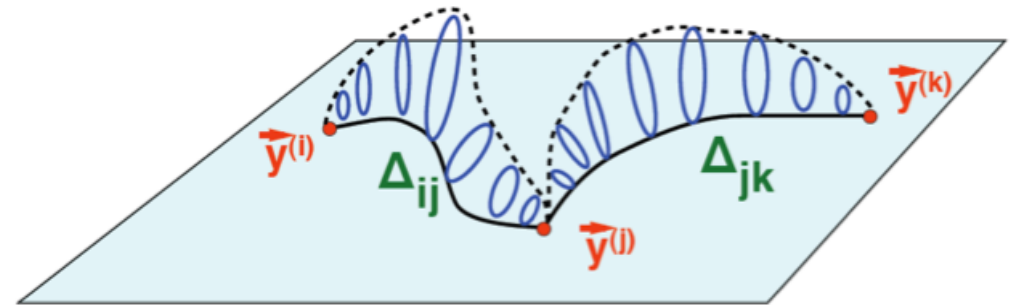
- Where is the **BH mass** ?

$$E = \dots + F_{12} F^{12} + \dots$$

- BH angular momentum

$$J = E \times B = \dots + F_{01} F_{12} + \dots$$

2-cycles + magnetic flux



Bubbling Geometries

Black Hole Solitons

beautiful GR story behind

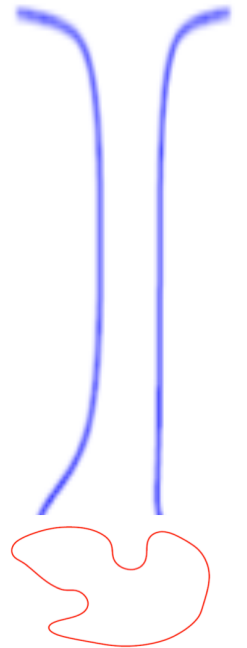
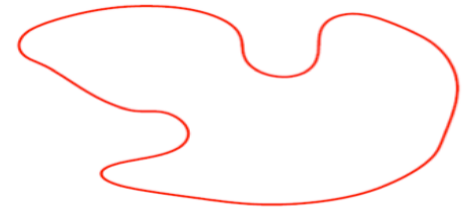
Gibbons, Warner

The charge is dissolved in magnetic fluxes. No singular sources.

Klebanov-Strassler

Add wiggles - increase entropy

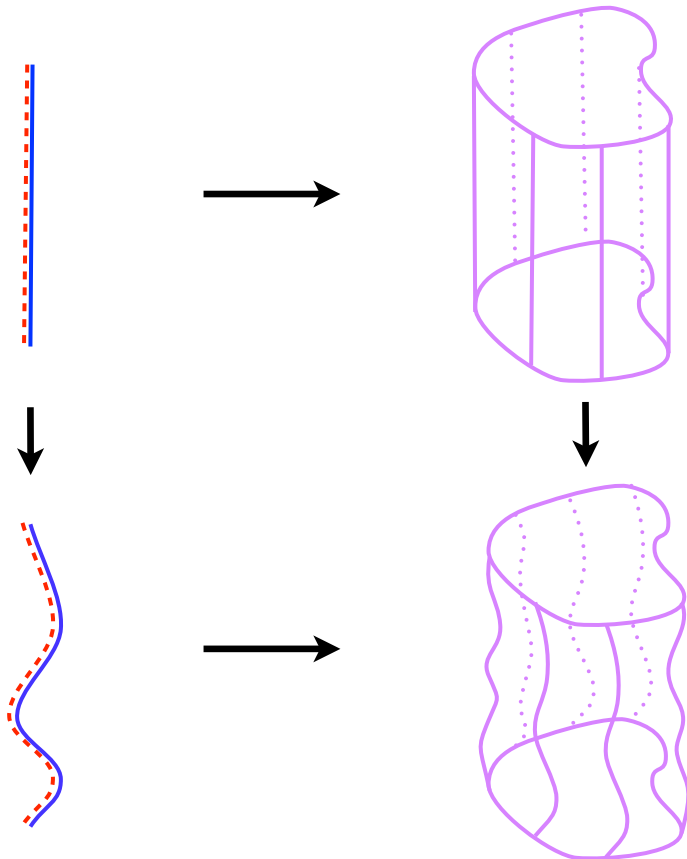
- Add supertubes
 - supersymmetric brane configs
 - arbitrary shape Mateos, Townsend
- Construct backreacted solution
 - Taub-NUT Page Green's functions (painful)
- Smooth !
 - exactly as in flat space
Lunin, Mathur; Emparan, Mateos, Townsend
Lunin, Maldacena, Maoz
- Entropy: $S \sim (Q^{5/2})^{1/2}$
- Huge but not yet black-hole-like ($Q^{3/2}$)
- Need more degrees of freedom !



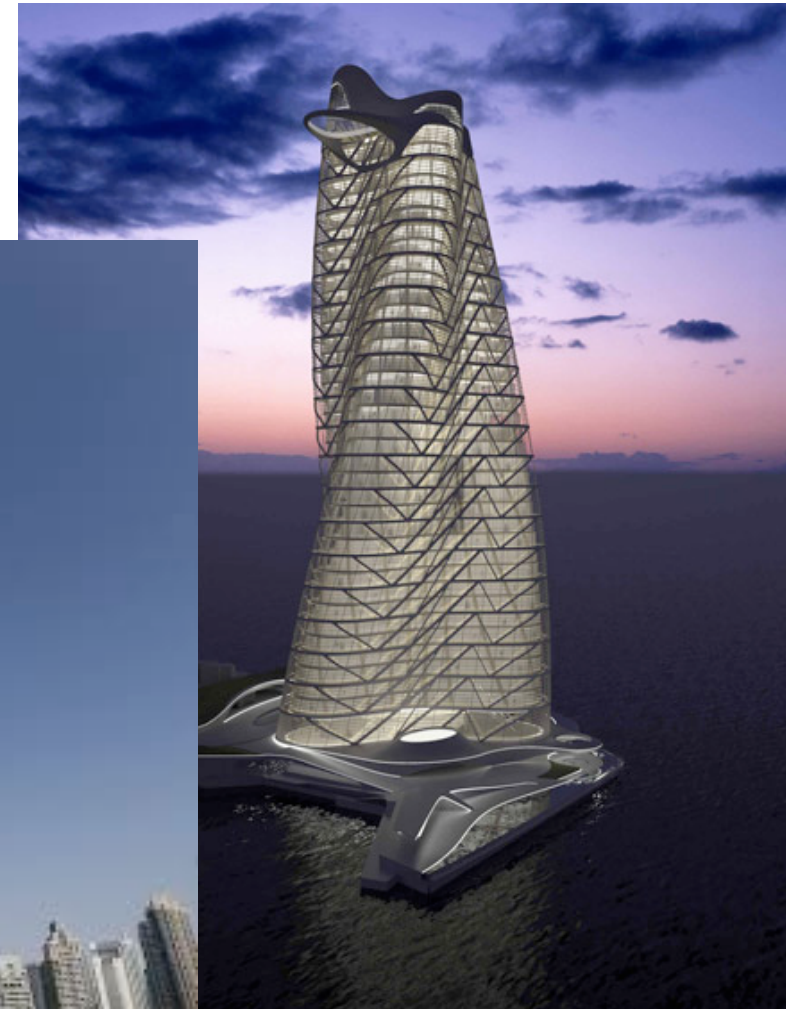
Get even more entropy

Bena, de Boer, Shigemori, Warner

- **Supertubes (locally 16 susy)** - 8 functions of **one** variable ($c = 8$)
- **Superstrata (locally 16 susy)** - 4 functions of **two** variables ($c = \infty$)
- Double supertube transition:



Should be
Smooth !!!



Largest family of solutions known to mankind

Arbitrary functions of two variables: $\infty \times \infty$ parameters
 Bena, Giusto, Russo, Shigemori, Warner

$$\begin{aligned}
 ds_{10}^2 &= \frac{1}{\sqrt{\alpha}} ds_6^2 + \sqrt{\frac{Z_1}{Z_2}} d\hat{s}_4^2, \\
 ds_6^2 &= -\frac{2}{\sqrt{\mathcal{P}}} (dv + \beta) \left[du + \omega + \frac{\mathcal{F}}{2} (dv + \beta) \right] + \sqrt{\mathcal{P}} ds_4^2, \\
 e^{2\Phi} &= \frac{Z_1^2}{\mathcal{P}}, \\
 B &= -\frac{Z_4}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_4 \wedge (dv + \beta) + \delta_2, \\
 C_0 &= \frac{Z_4}{Z_1}, \\
 C_2 &= -\frac{Z_2}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_1 \wedge (dv + \beta) + \gamma_2, \\
 C_4 &= \frac{Z_4}{Z_2} \widehat{\text{vol}}_4 - \frac{Z_4}{\mathcal{P}} \gamma_2 \wedge (du + \omega) \wedge (dv + \beta) + x_3 \wedge (dv + \beta) + \mathcal{C}, \\
 C_6 &= \widehat{\text{vol}}_4 \wedge \left[-\frac{Z_1}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) + a_2 \wedge (dv + \beta) + \gamma_1 \right] \\
 &\quad - \frac{Z_4}{\mathcal{P}} \mathcal{C} \wedge (du + \omega) \wedge (dv + \beta), \\
 \alpha &\equiv \frac{Z_1 Z_2}{Z_1 Z_2 - Z_4^2}, \quad \mathcal{P} \equiv Z_1 Z_2 - Z_4^2.
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{Rr}{\sqrt{2} k_2 (m_1^2 - 1)} \frac{m_1 (k_2 + m_1 + 1) \Delta_{k_2+m_1-1, m_1-1} + (k_2 + m_1 - 1) \Delta_{k_2+m_1}}{(r^2 + a^2)^2} \\
 &= \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2 \sin \theta \cos \theta} \left[2(m_1 - 1) \Delta_{k_2+m_1-3, m_1-1} \right. \\
 &\quad + (m_1 - 1)(m_1 - 2) \Delta_{k_2+m_1-1, m_1-1} + m_1 (k_2 - 2) \Delta_{k_2+m_1-1, m_1+1} \\
 &\quad \left. - m_1 (m_1 - 1) \Delta_{k_2+m_1+1, m_1-1} + (m_1^2 (k_2 - 1) + 1) \Delta_{k_2+m_1+1, m_1+1} \right], \\
 &= -\frac{R}{\sqrt{2}} \frac{\Delta_{k_2+m_1+1, m_1+1}}{\Sigma} \sin^2 \theta - \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2} \left[2(m_1 - 1) \Delta_{k_2+m_1-3, m_1-1} \right. \\
 &\quad + (m_1^2 - 2m_1 + k_2 - 1) \Delta_{k_2+m_1-1, m_1-1} + m_1 (k_2 - 2) \Delta_{k_2+m_1-1, m_1+1} \\
 &\quad + m_1 (k_2 - m_1 - 1) \Delta_{k_2+m_1+1, m_1-1} + (k_2 (m_1^2 + m_1 - 1) - m_1 (m_1 + 1)) \Delta_{k_2+m_1+1, m_1+1} \\
 &\quad \left. - \frac{R}{\sqrt{2}} \frac{\Delta_{k_2+m_1+1, m_1+1}}{\Sigma} \cos^2 \theta - \frac{R}{\sqrt{2} k_2 (m_1^2 - 1) a^2} \left[(k_2 - 1)(m_1 - 1) \Delta_{k_2+m_1-1, m_1-1} \right. \right. \\
 &\quad - 2(m_1 - 1) \Delta_{k_2+m_1-3, m_1-1} - (m_1 - 1)(m_1 - 2) \Delta_{k_2+m_1-1, m_1-1} \\
 &\quad - (m_1 - 1)(k_2 - 3) \Delta_{k_2+m_1-1, m_1+1} + m_1 (m_1 - 1) \Delta_{k_2+m_1+1, m_1-1} \\
 &\quad \left. \left. - (m_1 - 1)(m_1 (k_2 - 1) + 1) \Delta_{k_2+m_1+1, m_1+1} \right] \right].
 \end{aligned}$$

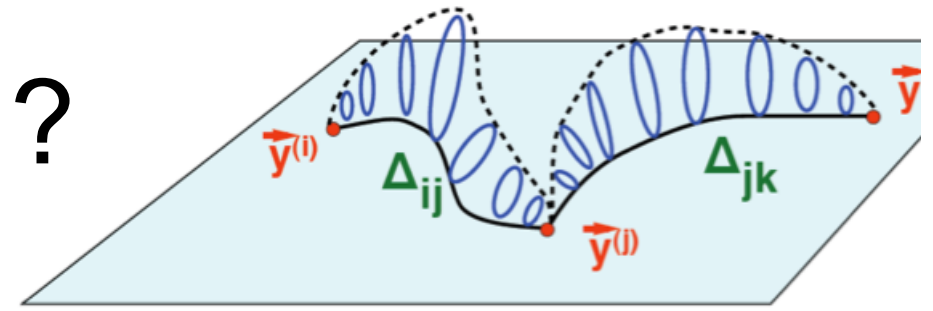
Habemus Superstratum !!!

What took us so long ?

- Superstrata conjectured in 2011
constructed in 2015
- 5D microstates with GH bubbles: $U(1)^3$
- Oscillations \rightarrow singularities
- Precision Holography: Skenderis, Taylor, Kanitscheider
- Open string emission: Giusto, Russo, Turton
- *There is another Skywalker !*
- At least $U(1)^4$
- Metric depends on $Z_1 Z_2 - Z_4^2$ Coiffuring
Bena, Ross, Warner
- Singularities cancel - solution smooth !!!



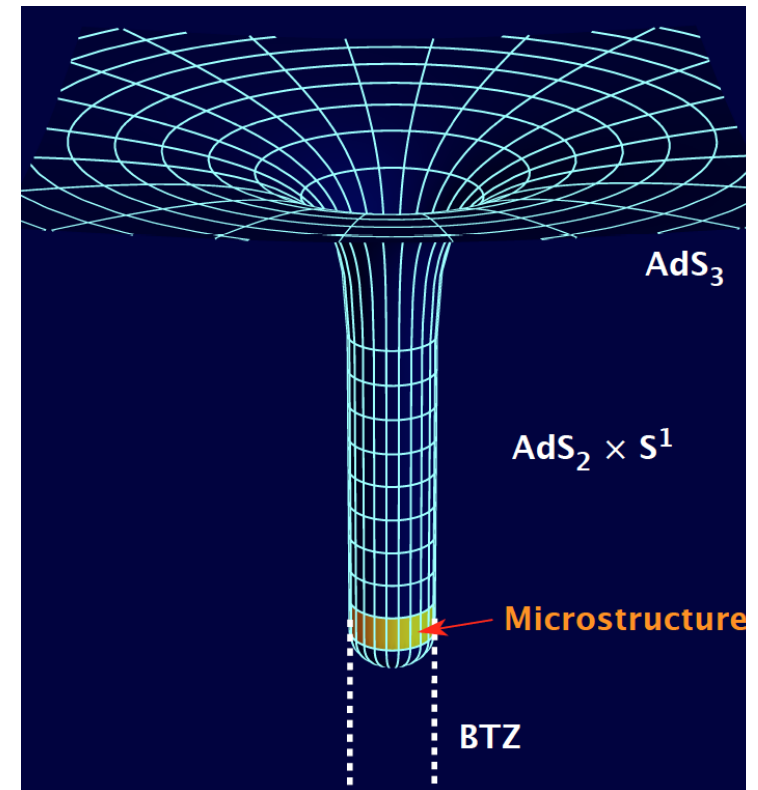
Why not collapsing ?



- 5(+6)d : smooth solutions + **quantized** magnetic flux on topologically-nontrivial **2-cycles**
 - cycles smaller \rightarrow increases energy
 - bubbling = **only** mechanism to avoid collapse in semiclassical limit Gibbons, Warner
 - If **any** state in the **e^S -dimensional** BH Hilbert space has a semiclassical limit, it **must** be a microstate geometry !
- 4(+6)d : multicenter solutions Denef
 - smooth GH centers with negative charge \rightarrow centers with **negative D6 charge** and **negative mass**
 - common in String Theory (e.g. orientifolds); **nowhere else**
 - **Highly unusual** matter from a 4d perspective
 - Usual matter does not hang around, just falls in BH

Deep superstrata

- BH microstates with GH bubbles - **very large J**
- Typically $\sim 99\%$ of c.c. bound Heidmann
- With a lot of pain $\sim 85\%$
Bena, Wang, Warner '06; Bena, Heidmann, Ramírez '17
- Build deep superstrata:
J can be **arbitrarily small**
Bena, Giusto, Martinec Russo, Shigemori, Turton, Warner '16 (PRL editor's selection)
- First BTZ microstates
- **CFT dual state known**
- More recent **low-J** solutions
with **3 GH centers + supertube**
Bena, Heidmann, Ramírez
or with **1001 GH bubbles**
Ávila, Ramírez, Ruipérez



Superstrata

Entropy:

- D1-D5 supertube - **dimension** of moduli space
 - classically: **dimension** = ∞
 - quantize: **dimension** = $4 N_1 N_5$ = number of **momentum carriers**
- Counting (+ fermions) (à la Maldacena Strominger Witten)
 $S = 2\pi (N_1 N_5 N_p)^{1/2} !!!$ Bena, Shigemori, Warner

It remains to dot the i's and cross the t's :

- We have AdS-CFT duals. Solutions **more and more messy** as one approaches typical states (long strings). **Recursive construction**
- **D1-D5 CFT** - fractional momentum carriers. Have some, not all.
- Fluxes + warping: **Small & Crumply** \longrightarrow **Big & Fluffy & Smooth**
- Are typical microstates spanned by **smooth solutions** ?

Quantum Gravity in AdS₂

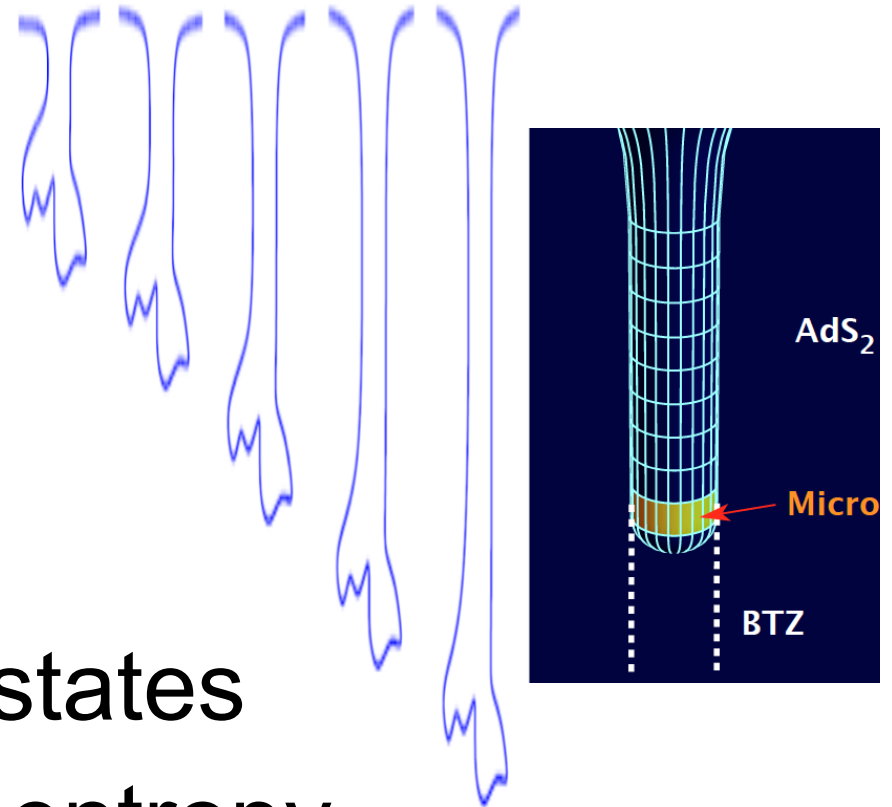
Bena, Heidmann, Turton, to appear

- Everybody & their brother & SYK & JT
- **AdS₂** - no finite-energy excitations
Maldacena, Michelson, Strominger
- backreaction of particle in AdS₂ either
 - destroys UV (work instead with *near-AdS₂*)
 - destroys IR - singularity
(? ↔ singularity in SYK 4-pt. function)
- Singularities in String Theory and AdS-CFT solved by **string and brane dynamics** involving **extra dimensions** 20 years of examples

Quantum Gravity in AdS_2

Bena, Heidmann, Turton, to appear

- Typical microstate geometries have **long AdS_2 throat**
- Limit when **length** $\rightarrow \infty$
- **Disconnect** from AdS_3
- Solutions above \rightarrow **asymptotically- AdS_2**
Bena, Heidmann, Turton
- **Same entropy** as microstates
- If superstrata count BH entropy, so do these solutions !

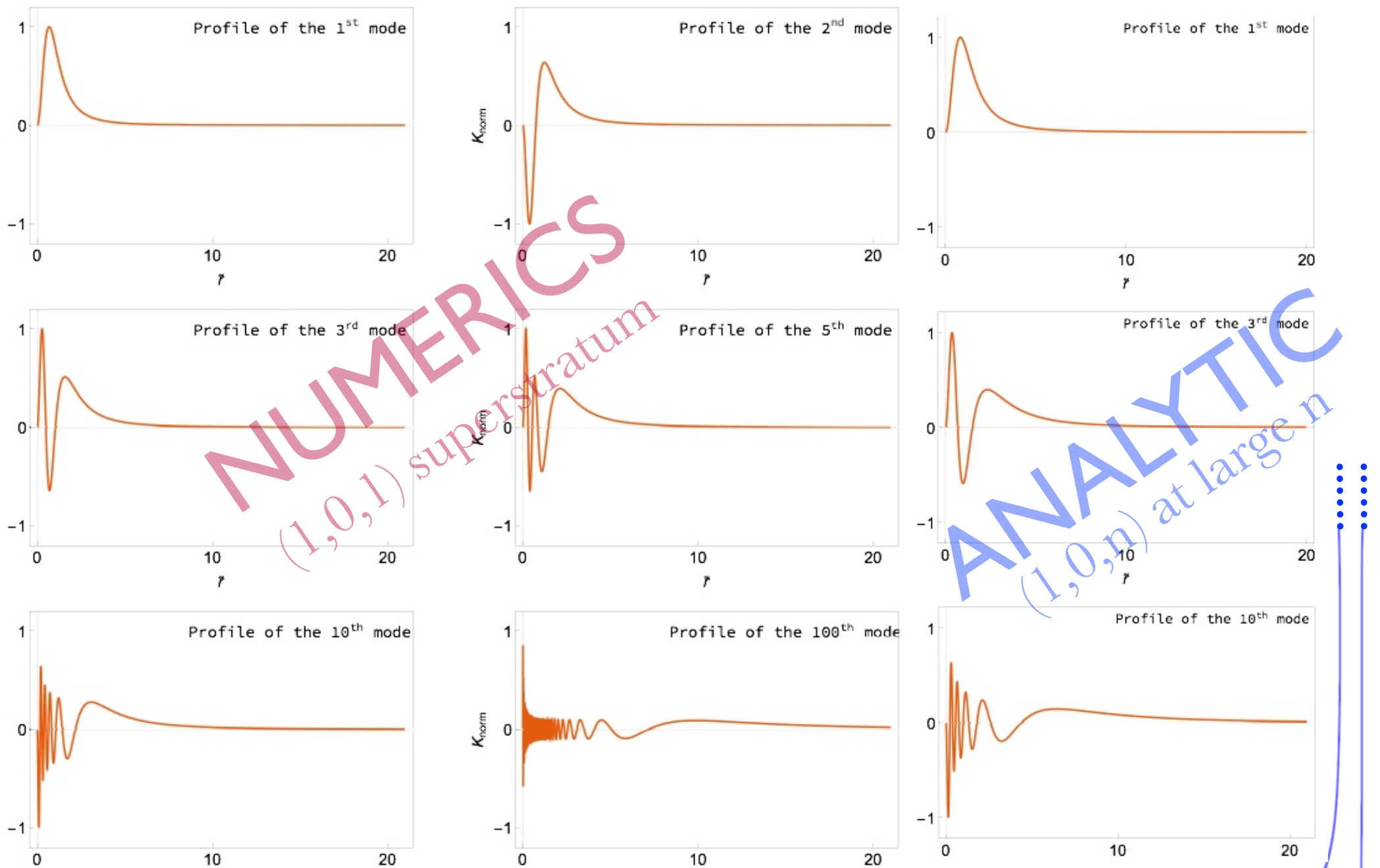


Quantum Gravity in AdS₂

Bena, Heidmann, Turton, to appear

- geometries with AdS₂ UV and IR cap
- BPS ground states of CFT₁ dual to AdS₂
- finite-energy time-dependent excitations →
Paulos
- CFT₁ has no conf.-invariant ground state !!!
- Empty Poincaré AdS₂ not dual to any ground state of CFT₁ (similar to Poincaré AdS₃)
- All CFT₁ ground states break conf. symmetry
- Tower of finite-energy excitations above each and every one of them



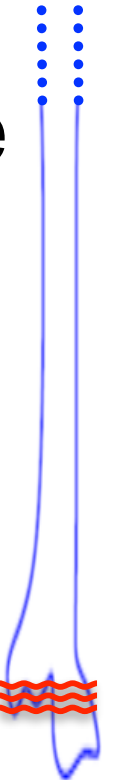


- Claims that CFT_1 does not have such excitations - looking at the wrong ground state

Quantum Gravity in AdS₂

Bena, Heidmann, Turton, to appear

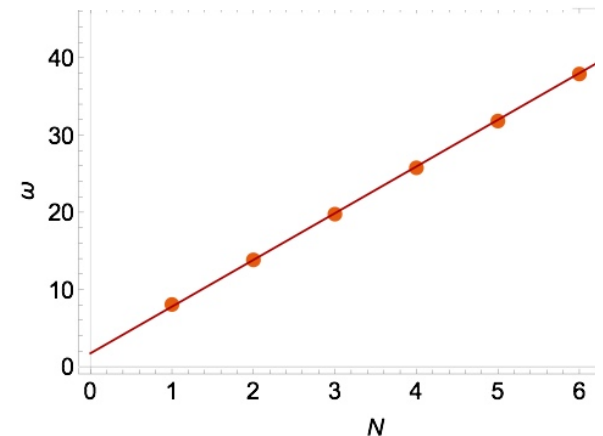
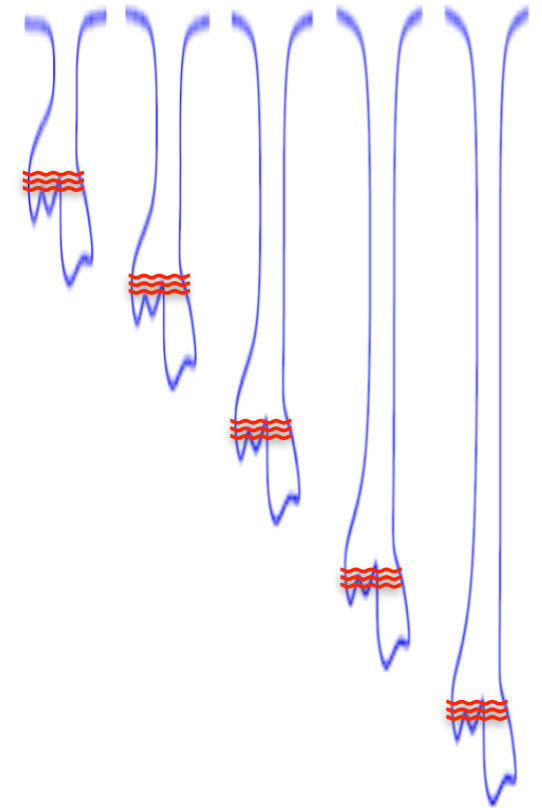
- Non-BPS excitations **do not destroy AdS₂ UV !**
IR cap is crucial
- Microstates of **AdS₂** non-extremal black hole
Castro, Grumiller, Larsen, McNees
- **Kosher** holography - **AdS₂ UV**
- SYK, **J-T**, near-AdS & friends = **non-Kosher:**
non-AdS UV, irrelevant ops, **IR** to **UV** flows
- Can one link the two approaches ?
 - see **absence of conf. symmetry** of ground states from SYK ?



Gluing back to AdS_3

Bena, Heidmann, Turton, to appear

- AdS_2 microstates: $J=0$
- J comes from AdS_2-AdS_3 gluing
- Deep microstate geometries fit inside $AdS_2 \rightarrow$ genuine BH states
Sen
- AdS_3 mass gap depends on length
- Length of AdS_2 throat quantized
CFT dual, deBoer & al
- smallest gap = $2 \times 1/N_1 N_5$
CFT₁ finite-energy excitations \rightarrow
CFT₂ excitations above gap



SUSY microstates – the story:

- We have a huge number of them
 - Arbitrary continuous functions of 2 variables
 - Smooth solutions. 4 scales !
 - Superstrata reproduce black hole entropy ☺
Bena, Shigemori, Warner
- Dual to CFT states in typical sector
 - This is where BH states live too ☺**F**
 - **CFT₁** has no conformally-invariant ground state !
hence **BH** microstates have no horizon
Bena, Wang, Warner; Bena, Heidmann, Turton
- Two non-backreacted calculations:
 - BH entropy - scaling multicenter config ☺
Denef, Moore; Denef, Gaiotto, Strominger, Van den Bleeken, Yin
 - Higgs-Coulomb map.
Bena, Berkooz, de Boer, El Showk, Van den Bleeken; Lee, Wang, Yi

Strominger - Vafa

$$S = S_{\text{BH}}$$

Black Hole Deconstruction

Denef, Gaiotto, Strominger,
Van den Bleeken, Yin (2007)

$$S \sim S_{\text{BH}}$$

Black
Holes

Effective coupling (g_s)

Multicenter Quiver QM

Denef, Moore (2007)
Bena, Berkooz, de Boer, El Showk,
Van den Bleeken.

Smooth Horizonless
Microstate Geometries

$$S \sim S_{\text{BH}}$$

Size grows

No Horizon

Punchline:

Typical states **grow** as G_N increases
Horizon never forms

CFT_1 has **no conformally-invariant ground state**

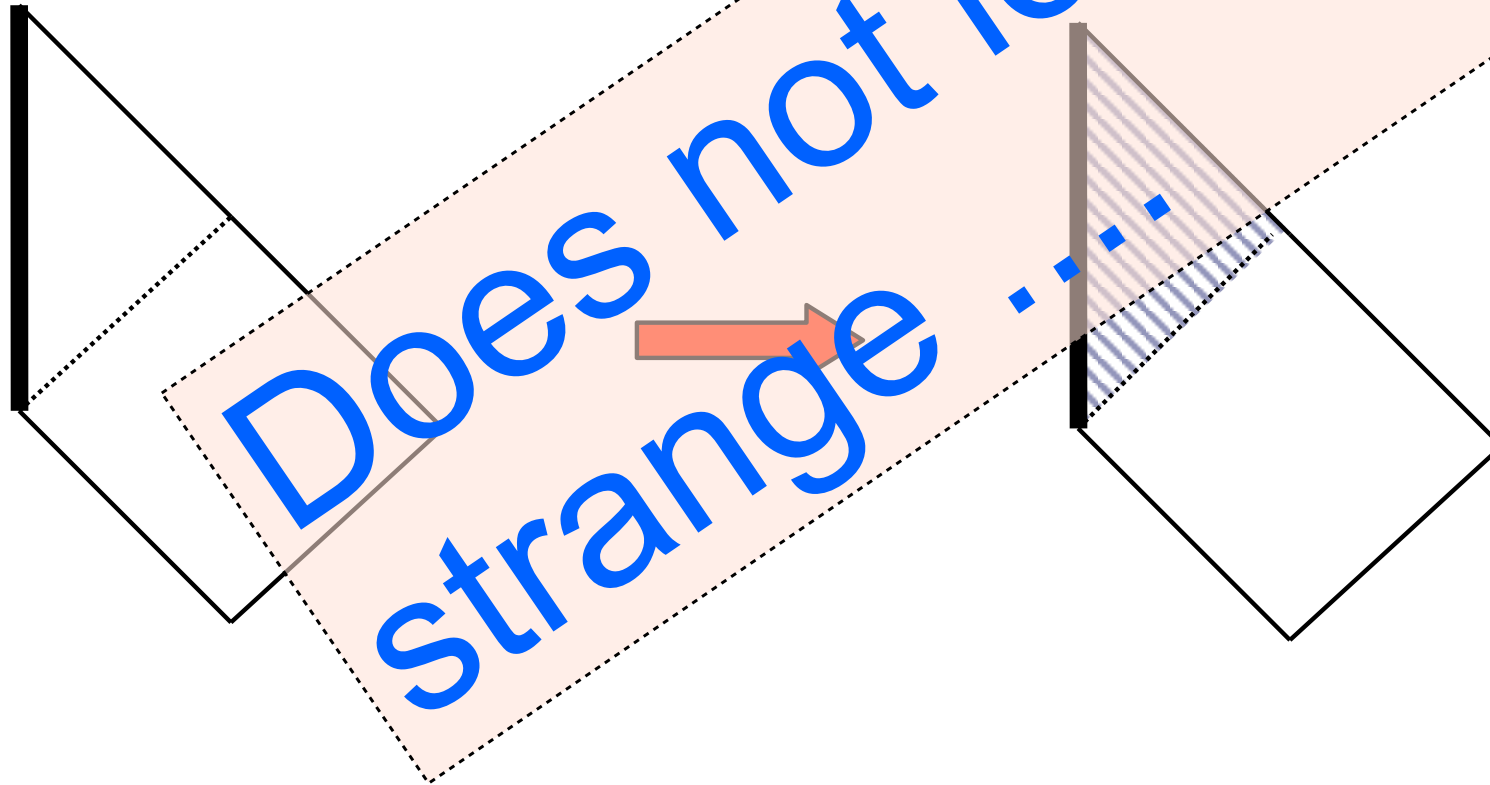
Quantum effects from singularity **extend to horizon**

Similar story for **non-SUSY extremal** black holes

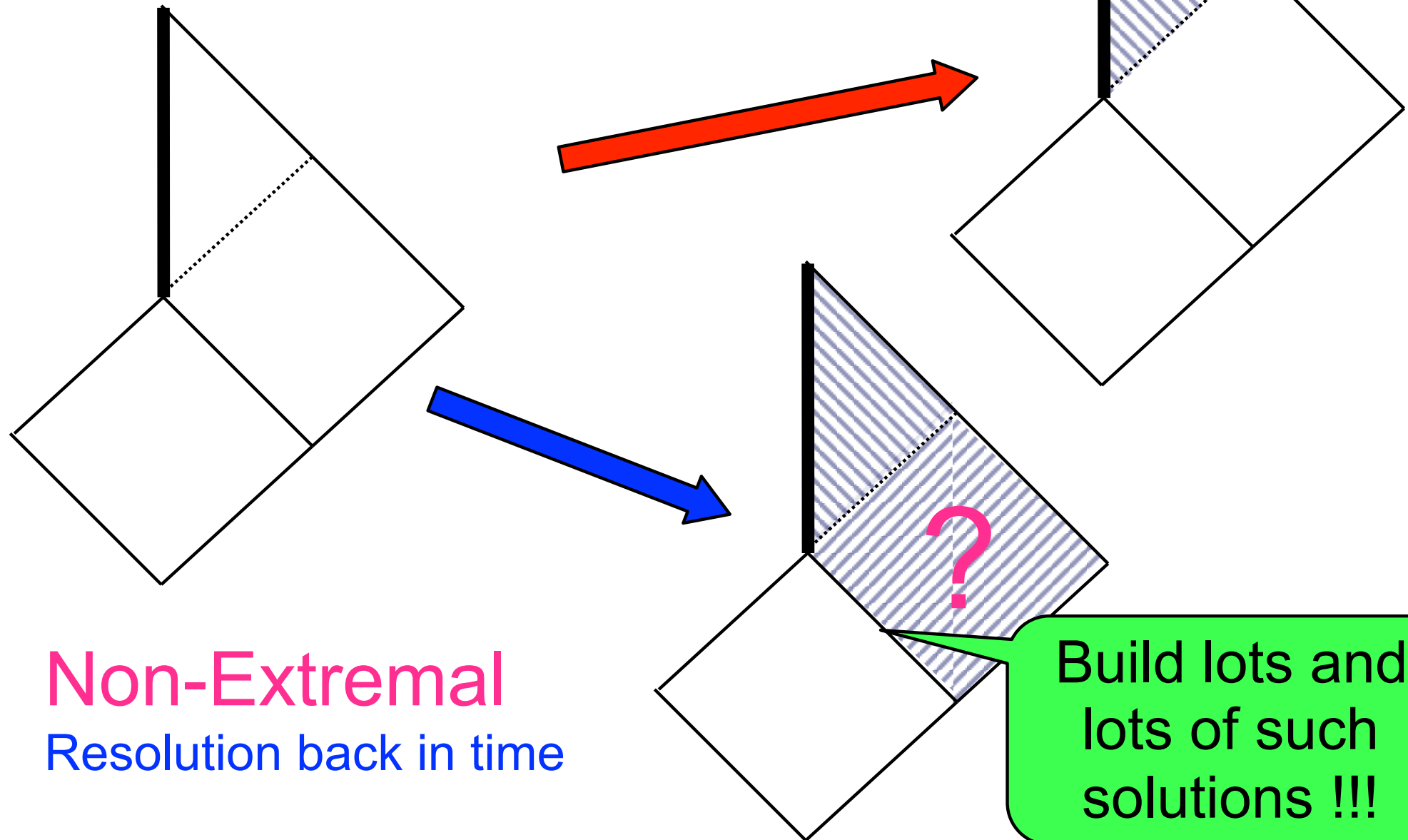
All based on **almost-BPS** ansatz Goldstein, Katmadas

BPS Black Hole = Extremal

- This is **not so strange**
- Horizon **in causal future** of singularity
- **Time-like singularity** resolved by (stringy) low-mass modes extending to horizon



The really big deal fuzzball, firewall



Non-Extremal
Resolution back in time

Build lots and
lots of such
solutions !!!

Very few known. JMaRT. Extremely hard to build...

– Coupled nonlinear 2'nd order PDE's do not factorize

Do not pray to the saint who
does not help you !

Romanian proverb

- Idea: perturbative construction - near-BPS
- Add **antibranes** to BPS bubbling sols.
Kachru, Pearson, Verlinde
- Metastable minima
Bena, Puhm, Vercoocke
- Decay to susy minima:
brane-flux annihilation - Hawking radiation
- Microstates of **near-extremal BH**

Very few known. JMaRT. Extremely hard to build...

– Coupled nonlinear 2nd order PDE's do not factorize

When a bird is blind, God sometimes makes its nest ! another Romanian proverb

- For some solutions the 2nd order PDE's do factorize !!!
- We can build analytically certain classes of non-extremal solutions
- Add extra cycles to JMART
- Method can get us far from extremality.
- How far ? How generic ? Antibranes ?

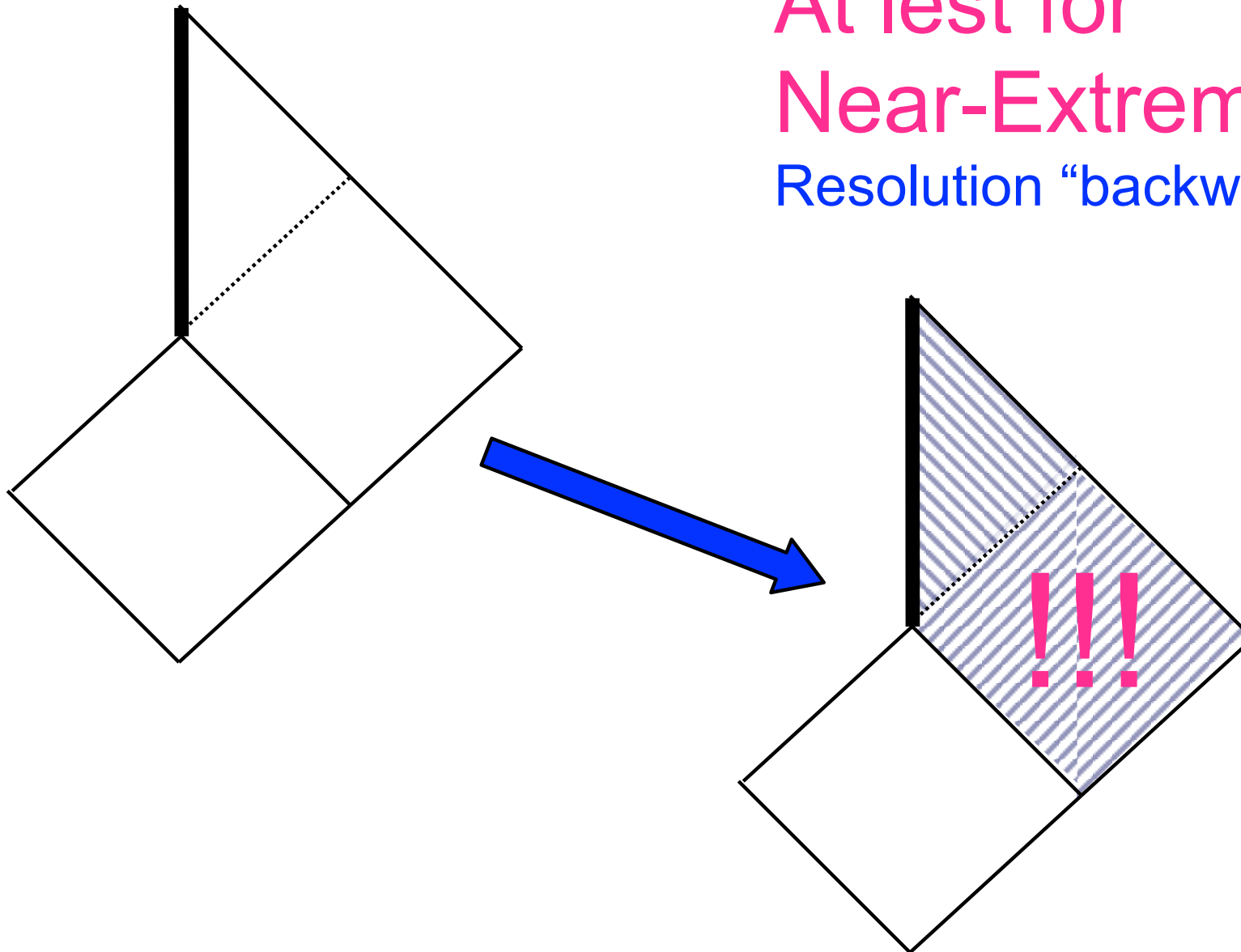
Bossard, Katmadas

Bena, Bossard, Katmadas, Turton

Bossard, Katmadas, Turton

The really big deal

At least for
Near-Extremal
Resolution “backwards in time!”



Pure BH states have no horizon - 4 approaches:

(1) Information-theory arguments Mathur 2009, AMPS, etc

- secondary question: firewall ? burn or sail through ?

(2) Generic AdS-CFT Skenderis Taylor, AMPS2 (Papadodimas Raju against)

- nontrivial vevs \Rightarrow no spherical symmetry \Rightarrow no horizon
- **CFT₁ no conf-invariant ground state** Bena Heidmann, Turton

(3) Follow microstates from weak to strong coupling

- **BH deconstruction, String emission, Higgs-Coulomb map**
Denef, Gaiotto, Strominger, Van den Bleeken, Yin, Giusto, Russo, Turton
Bena, Berkooz, de Boer, El Showk, Van den Bleeken; Lee, Wang, Yi

(4) Lots of BH microstate geometries = Hair !!!

- One mechanism in three hypostases:
Bubbling \Leftrightarrow **Brane polarization** \Leftrightarrow **NonAbelian**
- Can get BH entropy

A few questions

- **Would all microstates be classical ?**
 - Only constructions that include gravity and one can trust.
 - **Hovering mechanism extrapolates** \Rightarrow brane polarization, non-Abelian
 - Typical states: many small bubbles (size $\sim \ell_P$), or few big bubbles ?
 - Larger bubbles - more entropy Denef, Moore; Bena, Shigemori, Warner
- **Don't people in Saclay say antibranes are bad?**
 - **Tachyonic !** Bad for cosmology, **but not for BH !**
 - Instabilities in fact **expected** for non-extremal black hole microstates; **JMaRT (+ bubbles)** has them Myers & al
 - D1-D5: **BPS left-movers** + **right movers** Mathur
- **What about non-linear instabilities ?** Eperon, Santos, Reall
 - first-order backreaction of non-BPS perturbation;
D1-D5 **right movers** \Rightarrow Closed string emission
 - **Moduli space of classical solutions.** non-BPS \Rightarrow Motion
Bena, Pasini Marolf, Michel, Puhm

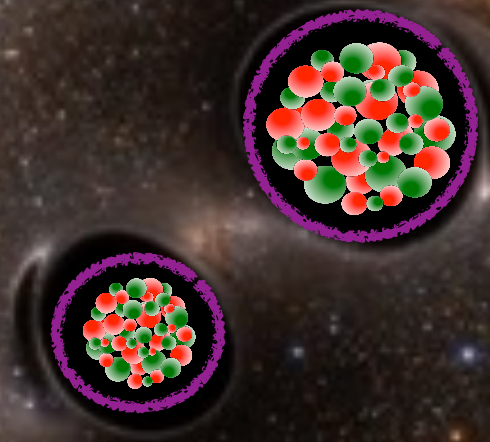
A few questions

- Can you fall through horizon **drinking your coffee** ? (as GR textbooks say)
- Do you rather go **splat** at the horizon scale?
- 3 options:
 - Analyze ∞ **density shells** / membranes / stuff carrying d.o.f. @ horizon (kept from collapsing by the **Tooth Fairy**)
 - Modify gravity by **weird terms** and analyze horizon
 - Use actual solutions of String Theory
- Answer likely depends on E_{gap} , λ_T
- **Known bubbling solutions** or **polarized branes** have no intention to let you fall through unharmed

How can we observe this ?

Universal feature:

- Low-mass degrees of freedom at horizon.



LIGO, eLISA:

Extra dissipation - different gravitational waves
Distortion of the Kerr multipole moments

Summary and Future Directions

- String theory configurations that **hover above horizon**.
Topology + fluxes \Leftrightarrow **brane polarization** \Leftrightarrow **nonabelian d.o.f.**
- **BPS black hole microstates** = horizonless solitons
 - **low-mass modes** affect **large (horizon) scales**
 - Convergence of many research directions
 - BPS **superstrata** - 2 variables - **Black Hole Entropy !**
- Kosher **AdS₂** holography. **CFT₁** no conf. ground state !
- Extend to **non-extremal** black holes
 - **Near**-extremal
 - Metastable supertubes Bena, Puhm, Vernocke
 - **Far** from extremality — 2'nd order nonlinear coupled PDE
 - **Systematic construction** Bena, Bossard, Katmadas, Turton
 - Others: numerics? inverse scattering? blackfolds?
 - Maybe start thinking about **experimental** consequences ?
 - Gravity waves
 - Supermassive BH formation easier

Connection with T-branes

Bena, Blåbäck, Savelli, Zoccarato

$$F^{(0,2)} = 0,$$

$$\bar{\partial}_{\bar{A}} \Phi = 0,$$

$$\omega \wedge F_2 = [\Phi, \Phi^\dagger]$$

$$A_x = \frac{1}{2}(\Phi_1 + i\Phi_2)$$

$$A_y = \frac{1}{2}(\Phi_3 + i\Phi_4)$$

$$\Phi = \frac{1}{2}(\Phi_5 - i\Phi_6)$$

Constant worldvolume fields
T-dualize

$$F^{(0,2)} = -i[A_x, A_y] = 0 \iff \begin{cases} [\Phi_1, \Phi_3] = [\Phi_2, \Phi_4] \\ [\Phi_1, \Phi_4] = [\Phi_3, \Phi_2] \end{cases}$$

$$\bar{\partial}_{\bar{A}_x} \Phi = 0 = -i[A_{\bar{x}}, \Phi] = 0 \iff \begin{cases} [\Phi_1, \Phi_5] = [\Phi_2, \Phi_6] \\ [\Phi_1, \Phi_6] = [\Phi_5, \Phi_2] \end{cases}$$

$$\bar{\partial}_{\bar{A}_y} \Phi = 0 = -i[A_{\bar{y}}, \Phi] = 0 \iff \begin{cases} [\Phi_3, \Phi_5] = [\Phi_4, \Phi_6] \\ [\Phi_3, \Phi_6] = [\Phi_5, \Phi_4] \end{cases}$$

$$\omega \wedge F_2 - [\Phi, \Phi^\dagger] = [A_x, A_{\bar{x}}] + [A_y, A_{\bar{y}}] - [\Phi, \Phi^\dagger] \iff [\Phi_1, \Phi_2] + [\Phi_3, \Phi_4] + [\Phi_5, \Phi_6]$$

Connection with T-branes

Bena, Johan Blåbäck, Savelli, Zoccarato

Solutions with infinite matrices:

$$D_i = \bigotimes_{j=1}^6 ((1 - \delta_{ij})\mathbb{I}_M + \delta_{ij}D) , X_i = \bigotimes_{j=1}^6 ((1 - \delta_{ij})\mathbb{I}_M + \delta_{ij}X) \quad [D, X] = i\mathbb{I}_M$$

$$\Phi_1 = D_1 - \frac{1}{2}\sqrt{\alpha_1}X_1X_3$$

$$\Phi_2 = D_2 - \frac{1}{2}\sqrt{\alpha_1}X_1X_4 - \frac{1}{2}\sqrt{\alpha_2}X_1X_2$$

$$\Phi_3 = D_3 - \frac{1}{2}\sqrt{\alpha_3}X_3X_5$$

$$\Phi_4 = D_4 - \frac{1}{2}\sqrt{\alpha_3}X_3X_6 + \frac{1}{2}\sqrt{\alpha_2}X_3X_4$$

$$\Phi_5 = D_5 + \frac{1}{2}\sqrt{\alpha_2}X_1X_5$$

$$\Phi_6 = D_6 + \frac{1}{2}\sqrt{\alpha_2}X_2X_5 + \frac{1}{2}\sqrt{\alpha_3}X_5X_6$$

D0	-	-	-	-	-	-
D4	×	×	×	×	-	-
D4	×	×	-	-	×	×
D4	-	-	×	×	×	×

D0 description of 4D susy BH

AdS₂

New microscopic counting !