Gravity and the planar spin-2 Schroedinger equation

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work done in collaboration with Jan Rosseel and Paul Townsend

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Outline

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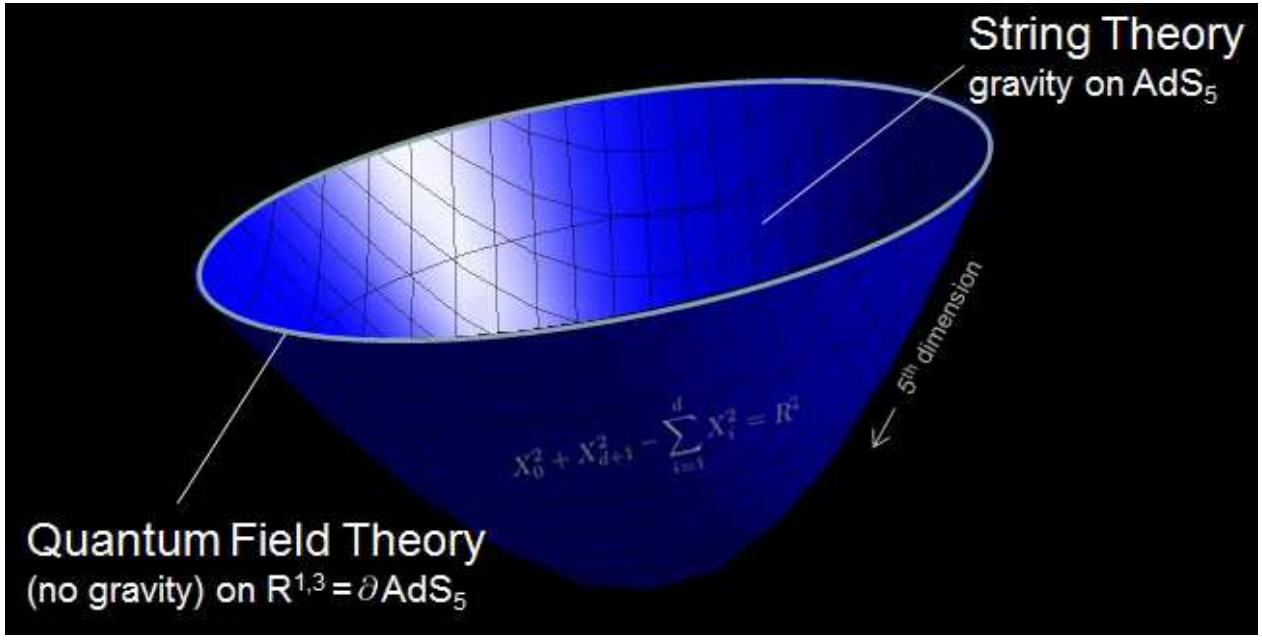
Holography



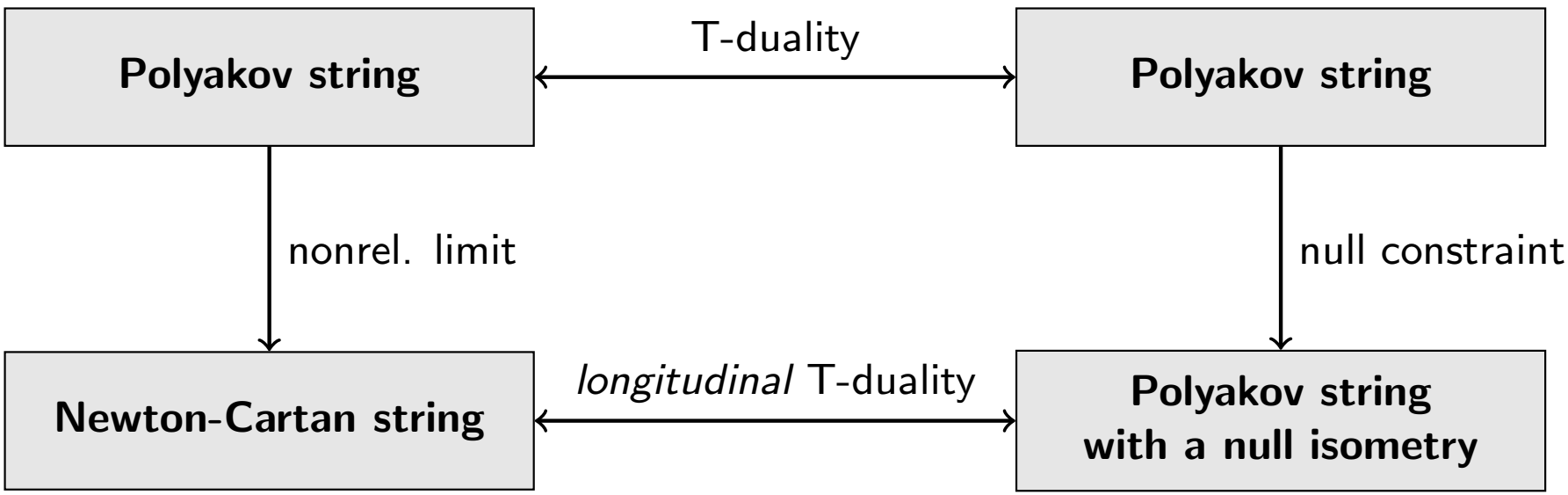
Gravity is not only used to describe the gravitational force!

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

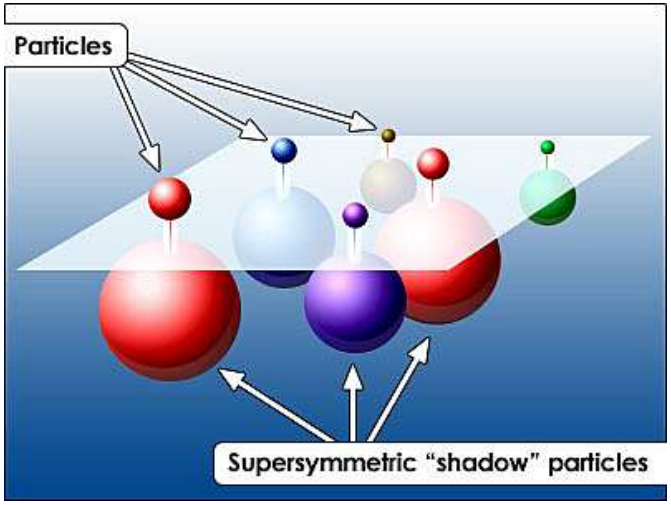
The Newton-Cartan String

Gomis, Ooguri (2001); Gomis, Yan + E.B., in preparation

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{-\tau}\,\tau^{\alpha\beta} h_{\alpha\beta} + \frac{T}{2}\int d^2\sigma\,\epsilon^{\alpha\beta} b_{\alpha\beta}$$



Supersymmetry



supersymmetry allows to apply powerful localization techniques to exactly calculate partition functions of (non-relativistic) supersymmetric field theories

Pestun (2007); Festuccia, Seiberg (2011), Pestun, Zabzine (2016)

Curved Background

$$\{\tau_\mu\,,e_\mu{}^a\,,m_\mu\,,\psi_{\mu\pm}\}\;:\qquad\text{on-shell}$$

Andringa, Rosseel, Sezgin + E.B. (2013)

$$\{\tau_\mu\,,e_\mu{}^a\,,m_\mu\,,\psi_{\mu\pm};S\}\;:\qquad\text{zero torsion off-shell}$$

flat background

Knodel, lisbao and Liu (2016)

$$\{\tau_\mu\,,e_\mu{}^a\,,m_\mu\,,\psi_{\mu\pm};A_\mu\,,A\,,F\,,\psi\}\;:\qquad\text{twistless torsion off-shell}$$

Chatzistavrakidis, Lahnsteiner, Romano, Rosseel + E.B., in preparation

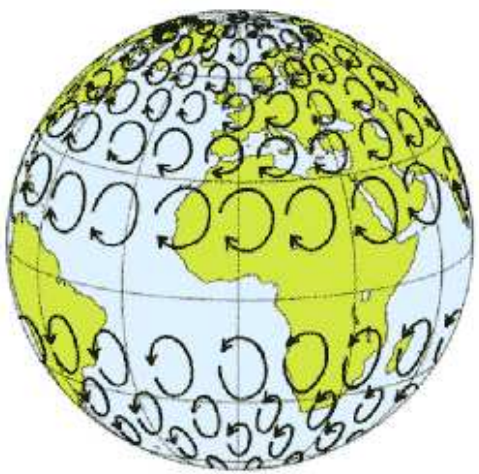
curved background

Condensed Matter

Effective Field Theory (EFT) coupled to NC background fields

serve as response functions and lead to restrictions on EFT

compare to



Coriolis force

Luttinger (1964), Greiter, Wilczek, Witten (1989), Son (2005, 2012), Can, Laskin, Wiegmann (2014)

Jensen (2014), Gromov, Abanov (2015), Gromov, Bradlyn (2017)

what about non-relativistic matter?

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Motivation

special feature FQH Effect: existence of a gapped collective non-rel. parity non-invariant helicity-2 excitation, known as the GMP mode

Girvin, MacDonald and Platzman (1985)

recent proposal for a non-relativistic spatially covariant bimetric EFT describing non-linear dynamics of this massive spin-2 GMP mode

Haldane (2011), Gromov, Geraedts, Bradlyn (2017), Gromov, Son (2017), Nguyen, Gromov, Son (2017)

in a linearized approximation around a flat background this gives rise to a single spin-2 Planar Schrödinger Equation

$$i\hbar\dot{\Psi} + \frac{\hbar^2}{2m}\nabla^2\Psi = 0$$

Key Question

Rosseel, Townsend + E.B. (2018)

has this single helicity 2

Planar Schrödinger Equation

a (massive) gravity origin?

The 'force limit' of spin 0

$$\frac{1}{c^2}\ddot{\Phi} - \nabla^2\Phi + \left(\frac{mc}{\hbar}\right)^2 \Phi = 0$$

Take the non-relativistic limit $c \to \infty$ keeping $\lambda = \hbar/mc$ fixed $\quad \to$

$$\nabla^2\Phi = \frac{1}{\lambda^2}\Phi$$

no massive spin 0 particle!

N.B. The limit can also be taken in an arbitrary background

The 'particle limit' of complex spin 0

$$\frac{1}{c^2}\ddot{\Phi} - \nabla^2\Phi + \left(\frac{mc}{\hbar}\right)^2 \Phi = 0$$

To avoid infinities we redefine

$$\Phi = e^{-\frac{i}{\hbar}(mc^2)t}\psi$$

so that the Klein-Gordon equation becomes

$$-\frac{1}{2mc^2}\left(i\hbar\frac{d}{dt}\right)^2\psi - i\hbar\dot{\psi} - \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

and the $c \to \infty$ limit yields the Schrödinger equation

$$i\hbar\dot{\psi} + \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

General Feature

one complex massive helicity mode $\Leftrightarrow$ one Schrödinger Equation

Alternative Particle Limit of 3D Real Proca

- make time-space decomposition $A_\mu = \left(A_0, \vec{A}\right)$
- eliminate auxiliary field A_0
- rescale $\vec{A} \to \vec{B}$ and define $B = \frac{1}{\sqrt{2}}\left(B_1 + iB_2\right) \quad \Rightarrow$

$$\mathcal{L} = \frac{1}{c^2}\dot{B}^\star \dot{B} + B^\star \nabla^2 B - \left(\frac{mc}{\hbar}\right)^2 B^\star B$$

$$\text{redefine} \quad B = e^{-\frac{i}{\hbar}(mc^2)t}\Psi[1] : \quad \text{breaks parity} \quad \Rightarrow$$

$$i\hbar\dot{\Psi}[1] + \frac{\hbar^2}{2m}\nabla^2\Psi[1] = 0$$

single planar spin-1 Schroedinger equation

From Spin-1 to Spin-2

A_μ: $3 = 1+2$ under spatial SO(2):

$$A_0 \quad \text{and} \quad A_1 + iA_2$$

$f_{\mu\nu}$ with $\eta^{\mu\nu} f_{\mu\nu} = 0$: 5=1+2+2 under spatial SO(2):

$$f_{11} + f_{22}, \quad f_{01} + if_{02} \quad \text{and} \quad \frac{1}{2}\left(f_{11} - f_{22}\right) + if_{12} \quad \Rightarrow$$

single planar spin-2 Schroedinger equation

Same result can be obtained from a SS null-reduction of 4D GR

Towards Interactions: special features of 3D

J. Rosseel, P. Townsend + E.B., work in progress

- 'taking the square-root':

$$\Box - m^2 = O(m)O(-m) \quad \text{with} \quad [O(m)]_\mu{}^\rho = \epsilon_\mu{}^{\tau\rho}\partial_\tau + m\delta_\mu{}^\rho$$

- 'boosting up the derivatives':

$$\partial^\mu A_\mu = 0 \quad \rightarrow \quad A_\mu = \epsilon_\mu{}^{\nu\rho}\partial_\rho B_\sigma$$

- 'CS-like' formulation:

$$L = \tfrac{1}{2} g_{rs} a^r \cdot da^s + \tfrac{1}{6} f_{rst} a_r \cdot \left(a^s \times a^t\right) \qquad r = 1, \cdots, N$$

- take real limit or complex limit followed by self-duality truncation?

Non-relativistic 3D Chern-Simons Like Gravity

- The 3D Galilei and Bargmann algebras do not allow an invariant bilinear form

- Precisely in 3D there exists a so-called Extended Bargmann Algebra with two central extensions and an invariant bilinear from. The second central extension is related to spin

Jackiw, Nair (2000)

- can one use two such algebras to construct a CS-like bi-metric gravity theory describing the non-linear dynamics of a massive spin 2 particle instead of a massive deformation of Poisson's equation?

Outline

Comment

- If a non-relativistic limit exists, it may lead to interesting connections between 3D gravity and condensed matter concerning
  - higher derivatives
  - higher spins

Golkar, Dung Xuan Nguyen, Roberts, Son (2016)

Take Home Message

Newton-Cartan Geometry leads to fruitful interactions between holography, effective field theory and supersymmetry. It even has connections with engineering!