Non-perturbative decay of non-Abelian hair

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based on

PAC and Tomás Ortín, JHEP 1712 (2017) 091, arXiv: 1710.05052

Geometry, Duality and Strings
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Motivation

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- Framework: continuation of Pedro's and Tomas' talks: black holes in Heterotic string with α' corrections
- Let us remember the entropy and mass of black holes at order $\mathcal{O}(\alpha')$
  PAC, Meessen, Ortin, Ramirez

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\begin{aligned}
S &= 2\pi\sqrt{N_{S5}N_{F1}N_W}\,(1+8/N_{S5}) \\
M &= \frac{R_z^2}{\ell_s^2 g_s^2}N_{S5} + \frac{R_z}{\ell_s^2}N_{F1} + \frac{1}{R_z}N_W\,(1+16/N_{S5})
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This solution contains

N_{S5} solitonic 5-branes

N_{F1} fundamental strings

N_W units of momentum

$N_{G5} = \mathbf{1}$ **gauge 5-brane**

- The gauge 5-brane is important because it cancels part of the α' corrections
- But there is an obvious reason why this black hole might be unstable

Motivation

What if we remove the gauge 5-brane?

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Even better: if we exchange the gauge 5-brane by 8 S5 branes

$$N'_{G5} = 0\,, \quad N'_{S5} = N_{S5} + 8$$

the mass and charges remain unchanged, but the entropy increases

$$M' = M\,, \quad Q' = Q, \quad S' = S + 8\pi\sqrt{\frac{N_{F1}N_W}{N_{S5}}}$$

- The purely Abelian solution is thermodynamically preferred
- Spontaneous decay $|A|^2 \sim e^{\Delta S}$???
- Not so easy: our solution is protected by **supersymmetry and topology**
- **But non-perturbative decay is still possible**

Interpolating solution

Interpolating solution

Let us consider the following solution of Heterotic String Effective action at $\mathcal{O}(\alpha')$:

Interpolating solution $\mathrm{AdS}_3 \times S_3 \times T^4 \rightarrow \mathrm{AdS}_3' \times S_3' \times T^4$

$$ds^2 = \frac{2\rho^2}{Q_-} du dv - \frac{Q_+}{Q_-} du^2 - R^2(\rho) \left(\frac{d\rho^2}{\rho^2} + d\Omega^2_{(3)} \right) - dy^i dy^i \,,$$

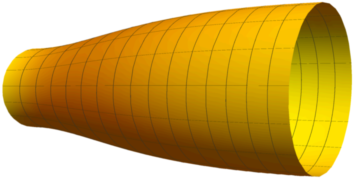
$$e^{2\phi} = e^{2\phi_\infty} \frac{R^2(\rho)}{Q_-} \,, \quad A^A = -\frac{\rho^2}{\kappa^2 + \rho^2} v_L^A \,,$$

$$B = -\frac{\rho^2}{Q_-} dv \wedge du - \frac{Q_0}{4} \cos\theta d\psi \wedge d\varphi \,,$$

where $R^2(\rho)$ is the function

$$R^2(\rho) = Q_0 + 8\alpha' \frac{\rho^2(\rho^2 + 2\kappa^2)}{(\kappa^2 + \rho^2)^2}$$

$$R^2(0) = Q_0, \; R^2(\infty) = Q_0 + 8\alpha'$$



Interpolating solution

Each AdS$_3 \times S_3 \times T^4$ factor represents the near-horizon geometry of a black hole

- At $\rho = 0$: $R(0)^2 = Q_0$ $A_0^A = 0$

  Near-horizon of black hole with

$$Q_0 = \alpha' N_{S5}\,, \quad Q_- = \alpha' g_s^2 N_{F1}\,, \quad Q_+ = \frac{g_s^2 \alpha'^2}{R_z^2} N_W$$

- At infinity: $R(\infty)^2 = Q_0 + 8\alpha'$ $A_\infty^A = -v_L^A = \text{pure gauge}$

We must gauge-transform $A_\infty^A \to A_\infty^{A\prime} = 0$. After this gauge transformation, the 2-form B takes the form

$$B \to B' = -\frac{\rho^2}{Q_-} dv \wedge du - \frac{Q_0 + 8\alpha'}{4} \cos\theta d\psi \wedge d\varphi\,,$$

8 S5-branes have been generated at infinity $N'_{S5} = N_{S5} + 8$

Interpolating solution

- $\rho = 0$ is the near-horizon of a black hole with N_{S5} S5 branes
- $\rho \to \infty$ is the near-horizon of a black hole with $N_{S5} + 8$ S5 branes

This solution represents the decay of a gauge 5-brane into 8 S5 branes

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Problem: the solution is Lorentzian

We would need an **Euclidean version** of the solution, *i.e.* we need to **Wick-rotate it**.

In such case, the transition probability amplitude could be estimated as

$$\mathcal{Z} \sim e^{-S_E}$$

The rest of the talk will be devoted to find such Wick rotation

Construction of the Euclidean Instanton

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Wick rotation of generic spacetimes is a far from trivial problem. *e.g.* Visser

It is well-known that Wick rotation is problematic in extremal solutions

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- **Magnetically charged RN BH.** The magnetic charge is not rotated. There is an Euclidean extremal black hole, but the Euclidean action yields a wrong value for the entropy Hawking, Horowitz, Ross

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- **Kerr BH**. Wick rotation requires $a \to ia$. There is no Euclidean extremal Kerr BH.

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Wick rotation of extremal solutions either doesn't exist — there are too few parameters— or it doesn't give the right result.

It is convenient to work with the non-extremal solution and afterwards take the extremal limit

Construction of the Euclidean Instanton

Problem 1: dyonic 2-form

$$B = -\frac{\rho^2}{Q_-} dv \wedge du - \frac{Q_0}{4} \cos\theta d\psi \wedge d\varphi \,,$$

- $Q_- \to iQ_-$? It does not work

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- We get an auxiliar theory with a 2-form and a 6-form, with corresponding field strengths H' and $\tilde{H}$
- Any solution of this action satisfying the constraint $H \wedge \tilde{H} = 0$ is a solution of the original Heterotic Supergravity with

$$H = H' + e^{2\phi} \star \tilde{H} \,.$$

Construction of the Euclidean Instanton

Expressed in these variables, the solution reads

$$ds^2 = \frac{2\rho^2}{Q_-}dudv - \frac{Q_+}{Q_-}du^2 - R^2(\rho)\left(\frac{d\rho^2}{\rho^2} + d\Omega^2_{(3)}\right) - dy^i dy^i \,,$$

$$e^{2\phi} = e^{2\phi_\infty}\frac{R^2(\rho)}{Q_-}\,,$$

$$B' = -\frac{Q_0}{4}\cos\theta d\psi \wedge d\varphi\,,$$

$$\tilde{B} = -\frac{e^{-2\phi_\infty}Q_-}{4}\cos\theta d\psi \wedge d\varphi \wedge dy^6 \wedge dy^7 \wedge dy^8 \wedge dy^9\,,$$

$$A^A = -\frac{\rho^2}{\kappa^2+\rho^2}v^A_L\,.$$

All the forms are now magnetic and unaffected by Wick rotation.
We only need to care about the metric

Problem 2: Wick-rotation of the metric

After the change of variables $v = (t-x)/\sqrt{2}$, $u = (t+x)/\sqrt{2}$ we get

$$ds^2 = \frac{1}{Q_-}\left[\left(\rho^2 - \frac{Q_+}{2}\right)dt^2 - Q_+ dt dx - \left(\rho^2 + \frac{Q_+}{2}\right)dx^2\right] - R^2(\rho)\left(\frac{d\rho^2}{\rho^2} + d\Omega^2_{(3)}\right) - dy^i dy^i$$

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- When $Q_+ = 0$: $t = -i\tau$, but this case is not interesting

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- When $Q_+ \neq 0$ there is no way to Wick-rotate this metric
- The problem is related to extremality: we have too few parameters
- Idea: deform the metric by adding a non-extremality parameter

Construction of the Euclidean Instanton

Non-Extremal Off-Shell (NEOS) deformation

$$ds^2_{\text{NEOS}} = \frac{1}{Q_-}\left[\left((\rho+\rho_0)^2 - \frac{Q_+}{2}\right)dt^2 - a\,dt\,dx - \left((\rho+\rho_0)^2 + \frac{Q_+}{2}\right)dx^2\right]$$
$$- R^2(\rho)\left[\frac{(\rho+\rho_0)^2 d\rho^2}{(\rho+\rho_0)^4 - \rho_0^4} + d\Omega^2_{(3)}\right] - dy^i dy^i \,, \text{ where } \quad \rho_0^2 \equiv \frac{1}{2}\sqrt{Q_+^2 - a^2}$$

It is not a solution of the equations of motion, but it fulfills a series of properties

- Continuous deformation of the solution (for $a = Q_+$ we recover our original solution)
- It is non-extremal $T_H = \beta^{-1} = \frac{1}{2\pi}\sqrt{\frac{(Q_+^2 - a^2)^{1/2}}{Q_0 Q_-}}$
- Interpolates between two same $\text{AdS}_3 \times S_3 \times T^4$ geometries as the original metric
- For arbitrary a the NEOS metric can be Wick-rotated into a Euclidean metric by performing $t = -i\tau$, $a = i\aleph$

Construction of the Euclidean Instanton

We use this metric in order to compute the Euclidean action

- For arbitrary a we Wick rotate the metric $t = -i\tau$, $a = i\aleph$
- We compute the Euclidean action (with GHY term + counterterms) as a function of $\aleph$, $S_E(\aleph)$

$$S_E(\aleph) = \frac{\pi R_z \sqrt{Q_0 Q_+ Q_-}}{2 l_s^4} \frac{\Delta(\aleph)}{\rho_0^2}$$, where $\Delta(\aleph)$ is an analytic function

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- We remove Wick rotation $\aleph = -ia$ and take the extremal limit $a \to Q_+$

$$\lim_{a \to Q_+} \frac{\Delta(-ia)}{\rho_0^2} = g_s^2 \log\left(1 + \frac{8\alpha'}{Q_0}\right) \approx \frac{8\alpha' g_S^2}{Q_0}$$

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In terms of the stringy charges, we get

$$S_E = 4\pi \sqrt{\frac{N_{F1} N_W}{N_{S5}}}$$

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Therefore, the transition probability amplitude is

$$|\mathcal{A}|^2 \sim e^{-8\pi\sqrt{\frac{N_{F1}N_W}{N_{S5}}}}$$

This result coincides with $e^{-\Delta S}$, where ΔS is the change of Black hole entropy in the process:

$$\Delta S = 8\pi\sqrt{\frac{N_{F1}N_W}{N_{S5}}}$$

- Note the change in the sign wrt statistical mechanics prediction $\sim e^{\Delta S}$. **Topology suppresses the decay.**
- Validity of ST background requires $N_W >> N_{F1} >> N_{S5}$: the gauge 5-brane is long lived
- The decay rate increases when N_{S5} is larger. **S5-branes are hungry for gauge 5-branes.**

Conclusions

- We argued that non-Abelian hair in α'-corrected black holes might be unstable
- We found a tunneling process by which the non-Abelian hair could decay
- We introduced a new method to Wick-rotate Lorentzian metrics: the NEOS deformation. Using this method we computed the Euclidean action of the tunneling process
- On general circumstances the tunneling amplitude is tiny and the non-Abelian hair is long-lived

Conclusions

Thank you for your attention

Conclusions

Heterotic supergravity

$$S=\frac{g_s^2}{16\pi G_N^{(10)}}\int d^{10}x\sqrt{|g|}e^{-2\phi}\left[R-4(\partial\phi)^2+\tfrac{1}{2\cdot 3!}H^2-\alpha' F^A F^A\right]$$

Dual action

$$\tilde{S}=\frac{g_s^2}{16\pi G_N^{(10)}}\int d^{10}x\sqrt{|g|}\left\{e^{-2\phi}\left[R-4(\partial\phi)^2+\frac{1}{2\cdot 3!}H^2-\alpha' F^A F^A\right]\right.$$

$$\left.+\frac{e^{2\phi}}{2\cdot 7!}\tilde{H}^2-\frac{\alpha'}{2\cdot 6!\sqrt{|g|}}\epsilon^{\mu_1\cdots\mu_6\alpha\beta\gamma\delta}\tilde{B}_{\mu_1\cdots\mu_6}F^A{}_{\alpha\beta}F^A{}_{\gamma\delta}\right\}$$

$$H\wedge\tilde{H}=0\,,\qquad H_{\text{Het.}}=H+e^{2\phi}\star\tilde{H}\,.$$

Conclusions

Euclidean action + boundary terms

$$S_E = \frac{g_s^2}{16\pi G_N^{(10)}} \int_{\mathcal{M}} d^{10}x \sqrt{|g_E|} \left\{ e^{-2\phi} \left[R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 + \alpha' F^A F^A \right] \right.$$

$$\left. + \frac{e^{2\phi}}{2\cdot 7!} \tilde{H}^2 + \alpha' \frac{\epsilon^{\mu_1 \cdots \mu_6 \alpha\beta\gamma\delta}}{2\cdot 6! \sqrt{|g_E|}} \tilde{B}_{\mu_1 \cdots \mu_6} F^A{}_{\alpha\beta} F^A{}_{\gamma\delta} \right\}$$

$$+ \frac{g_s^2}{8\pi G_N^{(10)}} \int_{\partial\mathcal{M}} d^9x \sqrt{|h_E|} \left[- \frac{e^{2\phi}}{2\cdot 6!} n^\mu (\tilde{H} \cdot \tilde{B})_\mu - e^{-2\phi} (K - K_0) \right]$$