Holographic interpretation of non-Abelian T-duals

Based on: 1705.09661, 1609.09061

In collaboration with:
G. Itsios, Y. Lozano, N.T. Macpherson, Carlos Núñez

Jesús Montero

Universidad de Oviedo

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

Geometry, Duality and Strings 2018
Universidad de Murcia. May 24th, 2018.

Outline

1. Introduction: (Non-)Abelian T-duality
2. $\mathcal{N} = 1$ (N)ATD of Klebanov-Witten as mass deformations
3. Conclusions

Non-Linear Sigma Model formalism of T-duality

- 2D CFT associated to string in $D = 10$ target space:

$$S = \int d^2\sigma \, (G_{mn} + B_{mn}) \, \partial_+ X^m \partial_- X^n$$

- Assume $U(1)$ isometry $X^m = (\theta, X^\mu)$, gauge it $\partial_\pm \to \partial_\pm + A_\pm$, and add Lagrange multiplier $\tilde{\theta} \in u(1)$ to enforce a flat connection: [Roček, Verlinde '91]

$$+\tilde{\theta} \int (\partial_+ A_- - \partial_- A_+)$$

- Solving for $A = A(\partial\tilde{\theta}, \partial X^\mu) \implies$ dual action with different target space:

$$\hat{S} = \int d^2\sigma \, (\hat{G}_{mn} + \hat{B}_{mn}) \, \partial_+ \hat{X}^m \partial_- \hat{X}^n \, , \qquad \hat{X}^m = (\tilde{\theta}, X^\mu)$$

- Transformation rules for dilaton and RR sector can also be derived:

$$F_p \longrightarrow F_{p\pm 1} \qquad\qquad (\mathrm{IIA} \longleftrightarrow \mathrm{IIB})$$

[Hassan '99]

Non-Linear Sigma Model formalism of T-duality

- 2D CFT associated to string in $D = 10$ target space:

$$S = \int d^2\sigma \, (G_{mn} + B_{mn}) \, \partial_+ X^m \partial_- X^n$$

- Assume $U(1)$ isometry $X^m = (\theta, X^\mu)$, gauge it $\partial_\pm \to \partial_\pm + A_\pm$, and add Lagrange multiplier $\tilde{\theta} \in u(1)$ to enforce a flat connection: [Roček, Verlinde '91]

$$+\tilde{\theta} \int (\partial_+ A_- - \partial_- A_+)$$

- Solving for $A = A(\partial\tilde{\theta}, \partial X^\mu) \implies$ dual action with different target space:

$$\hat{S} = \int d^2\sigma \, (\hat{G}_{mn} + \hat{B}_{mn}) \, \partial_+ \hat{X}^m \partial_- \hat{X}^n \,, \qquad \hat{X}^m = (\tilde{\theta}, X^\mu)$$

- Transformation rules for dilaton and RR sector can also be derived:

$$F_p \longrightarrow F_{p\pm1}, F_{p\pm3} \qquad (\mathrm{IIA} \longleftrightarrow \mathrm{IIB})$$ [Sfetsos, Thompson'10]

NATD: $SU(2)$-isometry (S^3 in target space) and 3 Lagrange mult. $\in su(2)$.

[de la Ossa, Quevedo '92]

Dealing with non-trivial worldsheets

ATD case: Cancel $\oint A$ holonomies (arising from partial integration) for any number of holes, making $\tilde{\theta}$ periodic,

$$\int d\theta \wedge d\tilde{\theta} = (2\pi)^2 \quad \Longrightarrow \quad S^1 \xrightarrow{ATD} \tilde{S}^1$$

NATD: Non-commutativity ruins the argument for non-spherical WS's,

$$\int \tilde{\theta} \left(\partial_+ A_- - \partial_- A_+ - [A_+, A_-] \right)$$

- NATD not shown to be a full string theory symmetry.
- Lagrange multipliers $\in su(2) \approx \mathbb{R}^3$:

compact $S^3 \xrightarrow{NATD}$ non-compact $\mathbb{R}^+ \times S^2$ locally

- Interior space of finite volume required for consistent dual SCFT. How can we implement a "cut-off"? Need extra information not provided by NATD!

[Lozano, Núñez '16]

NATD: Supersymmetric solution generating technique

New explicit solutions

- IIB $\mathcal{N} = 2$ $AdS_6 \times S^2$ from the unique Brandhuber-Oz $AdS_6 \times S^4$.
  Motivated classifications by [Tomasiello et al. '14], [D'Hoker et al. '16 & '17].

  [Lozano, Ó Colgáin, Rodríguez-Gómez, Sfetsos '12]

- 11D $\mathcal{N} = 2$ AdS_4 with magnetic G_4

  [Lozano, Macpherson, JM '15]

- 11D $\mathcal{N} = (0,4)$ $AdS_3 \times S^2$

  [Lozano, Macpherson, JM, Colgáin '15]

Interest of new solutions...

1. Motivate new classifications.
2. Identify them as explicit examples of existing supersymmetric classifications.
3. If outsiders, generalize the classification using Killing spinor techniques.

[Kelekci, Lozano, JM, Colgáin, Park '16]

NATD: Supersymmetric solution generating technique

New explicit solutions

- IIB $\mathcal{N} = 2$ $AdS_6 \times S^2$ from the unique Brandhuber-Oz $AdS_6 \times S^4$. **Motivated classifications** by [Tomasiello et al. '14], [D'Hoker et al. '16 & '17].

  [Lozano, Ó Colgáin, Rodríguez-Gómez, Sfetsos '12]

- 11D $\mathcal{N} = 2$ AdS_4 with magnetic G_4 1st explicit solution since the uplift from 7D SUGRA of [Pernici, Sezgin '85]. [Lozano, Macpherson, JM '15]

- 11D $\mathcal{N} = (0, 4)$ $AdS_3 \times S^2$

  [Lozano, Macpherson, JM, Colgáin '15]

Interest of new solutions. . .

1. Motivate new classifications.
2. Identify them as explicit examples of existing supersymmetric classifications.
3. If outsiders, generalize the classification using Killing spinor techniques.

   [Kelekci, Lozano, JM, Colgáin, Park '16]

NATD: Supersymmetric solution generating technique

New explicit solutions

- IIB $\mathcal{N} = 2$ $AdS_6 \times S^2$ from the unique Brandhuber-Oz $AdS_6 \times S^4$.
  Motivated classifications by [Tomasiello et al. '14], [D'Hoker et al. '16 & '17].

  [Lozano, Ó Colgáin, Rodríguez-Gómez, Sfetsos '12]

- 11D $\mathcal{N} = 2$ AdS_4 with magnetic G_4 1st explicit solution since the uplift from 7D SUGRA of [Pernici, Sezgin '85]. [Lozano, Macpherson, JM '15]

- 11D $\mathcal{N} = (0, 4)$ $AdS_3 \times S^2$ with G_4 flux beyond the Ansatz of known class by [Kim et al. '07] [Lozano, Macpherson, JM, Colgáin '15]

Interest of new solutions. . .

1. Motivate new classifications.
2. Identify them as explicit examples of existing supersymmetric classifications.
3. If outsiders, generalize the classification using Killing spinor techniques.

[Kelekci, Lozano, JM, Colgáin, Park '16]

Our purpose for this talk

[Itsios, Lozano, JM, Núñez '17]

Propose a 4D $\mathcal{N}=1$ linear quiver gauge theory dual to the NATD of Klebanov-Witten, assuming the following diagram,

$$
\begin{array}{llcl}
\mathcal{N}=2: & AdS_5\times S^5/\mathbb{Z}_2 & \longrightarrow & \text{NATD of } AdS_5\times S^5/\mathbb{Z}_2 \\
 & \Big\downarrow \text{mass} & & \Big\downarrow \text{mass} \\
\mathcal{N}=1: & AdS_5\times T^{1,1} & \longrightarrow & \text{NATD of } AdS_5\times T^{1,1}
\end{array}
$$

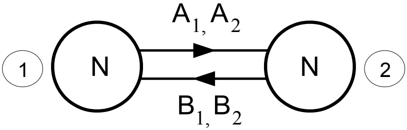
Motivated by: $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32} \longrightarrow \dfrac{c_{NA\ AdS_5\times T^{1,1}}}{c_{NA\ AdS_5\times S^5/\mathbb{Z}_2}} \approx \dfrac{27}{32}$

Outline

1. Introduction: (Non-)Abelian T-duality
2. $\mathcal{N} = 1$ (N)ATD of Klebanov-Witten as mass deformations
3. Conclusions

$\mathcal{N} = 1$ Klebanov-Witten theory ('97)

$SU(N) \times SU(N)$ gauge group with bifundamental matter fields A_1, A_2 and B_1, B_2, transforming in the $(N, \bar{N})$ and $(\bar{N}, N)$ representations of $SU(N)$, respectively:



Theory living on N D3-branes at the tip of the conifold with base $T^{1,1}$

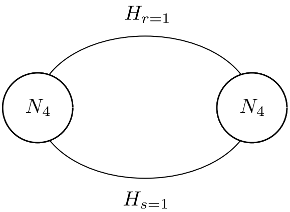
AdS/CFT dual to $AdS_5 \times T^{1,1}$

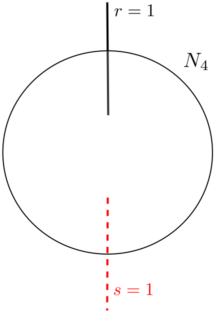
$$ds^2_{T^{1,1}} = \lambda_1^2 ds^2(S_1^2) + \lambda_2^2 ds^2(S_2^2) + \lambda^2(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2)^2 \ ,$$

$$F_5 = \frac{4}{L}\Big(\mathrm{Vol}(AdS_5) - L^5 \mathrm{Vol}(T^{1,1})\Big)$$

IIA Abelian T-dual of KW on $S^1 \hookrightarrow T^{1,1} \to S^2 \times S^2$

Electric F_6 for D4-branes, magnetic B_2 along $\mathbf{S}_1^2$ **for NS5-branes** and along S_2^2 for NS5′-branes, dual to $\mathcal{N} = 1$ theory. [Uranga '98], [Dasgupta, Mukhi '98]

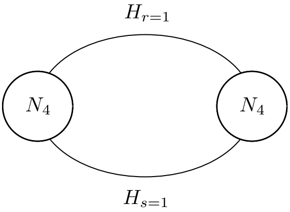


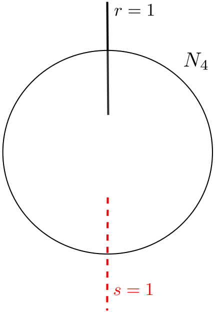


	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x			x			
NS5	x	x	x	x	**x**	**x**				
NS5'	x	x	x	x				x	x	

IIA Abelian T-dual of KW on $S^1 \hookrightarrow T^{1,1} \to S^2 \times S^2$

Electric F_6 for D4-branes, magnetic B_2 along $\mathbf{S}^2_1$ **for NS5-branes** and along S^2_2 for NS5′-branes, dual to $\mathcal{N} = 1$ theory. [Uranga '98], [Dasgupta, Mukhi '98]





Mass deformation $\longleftrightarrow$ relative rotation of NS5-branes

- NS5 $\parallel$ NS5′: $\mathcal{N} = 2$ $AdS_5 \times S^5/\mathbb{Z}_2$.
- NS5 $\perp$ NS5′: $\mathcal{N} = 1$ Klebanov-Witten.

IIA NATD of KW on $S^3 \hookrightarrow T^{1,1} \rightarrow S^2$: brane content

- Electric components for F_6, F_8 suggest D4, D6 branes.
- Large gauge transformations are introduced to keep $\frac{1}{4\pi^2\alpha'} \int_{\Sigma_2} B_2 \in [0,1[$

$$\rho \in [\rho_n, \rho_{n+1}) \; : \qquad B_2 \longrightarrow B_2 - n\pi\alpha' \, \mathrm{Vol}(\Sigma_2)$$

- In the presence of B_2, Page charges are quantized (and get shifted by LGTs):

$$N_6 = \frac{1}{2\,\kappa_{10}^2\, T_6} \int_{S_1^2} F_2 = \frac{2}{27} \frac{L^4}{g_s^2 \alpha'^2}$$

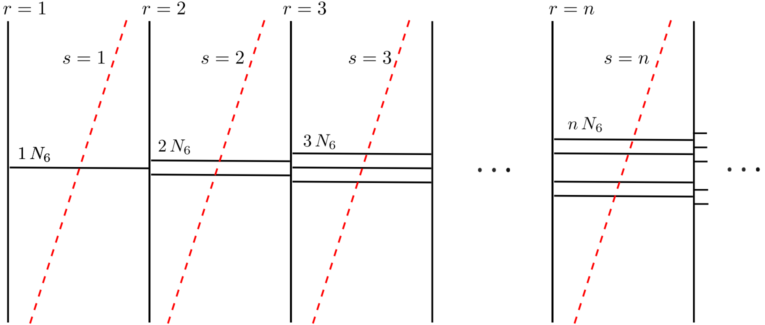
$$N_4 = \frac{1}{2\,\kappa_{10}^2\, T_4} \int_{S_1^2 \times S^2} \left(F_4 - F_2 \wedge B_2\right) = n\, N_6$$

$$N_{NS5} = \frac{1}{2\kappa_{10}^2\, T_{NS5}} \int_{S^2} \int_0^{n\,\pi} d\rho \;\; H_3 = n \, , \qquad N_{NS5'} = \frac{1}{2\kappa_{10}^2\, T_{NS5}} \int_{S_1^2} \int_0^{m\,\pi} d\rho \;\; H_3 = m$$

- Two different 2-cycles allow for two different sets of LGTs, but only those around S^2 shift the colour D4-charge.

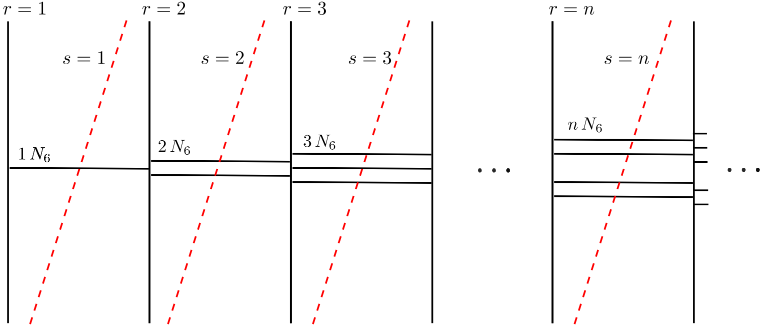
NATD on $S^3 \hookrightarrow T^{1,1} \to S^2$: Brane set-up

[Itsios, Núñez, Sfetsos, Thompson '13]
[Itsios, Lozano, JM, Núñez '17]

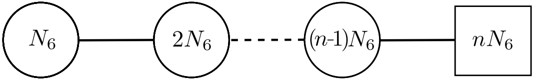


- D4-brane charge created each time an NS5-brane (not NS5′) is crossed.
- $\mathcal{N} = 1$ and $\mathcal{N} = 2$ vector multiplets connected by bifundamentals.
- Realizes a 4d $\mathcal{N} = 1$ linear quiver gauge theory generalizing [Bah, Bobev '13].
- A priori infinite brane set-up and quiver!

[Itsios, Núñez, Sfetsos, Thompson '13]
[Itsios, Lozano, JM, Núñez '17]



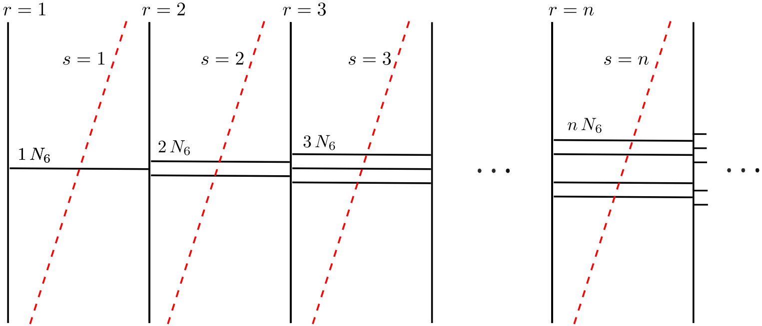
- Propose a "regularised quiver": Choose finite n for cut-off by means of flavours. NATD solution should arise for $\rho << \rho_{\text{cut-off}}$.



- For a hard cut-off, regularized quiver has vanishing scaling dimensions.

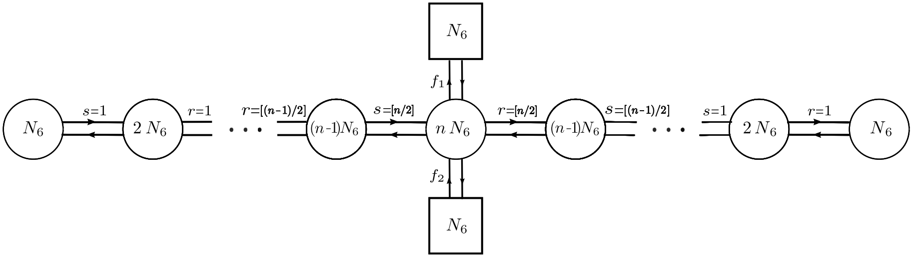
NATD on $S^3 \hookrightarrow T^{1,1} \to S^2$: Brane set-up

[Itsios, Núñez, Sfetsos, Thompson '13]
[Itsios, Lozano, JM, Núñez '17]



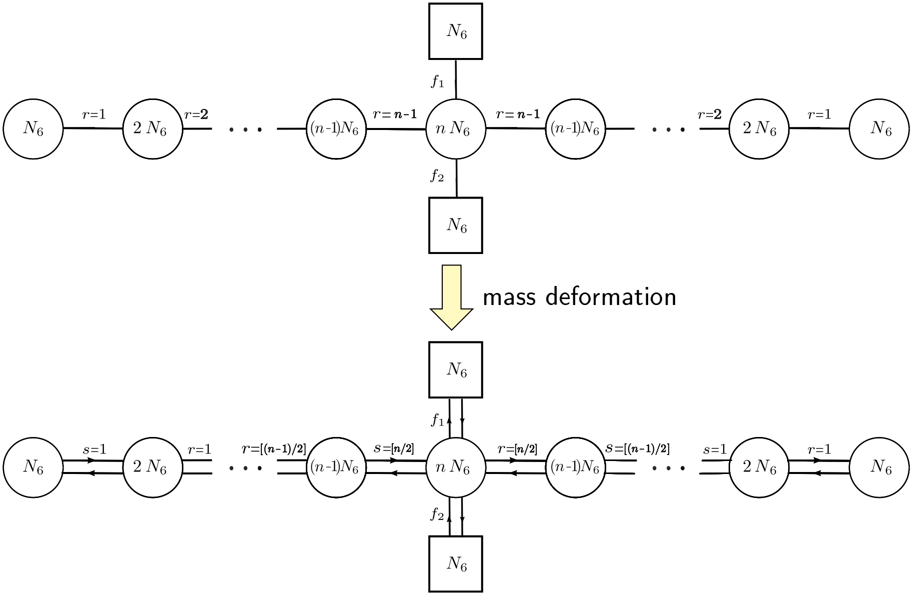
- Propose a "regularised quiver": Choose finite n for cut-off by means of flavours. NATD solution should arise for $\rho << \rho_{\text{cut-off}}$.
- For a hard cut-off, regularized quiver has vanishing scaling dimensions.
- Try brane re-ordering? Require Seiberg self-duality and the vanishing of the beta functions and R-symmetry anomalies.

4D $\mathcal{N} = 1$ strongly coupled field theory proposal



- Only $\mathcal{N} = 1$ vectors: alternating "r" hypers (NS5) and "s" hypers (NS5$'$).
- Quiver *completed* at finite n with flavors: semi-infinite D4s or transversal D6s.
- Same anomalous dimensions as KW. Scaling dimensions compatible with quartic superpotential.
- Central charges consistent with mass-triggered RG flow from a version of the [Lozano, Núñez '16] $\mathcal{N} = 2$ theory dual to the NATD of $AdS_5 \times S^5/\mathbb{Z}_2$.

The mass deformation for the $\mathcal{N} = 2$ $AdS_5 \times S^5/\mathbb{Z}_2$ theory



Central charges for the $\mathcal{N}=2$ and $\mathcal{N}=1$ theories

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \quad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

Tachikawa-Wecht: 4D $\mathcal{N}=1$ IR from mass-deformed $\mathcal{N}=2$ UV

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\,n^3\,N_6^2 \approx c_{NATD\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\,n^3\,N_6^2 \approx c_{NATD\ AdS_5\times T^{1,1}}$$

Central charges for the $\mathcal{N}=2$ and $\mathcal{N}=1$ theories

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \quad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

Tachikawa-Wecht: 4D $\mathcal{N}=1$ IR from mass-deformed $\mathcal{N}=2$ UV

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\, n^3\, N_6^2 \approx c_{NATD\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\, n^3\, N_6^2 \approx c_{NATD\ AdS_5\times T^{1,1}}$$

Central charges for the $\mathcal{N}=2$ and $\mathcal{N}=1$ theories

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \quad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

Tachikawa-Wecht: 4D $\mathcal{N}=1$ IR from mass-deformed $\mathcal{N}=2$ UV

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\,n^3\,N_6^2 \approx c_{NATD\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\,n^3\,N_6^2 \approx c_{NATD\ AdS_5\times T^{1,1}}$$

Outline

Non-Abelian T-duality

- It can be used to generate new solutions of type II SUGRA and motivate/probe/challenge supersymmetric classifications.
- AdS/CFT: regularised quiver $\longleftrightarrow$ completed geometry. New CFTs arise:

  D3 $\rightarrow$ (D4,NS5,NS5')   (D2,D6) $\rightarrow$ (D3,D5,NS5)

- NATD SUGRA solution as a local realization of the completed geometry.

Open problems

1. NATD KW: Completion of the gravity solution requires backreacted D6 flavour branes. [Bah '13], [Tomasiello et al. '15 & '17]
2. IIA $AdS_6 \times S^4$ (D4,D8) $\longrightarrow$ IIB $AdS_6 \times S^2$ (D5,NS5,D7) (ongoing work)
   - Possible 5d fixed point of (p, q) 5- and 7-branes webs.
   - Extension of the IIB $AdS_6 \times S^2$ class by [D'Hoker et al. '16 & '17]?
   - Varying axio-dilaton: $AdS_6 \times S^2 \times CY_2$ F-theory origin?

Non-Abelian T-duality

- It can be used to generate new solutions of type II SUGRA and motivate/probe/challenge supersymmetric classifications.
- AdS/CFT: regularised quiver $\longleftrightarrow$ completed geometry. New CFTs arise:

  D3 $\rightarrow$ (D4,NS5,NS5')   (D2,D6) $\rightarrow$ (D3,D5,NS5)

- NATD SUGRA solution as a local realization of the completed geometry.

Open problems

1. NATD KW: Completion of the gravity solution requires backreacted D6 flavour branes. [Bah '13], [Tomasiello et al. '15 & '17]
2. IIA $AdS_6 \times S^4$ (D4,D8) $\longrightarrow$ IIB $AdS_6 \times S^2$ (D5,NS5,D7) (ongoing work)
   - Possible 5d fixed point of (p, q) 5- and 7-branes webs.
   - Extension of the IIB $AdS_6 \times S^2$ class by [D'Hoker et al. '16 & '17]?
   - Varying axio-dilaton: $AdS_6 \times S^2 \times CY_2$ F-theory origin?

Thank you for your attention!

IIA Abelian T-dual of $AdS_5 \times S^5$

$AdS_5 \times S^5$ solution (S^5 as $S^1 \times S^3$ over an interval)

$$ds^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + L^2\cos^2\alpha\left(d\theta^2 + \sin^2\theta\, d\phi^2 + (d\psi + \cos\theta\, d\phi)^2\right)$$

$$F_5 = \frac{2}{L}(1+\star_{10})\mathrm{Vol}(AdS_5) = (1+\star_{10})8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge \mathrm{Vol}(S^3)$$

$\alpha\in[0,\pi/2]$, $\beta,\phi\in[0,2\pi]$, $\theta\in[0,\pi]$, $\psi\in[0,4\pi]$

Abelian T-dual on $S^1 \hookrightarrow S^3 \to S^2$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\psi^2}{L^2\cos^2\alpha} + L^2\cos^2\alpha\, ds^2(S^2)$$

$$B_2 = \cos\theta\, d\phi\wedge d\psi \sim \psi\,\mathrm{Vol}(S^2), \qquad e^{-2\Phi} = L^2\cos^2\alpha$$

$$F_4 = 8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge\mathrm{Vol}(S^2)$$

- The S^1 survives the duality: $\psi\in[0,4\pi]$ goes to $[0,\pi]$.

IIA Abelian T-dual of $AdS_5 \times S^5$

$AdS_5 \times S^5$ solution (S^5 as $S^1 \times S^3$ over an interval)

$$ds^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + L^2\cos^2\alpha\left(d\theta^2 + \sin^2\theta\, d\phi^2 + (d\psi + \cos\theta\, d\phi)^2\right)$$

$$F_5 = \frac{2}{L}(1 + \star_{10})\mathrm{Vol}(AdS_5) = (1 + \star_{10})8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge\mathrm{Vol}(S^3)$$

$$\alpha\in[0,\pi/2],\ \ \beta,\phi\in[0,2\pi],\ \ \theta\in[0,\pi],\ \ \psi\in[0,4\pi]$$

Abelian T-dual on $S^1 \hookrightarrow S^3 \to S^2$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\psi^2}{L^2\cos^2\alpha} + L^2\cos^2\alpha\, ds^2(S^2)$$

$$B_2 = \cos\theta\, d\phi\wedge d\psi\ \sim\ \psi\,\mathrm{Vol}(S^2), \qquad e^{-2\Phi} = L^2\cos^2\alpha$$

$$F_4 = 8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge\mathrm{Vol}(S^2)$$

- The S^1 survives the duality: $\psi\in[0,4\pi]$ goes to $[0,\pi]$.

IIA Abelian T-dual of $AdS_5 \times S^5$

$AdS_5 \times S^5$ solution (S^5 as $S^1 \times S^3$ over an interval)

$$ds^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + L^2\cos^2\alpha\left(d\theta^2 + \sin^2\theta\, d\phi^2 + (d\psi + \cos\theta\, d\phi)^2\right)$$

$$F_5 = \frac{2}{L}(1+\star_{10})\mathrm{Vol}(AdS_5) = (1+\star_{10})8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge \mathrm{Vol}(S^3)$$

$$\alpha\in[0,\pi/2],\ \ \beta,\phi\in[0,2\pi],\ \ \theta\in[0,\pi],\ \ \psi\in[0,4\pi]$$

Abelian T-dual on $S^1 \hookrightarrow S^3 \to S^2$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\psi^2}{L^2\cos^2\alpha} + L^2\cos^2\alpha\, ds^2(S^2)$$

$$B_2 = \cos\theta\, d\phi\wedge d\psi \ \sim\ \psi\,\mathrm{Vol}(S^2), \qquad e^{-2\Phi} = L^2\cos^2\alpha$$

$$F_4 = 8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge\mathrm{Vol}(S^2)$$

- The S^1 survives the duality: $\psi\in[0,4\pi]$ goes to $[0,\pi]$.

IIA Abelian T-dual of $AdS_5 \times S^5$

Abelian T-dual on $S^1 \hookrightarrow S^3 \to S^2$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\psi^2}{L^2\cos^2\alpha} + L^2\cos^2\alpha\, ds^2(S^2)$$

$$B_2 = \cos\theta\, d\phi \wedge d\psi \;\sim\; \psi\,\mathrm{Vol}(S^2), \qquad e^{-2\Phi} = L^2\cos^2\alpha$$

$$F_4 = 8L^4 \sin\alpha\, \cos^3\alpha\, d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^2)$$

- Shrinking S^3 at $\alpha = \pi/2$ gives rise to singularity: localized NS5-branes,

$$N_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{S^2} \int_0^{\pi} H_3 = 1$$

- Electric F_6 sourced by D4-branes,

$$N_4 = \frac{1}{2\,\kappa_{10}^2\, T_4} \int_{(\alpha,\beta)\times S^2} F_4$$

IIA Abelian T-dual of $AdS_5 \times S^5$

Abelian T-dual on $S^1 \hookrightarrow S^3 \to S^2$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\psi^2}{L^2\cos^2\alpha} + L^2\cos^2\alpha\, ds^2(S^2)$$

$$B_2 = \cos\theta\, d\phi \wedge d\psi \;\sim\; \psi\,\mathrm{Vol}(S^2), \qquad e^{-2\Phi} = L^2\cos^2\alpha$$

$$F_4 = 8L^4 \sin\alpha\, \cos^3\alpha\, d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^2)$$

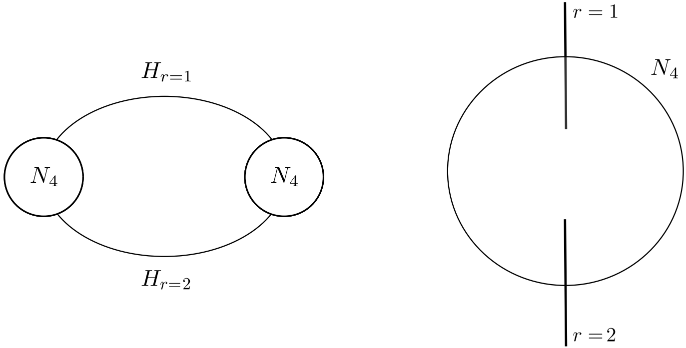
- Orbifolding $S^5/\mathbb{Z}_n$: take $\psi \in [0, n\,\pi]$ in the ATD.
- The number N_4 of D4-branes remains invariant, but

$$N_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{S^2}\int_0^{n\,\pi} H_3 = n$$

- For $n = 2$, we get N_4 D4-branes between two NS5-branes, giving rise to a 4d $\mathcal{N} = 2$ theory. [Alishahiha, Oz'99]

- N_4 D4-branes between two NS5-branes, dual to 4d $\mathcal{N} = 2$ theory.

[Alishahiha, Oz'99]



	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x			x			
NS5	x	x	x	x	x	x				

IIA non-Abelian T-dual of $AdS_5 \times S^5$

[Sfetsos, Thompson '10] [Lozano, Núñez '16]

$AdS_5 \times S^5$ solution (S^5 as $S^1 \times S^3$ over an interval)

$$ds^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + L^2\cos^2\alpha\, ds^2(S^3)$$

$$F_5 = \frac{2}{L}(1+\star_{10})\mathrm{Vol}(AdS_5) = (1+\star_{10})8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta\wedge \mathrm{Vol}(S^3)$$

$\alpha \in [0,\pi/2]$, $\beta,\phi \in [0,2\pi]$, $\theta \in [0,\pi]$, $\psi \in [0,4\pi]$

NATD of $AdS_5 \times S^5$

$$d\hat{s}^2 = ds^2_{AdS_5} + 4L^2\left(d\alpha^2 + \sin^2\alpha\, d\beta^2\right) + \frac{d\rho^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha\,\rho^2}{\rho^2 + L^4\cos^2\alpha} ds^2(S^2)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2), \quad e^{-2\Phi} = L^2\cos^2\alpha\,(L^4\cos^4\alpha + \rho^2)$$

$$F_2 = 8L^4\sin\alpha\,\cos^3\alpha\, d\alpha\wedge d\beta, \quad F_4 = F_2\wedge B_2$$

- Shrinking S^3 at $\alpha = \frac{\pi}{2}$ gives rise to a singularity (NS5-branes) in the NATD.

Large gauge transformations and Page charges

- Tip of a cone with S^2 boundary: non-vanishing holonomies of B_2,

$$b_0(\rho) = \frac{1}{4\pi^2}\int_{S^2} B_2 \quad \Rightarrow \qquad |b_0| \in [0,1]$$

Large gauge transformations

$$b_0(\rho = n\,\pi) = n \qquad\qquad B_2 \longrightarrow\; B_2 - n\,\pi\,\mathrm{Vol}(S^2)$$

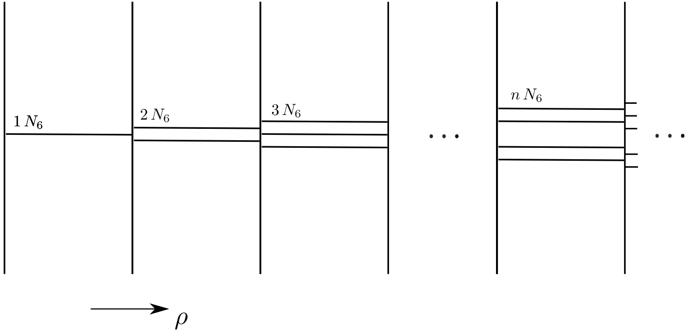
$$\rho \in \Big[n\,\pi, (n+1)\pi\Big)$$

- Number of branes (Page charges) shifted under LGTs:

$$N_6 = \frac{1}{2\,\kappa_{10}^2\; T_6}\int_{(\alpha,\beta)} F_2\,, \qquad N_4 = \frac{1}{2\,\kappa_{10}^2\; T_4}\int_{(\alpha,\beta)\times S^2}\big(F_4 - F_2\wedge B_2\big) = n\,N_6$$

$$N_{NS5} = \frac{1}{2\kappa_{10}^2\, T_{NS5}}\int_{S^2}\int_0^{n\,\pi} H_3 = n$$

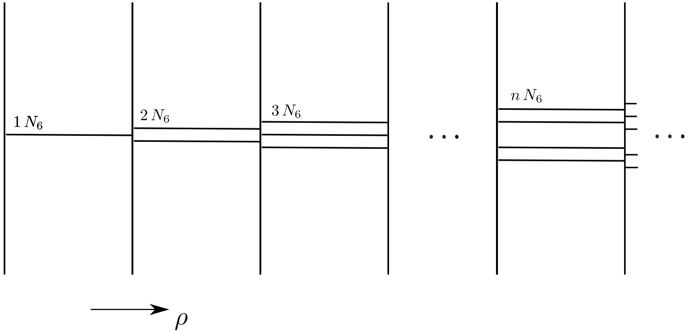
NATD of $AdS_5 \times S^5$: brane picture



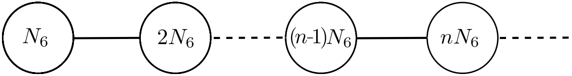
- Infinite set-up of $N_4 = n\,N_6$ D4-branes stacks between NS5-branes.

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x			x			
NS5	x	x	x	x	x	x				

NATD of $AdS_5 \times S^5$: brane picture

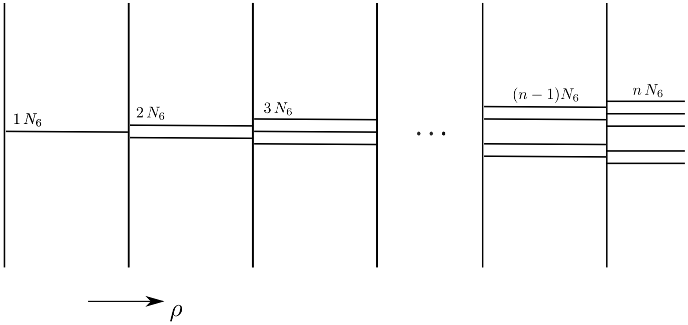


- Dual to a 4d $\mathcal{N} = 2$ theory in a strongly coupled fixed point.

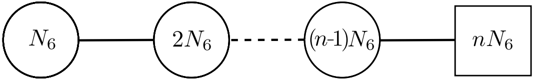


- A priori infinite brane set-up and quiver!

Regularised field theory for the NATD of $AdS_5 \times S^5$



- Propose a "regularised quiver": Choose finite n for cut-off by means of flavours:

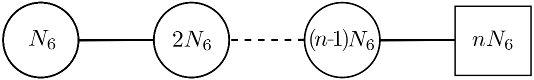


- NATD solution should arise for $\rho << \rho_{\text{cut-off}}$.

Regularised field theory for the NATD of $AdS_5 \times S^5$

[Lozano, Núñez '16]

- Simplest consistent *regularised* linear quiver given by



- Field theoretical central charge consistent with holographical counterpart $c^{(0,n)}_{\mathrm{NATD}\ AdS_5 \times S^5}$ computed up to the cut-off $n = N_5$:

$$c_{\mathcal{N}=2} = \frac{1}{12}(2n_v + n_h) = \frac{1}{12} n^3 N_6^2 + \mathcal{O}(n^2)$$

- Embedding in the 4d $\mathcal{N} = 2$ Gaiotto-Maldacena class as a superposition of Maldacena-Núñez solutions provides all the information required to construct the **regular, completed** gravity background.

- Singular NATD gravity solution would arise as a local realization of the completed background.

NATD on $S^3 \hookrightarrow T^{1,1} \to S^2$: IIA NS-NS sector

IIA NS-NS sector (rescaling $\rho \longrightarrow \frac{L^2}{\alpha'}\rho$)

$$d\hat{s}^2 = ds^2_{AdS_5} + L^2\lambda_1^2\left(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\right) + \frac{L^2}{Q}\Big[\lambda_1^4\,\left(\cos\chi\; d\rho - \rho\sin\chi\; d\chi\right)^2$$

$$+\lambda^2\lambda_1^2\left(\sin\chi\; d\rho + \rho\cos\chi\; d\chi\right)^2 + \lambda^2\lambda_1^2\rho^2\sin^2\chi\,\left(d\xi + \cos\theta_1 d\phi_1\right)^2 + \rho^2 d\rho^2\Big],$$

$$B_2 = \frac{L^2\rho^2\sin\chi}{2Q}\Big[(\lambda^2 - \lambda_1^2)\sin 2\chi\; d\xi\wedge d\rho + 2P\,\rho\; d\xi\wedge d\chi\Big]$$

$$-\frac{L^2\lambda^2\cos\theta_1}{Q}\Big[(\lambda_1^4 + \rho^2)\cos\chi d\rho\wedge d\phi_1 - \lambda_1^4\,\rho\sin\chi\; d\chi\wedge d\phi_1\Big],$$

$$e^{-2\hat{\Phi}} = \frac{L^6\;Q}{g_s^2\;\alpha'^3}, \qquad Q = \lambda^2\lambda_1^4 + \rho^2(\lambda^2\cos^2\chi + \lambda_1^2\sin^2\chi)$$

- Solution originally derived in [Itsios, Núñez, Sfetsos, Thompson '12].
- Regular solution with uncompact "inner" space.

NATD on $S^3 \hookrightarrow T^{1,1} \to S^2$: IIA brane content

- Electric components for F_6, F_8 suggest D4, D6 branes.
- In the presence of B_2, Page charges and LGTs must be considered,

$$\rho \in [\rho_n, \rho_{n+1}) \; : \qquad B_2 \longrightarrow B_2 - n\pi\alpha' \,\mathrm{Vol}(\Sigma_2)$$

- Two different 2-cycles allow for two different sets of LGTs:

$$N_6 = \frac{1}{2\,\kappa_{10}^2\, T_{D6}} \int_{S_1^2} F_2 = \frac{2}{27}\frac{L^4}{g_s^2\,\alpha'^2}\,,$$

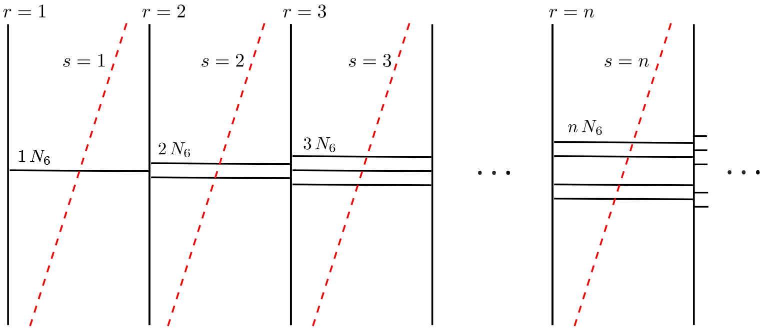
$$N_4 = \frac{1}{2\,\kappa_{10}^2\, T_{D4}} \int_{S_1^2\times(\chi,\xi)} \big(F_4 - F_2\wedge B_2\big) = n\,N_6$$

$$N_{NS5} = \frac{1}{2\kappa_{10}^2\, T_{NS5}} \int_{(\chi,\xi)} \int_0^{\rho_n} d\rho\, H_3 = n$$

$$N_{NS5'} = \frac{1}{2\kappa_{10}^2\, T_{NS5}} \int_{S_1^2} \int_0^{m\pi} d\rho\, H_3 = m$$

- As in the $\mathcal{N}=2$ case, we want to recover ATD in the large ρ limit.

Brane set-up of the NATD of KW



- D4-brane charge created each time an NS5-brane (not NS5 $'$) is crossed.

Abelian vs. non-Abelian

Abelian T-duality ($U(1)$ isometry)

- Duals keep the $U(1)$ isometry. Invertible transformation (two T-dualities in a row give back original background).
- T-duality is a symmetry of full (perturbative) string theory.

$$\theta \in [0, 2\pi] \quad \longrightarrow \quad \tilde{\theta} \in [0, 2\pi]$$

Non-Abelian T-duality (e.g. $SU(2)$ isometry)

- $SU(2)$ isometry no longer appears in the metric. Non-invertible transformation: cannot dualise twice to recover the original background.
- Only proven to be a symmetry at tree level (SUGRA).

$$g \in SU(2) \quad \longrightarrow \quad \{v_1, v_2, v_3\} \in \mathbb{R}^3 \quad \text{(non-compact!)}$$

Spinors and RR sector transforming under NATD

- Left and right movers transform differently under T-duality:

$$\partial_\pm \hat{X}^m = (M_\pm)^m_{\ n} \partial_\pm X^n$$

- [Hassan '99] Covariance of the dual background under the associated Lorentz transformation $\Lambda = M_+^{-1} M_-$, with spinor representation:

$$\Omega^{-1}\Gamma^a\Omega = \Lambda^a_{\ b}\Gamma^b$$

- Killing spinor $\epsilon = (\epsilon_1, \epsilon_2)^t$ with ϵ_1, ϵ_2 Majorana-Weyl spinors:

$$\hat{\epsilon}_1 = \epsilon_1 \,, \quad \hat{\epsilon}_2 = \Omega.\epsilon_2$$

- Rotation of the spinors reduces structure group of the interior space, e.g. G_2 to $SU(3)$, or $SU(3)$ to $SU(2)$.

Spinors and RR sector transforming under NATD

- SUSY preservation if Kosmann spinorial Lie derivative vanishes along the isometries on which we dualise:

$$\mathcal{L}_k \,\epsilon = k^a D_a \epsilon + \frac{1}{8}(dK)_{ab}\Gamma^{ab}\,\epsilon = 0$$

This usually imposes additional constraints on the Killing spinors: not to depend on dualization directions $\implies$ reduced supersymmetry!

NATD typically reduces G-structure, even if SUSY is partially broken!

- [Sfetsos, Thompson '10] Transformation rule for RR fields using flux polyform $F = \sum_n F_n$ (n even/odd for IIA/IIB) rewritten as bispinor:

$$\left.\begin{aligned} P &= \tfrac{1}{2} e^{\Phi} \textstyle\sum_k \tfrac{1}{k!} F_{\mu_1 \ldots \mu_k} \Gamma^{\mu_1 \cdots \mu_k} = \tfrac{1}{2} e^{\Phi} \not{F} \\ \hat{P} &= \tfrac{1}{2} e^{\hat{\Phi}} \textstyle\sum_k \tfrac{1}{k!} \hat{F}_{\mu_1 \ldots \mu_k} \Gamma^{\mu_1 \cdots \mu_k} = \tfrac{1}{2} e^{\hat{\Phi}} \not{\hat{F}} \end{aligned}\right\} \implies \boxed{\hat{P} = P.\Omega^{-1}}$$

Brane contents for the NATD solution

Branes of the original background either ...

- do not wrap the dualised S^3:

$$D_p \longrightarrow \begin{cases} D_{p+1} & \text{also extended along } \rho \\ D_{p+3} & \text{also extended along } \rho \times S^2 \end{cases}$$

- or do wrap the dualised S^3 directions:

$$D_p \longrightarrow \begin{cases} D_{p-1} & \text{still wrapping the remaining } S^2 \\ D_{p-3} & \text{not wrapping any of the dual directions} \end{cases}$$

- Usual pattern:

$$\mathrm{NATD}(\mathrm{AdS}_{\mathrm{p+1}}) \quad \longleftrightarrow \quad \mathrm{Dp}-\mathrm{D(p+2)}-\mathrm{NS5}$$

$U(1)$ Hopf fiber T-dual is related to the $SU(2)$ non-Abelian T-dual as

$$\lim_{\rho\to\infty}\begin{pmatrix} ds^2 \\ B_2 \\ e^{-\Phi} \\ e^{\Phi}F \\ \epsilon_{1,2}\end{pmatrix}_{NATD} = \begin{pmatrix} ds^2 \\ B_2 \\ \rho\, e^{-\Phi} \\ e^{\Phi}F \\ \epsilon_{1,2}\end{pmatrix}_{ATD}$$