Three-charge black hole with α' corrections

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Preliminaries: What is this about?

In this talk I present the first-order corrections to the **three-charge black hole solution in heterotic string** effective theory.

This solution became universally famous after the Strominger-Vafa **microscopic** computation of the **entropy**, which **agrees with** the **Bekenstein-Hawking** entropy.

$$S_{CFT} = S_{SUGRA} = 2\pi\sqrt{N_{S5}N_{F1}N_W} \qquad \text{(for } \alpha' = 0\text{)}$$

This computation agrees at zeroth-order in α' expansion. What happens **beyond leading order**?

The microscopic result from the CFT is known to all orders in α', obtained by Castro and Murthy.

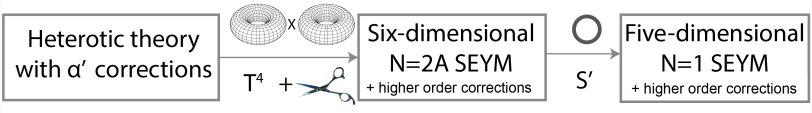
$$S_{CFT} = 2\pi\sqrt{(N_{S5}+16)\,N_{F1}N_W}$$

Studies of near-horizon solutions point in the right direction, but more information is needed. (Sahoo, Sen)

The three-charge black hole

Compactification scheme, relates to $\mathcal{N} = 1$, $d = 5$ SEYM theory:

Cano, Meessen, Ortin, PFR



Cano, Ortin, Santoli

Fundamental objects in the solution

	v	1	2	3	4	u	ρ	θ	ϕ	ψ
NS5	X	X	X	X	X	X	-	-	-	-
F1	X	$\sim$	$\sim$	$\sim$	$\sim$	X	-	-	-	-
W	X	$\sim$	$\sim$	$\sim$	$\sim$	X	-	-	-	-
G5	X	X	X	X	X	X	-	-	-	-

The three-charge black hole at zeroth order in α'

Bosonic field configuration, metric g_{μ}, Kalb-Ramond 2-form $B_{\mu\nu}$ with field strength H and dilaton ϕ,

$$\begin{aligned} ds^2 &= \frac{2}{\mathcal{Z}_-} du \left(dv - \tfrac{1}{2}\mathcal{Z}_+ du \right) - \mathcal{Z}_0 (d\rho^2 + \rho^2 d\Omega^2_{(3)}) - dy^i dy^i \,, \\ H &= d\mathcal{Z}_-{}^{-1} \wedge du \wedge dv - \frac{\rho^3 (\mathcal{Z}_0)'}{8} \sin\theta d\theta \wedge d\psi \wedge d\phi \,, \\ e^{-2\phi} &= e^{-2\phi_\infty} \frac{\mathcal{Z}_-}{\mathcal{Z}_0} \,, \end{aligned}$$

Completely specified by three harmonic functions in $\mathbb{R}^4$,

$$\mathcal{Z}_{0+-} = 1 + \frac{Q_{0+-}}{\rho^2} \,.$$

$$\mathcal{Q}_0 = \alpha' N_{S5} \,, \qquad \mathcal{Q}_- = \alpha' g_s^2 N_{F1} \,, \qquad \mathcal{Q}_+ = \left(\frac{\alpha' g_s}{R_u} \right)^2 N_W \,.$$

Review of the three-charge black hole

The metric naturally describes a 5-dimensional black hole,

$$ds^2 = (\mathcal{Z}_0\mathcal{Z}_+\mathcal{Z}_-)^{-2/3}dt^2 - (\mathcal{Z}_0\mathcal{Z}_+\mathcal{Z}_-)^{1/3}(d\rho^2 + \rho^2 d\Omega^2_{(3)})\,,$$

$AdS_2 \times S^3$ near-horizon geometry, with area of horizon

$$A_H = 2\pi^2\sqrt{Q_0 Q_+ Q_-}\,,$$

The ADM mass is just

$$M = \frac{\pi}{4G_N^{(5)}}\left[Q_0 + Q_- + Q_+\right]\,.$$

There is **no hair**, completely determined by its conserved charges, which are related to the presence of **solitonic 5-branes** (Q_0), **strings** (Q_-) and **momentum** (Q_+).

Let's add α' corrections

The action is

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 \right.$$

$$\left. -\alpha' \left[F^A{}_{\mu\nu} F^{A\mu\nu} + R_{(-)\,\mu\nu}{}^a{}_b R_{(-)}{}^{\mu\nu\, b}{}_a \right] \right\} ,$$

with

$$\begin{aligned} F^A &= dA^A + \tfrac{1}{2}\epsilon^{ABC} A^B \wedge A^C , \\ R_{(-)}{}^a{}_b &= d\Omega_{(-)}{}^a{}_b - \Omega_{(-)}{}^a{}_c \wedge \Omega_{(-)}{}^c{}_b , \\ \Omega_{(-)}{}^a{}_b &= \omega^a{}_b - \tfrac{1}{2} H_\mu{}^a{}_b dx^\mu , \\ H &= dB + 2\alpha'(\omega^{\mathrm{YM}} + \omega^{\mathrm{L}}_{(-)}) , \end{aligned}$$

$$\begin{aligned} \omega^{\mathrm{YM}} &= F^A \wedge A^A - \tfrac{1}{3!}\epsilon^{ABC} A^A \wedge A^B \wedge A^C , \\ \omega^{\mathrm{L}}_{(-)} &= R_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_a + \tfrac{1}{3}\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_c \wedge \Omega_{(-)}{}^c{}_a . \end{aligned}$$

Low energy Heterotic String

Anomaly cancellation mechanism, $Tr(F \wedge F)$ and $Tr(R \wedge R)$ contributions cancel between them. This suggest the spin connection $\omega^a{}_b$ is treated at the same level than the vector A^A.

However, it is more convenient to work with a **torsionful spin connection** $\Omega_{(-)}{}^a{}_b$, which builds a $SO(1,9)$ Super Yang-Mills multiplet. (Bergshoeff, de Roo)

$$\Omega_{(-)}{}^a{}_b \equiv \Omega^a{}_b - \frac{1}{2} H^a{}_b \,,$$

Enhancement of the gauge group to $SU(2) \times SO(1,9)$. Notice, however, that the $SO(9,1)$ factor is not independent!

Problem! The recursive definition of H

$$H = dB + 2\alpha' (\omega_{YM} - \omega_L) \,,$$

introduces an infinite tower of corrections to recover supersymmetry.

Supersymmetric field configuration

Structure of fields for supersymmetric configurations, (T. Ortin talk)

$$
\begin{aligned}
ds^2 &= \frac{2}{\mathcal{Z}_-} du\left(dv - \tfrac{1}{2}\mathcal{Z}_+ du\right) - \mathcal{Z}_0\left(d\rho^2 + \rho^2 d\Omega^2_{(3)}\right) - dy^i dy^i\,, \quad i = 1, \ldots, 4\,,\\
H &= d\mathcal{Z}_-^{-1} \wedge du \wedge dv - \frac{\rho^3 \mathcal{Z}_0'}{8} \sin\theta d\theta \wedge d\psi \wedge d\phi\,,\\
e^{-2\phi} &= e^{-2\phi_\infty} \frac{\mathcal{Z}_-}{\mathcal{Z}_0}\,,\\
A^A &= -\frac{\rho^2}{(\kappa^2 + \rho^2)} v_L^A\,, \qquad \textit{BPST instanton}
\end{aligned}
$$

How do supersymmetric configurations solve the equations of motion?

Solving the e.o.m.

Write the general eqs.

- Yang-Mills equations:

$$\alpha' e^{2\phi}\mathfrak{D}\left(e^{-2\phi}\star F^A\right) = 0\,.$$

  Solved for self-dual F^A on hyperKähler base. Instanton.

- Kalb-Ramond 2-form equation

$$d\left(e^{-2\phi}\star H\right) = 0 \qquad \rightarrow \qquad \star_{(4)}\nabla^2\mathcal{Z}_- = 0.$$

- Einstein equations

$$R_{\mu\nu} - 2\nabla_\mu\partial_\nu\phi + \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} - 2\alpha'\left(F^A{}_{\mu\rho}F^A{}_\nu{}^\rho + R_{(-)\,\mu\rho}{}^a{}_b R_{(-)\,\nu}{}^{\rho\,b}{}_a\right) = 0\,,$$

  Only uu component is relevant

$$\rightarrow \qquad \nabla^2\mathcal{Z}_+\cdot\left(\frac{1}{4\mathcal{Z}_0\mathcal{Z}_-}\right) = \mathbf{2\alpha' R_{(-)\,u\rho}{}^a{}_b R_{(-)\,u}{}^{\rho\,b}{}_a}\,.$$

- Dilaton equation automatically satisfied.

Solving the e.o.m.

- Bianchi identity of Kalb-Ramond 2-form

$$dH = 2\alpha' F^A \wedge F^A \quad \rightarrow \quad \star_{(4)} \nabla^2 \mathcal{Z}_0 = \mathbf{2}\alpha' \left[\mathbf{F^A} \wedge \mathbf{F^A} + \mathbf{R_{(-)}{}^a{}_b} \wedge \mathbf{R_{(-)}{}^b{}_a} \right],$$

In conclusion, the solution is specified in terms of the functions

$$\mathcal{Z}_0 = \mathcal{Z}_0^{(0)} + \alpha' \mathbf{f_0} + \mathcal{O}(\alpha'^2),$$

$$\mathcal{Z}_- = \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2),$$

$$\mathcal{Z}_+ = \mathcal{Z}_+^{(0)} + \alpha' \mathbf{f_+} + \mathcal{O}(\alpha'^2),$$

with

$$\mathcal{Z}_{0,+,-}^{(0)} = 1 + \frac{\mathcal{Q}_{0,+,-}}{\rho^2}.$$

Correction induced by Yang-Mills fields

The BPST instanton, particular case of **'t Hooft ansatz**

$$A^A = -\frac{\rho^2}{(\kappa^2+\rho^2)} v_L^A \qquad \rightarrow \qquad A^A = \mathbb{M}^L_{mp}\partial_p \log \mathcal{Z}_{\mathrm{YM}} dx^m$$

with $\mathbb{M}^L_{mp}$ are $su(2)_L$ generators in $so(4) \cong su(2)_L \oplus su(2)_R$,

$$(\mathbb{M}_{mq})_{np} = (\mathbb{M}_{np})_{mq} \equiv 2\delta_{n[m}\delta_{q]p}\,, \qquad \mathbb{M}^{R,L}_{mq} \equiv \tfrac{1}{2}\left(\mathbb{M}_{mq} \pm \tfrac{1}{2}\varepsilon_{mqrs}\mathbb{M}_{rs}\right)\,.$$

and

$$\mathcal{Z}_{\mathrm{YM}} = 1 + \frac{\kappa^2}{\rho^2}$$

Then,

$$2\alpha' F^A \wedge F^A = 2\alpha' \star_{(4)} \nabla^2 \left[\frac{4}{\rho^2} - (\partial \log \mathcal{Z}_{\mathrm{YM}})^2\right] = \star_{(4)} \nabla^2 \left[8\alpha' \frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2}\right]$$

Deciphering the torsionful spin connection

As already pointed out, $\Omega_{(-)}$ is a local connection; a 1-form taking values in $so(1,9)$.

First, the ansatz breaks $so(1,9) \to so(1,5)$.

Moreover, when explicitly computed, it is possible to distinguish **two type of contributions** $\Omega_{(-)} = \Omega_{(-)}^{SO(1,2)} + \Omega_{(-)}^{SO(4)}$.

Let us first focus on the $so(4) \cong su(2)_L \oplus su(2)_R$ sector, showing the details of the computation.

Spin connection (before adding torsion) obtained from a metric conformal to flat space:

$$\mathcal{Z}_0 \left(d\rho^2 + \rho^2 d\Omega_{(3)}^2 \right)$$

Cartan structure equation: $de^a = \Omega^a{}_b \wedge e^b$.

Deciphering the torsionful spin connection

Origin of the 't Hooft ansatz for instantons on $\mathbb{R}^4$, based on Attiyah-Hitchin-Singer theorem.

Consider the metric

$$ds^2 = \mathcal{Z}_{YM}^2 \left(d\rho^2 + \rho^2 d\Omega_{(3)}^2 \right) , \qquad e^a = \mathcal{Z}_{YM} dx^a$$

It has vanishing Ricci scalar if $\mathcal{Z}_{YM}$ is a harmonic function. From Cartan structure equations:

$$de^a = \Omega^a{}_b \wedge e^b , \qquad d(log \mathcal{Z}_{YM}) \wedge dx^a = \Omega^a{}_b \wedge dx^b$$

Projecting the $so(4)$-valued connection on the $su(2)_L$ subalgebra, we get

$$\Omega^a{}_b|_{su(2)_L} = (\mathbb{M}^L_{mp})^a{}_b \partial_p \log \mathcal{Z}_{\mathrm{YM}} dx^m$$

This can be generalized to **any hyperKähler space!**

Deciphering the torsionful spin connection

Our case: $\Omega^a{}_b|_{su(2)_L}$ component itself is not an instanton!

Our 4-dimensional metric is of the form

$$\mathcal{Z}_0 \left(d\rho^2 + \rho^2 d\Omega^2_{(3)} \right)$$

Missing factor of 2! Moreover, $\Omega^a{}_b|_{su(2)_R}$ can be anything.

But remember! Heterotic string effective action adds **torsion** to the spin connection through the 3-form H. This introduces a **magical cancellation**, such that

$$\Omega_{(-)}{}^a{}_b|_{su(2)_L} = (\mathbb{M}^L_{mp})^a{}_b \partial_p \log \mathcal{Z}_0 dx^m \,, \qquad \Omega_{(-)}{}^a{}_b|_{su(2)_R} = 0 \,.$$

Which implies that **the $so(4)$ part of the spin connection is nothing but a BPST instanton**, to first order in α'.

Deciphering the torsionful spin connection

The torsionful spin connection modifies $\mathcal{Z}_0$ as

$$-8\alpha' \left[\frac{\rho^2 + 2\mathcal{Q}_0}{(\rho^2 + \mathcal{Q}_0)^2} \right] + \mathcal{O}(\alpha'^2) \,,$$

Notice the curvature squared term has the **wrong sign in the action!** This means it behaves as a **source of negative-energy**.

The remaining part of the torsionful spin connection, $\Omega_{(-)}|_{so(1,2)}$, decouples from the e.o.m. except at the *uu* component of Einstein equations. It is solvable, giving a term

$$+16\alpha' \frac{\mathcal{Q}_+(\rho^2 + \mathcal{Q}_0 + \mathcal{Q}_-)}{\mathcal{Q}_0(\rho^2 + \mathcal{Q}_0)(\rho^2 + \mathcal{Q}_-)} + \mathcal{O}(\alpha'^2) \,,$$

Which is positive, related to character of $so(1,2)$.

Technical observation: **Similar to a non-Abelian dyon**! (PFR)

The α'-corrected solution

The solution is then specified by the functions

$$\begin{aligned}
\mathcal{Z}_0 &= \mathcal{Z}_0^{(0)} + \mathbf{8}\alpha' \left[\frac{\rho^2 + \mathbf{2}\kappa^2}{(\rho^2+\kappa^2)^2} - \frac{\rho^2 + \mathbf{2}\mathcal{Q}_0}{(\rho^2+\mathcal{Q}_0)^2} \right] + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_- &= \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_+ &= \mathcal{Z}_+^{(0)} + \mathbf{16}\alpha' \frac{\mathcal{Q}_+(\rho^2 + \mathcal{Q}_0 + \mathcal{Q}_-)}{\mathcal{Q}_0(\rho^2+\mathcal{Q}_0)(\rho^2+\mathcal{Q}_-)} + \mathcal{O}(\alpha'^2)\,,
\end{aligned}$$

Again, the solution describes a collection of N_{S5} **solitonic 5-branes,** N_{F1} **strings carrying** N_W **momentum** wrapping coordinate u and **one gauge 5-brane**.

Moreover, the torsionful spin connection contributes as **one BPST anti-instanton** and **delocalized momentum**.

The α'-corrected solution

Taking into account all the contributions, the mass is

$$M = \frac{\pi}{4G_N^{(5)}}\left[\mathcal{Q}_0 + (\mathbf{8}\alpha' - \mathbf{8}\alpha') + \mathcal{Q}_- + \mathcal{Q}_+(1 + \mathbf{16}\alpha'/\mathcal{Q}_\mathbf{0})\right] .$$

While the (5-dimensional) area receives no corrections

$$A_\mathrm{H} = 2\pi^2\sqrt{\mathcal{Q}_0\mathcal{Q}_+\mathcal{Q}_-}\,,$$

However the entropy, computed using Wald's formula, gets modified

$$S = -2\pi\int_\mathrm{H} d^3x\sqrt{|h|}\,\frac{\partial\mathcal{L}_{(5)}}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd}\,,$$

$$\mathcal{L}_{(10)} = \frac{g_s^2}{16\pi G_N^{(10)}}e^{-2\phi}\left\{\mathbf{R} - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!}\mathbf{H^2}\right.$$

$$\left. -\alpha'\left[F^A{}_{\mu\nu}F^{A\mu\nu} + \mathbf{R}_{(-)\,\mu\nu}{}^{\mathbf{a}}{}_{\mathbf{b}}\mathbf{R}_{(-)}{}^{\mu\nu\,\mathbf{b}}{}_{\mathbf{a}}\right]\right\}\,,$$

The α'-corrected solution: Entropy

Wald's formula requires the compactification of the whole heterotic action to 5 dimensions. Can we avoid it?

Certainly, $\mathcal{L}_{(5)} \sim \mathcal{L}_{(10)}$. Moreover, we can track the 5-dimensional Riemann in $\mathcal{L}_{(10)}$,

$$d\hat{s}^2 = e^{\phi-\phi_\infty}\left[(k/k_\infty)^{-2/3}ds^2 - k^2\mathcal{A}^2\right] - dy^i dy^i \,,$$

so

$$\hat{e}^a{}_\mu = e^{(\phi-\phi_\infty)/2}(k/k_\infty)^{-1/3}e^a{}_\mu\,, \qquad \hat{R}_{abcd} = e^{-(\phi-\phi_\infty)}(k/k_\infty)^{2/3}R_{abcd} + \ldots$$

We can rewrite Wald's entropy formula as

$$S = -2\pi\int_{\mathrm{H}\times\mathrm{S}^1\times\mathrm{T}^4} d^8\hat{x}\frac{\sqrt{|\hat{g}|}}{\sqrt{f}}\, e^{-(\phi-\phi_\infty)}(k/k_\infty)^{2/3}\frac{\partial\mathcal{L}_{(10)}}{\partial\hat{R}_{abcd}}\epsilon_{ab}\epsilon_{cd}\,,$$

which can be applied directly in 10 dimensions.

The α'-corrected solution: Entropy

Three type of contributions:

- The term R gives $S^{(0)} = 2\pi\sqrt{N_{\mathrm{S5}} N_{\mathrm{F1}} N_{\mathrm{W}}}$.
- The term $R_{(-)\,\mu\nu}{}^{a}{}_{b} R_{(-)}{}^{\mu\nu\ b}{}_{a}$ gives $S_1^{(1)} = 0$.
- The term H^2 gives $S_2^{(1)} = \frac{8}{N_{\mathrm{S5}}} 2\pi\sqrt{N_{\mathrm{S5}} N_{\mathrm{F1}} N_{\mathrm{W}}}$.

$$
\begin{aligned}
S &= 2\pi\sqrt{N_{\mathrm{S5}} N_{\mathrm{F1}} N_{\mathrm{W}}}\,(1 + \mathbf{8}/\mathbf{N}_{\mathrm{S5}}) \\
&\sim 2\pi\sqrt{(N_{\mathrm{S5}} + \mathbf{16}) N_{\mathrm{F1}} N_{\mathrm{W}}}\,, \qquad \textit{CFT result!! (Castro, Murthy)}
\end{aligned}
$$

Final remarks

We have found **analytically** the corrections of first order in α' to the Strominger-Vafa black hole in heterotic theory.

The modification of the entropy is in **agreement with CFT results**.

Negative energy sources. This contribution can be compensated when Yang-Mills fields are present.

Notice that the apperance of **non-Abelian effects is unavoidable**, even if Yang-Mills fields are turned off.

More **general results**: KK monopoles and multicenter solutions. Dyonic solutions in progress.

THANKS FOR YOUR ATTENTION