Microstate geometries and the CTCs problem

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GEOMETRY, DUALITIES AND STRINGS

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Outline

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Motivation

- One of the greatest successes of String Theory is the microscopic interpretation of the black hole entropy

  [Strominger, Vafa '96]

- Description of such microstates at finite gravitational coupling? Horizon?

- It has been argued that only modifications at the horizon scale are compatible with Quantum Mechanics

  [Mathur, Lunin '01]

- Only progress done in the context of the "microstate geometries programme", whose aim is to construct horizonless solutions of SUGRA theories

  [Bena, Warner '06, Berglund, Gimon, Levi '06, ...]

Multi-GH microstate geometries

There are two ways to construct these solutions:

1. Using superstrata technology
   [Bena, Martinec, Russo, Shigemori, Turton, Warner '15]
2. Via multicenter Gibbons-Hawking metrics

I will focus on the latter possibility

- For supersymmetric configurations, we have solution-generating techniques at our disposal
- Limited explicit examples (CTCs, angular momentum oversaturating the c.c. bound)

The CTCs problem

I will focus on 5-dimensional timelike supersymmetric solutions. The general form of the metrics that I will consider is

$$ds^2 = f^2(dt + \hat{\omega})^2 - f^{-1} h_{\underline{mn}} dx^m dx^n$$

where the 1-form $\hat{\omega}$ must satisfy a differential equation (one of the BPS eqs)

- Closed timelike curves appear if the integrability condition is not satisfied at the GH centers. If this happens, $\hat{\omega}$ is not regular, what implies that the time coordinate t must be compact in order to avoid Dirac-Misner singularities

- Even when integrability condition is solved, closed timelike curves usually appear. This is one of the main difficulties when trying to construct explicit solutions using multicenter GH metrics

Plan of the talk

1. I will shortly review the solution generating technique to construct generic timelike supersymmetric solutions of $\mathcal{N} = 1, d = 5$ SUGRA when a $SU(2)$ gauging is present
2. As a particular example, I will consider microstate geometries with the same asymptotics than a non-Abelian generalization of the BMPV black hole (see Pedro's talk)
3. Finally, I will focus on the main result of our paper: We give an algebraic condition that tells if the solution has CTCs or not, which enormously simplify the task of constructing explicit solutions

Timelike supersymmetric solutions of $\mathcal{N}=1, d=5$ gauged supergravity

(Additional assumption: triholomorphic isometry)

- Metric:

$$
\begin{aligned}
ds^2 &= f^2(dt+\hat{\omega})^2 - f^{-1} d\hat{s}^2_{GH} \\
d\hat{s}^2_{GH} &= H^{-1}(dz+\chi)^2 + H dx^s dx^s
\end{aligned}
$$

- Vector fields:

$$
\begin{aligned}
A^I &= h^I f(dt+\hat{\omega}) + \hat{A}^I \\
\hat{A}^I &= -H^{-1}\Phi^I(dz+\chi) + \breve{A}^I
\end{aligned}
$$

- Scalars:

$$
h_I/f = L_I + 3C_{IJK}\Phi^J\Phi^K H^{-1}
$$

- 1-form $\hat{\omega}$:

$$
\hat{\omega} = \omega_5(dz+\chi) + \breve{\omega}
$$

All functions are independent of t and z.

BPS equations

The BPS eqs become the following set of diff. eqs on $\mathbb{E}^3$:

$$\begin{aligned}
\star_3 dH &= d\chi \\
\star_3 \breve{\mathfrak{D}}\Phi^I &= \breve{F}^I \qquad \text{(Bogomol'nyi eqs)} \\
\breve{\mathfrak{D}}^2 L_I &= g^2 f_{IJ}{}^L f_{KL}{}^M \Phi^J \Phi^K L_M \\
\star_3 d\breve{\omega} &= HdM - MdH + \frac{1}{2}(\Phi^I \breve{\mathfrak{D}} L_I - L_I \breve{\mathfrak{D}}\Phi^I) \\
\omega_5 &= M + \frac{1}{2} L_I \Phi^I H^{-1} + C_{IJK}\Phi^I \Phi^J \Phi^K H^{-2}
\end{aligned}$$

which can be solved by 10 harmonic functions on $\mathbb{E}^3$ $(H, M, \Phi^i, L_i, P, Q)$

Non-Abelian sector

The equations in the non-Abelian sector are also satisfied by two harmonic functions: $P(\vec{x})$ and $Q(\vec{x})$

$$A^A = -\sqrt{3} h^A f(dt + \hat{\omega}) + \hat{A}^A$$
$$\hat{A}^A = -2\sqrt{6}\left[-\Phi^A H^{-1}(dz + \chi) + \breve{A}^A\right]$$

Magnetic part $\hat{A}^A$:

$$\Phi^A = -\frac{1}{\breve{P}}\frac{\partial P}{\partial x^s}\delta^A_s \qquad \breve{A}^A = \epsilon^A{}_{rs}\frac{1}{\breve{g}P}\frac{\partial P}{\partial x^r}dx^s$$

It can be written by using a (generalized) 't Hooft ansatz

$$\hat{A}^A = \bar{\eta}^A_{mn}\partial_n \log P\, v^m$$

[Chimento, Meessen, Ortín, Ramírez and AR '18]

Electric part:

$$L_A = -\frac{1}{P}\frac{\partial Q}{\partial x^s} \qquad \text{(dyon)}$$

[Ramírez '16]

Multicenter solutions

A very big class of supersymmetric solutions describing non-Abelian multicenter BHs, black rings, MGs, etc... can be captured by taking generic multicenter functions:

[Meessen, Ortín, Ramírez '15]
[Ortín, Ramírez '16]
[Cano, Meessen, Ortín, Ramírez '17]
...

$$
\begin{aligned}
H &= \sum_a \frac{q_a}{r_a} \qquad M = m_0 + \frac{m_a}{r_a} \qquad L_i = l_0^i + \sum_a \frac{l_a^i}{r_a} \qquad \Phi^i = \sum_a \frac{k_a^i}{r_a} \\
P &= 1 + \sum_a \frac{\lambda_a}{r_a} \qquad Q = \sum_a \frac{\sigma_a \lambda_a}{r_a}
\end{aligned}
$$

Microstate geometries on a Gibbons-Hawking base

It is possible to find horizonless configurations if the parameters of the harmonic functions are related through

$$l_a^0 = -\frac{1}{q_a}\left(k_a^1 k_a^2 - \frac{1}{2g^2}\right) \qquad l_a^{1,2} = -\frac{k_a^0 k_a^{2,1}}{q_a}$$

$$\sigma_a = \frac{k_a^0}{q_a} \qquad m_a = \frac{k_a^0}{2q_a^2}\left(k_a^1 k_a^2 - \frac{1}{2g^2}\right)$$

Asymptotic flatness demands

$$l_0^0 l_0^1 l_0^2 = 1 \qquad m_0 = -\frac{1}{2}\sum_{i,a} l_0^i k_a^i$$

Therefore, asymptotically flat horizonless configurations are specified by $4(n-1)$ charge parameters (k_a^i and q_a), 2 moduli parameters and n non-Abelian *hair* parameters λ_a and $3(n-2)$ coordinates of the centers (which are not really independent because they must satisfy $n(n-1)(n-2)/2$ triangular inequalities...)

Ambipolar GH spaces

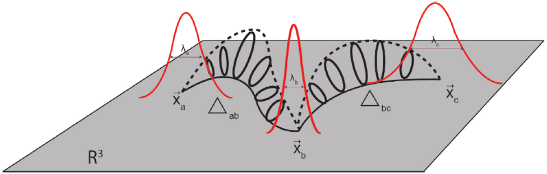
$$d\hat{s}^2 = H^{-1}(dz+\chi)^2 + H dx^s dx^s \qquad H = \sum_a \frac{q_a}{r_a} \quad q_a \in \mathbb{Z}$$

Asymptotically flat solutions $\Rightarrow \sum_a q_a = 1$

1. It alternates from $++++$ to $----$ signature (but the full 5-dim metric has the right signature)
2. Non-trivial topology (non-contractible 2-cycles Δ_{ab})- crucial to have asymptotic charges without brane sources

Fluxes

$$\Pi^i_{ab} = \int_{\Delta_{ab}} F^i = \frac{k^i_b}{q_b} - \frac{k^i_a}{q_a}$$



Bubble equations

I will discuss the Abelian case, since the introduction of non-Abelian fields is not relevant for the discussion

- The integrability condition of $\breve{\omega}$ leads to

$$\sum_{b \neq a} \frac{q_a q_b}{r_{ab}} \Pi^0_{ab} \Pi^1_{ab} \Pi^2_{ab} = \sum_{b,i} q_a q_b l^i_0 \Pi^i_{ab}$$

  which are known as the **bubble equations**

- These $n-1$ equations relate the size of the bubbles and the charge parameters

- If the bubble equations are satisfied, it is guaranteed that $\hat{\omega}$ is regular everywhere and therefore Dirac-Misner singularities are avoided

Solving the bubble equations I

- Traditionally seen as constrains for the sizes of the bubbles (solved for the distances)

- We can look at them as equations constraining the fluxes (say $\Pi^2 \sim X$) once we give as input data the size of the bubbles r_{ab}, the GH charges q_a and the remaining fluxes Π^0 and Π^1

$$\mathcal{M}\left(q_a, r_{ab}, \Pi^0, \Pi^1\right) X = B\left(q_a, r_{ab}, \Pi^0, \Pi^1\right)$$

- The quantization condition for the fluxes

$$k_a^i = \frac{(2\pi)^2 \, l_P^3}{V_i} n_a^i \qquad\qquad n_a^i \in \mathbb{Z}$$

[Berglund, Simon, Levi]

is satisfied if the fluxes are given by rational numbers, what forces us to consider set of points whose distances are also given by rational numbers

Solving the bubble equations II

The components of $\mathcal{M}^2$ and B^2 are

$$\mathcal{M}^2_{\underline{ab}} = \alpha^2_{(\underline{a}+1)(\underline{b}+1)} - \delta^{\underline{b}}_{\underline{a}} \sum_{c=1}^{n} \alpha^2_{(\underline{a}+1)c} \qquad B^2_{\underline{a}} = \beta^2_{(\underline{a}+1)}$$

where

$$\alpha^2_{ab} = \begin{cases} \frac{q_a q_b}{r_{ab}} \left(\Pi^0_{ab}\Pi^1_{ab} - l^2_0 r_{ab} \right) & a \neq b \\ 0 & a = b \end{cases}$$

and

$$\beta^2_a = \sum_{b=1}^{n} q_a q_b \left(l^0_0 \Pi^0_{ab} + l^1_0 \Pi^1_{ab} \right)$$

Defining $X^2_{\underline{b}} = \Pi^2_{1(\underline{b}+1)}$, the bubble eqs read: $\sum_b \mathcal{M}^2_{\underline{ab}} X^2_{\underline{b}} = B_{\underline{a}}$

Alternatively, one can take the 0-fluxes or the 1-fluxes as the unknowns, what defines the matrices $\mathcal{M}^0$ and $\mathcal{M}^1$. The three matrices play the same role

Absence of CTCs

In a slice of constant t

$$-ds_4^2 = \frac{\mathcal{I}}{f^{-2}H^2}\left(dz + \chi - \frac{\omega_5 H^2}{\mathcal{I}}\breve{\omega}\right)^2 + f^{-1}H\left(r^2\sin\theta^2 d\phi^2 - \frac{\breve{\omega}^2}{\mathcal{I}}\right) + f^{-1}H\left(dr^2 + r^2 d\theta^2\right)$$

Condition to avoid CTCs:

$$\mathcal{I} = f^{-3}H - \omega_5^2 H^2 > 0 \quad \Rightarrow \quad f^{-3}H^3 = Z_0 H\, Z_1 H\, Z_2 H > 0$$

- Near the centers and asymptotically the first term $f^{-3}H$ dominates since $\omega_{5,\infty} = 0$ and $\lim_{r_a \to 0} \omega_5 \sim \mathcal{O}(r_a)$
- **Claim**: $f^{-3}H$ also dominates in intermediate regions and the positivity of $\mathcal{I}$ is guaranteed if $f^{-3}H > 0$
- Since Z_i changes sign when H does, $f^{-3}H > 0$ implies $Z_i H > 0$

Relation with the bubble equations

In particular, it must happen that $\lim_{r_a \to 0} Z_i H > 0$. Explicitly,

$$\lim_{r_a \to 0} Z_i H = \frac{1}{r_a} \left[l_0^i q_a - \frac{|\epsilon_{ijk}|}{2} \sum_{\substack{b=1 \\ b \neq a}}^{n} \frac{q_a q_b}{r_{ab}} \Pi_{ab}^j \Pi_{ab}^k \right] + \mathcal{O}(r_a^0) \qquad \epsilon_{012} = +1$$

Remarks:

- The terms between brackets are very similar to the diagonal elements of the matrices $\mathcal{M}^i$

$$\mathcal{M}^i_{(a-1)(a-1)} = l_0^i q_a (1 - q_a) - \frac{|\epsilon_{ijk}|}{2} \sum_{\substack{b=1 \\ b \neq a}}^{n} \frac{q_a q_b}{r_{ab}} \Pi_{ab}^j \Pi_{ab}^k$$

- Indeed, the positivity of the diagonal elements implies the positivity of the divergences of $Z_i H$

Simplest case: 2 centers

In order to gain some insight, we studied 2-center configurations. We found that

1. It is possible to show that $Z_i H > 0$ everywhere if $\mathcal{M}^i > 0$ are positive (still not useful since $\mathcal{M}^0$ and $\mathcal{M}^1$ depend on Π^2, obtained by solving the bubble eqs)

2. If one of the matrices, let's say $\mathcal{M}^2$, is positive and the bubble equations are satisfied, so are the rest

3. If the parameters are choosen in such a way that $Z_2 H > 0$ but $\mathcal{M}^2 < 0$, at some point $Z_0 H$ and $Z_1 H$ become negative

Conclusion: The positivity of $\mathcal{M}^2$ is a necessary and sufficient condition for the positivity of $Z_i H$ and also (if we trust in the previous claim) of the quartic invariant $\mathcal{I}$

An algebraic criterion to avoid general CTCs

For the multicenter case, we conjectured that

A given configuration is free of CTCs if and only if all the eigenvalues of the matrix $\mathcal{M}^2$ are positive

- The conjecture has been tested in more than 100.000 solutions with pseudo-random parameters
- It is possible to design an algorithmic procedure to explore the full space of solutions in a systematic way
- Examples with exotic properties (large number of centers, very low angular momentum, etc...)

[Ávila, Ramírez and AR '18]

Conclusions and future directions

- We give an algebraic criterion to discern if a solution has closed timelike curves or not
- This help us enormously in the task of constructing of explicit solutions
- Physics behind ?
- It would be very interesting to find and understand the CFT dual descriptions of these solutions
- ...

Thanks for your attention!

Extra material

The theory: $\mathcal{N}=1$, $d=5$ $SU(2)$-gauged supergravity

Action (bosonic sector)

$$S=\int d^5x\sqrt{|g|}\,\Big\{R+\tfrac{1}{2}g_{xy}\mathfrak{D}_\mu\phi^x\mathfrak{D}^\mu\phi^y-\tfrac{1}{4}a_{IJ}F^{I\,\mu\nu}F^J{}_{\mu\nu}-\tfrac{1}{4}C_{IJK}\frac{\varepsilon^{\mu\nu\rho\sigma\lambda}}{\sqrt{|g|}}\Big[F^I{}_{\mu\nu}F^J{}_{\rho\sigma}A^K{}_\lambda$$
$$-\tfrac{1}{2}gf_{LM}{}^IF^J{}_{\mu\nu}A^K{}_\rho A^L{}_\sigma A^M{}_\lambda+\tfrac{1}{10}g^2f_{LM}{}^If_{NP}{}^JA^K{}_\mu A^L{}_\nu A^M{}_\rho A^N{}_\sigma A^P{}_\lambda\Big]\Big\}$$

- $ST[2,6]$ model - $C_{0xy}=\frac{1}{6}\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&-1&0&0\\0&0&0&-1&0\\0&0&0&0&-1\end{pmatrix}$
- Field content- Gravity multiplet $(e^a{}_\mu, A^0_\mu, \psi^i_\mu)$+5 vector multiplets $(\phi^x, A^x_\mu, \lambda^{xi})$
- Non-Abelian gauging: three of the vector fields (A^A w/ $A=3,4,5$) gauge a $SU(2)$ subgroup of the isometries of the scalar manifold
- This model can be obtained from $\mathcal{N}=1$, $d=10$ Heterotic supergravity upon compactifcation on a 5-torus followed by a truncation of some of the Kaluza-Klein massless modes

[Cano, Ortín, Santoli '16]

Timelike supersymmetric solutions

Timelike supersymmetric solutions of this theory are of the form

$$
\begin{aligned}
ds^2 &= f^2(dt+\hat{\omega})^2 - f^{-1}d\hat{s}^2_{HK} \ , \\
A^I &= -\sqrt{3}h^I f(dt+\hat{\omega}) + \hat{A}^I \ , \\
\phi^x &= h_x/h_0 \ ,
\end{aligned}
$$

with

$$
h^I \equiv 27 C^{IJK} h_J h_K \ , \qquad h^I h_I = 1 \ .
$$

These *base-space fields* satisfy the following set of BPS equations

$$
\begin{aligned}
\hat{F}^I &= \hat{\star}_4 \hat{F}^I \ , \qquad \text{(Instanton)} \\
\hat{\mathfrak{D}}^2 (h_I/f) &= \frac{1}{6} C_{IJK} \hat{F}^J \cdot \hat{F}^K \ , \\
d\hat{\omega} + \hat{\star}_4 d\hat{\omega} &= \frac{\sqrt{3}}{2}(h_I/f)\hat{F}^I \ .
\end{aligned}
$$

[Bellorín, Ortín, '07]

Still hard to solve...

Asymptotic charges

Abelian charges:

$$\begin{aligned}\mathcal{Q}_0 &= -\sum_{a,b,c} q_a q_b q_c \Pi^1_{ab}\Pi^2_{ac} + \frac{1}{2g^2}\sum_a \frac{1}{q_a} \\ \mathcal{Q}_{1(2)} &= -\sum_{a,b,c} q_a q_b q_c \Pi^0_{ab}\Pi^{1(2)}_{ac}\end{aligned}$$

Angular momentum:

$$J = -\frac{1}{2}\sum_{a,b,c,d} q_a q_b q_c q_d \Pi^0_{ab}\Pi^1_{ac}\Pi^2_{ad} + \frac{1}{4g^2}\sum_{a,b}\frac{q_b\Pi^0_{ab}}{q_a}$$

Mass:

$$\mathcal{M} = \frac{\pi}{G_N^{(5)}}\left(\frac{\mathcal{Q}_0}{l_0^0} + \frac{\mathcal{Q}_1}{l_0^1} + \frac{\mathcal{Q}_2}{l_0^2}\right)$$

Entropy (of the corresponding BH solution):

$$S = \frac{\pi^2}{2G_N^{(5)}}\sqrt{\mathcal{Q}_0\mathcal{Q}_1\mathcal{Q}_2 - J^2}$$

Scaling solutions

Scaling solutions are those for which the centers can be brought arbitrarily close without modifying significantly the asymptotic charges. Let's consider those that preserve the form of the distribution

$$r_{ab} = \mu d_{ab}$$

($\mu \to 0$ is the scaling limit)

$$\left(\bar{\mathcal{M}}^2 + \mu \mathring{\mathcal{M}}^2\right) X^2 = \bar{B}^2 + \mu \mathring{B}^2$$

- In the Abelian limit $g \to \infty$, $\bar{B}^2 \to 0$. The majority of the Abelian solutions cannot be scaled (only those at special points of the parameter space: $\det \bar{M}^2 = 0$)
- When non-Abelian fields are present, the solutions can always be scaled since $\bar{B}^2 \neq 0$