Weaving the Exotic Web

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Based on a collaboration with

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and **Tetsuji Kimura** (Nihon U), arxiv:1805.12117.

Geometry, Duality and Strings 2018

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Before explaining our motivation,
I will review

Exotic brane?

[Elitzur, Giveon, Kutasov, Rabinovici ‘97;
Blau, O′Loughlin ′97;
Obers, Pioline ′99; Eyras, Lozano ′00;
Lozano-Tellechea, Ortin ′00]

Standard branes

In type II string theories or M-theory, there are many supersymmetric branes:

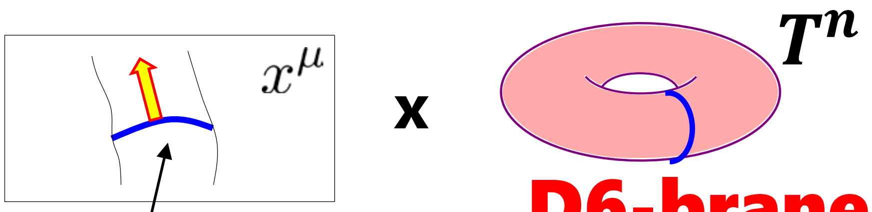
Type II: P, F1, Dp, NS5.

M-theory: P, M2, M5.

Wrapped brane

Type II string / n-torus:

We can consider a wrapped brane.



4-brane

D6-brane wrapped on a 2-cycle

$$T_4 \equiv \underbrace{\boxed{\frac{1}{g_s\, l_s^7}}}_{\text{D6 tension}} \times \underbrace{(R_1 R_2)}_{\text{area}} = \frac{1}{g_s\, l_s^5}\left(\frac{R_1 R_2}{l_s^2}\right)$$

Wrapped brane

Under a T-duality

$$\boxed{g_s \to \frac{l_s}{R_y} g_s, \quad R_y \to \frac{l_s^2}{R_y}}$$

$$\begin{tikzcd}
T_4 = \frac{1}{g_s\, l_s^5}\left(\frac{R_1 R_2}{l_s^2}\right) \arrow[r, "T_3"] \arrow[d, "T_2"] & T_4 = \frac{1}{g_s\, l_s^5}\left(\frac{R_1 R_2 R_3}{l_s^3}\right) \\
T_4 = \frac{1}{g_s\, l_s^5}\left(\frac{R_1}{l_s}\right) &
\end{tikzcd}$$

Tension of D7 wrapped on a 3-cycle

Tension of D5 wrapped on a 1-cycle

Wrapped brane

Under a T-duality

$$\boxed{g_s \to \frac{l_s}{R_y}\, g_s\,, \qquad R_y \to \frac{l_s^2}{R_y}}$$

$$T_4 = \frac{1}{g_s\, l_s^5}\left(\frac{R_1 R_2}{l_s^2}\right) \xrightarrow{T_3} T_4 = \frac{1}{g_s\, l_s^5}\left(\frac{R_1 R_2 R_3}{l_s^3}\right)$$

Tension of D7 wrapped on a 3-cycle

S-duality

$$\boxed{l_s^2 \to g_s\, l_s^2\,, \qquad g_s \to g_s^{-1}}$$

$$T_4 = \frac{1}{g_s^3\, l_s^5}\left(\frac{R_1 R_2 R_3}{l_s^3}\right)$$

cubic

Tension of NS7 (Q7) wrapped on a 3-cycle

Exotic branes

$$T_4 = \frac{1}{g_s^3\, l_s^5}\left(\frac{R_1R_2R_3}{l_s^3}\right)$$

Tension of NS7 (Q7) wrapped on a 3-cycle

cubic

$$\Big\downarrow\, T_4$$

$$T_4 = \frac{1}{g_s^3\, l_s^5}\left(\frac{R_1R_2R_3}{l_s^3}\right)\left(\frac{R_4}{l_s}\right)^2$$

Tension of KK8 wrapped on a 3-cycle

quadratic

$$\Big\downarrow\, T_5$$

$$T_4 = \frac{1}{g_s^3\, l_s^5}\left(\frac{R_1R_2R_3}{l_s^3}\right)\left(\frac{R_4R_5}{l_s^2}\right)^2$$

Tension of ??

Exotic branes

We follow the notation of [Obers, Pioline, hep-th/9809039].

If a p-brane has the tension

$$T_p = \frac{1}{g_s^n\, l_s}\left(\frac{R_{n_1}\cdots R_{n_{b-p}}}{l_s^b}\right)\left(\frac{R_{m_1}\cdots R_{m_{c_2}}}{l_s^{c_2}}\right)^2\cdots\left(\frac{R_{\ell_1}\cdots R_{\ell_{c_s}}}{l_s^{c_s}}\right)^s$$

we call the brane a $b_n^{(c_s,\dots,c_2)}$-brane.

Type II / T^7

$n = 0, 1, \dots, 11$

$s = 2, \dots, 6$

For $n = 2, \dots, 11$, the branes other than 5_2 and 5_2^1 are usually called exotic branes.

NS5 **KKM**

Exotic branes

Type IIB / T^7

**[T.Kimura,
J.J. Fernandez-Melgarejo, YS]**

$$\begin{gathered}
\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,\ 6_3^{(1,1)}\,, \\
4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,\ 2_4^{(3,3)}\,, \\
1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,\ 2_5^{(1,5)}\,, \\
2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,\ 2_5^{(1,0,4,2)}\,, \\
2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,\ 2_5^{(3,1,3)}\,, \\
2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,\ 3_6^{(3,2,1)}\,, \\
3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,\ 1_7^{(3,4,0)}\,, \\
1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ 2_7^{(1,0,1,5,0)}\,,\ 2_7^{(1,0,3,3,0)}\,,\ 2_7^{(1,0,5,1,0)}\,; \\
2_7^{(1,3,3)}\,,\ 2_7^{(3,1,3)}\,,\ 2_7^{(1,0,4,2)}\,,\ 2_7^{(1,2,2,2)}\,,\ 2_7^{(1,4,0,2)}\,,\ 2_7^{(2,1,3,1)}\,,\ 2_7^{(2,3,1,1)}\,,\ 2_7^{(3,0,4,0)}\,, \\
2_7^{(3,2,2,0)}\,,\ 2_7^{(3,4,0,0)}\,,\ 1_8^{(7,0,0)}\,,\ 2_8^{(1,0,6,0)}\,,\ 2_8^{(3,0,4,0)}\,,\ 2_8^{(5,0,2,0)}\,,\ 2_8^{(7,0,0,0)}\,,\ 2_8^{(3,3,1)}\,, \\
2_8^{(1,3,2,1)}\,,\ 2_8^{(2,3,1,1)}\,,\ 2_8^{(3,3,0,1)}\,,\ 2_8^{(1,0,3,3,0)}\,,\ 2_8^{(1,1,3,2,0)}\,,\ 2_8^{(1,2,3,1,0)}\,,\ 2_8^{(1,3,3,0,0)}\,, \\
2_9^{(1,4,2,0)}\,,\ 2_9^{(3,2,2,0)}\,,\ 2_9^{(5,0,2,0)}\,,\ 2_9^{(1,0,5,1,0)}\,,\ 2_9^{(1,2,3,1,0)}\,,\ 2_9^{(1,4,1,1,0)}\,,\ 2_9^{(2,1,4,0,0)}\,, \\
2_9^{(2,3,2,0,0)}\,,\ 2_9^{(2,5,0,0,0)}\,,\ 2_{10}^{(3,4,0,0)}\,,\ 2_{10}^{(1,3,3,0,0)}\,,\ 2_{10}^{(2,3,2,0,0)}\,,\ 2_{10}^{(3,3,1,0,0)}\,,\ 2_{10}^{(4,3,0,0,0)}\,, \\
2_{11}^{(7,0,0,0)}\,,\ 2_{11}^{(2,5,0,0,0)}\,,\ 2_{11}^{(4,3,0,0,0)}\,,\ 2_{11}^{(6,1,0,0,0)}\,.
\end{gathered}$$

defect branes, domain-wall branes, space-filling branes

Exotic branes

[T.Kimura,
J.J. Fernandez-Melgarejo, YS]

Type IIA / T^7

$$\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D0}\,,\ \mathrm{D2}\,,\ \mathrm{D4}\,,\ \mathrm{D6}\,,\ 8_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 6^1_3\,,\ 4^3_3\,,\ 2^5_3\,,\ 0^7_3\,,$$

$$7^{(1,0)}_3\,,\ 5^{(1,2)}_3\,,\ 3^{(1,4)}_3\,,\ 1^{(1,6)}_3\,,\ 6^{(2,1)}_3\,,\ 4^{(2,3)}_3\,,\ 2^{(2,5)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,,$$

$$3^{(2,3)}_4\,,\ 2^{(3,3)}_4\,,\ 1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 8^{(1,0)}_4\,,\ 6^{(3,0)}_4\,,\ 4^{(5,0)}_4\,,$$

$$2^{(7,0)}_4\,,\ 2^6_5\,,\ 2^{(2,4)}_5\,,\ 2^{(4,2)}_5\,,\ 2^{(6,0)}_5\,,\ 1^{(1,1,5)}_5\,,\ 1^{(1,3,3)}_5\,,\ 1^{(1,5,1)}_5\,,\ 2^{(1,0,1,5)}_5\,,\ 2^{(1,0,3,3)}_5\,,$$

$$2^{(1,0,5,1)}_5\,,\ 5^{(1,3)}_5\,,\ 5^{(3,1)}_5\,,\ 4^{(1,0,4)}_5\,,\ 4^{(1,2,2)}_5\,,\ 4^{(1,4,0)}_5\,,\ 3^{(2,1,3)}_5\,,\ 3^{(2,3,1)}_5\,,\ 2^{(3,0,4)}_5\,,$$

$$2^{(3,2,2)}_5\,,\ 2^{(3,4,0)}_5\,,\ 1^{(4,3)}_6\,,\ 1^{(1,4,2)}_6\,,\ 1^{(2,4,1)}_6\,,\ 1^{(3,4,0)}_6\,,\ 3^{(2,4)}_6\,,\ 3^{(1,2,3)}_6\,,\ 3^{(2,2,2)}_6\,,$$

$$3^{(3,2,1)}_6\,,\ 3^{(4,2,0)}_6\,,\ 2^{(1,0,2,4)}_6\,,\ 2^{(1,1,2,3)}_6\,,\ 2^{(1,2,2,2)}_6\,,\ 2^{(1,3,2,1)}_6\,,\ 2^{(1,4,2,0)}_6\,,\ 1^{(7,0)}_7\,,$$

$$1^{(2,5,0)}_7\,,\ 1^{(4,3,0)}_7\,,\ 1^{(6,1,0)}_7\,,\ 3^{(1,5,0)}_7\,,\ 3^{(3,3,0)}_7\,,\ 3^{(5,1,0)}_7\,,\ 2^{(1,0,0,6,0)}_7\,,\ 2^{(1,0,2,4,0)}_7\,,$$

$$2^{(1,0,4,2,0)}_7\,,\ 2^{(1,0,6,0,0)}_7\,,\ 2^{(4,3)}_7\,,\ 2^{(2,2,3)}_7\,,\ 2^{(4,0,3)}_7\,,\ 2^{(1,1,3,2)}_7\,,\ 2^{(1,3,1,2)}_7\,,\ 2^{(2,0,4,1)}_7\,,$$

$$2^{(2,2,2,1)}_7\,,\ 2^{(2,4,0,1)}_7\,,\ 2^{(3,1,3,0)}_7\,,\ 2^{(3,3,1,0)}_7\,,\ 1^{(7,0,0)}_8\,,\ 2^{(7,0)}_8\,,\ 2^{(2,0,5,0)}_8\,,\ 2^{(4,0,3,0)}_8\,,$$

$$2^{(6,0,1,0)}_8\,,\ 2^{(3,3,1)}_8\,,\ 2^{(1,3,2,1)}_8\,,\ 2^{(2,3,1,1)}_8\,,\ 2^{(3,3,0,1)}_8\,,\ 2^{(1,0,3,3,0)}_8\,,\ 2^{(1,1,3,2,0)}_8\,,\ 2^{(1,2,3,1,0)}_8\,;$$

$$2^{(1,3,3,0,0)}_8\,,\ 2^{(5,2,0)}_9\,,\ 2^{(2,3,2,0)}_9\,,\ 2^{(4,1,2,0)}_9\,,\ 2^{(1,1,4,1,0)}_9\,,\ 2^{(1,3,2,1,0)}_9\,,\ 2^{(1,5,0,1,0)}_9\,,$$

$$2^{(2,0,5,0,0)}_9\,,\ 2^{(2,2,3,0,0)}_9\,,\ 2^{(2,4,1,0,0)}_9\,,\ 2^{(3,4,0,0)}_{10}\,,\ 2^{(1,3,3,0,0)}_{10}\,,\ 2^{(2,3,2,0,0)}_{10}\,,\ 2^{(3,3,1,0,0)}_{10}\,,$$

$$2^{(4,3,0,0,0)}_{10}\,,\ 2^{(1,6,0,0,0)}_{11}\,,\ 2^{(3,4,0,0,0)}_{11}\,,\ 2^{(5,2,0,0,0)}_{11}\,,\ 2^{(7,0,0,0,0)}_{11}\,.$$

defect branes, domain-wall branes, space-filling branes

Exotic branes

In the paper,
we also studied M-theory branes.

$$\boxed{R_{\mathrm{M}} = g_s l_s \,, \qquad l_p = g_s^{1/3}\, l_s \,.}$$

$$\mathrm{P}\,,\ \mathrm{M2}\,,\ \mathrm{M5}\,,\ \mathrm{KKM}\,,\ 5^3_{12}\,,\ 8^{(1,0)}_{12}\,,\ 2^6_{15}\,,\ 5^{(1,3)}_{15}\,,\ 0^{(1,7)}_{18}\,,\ 3^{(2,4)}_{18}\,,\ 6^{(3,1)}_{18}\,,\ 5^{(1,0,4)}_{18}\,,$$

$$2^{(4,3)}_{21}\,,\ 1^{(1,1,6)}_{21}\,,\ 4^{(1,2,3)}_{21}\,,\ 2^{(7,0)}_{24}\,,\ 1^{(1,4,3)}_{24}\,,\ 4^{(1,5,0)}_{24}\,,\ 3^{(2,2,3)}_{24}\,,\ 2^{(1,0,2,5)}_{24}\,,$$

$$1^{(2,5,1)}_{27}\,,\ 3^{(3,3,1)}_{27}\,,\ 2^{(4,0,4)}_{27}\,,\ 2^{(1,1,3,3)}_{27}\,,\ 1^{(4,4,0)}_{30}\,,\ 3^{(5,2,0)}_{30}\,,\ 2^{(1,3,2,2)}_{30}\,,\ 2^{(2,0,5,1)}_{30}\,,\ 2^{(1,0,0,7,0)}_{30}\,,$$

$$1^{(7,1,0)}_{33}\,,\ 2^{(2,3,2,1)}_{33}\,,\ 2^{(1,0,3,4,0)}_{33}\,,\ 2^{(3,4,0,1)}_{36}\,,\ 2^{(4,1,3,0)}_{36}\,,\ 2^{(1,1,4,2,0)}_{36}\,,\ 2^{(1,3,3,1,0)}_{39}\,,\ 2^{(2,0,6,0,0)}_{39}\,;$$

$$2^{(1,6,0,1,0)}_{42}\,,\ 2^{(2,3,3,0,0)}_{42}\,,\ 2^{(3,4,1,0,0)}_{45}\,,\ 2^{(5,3,0,0,0)}_{48}\,,\ 2^{(8,0,0,0,0)}_{51}\,.$$

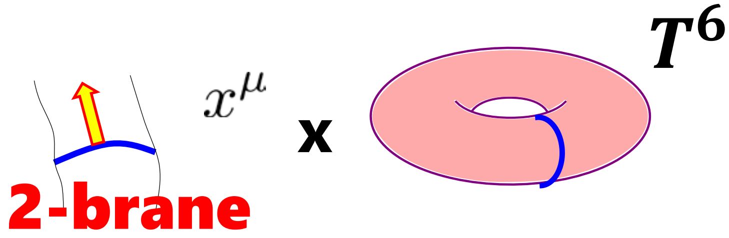
Here, I will concentrate only on type II branes.

Exotic branes

We have counted the number of p-branes in each dimensions:

For example,

$\underline{d = 4}$



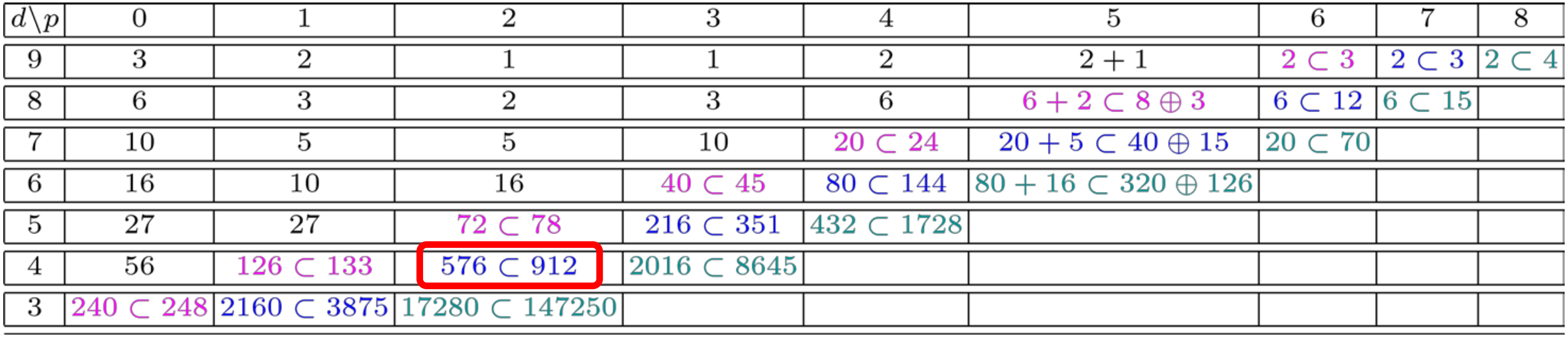
type IIB branes

D3 $(\mathbf{6})$, D5 $(\mathbf{20})$, D7 $(\mathbf{6})$, NS5 $(\mathbf{20})$, KKM $(\mathbf{60})$, 5^2_2 $(\mathbf{60})$, 5^3_2 $(\mathbf{20})$, 7_3 $(\mathbf{6})$, 5^2_3 $(\mathbf{60})$, 3^4_3 $(\mathbf{30})$, $6^{(1,1)}_3$ $(\mathbf{30})$, $4^{(1,3)}_3$ $(\mathbf{60})$, $2^{(1,5)}_3$ $(\mathbf{6})$, 5^3_4 $(\mathbf{20})$, $4^{(1,3)}_4$ $(\mathbf{60})$, $3^{(2,3)}_4$ $(\mathbf{60})$, $2^{(3,3)}_4$ $(\mathbf{20})$, $2^{(1,5)}_5$ $(\mathbf{6})$, $2^{(3,3)}_5$ $(\mathbf{20})$, $2^{(5,1)}_5$ $(\mathbf{6})$

In total, 576 branes!

Exotic branes

We have counted the number of p-branes:

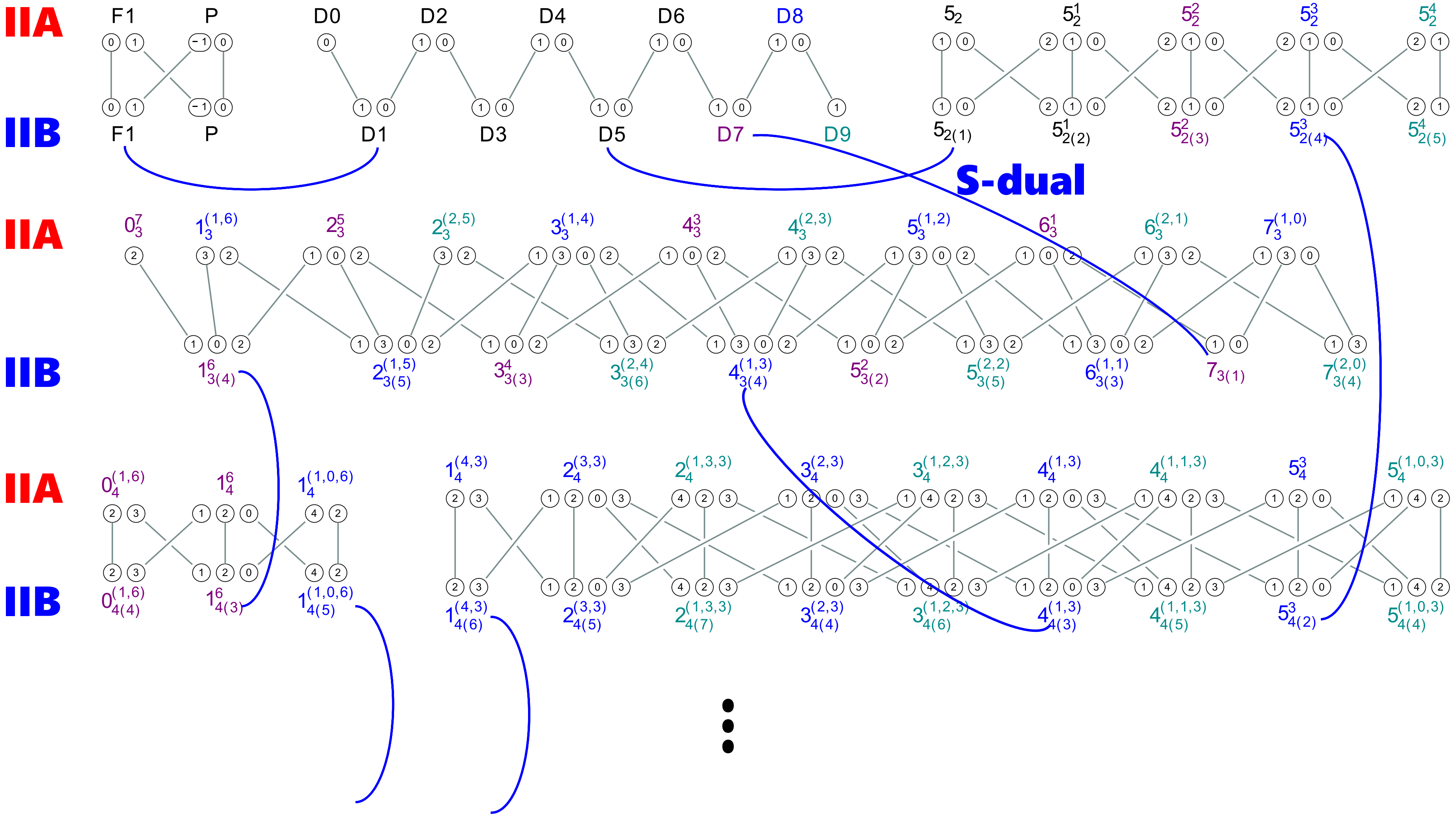


$d\backslash p$	0	1	2	3	4	5	6	7	8
9	3	2	1	1	2	$2+1$	$2 \subset 3$	$2 \subset 3$	$2 \subset 4$
8	6	3	2	3	6	$6+2 \subset 8 \oplus 3$	$6 \subset 12$	$6 \subset 15$	
7	10	5	5	10	$20 \subset 24$	$20+5 \subset 40 \oplus 15$	$20 \subset 70$		
6	16	10	16	$40 \subset 45$	$80 \subset 144$	$80+16 \subset 320 \oplus 126$			
5	27	27	$72 \subset 78$	$216 \subset 351$	$432 \subset 1728$				
4	56	$126 \subset 133$	$576 \subset 912$	$2016 \subset 8645$					
3	$240 \subset 248$	$2160 \subset 3875$	$17280 \subset 147250$						

↕ **Consistent**

**[A. Kleinschmidt,
“Counting supersymmetric branes,” arXiv:1109.2025]**

“Weaving the **Exotic Web**”



Motivation of our work

[Meessen, Ortin hep-th/9806120]

Supergravity solutions for the standard branes/**defect-branes** are known.

U-folds [de Boer-Shigemori '10; '12]

(globally non-geometric backgrounds)

$$\boxed{\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 7_3\,,\ 5^2_3\,,\ 3^4_3\,,\ 1^6_3\,,\ 6^{(1,1)}_3\,,$$

$$4^{(1,3)}_3\,,\ 2^{(1,5)}_3\,,\ 7^{(2,0)}_3\,,\ 5^{(2,2)}_3\,,\ 3^{(2,4)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,,\ 3^{(2,3)}_4\,,\ 2^{(3,3)}_4\,,$$

$$1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 9_4\,,\ 7^{(2,0)}_4\,,\ 5^{(4,0)}_4\,,\ 3^{(6,0)}_4\,,\ 2^{(1,5)}_5\,,$$

$$2^{(3,3)}_5\,,\ 2^{(5,1)}_5\,,\ 1^{(1,0,6)}_5\,,\ 1^{(1,2,4)}_5\,,\ 1^{(1,4,2)}_5\,,\ 1^{(1,6,0)}_5\,,\ 2^{(1,0,0,6)}_5\,,\ 2^{(1,0,2,4)}_5\,,\ 2^{(1,0,4,2)}_5\,,$$

$$2^{(1,0,6,0)}_5\,,\ 5^4_5\,,\ 5^{(2,2)}_5\,,\ 5^{(4,0)}_5\,,\ 4^{(1,1,3)}_5\,,\ 4^{(1,3,1)}_5\,,\ 3^{(2,0,4)}_5\,,\ 3^{(2,2,2)}_5\,,\ 3^{(2,4,0)}_5\,,\ 2^{(3,1,3)}_5\,,$$

$$2^{(3,3,1)}_5\,,\ \cdots$$

How about other branes?

domain-wall branes

space-filling branes

Domain-wall solutions

In type IIA, SUGRA solutions for D8 and KK8 branes are known.

[Meessen, Ortin hep-th/9806120]

$$\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D0}\,,\ \mathrm{D2}\,,\ \mathrm{D4}\,,\ \mathrm{D6}\,,\ \boxed{\mathrm{D8}}\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 6^1_3\,,\ 4^3_3\,,\ 2^5_3\,,\ 0^7_3\,;$$

$$\boxed{7^{(1,0)}_3}\,,\ 5^{(1,2)}_3\,,\ 3^{(1,4)}_3\,,\ 1^{(1,6)}_3\,,\ 6^{(2,1)}_3\,,\ 4^{(2,3)}_3\,,\ 2^{(2,5)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,;$$

$$3^{(2,3)}_4\,,\ 2^{(3,3)}_4\,,\ 1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 8^{(1,0)}_4\,,\ 6^{(3,0)}_4\,,\ 4^{(5,0)}_4\,;$$

$$2^{(7,0)}_4\,,\ 2^6_5\,,\ 2^{(2,4)}_5\,,\ 2^{(4,2)}_5\,,\ 2^{(6,0)}_5\,,\ 1^{(1,1,5)}_5\,,\ 1^{(1,3,3)}_5\,,\ 1^{(1,5,1)}_5\,,\ 2^{(1,0,1,5)}_5\,,\ 2^{(1,0,3,3)}_5\,;$$

$$2^{(1,0,5,1)}_5\,,\ 5^{(1,3)}_5\,,\ 5^{(3,1)}_5\,,\ 4^{(1,0,4)}_5\,,\ 4^{(1,2,2)}_5\,,\ 4^{(1,4,0)}_5\,,\ 3^{(2,1,3)}_5\,,\ 3^{(2,3,1)}_5\,,\ 2^{(3,0,4)}_5\,;$$

$$2^{(3,2,2)}_5\,,\ 2^{(3,4,0)}_5\,,\ \cdots\,.$$

However, they are solutions of deformed type IIA SUGRAs.

D8 … massive IIA

KK8 … another

non-covariant

Other domain-wall solutions are not obtained yet.

Domain-wall solutions

In this talk, I will show that
for all of the domain-wall branes,
their SUGRA backgrounds are
solutions of certain deformed SUGRAs.

(non-covariant)

$$
\begin{gathered}
\text{F1}\,,\ \text{P}\,,\ \text{D0}\,,\ \text{D2}\,,\ \text{D4}\,,\ \text{D6}\,,\ \text{D8}\,,\ \text{NS5}\,,\ \text{KKM}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 6^1_3\,,\ 4^3_3\,,\ 2^5_3\,,\ 0^7_3\,; \\
7^{(1,0)}_3\,,\ 5^{(1,2)}_3\,,\ 3^{(1,4)}_3\,,\ 1^{(1,6)}_3\,,\ 6^{(2,1)}_3\,,\ 4^{(2,3)}_3\,,\ 2^{(2,5)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,; \\
3^{(2,3)}_4\,,\ 2^{(3,3)}_4\,,\ 1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 8^{(1,0)}_4\,,\ 6^{(3,0)}_4\,,\ 4^{(5,0)}_4\,; \\
2^{(7,0)}_4\,,\ 2^6_5\,,\ 2^{(2,4)}_5\,,\ 2^{(4,2)}_5\,,\ 2^{(6,0)}_5\,,\ 1^{(1,1,5)}_5\,,\ 1^{(1,3,3)}_5\,,\ 1^{(1,5,1)}_5\,,\ 2^{(1,0,1,5)}_5\,,\ 2^{(1,0,3,3)}_5\,; \\
2^{(1,0,5,1)}_5\,,\ 5^{(1,3)}_5\,,\ 5^{(3,1)}_5\,,\ 4^{(1,0,4)}_5\,,\ 4^{(1,2,2)}_5\,,\ 4^{(1,4,0)}_5\,,\ 3^{(2,1,3)}_5\,,\ 3^{(2,3,1)}_5\,,\ 2^{(3,0,4)}_5\,; \\
2^{(3,2,2)}_5\,,\ 2^{(3,4,0)}_5\,,\ \cdots\,.
\end{gathered}
$$

Our strategy

We first construct SUGRA backgrounds for all of the domain-wall branes as solutions of Exceptional Field Theory (EFT).

Manifestly U-duality covariant SUGRA

**[West ’00; Berman, Perry ’11;
Berman, Godazgar, Perry, West ’12;
Hohm, Samtleben ’13; ....]**

(generalization of Double Field Theory)

Our strategy

We find that the EFT solutions have
a linear dependence on the winding-coordinate.

As I explain later, we can convert
the linear winding-coordinate dependence
into a deformation parameter of SUGRA.

⇩

EFT solutions reduce to solutions of deformed SUGRAs.

Mixed-symmetry potential

[Bergshoeff, Riccioni '10; '11; …]

Dp-brane … (electrically) couples to C_{p+1}

NS5-brane … couples to D_6

5^2_2-brane … couples to

a mixed-symmetry potential $D_{8,2}$

5^3_2-brane … couples to

a mixed-symmetry potential $D_{9,3}$

$p_3^{(1,7-p)}$-brane … couples to

a mixed-symmetry potential $E_{9,8-p,1}$

$\vdots$

set of antisymmetric indices

Mixed-symmetry potential

[Bergshoeff, Penas, Riccioni, Risoli,
arXiv:1508.00780]

In DFT, the mixed-symmetry potentials
are identified as the electric dual of the R-fluxes:
(which magnetically couple to exotic branes)

$$\begin{cases} \mathrm{d}\,\mathrm{D}_{9,3} = *_{10}(\text{R-flux } R^{3}) \\ \mathrm{d}\,\mathrm{E}_{9,8-p,1} = *_{10}\,(\text{R-flux } R^{8-p,1}) \end{cases}$$

We extend this correspondence to EFT, and obtained
more R-fluxes and mixed-symmetry potentials
that couple to all of the domain-wall branes.

Double Field Theory

In DFT, we introduce the doubled spacetime

$$(x^M) = (x^m,\, \tilde{x}_m)$$

P **F1**

All of the bosonic fields $(g_{mn},\, B_{mn},\, \Phi)$

are defined on the doubled space.

We organize them into

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & -B_{mp}\, g^{pn} \\ g^{np}\, B_{pn} & g^{mn} \end{pmatrix}, \qquad \mathrm{e}^{-2d} = \sqrt{-g}\, \mathrm{e}^{-2\Phi}\,.$$

generalized metric

T-duality-inv. dilaton

DFT

E.O.M. of DFT

$$\frac{1}{8}\,\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL}-\frac{1}{2}\,\partial_{(M|}\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{|N)L}+2\,\partial_M\partial_N d+\cdots=0\,,$$

$$\frac{1}{8}\,\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL}-\frac{1}{2}\,\mathcal{H}^{KL}\,\partial_L\mathcal{H}^{MN}\,\partial_N\mathcal{H}_{KM}+4\partial_M d\,\partial_N\mathcal{H}^{MN}+\cdots=0\,.$$

Covariant under O(n,n) T-duality rotations

$$\mathcal{H}_{MN}\;\rightarrow\;\Lambda_M{}^P\,\Lambda_N{}^Q\,\mathcal{H}_{PQ}\,,\qquad \partial_M d\rightarrow\Lambda_M{}^N\,\partial_N d\,,$$

$$x^M\;\rightarrow\;\Lambda^M{}_N\,x^N\,.$$

DFT

In terms of the usual supergravity fields

the T-duality rule is summarized as

$$g'_{ab} = g_{ab} - \frac{g_{ay}\, g_{by} - B_{ay}\, B'_{by}}{g_{yy}}\,, \qquad g'_{ay} = -\frac{B_{ay}}{g_{yy}}\,, \qquad g'_{yy} = \frac{1}{g_{yy}}\,;$$

$$B'_{ab} = B_{ab} - \frac{B_{ay}\, g_{by} - g_{ay}\, B_{by}}{g_{yy}}\,, \qquad B'_{ay} = -\frac{g_{ay}}{g_{yy}}\,, \qquad \mathrm{e}^{2\Phi'} = \frac{\mathrm{e}^{2\Phi}}{g_{yy}}\,.$$

$$+$$

Buscher's rule

$$x'^{a} = x^{a}\,, \qquad \tilde{x}'_{a} = \tilde{x}_{a}\,, \qquad \boxed{x'^{y} = \tilde{x}_{y}\,, \qquad \tilde{x}'_{y} = x^{y}\,.}$$

coordinate exchange

This maps a DFT solution to another DFT solution.

Example

5^2_2-brane solution: [Meessen, Ortin, hep-th/9806120; de Boer-Shigemori, arXiv:1004.2521]

$$\boxed{ds^2 = dx^2_{01\cdots5} + \frac{\tau_2}{|\tau|^2}\, dx^2_{67} + \tau_2\, dx^2_{89}\,, \quad e^{-2\Phi} = \frac{|\tau|^2}{\tau_2}\,, \quad B_2 = -\frac{\tau_1}{|\tau|^2}\, dx^6 \wedge dx^7\,.}$$

$$\tau = \tau_1 + i\,\tau_2\,, \quad \tau_1 = m\, x^8\,, \quad \tau_2 = h + m\,|x^9|$$

A solution of the usual SUGRA.

T_8

5^3_2-brane solution:

$$ds^2 = dx^2_{01\cdots5} + \frac{\tau_2}{|\tau|^2}\, dx^2_{67} + \tau_2^{-1}\, dx^2_8 + \tau_2\, dx^2_9\,, \quad e^{-2\Phi} = |\tau|^2\,, \quad B_2 = -\frac{\tau_1}{|\tau|^2}\, dx^6 \wedge dx^7\,.$$

$$\tau = \tau_1 + i\,\tau_2\,, \quad \tau_1 = m\, \tilde{x}_8\,, \quad \tau_2 = h + m\,|x^9|$$

A solution of DFT.

Non-geometric parameterization

To describe backgrounds of exotic branes
it is quite convenient to introduce the dual fields.

$$\begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & -B_{mp}\, g^{pn} \\ g^{np}\, B_{pn} & g^{mn} \end{pmatrix} = (\mathcal{H}_{MN}) = \begin{pmatrix} \tilde{g}_{mn} & -\tilde{g}_{mp}\, \beta^{pn} \\ \beta^{np}\, \tilde{g}_{pn} & (\tilde{g}^{-1} - \beta\, \tilde{g}\, \beta)^{mn} \end{pmatrix} ,$$

$$\sqrt{-g}\; \mathrm{e}^{-2\Phi} = \mathrm{e}^{-2d} = \sqrt{-\tilde{g}}\; \mathrm{e}^{-2\tilde{\phi}}$$

[Duff '89; Andriot, Larfors, Lust, Patalong, arXiv:1106.4015]

We can describe backgrounds with $(\tilde{g}_{mn},\, \beta^{mn},\, \tilde{\phi})$ **.**

Just a redefinition of $(g_{mn},\, B_{mn},\, \Phi)$ **.**

Non-geometric parameterization

5^2_2-brane solution:

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots5} + \tau_2^{-1}\,\mathrm{d}x^2_{67} + \tau_2\,\mathrm{d}x^2_{89}\,, \quad \mathrm{e}^{-2\tilde{\phi}} = \tau_2\,, \quad \beta^{67} = m\,x^8\,.$$

$$(\ \tau_2 = h + m\,|x^9|\ )$$

5^3_2-brane solution:

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots5} + \tau_2^{-1}\,\mathrm{d}x^2_{678} + \tau_2\,\mathrm{d}x^2_{9}\,, \quad \mathrm{e}^{-2\tilde{\phi}} = \tau_2^2\,, \quad \beta^{67} = m\,\tilde{x}_8\,.$$

$$(\ \tau_2 = h + m\,|x^9|\ )$$

Winding coordinate appears only in the β-field.

Moreover, the dependence is just linear.

Mixed-symmetry potentials

5^2_2-brane solution:

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots5} + \tau_2^{-1}\,\mathrm{d}x^2_{67} + \tau_2\,\mathrm{d}x^2_{89}\,,\qquad \mathrm{e}^{-2\tilde{\phi}} = \tau_2\,,\qquad \beta^{67} = m\,x^8\,.$$

$$\boxed{Q_m{}^{pq} \equiv \partial_m \beta^{pq}} \quad \Longrightarrow \quad Q_8{}^{67} = m\,.$$

$$\boxed{\mathrm{e}^{-2\tilde{\phi}}\,\tilde{g}_{pr}\,\tilde{g}_{qs}\,\tilde{*}_{10}\big(Q_m{}^{pq}\,\mathrm{d}x^m\big) \equiv \mathrm{d}D_{8,rs}\,.}$$

$$\Longrightarrow \quad D_{01234567,\,67} = -m\,\tau_2^{-1}\,.$$

5^3_2-brane solution:

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots5} + \tau_2^{-1}\,\mathrm{d}x^2_{678} + \tau_2\,\mathrm{d}x^2_{9}\,,\qquad \mathrm{e}^{-2\tilde{\phi}} = \tau_2^2\,,\qquad \beta^{67} = m\,\tilde{x}_8\,.$$

$$\boxed{R^{mpq} \equiv 3\,\tilde{\partial}^{[m}\beta^{pq]}} \quad \Longrightarrow \quad R^{678} = m\,.$$

$$\boxed{\mathrm{e}^{-2\tilde{\phi}}\,\tilde{g}_{m_1n_1}\,\tilde{g}_{m_2n_2}\,\tilde{g}_{m_3n_3}\,\tilde{*}_{10}R^{n_1n_2n_3} \equiv \mathrm{d}D_{9,n_1n_2n_3}\,.}$$

$$\Longrightarrow \quad D_{012345678,\,678} = -m\,\tau_2^{-1}\,.$$

DFT → EFT

In order to obtain SUGRA solutions for all of the exotic branes,

$$
\begin{aligned}
&\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,\ 6_3^{(1,1)}\,,\\
&4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,\ 2_4^{(3,3)}\,,\\
&1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,\ 2_5^{(1,5)}\,,\\
&2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,\ 2_5^{(1,0,4,2)}\,,\\
&2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,\ 2_5^{(3,1,3)}\,,\\
&2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,\ 3_6^{(3,2,1)}\,,\\
&3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,\ 1_7^{(3,4,0)}\,,\\
&1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ 2_7^{(1,0,1,5,0)}\,,\ 2_7^{(1,0,3,3,0)}\,,\ 2_7^{(1,0,5,1,0)}\,;\\
&2_7^{(1,3,3)}\,,\ 2_7^{(3,1,3)}\,,\ 2_7^{(1,0,4,2)}\,,\ 2_7^{(1,2,2,2)}\,,\ 2_7^{(1,4,0,2)}\,,\ 2_7^{(2,1,3,1)}\,,\ 2_7^{(2,3,1,1)}\,,\ 2_7^{(3,0,4,0)}\,,\\
&2_7^{(3,2,2,0)}\,,\ 2_7^{(3,4,0,0)}\,,\ 1_8^{(7,0,0)}\,,\ 2_8^{(1,0,6,0)}\,,\ 2_8^{(3,0,4,0)}\,,\ 2_8^{(5,0,2,0)}\,,\ 2_8^{(7,0,0,0)}\,,\ 2_8^{(3,3,1)}\,,\\
&2_8^{(1,3,2,1)}\,,\ 2_8^{(2,3,1,1)}\,,\ 2_8^{(3,3,0,1)}\,,\ 2_8^{(1,0,3,3,0)}\,,\ 2_8^{(1,1,3,2,0)}\,,\ 2_8^{(1,2,3,1,0)}\,,\ 2_8^{(1,3,3,0,0)}\,,\\
&2_9^{(1,4,2,0)}\,,\ 2_9^{(3,2,2,0)}\,,\ 2_9^{(5,0,2,0)}\,,\ 2_9^{(1,0,5,1,0)}\,,\ 2_9^{(1,2,3,1,0)}\,,\ 2_9^{(1,4,1,1,0)}\,,\ 2_9^{(2,1,4,0,0)}\,,\\
&2_9^{(2,3,2,0,0)}\,,\ 2_9^{(2,5,0,0,0)}\,,\ 2_{10}^{(3,4,0,0)}\,,\ 2_{10}^{(1,3,3,0,0)}\,,\ 2_{10}^{(2,3,2,0,0)}\,,\ 2_{10}^{(3,3,1,0,0)}\,,\ 2_{10}^{(4,3,0,0,0)}\,,\\
&2_{11}^{(7,0,0,0)}\,,\ 2_{11}^{(2,5,0,0,0)}\,,\ 2_{11}^{(4,3,0,0,0)}\,,\ 2_{11}^{(6,1,0,0,0)}\,.
\end{aligned}
$$

we need to extend DFT to EFT.

EFT

In EFT, we introduce the extended coordinates

$$(x^I) = (x^m,\ \tilde{x}_m,\ y^{\mathrm{D}}_m,\ y^{\mathrm{D}}_{m_1 m_2 m_3}\,,\ y^{\mathrm{S}}_{m_1\cdots m_5}\,,\ \cdots)$$

P F1 D1 D3 NS5

type IIB branes

generalized metric

$$(\mathcal{M}_{IJ}) = \begin{pmatrix} (g - B\,g^{-1}\,B + \cdots)_{mn} & -B_{mp}\,g^{pn} + \cdots & \cdots \\ g^{np}\,B_{pn} + \cdots & g^{mn} + \cdots & \cdots \\ \vdots & & \ddots \end{pmatrix} .$$

dilaton, R-R fields e.t.c.

Natural extension of DFT

Non-geometric parameterization

Similar to DFT, we can introduce the dual fields:

$$(\mathcal{M}_{IJ}) = \begin{pmatrix} (g - B\,g^{-1}\,B + \cdots)_{mn} & -B_{mp}\,g^{pn} + \cdots & \cdots \\ g^{np}\,B_{pn} + \cdots & g^{mn} + \cdots & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

Usual type IIB fields

$$= \begin{pmatrix} \tilde{g}_{mn} + \cdots & -\tilde{g}_{mp}\,\beta^{pn} + \cdots & \cdots \\ \beta^{np}\,\tilde{g}_{pn} + \cdots & (\tilde{g}^{-1} - \beta\,\tilde{g}\,\beta + \cdots)^{mn} & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

dual type IIB fields

$$(\tilde{g}_{mn},\ \tilde{\phi},\ \beta^{mn},\ \gamma^{m_1\cdots m_{2p}},\ \beta^{m_1\cdots m_6},\ \beta^{m_1\cdots m_7,\,n})$$

field redefinitions

EFT

T-duality:

constant matrix

[YS, Uehara arXiv:1701.07819]

$$\mathcal{M}_{IJ} = T_I{}^K\, T_J{}^L\, \mathcal{M}_{KL}\,, \qquad x^I = T^I{}_J\, x^J\,.$$

$$(x^I) = (x^m,\ \tilde{x}_m,\ y^{\text{D}},\ y^{\text{D}}_{m_1\cdots m_4}\,,\ y^{\text{S}}_{m_1\cdots m_5}\,,\ \cdots)$$

P **F1** **D0** **D4** **NS5**

type IIA branes

$$(\mathcal{M}_{IJ}) = \begin{pmatrix} (g - B\, g^{-1}\, B + \cdots)_{mn} & -B_{mp}\, g^{pn} + \cdots & \cdots \\ g^{np}\, B_{pn} + \cdots & g^{mn} + \cdots & \cdots \\ \vdots & & \ddots \end{pmatrix}.$$

type IIA fields

This T-duality maps a EFT solution to another EFT solution.

T-duality in EFT

In terms of the dual fields the T-duality rule becomes

$$\tilde{g}^{ab}_{(\mathrm{A})} = \tilde{g}^{ab} - \frac{\tilde{g}^{ay}\,\tilde{g}^{by} - \beta^{ay}\,\beta^{by}}{\tilde{g}^{yy}}\,, \quad \tilde{g}^{ay}_{(\mathrm{A})} = \frac{\beta^{ay}}{\tilde{g}^{yy}}\,, \quad \tilde{g}^{yy}_{(\mathrm{A})} = \frac{1}{\tilde{g}^{yy}}\,,$$

$$\beta^{ab}_{(\mathrm{A})} = \beta^{ab} + \frac{\beta^{ay}\,\tilde{g}^{by} - \tilde{g}^{ay}\,\beta^{by}}{\tilde{g}^{yy}}\,, \quad \beta^{ay}_{(\mathrm{A})} = \frac{\tilde{g}^{ay}}{\tilde{g}^{yy}}\,, \quad \mathrm{e}^{-2\tilde{\phi}_{(\mathrm{A})}} = \frac{\mathrm{e}^{-2\tilde{\phi}}}{\tilde{g}^{yy}}\,,$$

$$\gamma^{a_1\cdots a_{n-1}y}_{(\mathrm{A})} = \gamma^{a_1\cdots a_{n-1}} - (n-1)\,\frac{\gamma^{[a_1\cdots a_{n-2}|y|}\,\tilde{g}^{a_{n-1}]y}}{\tilde{g}^{yy}}\,,$$

$$\gamma^{a_1\cdots a_n}_{(\mathrm{A})} = \gamma^{a_1\cdots a_n y} + n\,\gamma^{[a_1\cdots a_{n-1}}\,\beta^{a_n]y} + n\,(n-1)\,\frac{\gamma^{[a_1\cdots a_{n-2}|y|}\,\beta^{a_{n-1}|y|}\,\tilde{g}^{a_n]y}}{\tilde{g}^{yy}}\,,$$

$$\beta^{a_1\cdots a_5 y}_{(\mathrm{A})} = \beta^{a_1\cdots a_5 y} + 5\,\eta^{[a_1\cdots a_4}\,\gamma^{a_5]y} + 5\,\eta^{[a_1 a_2 a_3|y|}\,\gamma^{a_4 a_5]} - \frac{45}{2}\,\gamma^{[a_1 a_2}\,\beta^{a_3 a_4}\,\gamma^{a_5]y}$$
$$- \frac{15}{2}\,\gamma^{[a_1 a_2}\,\gamma^{a_3 a_4}\,\beta^{a_5]y} + \frac{10\,\eta^{[a_1\cdots a_3|y|}\,\gamma^{a_4|y|}\tilde{g}^{a_5]y}}{\tilde{g}^{yy}} - \frac{15\,\gamma^{[a_1 a_2}\,\beta^{a_3|y|}\,\gamma^{a_4|y|}\,\tilde{g}^{a_5]y}}{\tilde{g}^{yy}}\,;$$

[YS, Uehara arXiv:1701.07819]

dual version of Buscher's rule

Coordinate exchange:

$$x^a \leftrightarrow x^a\,, \qquad \tilde{x}_a \leftrightarrow \tilde{x}_a\,, \qquad x^y \leftrightarrow \tilde{x}_y\,, \qquad y^{\mathrm{D}}_{a_1\cdots a_p} \leftrightarrow y^{\mathrm{D}}_{a_1\cdots a_p y}\,,$$

$$y^{\mathrm{S}}_{a_1\cdots a_4 b_1\cdots b_n y,\, b_1\cdots b_n} \leftrightarrow y^{\mathrm{S}}_{a_1\cdots a_4 b_1\cdots b_n y,\, b_1\cdots b_n}\,, \qquad y^{\mathrm{S}}_{a_1\cdots a_5 b_1\cdots b_n,\, b_1\cdots b_n} \leftrightarrow y^{\mathrm{S}}_{a_1\cdots a_5 b_1\cdots b_n y,\, b_1\cdots b_n y}\,,$$

$$y^{\mathrm{E}}_{a_1\cdots a_p b_1\cdots b_{7-p} y,\, b_1\cdots b_{7-p}} \leftrightarrow y^{\mathrm{E}}_{a_1\cdots a_p b_1\cdots b_{7-p} y,\, b_1\cdots b_{7-p} y}\,, \qquad \tilde{x}_{a b_1\cdots b_5 y,\, b_1\cdots b_5 y} \leftrightarrow \tilde{x}_{i b_1\cdots b_5 y,\, b_1\cdots b_5 y}\,;$$

$$\tilde{x}_{a_1\cdots a_6 y,\, a_1\cdots a_6} \leftrightarrow \tilde{x}_{a_1\cdots a_6 y,\, a_1\cdots a_6 y,\, y}\,, \qquad \tilde{x}_{a_1\cdots a_6 y,\, a_1\cdots a_6 y,\, a_6} \leftrightarrow \tilde{x}_{a_1\cdots a_6 y,\, a_1\cdots a_6 y,\, a_6}\,.$$

S-duality in EFT

For the dual fields in type IIB parameterization, the S-duality rules are summarized as

$$\tilde{g}'_{mn} = \sqrt{\mathrm{e}^{-2\tilde{\phi}} + \gamma^2}\,\tilde{g}_{mn}\,, \quad \mathrm{e}^{-\tilde{\phi}'} = \frac{\mathrm{e}^{-\tilde{\phi}}}{\mathrm{e}^{-2\tilde{\phi}} + \gamma^2}\,, \quad \gamma' = -\frac{\gamma}{\mathrm{e}^{-2\tilde{\phi}} + \gamma^2}\,,$$

$$\beta'^{mn} = -\gamma^{mn}\,, \quad \gamma'^{mn} = \beta^{mn}\,, \quad \gamma'^{m_1\cdots m_4} = \gamma^{m_1\cdots m_4} + 6\,\beta^{[m_1m_2}\,\gamma^{m_3m_4]}\,,$$

$$\gamma'^{m_1\cdots m_6} = -\beta^{m_1\cdots m_6} + 45\,\gamma^{[m_1m_2}\,\gamma^{m_3m_4}\,\beta^{m_5m_6]}\,,$$

$$\beta'^{m_1\cdots m_6} = \gamma^{m_1\cdots m_6} - 45\,\beta^{[m_1m_2}\,\beta^{m_3m_4}\,\gamma^{m_5m_6]}\,.$$

Coordinate permutations:

$$\tilde{x}'_m = -y^{\mathrm{D}}_m\,, \quad y'^{\mathrm{D}}_m = \tilde{x}_m\,, \quad y'^{\mathrm{D}}_{m_1\cdots m_5} = -y^{\mathrm{S}}_{m_1\cdots m_5}\,,$$

$$y'^{\mathrm{S}}_{m_1\cdots m_5} = y^{\mathrm{D}}_{m_1\cdots m_5}\,, \quad y'^{\mathrm{D}}_{m_1\cdots m_7} = y^{\mathrm{E}}_{m_1\cdots m_7}\,, \quad y'^{\mathrm{E}}_{m_1\cdots m_7} = y^{\mathrm{D}}_{m_1\cdots m_7}\,,$$

$$y'^{\mathrm{S}}_{m_1\cdots m_7,\,n_1n_2} = -y^{\mathrm{E}}_{m_1\cdots m_7,\,n_1n_2}\,, \quad y'^{\mathrm{E}}_{m_1\cdots m_7,\,n_1n_2} = y^{\mathrm{S}}_{m_1\cdots m_7,\,n_1n_2}\,,$$

$$y'^{\mathrm{E}}_{m_1\cdots m_7,\,n_1\cdots n_6} = -\tilde{x}_{m_1\cdots m_7,\,n_1\cdots n_6}\,, \quad \tilde{x}'_{m_1\cdots m_7,\,n_1\cdots n_6} = y^{\mathrm{E}}_{m_1\cdots m_7,\,n_1\cdots n_6}\,.$$

EFT solutions

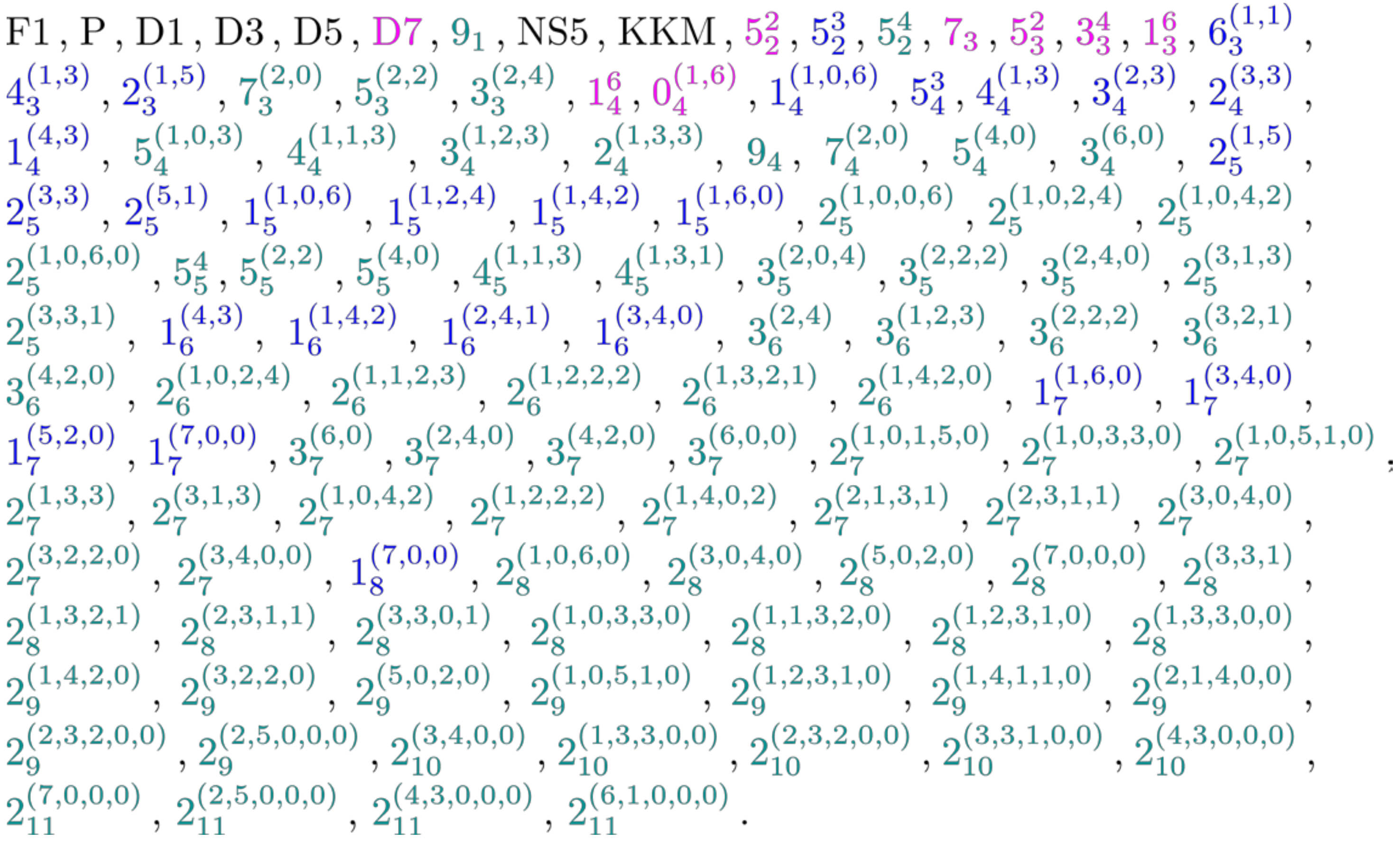
Using these duality rotations,

we can obtain EFT solutions for all of the branes:

$$
\begin{gathered}
\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 7_3\,,\ 5^2_3\,,\ 3^4_3\,,\ 1^6_3\,,\ 6^{(1,1)}_3\,, \\
4^{(1,3)}_3\,,\ 2^{(1,5)}_3\,,\ 7^{(2,0)}_3\,,\ 5^{(2,2)}_3\,,\ 3^{(2,4)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,,\ 3^{(2,3)}_4\,,\ 2^{(3,3)}_4\,, \\
1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 9_4\,,\ 7^{(2,0)}_4\,,\ 5^{(4,0)}_4\,,\ 3^{(6,0)}_4\,,\ 2^{(1,5)}_5\,, \\
2^{(3,3)}_5\,,\ 2^{(5,1)}_5\,,\ 1^{(1,0,6)}_5\,,\ 1^{(1,2,4)}_5\,,\ 1^{(1,4,2)}_5\,,\ 1^{(1,6,0)}_5\,,\ 2^{(1,0,0,6)}_5\,,\ 2^{(1,0,2,4)}_5\,,\ 2^{(1,0,4,2)}_5\,, \\
2^{(1,0,6,0)}_5\,,\ 5^4_5\,,\ 5^{(2,2)}_5\,,\ 5^{(4,0)}_5\,,\ 4^{(1,1,3)}_5\,,\ 4^{(1,3,1)}_5\,,\ 3^{(2,0,4)}_5\,,\ 3^{(2,2,2)}_5\,,\ 3^{(2,4,0)}_5\,,\ 2^{(3,1,3)}_5\,, \\
2^{(3,3,1)}_5\,,\ 1^{(4,3)}_6\,,\ 1^{(1,4,2)}_6\,,\ 1^{(2,4,1)}_6\,,\ 1^{(3,4,0)}_6\,,\ 3^{(2,4)}_6\,,\ 3^{(1,2,3)}_6\,,\ 3^{(2,2,2)}_6\,,\ 3^{(3,2,1)}_6\,, \\
3^{(4,2,0)}_6\,,\ 2^{(1,0,2,4)}_6\,,\ 2^{(1,1,2,3)}_6\,,\ 2^{(1,2,2,2)}_6\,,\ 2^{(1,3,2,1)}_6\,,\ 2^{(1,4,2,0)}_6\,,\ 1^{(1,6,0)}_7\,,\ 1^{(3,4,0)}_7\,, \\
1^{(5,2,0)}_7\,,\ 1^{(7,0,0)}_7\,,\ 3^{(6,0)}_7\,,\ 3^{(2,4,0)}_7\,,\ 3^{(4,2,0)}_7\,,\ 3^{(6,0,0)}_7\,,\ 2^{(1,0,1,5,0)}_7\,,\ 2^{(1,0,3,3,0)}_7\,,\ 2^{(1,0,5,1,0)}_7\,; \\
2^{(1,3,3)}_7\,,\ 2^{(3,1,3)}_7\,,\ 2^{(1,0,4,2)}_7\,,\ 2^{(1,2,2,2)}_7\,,\ 2^{(1,4,0,2)}_7\,,\ 2^{(2,1,3,1)}_7\,,\ 2^{(2,3,1,1)}_7\,,\ 2^{(3,0,4,0)}_7\,, \\
2^{(3,2,2,0)}_7\,,\ 2^{(3,4,0,0)}_7\,,\ 1^{(7,0,0)}_8\,,\ 2^{(1,0,6,0)}_8\,,\ 2^{(3,0,4,0)}_8\,,\ 2^{(5,0,2,0)}_8\,,\ 2^{(7,0,0,0)}_8\,,\ 2^{(3,3,1)}_8\,, \\
2^{(1,3,2,1)}_8\,,\ 2^{(2,3,1,1)}_8\,,\ 2^{(3,3,0,1)}_8\,,\ 2^{(1,0,3,3,0)}_8\,,\ 2^{(1,1,3,2,0)}_8\,,\ 2^{(1,2,3,1,0)}_8\,,\ 2^{(1,3,3,0,0)}_8\,, \\
2^{(1,4,2,0)}_9\,,\ 2^{(3,2,2,0)}_9\,,\ 2^{(5,0,2,0)}_9\,,\ 2^{(1,0,5,1,0)}_9\,,\ 2^{(1,2,3,1,0)}_9\,,\ 2^{(1,4,1,1,0)}_9\,,\ 2^{(2,1,4,0,0)}_9\,, \\
2^{(2,3,2,0,0)}_9\,,\ 2^{(2,5,0,0,0)}_9\,,\ 2^{(3,4,0,0)}_{10}\,,\ 2^{(1,3,3,0,0)}_{10}\,,\ 2^{(2,3,2,0,0)}_{10}\,,\ 2^{(3,3,1,0,0)}_{10}\,,\ 2^{(4,3,0,0,0)}_{10}\,, \\
2^{(7,0,0,0)}_{11}\,,\ 2^{(2,5,0,0,0)}_{11}\,,\ 2^{(4,3,0,0,0)}_{11}\,,\ 2^{(6,1,0,0,0)}_{11}\,.
\end{gathered}
$$

EFT solutions

In our paper, particularly, we obtained all of the domain-wall solutions in type II / M-theory.



$$
\begin{aligned}
&\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,\ 6_3^{(1,1)}\,,\\
&4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,\ 2_4^{(3,3)}\,,\\
&1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,\ 2_5^{(1,5)}\,,\\
&2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,\ 2_5^{(1,0,4,2)}\,,\\
&2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,\ 2_5^{(3,1,3)}\,,\\
&2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,\ 3_6^{(3,2,1)}\,,\\
&3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,\ 1_7^{(3,4,0)}\,,\\
&1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ 2_7^{(1,0,1,5,0)}\,,\ 2_7^{(1,0,3,3,0)}\,,\ 2_7^{(1,0,5,1,0)}\,;\\
&2_7^{(1,3,3)}\,,\ 2_7^{(3,1,3)}\,,\ 2_7^{(1,0,4,2)}\,,\ 2_7^{(1,2,2,2)}\,,\ 2_7^{(1,4,0,2)}\,,\ 2_7^{(2,1,3,1)}\,,\ 2_7^{(2,3,1,1)}\,,\ 2_7^{(3,0,4,0)}\,,\\
&2_7^{(3,2,2,0)}\,,\ 2_7^{(3,4,0,0)}\,,\ 1_8^{(7,0,0)}\,,\ 2_8^{(1,0,6,0)}\,,\ 2_8^{(3,0,4,0)}\,,\ 2_8^{(5,0,2,0)}\,,\ 2_8^{(7,0,0,0)}\,,\ 2_8^{(3,3,1)}\,,\\
&2_8^{(1,3,2,1)}\,,\ 2_8^{(2,3,1,1)}\,,\ 2_8^{(3,3,0,1)}\,,\ 2_8^{(1,0,3,3,0)}\,,\ 2_8^{(1,1,3,2,0)}\,,\ 2_8^{(1,2,3,1,0)}\,,\ 2_8^{(1,3,3,0,0)}\,,\\
&2_9^{(1,4,2,0)}\,,\ 2_9^{(3,2,2,0)}\,,\ 2_9^{(5,0,2,0)}\,,\ 2_9^{(1,0,5,1,0)}\,,\ 2_9^{(1,2,3,1,0)}\,,\ 2_9^{(1,4,1,1,0)}\,,\ 2_9^{(2,1,4,0,0)}\,,\\
&2_9^{(2,3,2,0,0)}\,,\ 2_9^{(2,5,0,0,0)}\,,\ 2_{10}^{(3,4,0,0)}\,,\ 2_{10}^{(1,3,3,0,0)}\,,\ 2_{10}^{(2,3,2,0,0)}\,,\ 2_{10}^{(3,3,1,0,0)}\,,\ 2_{10}^{(4,3,0,0,0)}\,,\\
&2_{11}^{(7,0,0,0)}\,,\ 2_{11}^{(2,5,0,0,0)}\,,\ 2_{11}^{(4,3,0,0,0)}\,,\ 2_{11}^{(6,1,0,0,0)}\,.
\end{aligned}
$$

R-fluxes

We also obtained R-fluxes, each of which magnetically couples to each of the domain-wall brane.

$$\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ 9_1\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,\ 6_3^{(1,1)}\,,$$

$$4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,\ 2_4^{(3,3)}\,,$$

$$1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,\ 2_5^{(1,5)}\,,$$

$$2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,\ 2_5^{(1,0,4,2)}\,,$$

$$2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,\ 2_5^{(3,1,3)}\,,$$

$$2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,\ 3_6^{(3,2,1)}\,,$$

$$3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,\ 1_7^{(3,4,0)}\,,$$

$$1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ 2_7^{(1,0,1,5,0)}\,,\ 2_7^{(1,0,3,3,0)}\,,\ 2_7^{(1,0,5,1,0)}\,;$$

$$2_7^{(1,3,3)}\,,\ 2_7^{(3,1,3)}\,,\ 2_7^{(1,0,4,2)}\,,\ 2_7^{(1,2,2,2)}\,,\ 2_7^{(1,4,0,2)}\,,\ 2_7^{(2,1,3,1)}\,,\ 2_7^{(2,3,1,1)}\,,\ 2_7^{(3,0,4,0)}\,,$$

$$2_7^{(3,2,2,0)}\,,\ 2_7^{(3,4,0,0)}\,,\ 1_8^{(7,0,0)}\,,\ 2_8^{(1,0,6,0)}\,,\ 2_8^{(3,0,4,0)}\,,\ 2_8^{(5,0,2,0)}\,,\ 2_8^{(7,0,0,0)}\,,\ 2_8^{(3,3,1)}\,,$$

$$2_8^{(1,3,2,1)}\,,\ 2_8^{(2,3,1,1)}\,,\ 2_8^{(3,3,0,1)}\,,\ 2_8^{(1,0,3,3,0)}\,,\ 2_8^{(1,1,3,2,0)}\,,\ 2_8^{(1,2,3,1,0)}\,,\ 2_8^{(1,3,3,0,0)}\,,$$

$$2_9^{(1,4,2,0)}\,,\ 2_9^{(3,2,2,0)}\,,\ 2_9^{(5,0,2,0)}\,,\ 2_9^{(1,0,5,1,0)}\,,\ 2_9^{(1,2,3,1,0)}\,,\ 2_9^{(1,4,1,1,0)}\,,\ 2_9^{(2,1,4,0,0)}\,,$$

$$2_9^{(2,3,2,0,0)}\,,\ 2_9^{(2,5,0,0,0)}\,,\ 2_{10}^{(3,4,0,0)}\,,\ 2_{10}^{(1,3,3,0,0)}\,,\ 2_{10}^{(2,3,2,0,0)}\,,\ 2_{10}^{(3,3,1,0,0)}\,,\ 2_{10}^{(4,3,0,0,0)}\,,$$

$$2_{11}^{(7,0,0,0)}\,,\ 2_{11}^{(2,5,0,0,0)}\,,\ 2_{11}^{(4,3,0,0,0)}\,,\ 2_{11}^{(6,1,0,0,0)}\,.$$

R-fluxes

We started from the standard R-flux:

$$R^{m_1m_2m_3}_{(2)} \equiv 3\,\tilde{\partial}^{[m_1}\beta^{m_2m_3]}$$

↔ **5^3_2-brane**

⇓ **chain of dualities**

$$R^{m_1\cdots m_6,\,n_1n_2n_3}_{(5)} \equiv \partial_{\mathrm{D}}^{n_1n_2n_3}\beta^{m_1\cdots m_6} + 3\,\partial_{\mathrm{S}}^{m_1\cdots m_6,\,[n_1}\gamma^{n_2n_3]} + 6\,\partial_{\mathrm{S}}^{[m_1\cdots m_5}\gamma^{m_6]n_1n_2n_3}$$
$$+ 3\,\partial_{\mathrm{E}}^{m_1\cdots m_6q,\,[n_1n_2}A_q^{n_3]} + (\text{non-linear terms like} \quad \beta^{\cdots}\,\partial_{\dots}\gamma^{\cdots})\,,$$

↔ **$2^{(3,3)}_5$-brane**

⇓ **chain of dualities**

$$R^{m_1\cdots m_7,\,n_1\cdots n_7,\,p}_{(7)} \equiv \partial_{\mathrm{E}}^{n_1\cdots n_7}\beta^{m_1\cdots m_7,\,p} - 7\,\partial_{\mathrm{E}}^{n_1\cdots n_7,\,p[m_1}\beta^{m_2\cdots m_7]}$$
$$+ 7\,\tilde{\partial}^{m_1\cdots m_7,\,[n_1\cdots n_6}\gamma^{n_7]p} + \tilde{\partial}^{m_1\cdots m_7,\,n_1\cdots n_7,\,p}\gamma$$
$$+ (\text{non-linear terms})$$

↔ **$1^{(1,6,0)}_7$-brane**

Example of EFT solution

$2_5^{(3,3)}$-brane solution:

$$\mathrm{d}\tilde{s}^2 = \tau_2^{3/2}\,\mathrm{d}x_{012}^2 + \tau_2^{-1/2}\,\mathrm{d}x_{345}^2 + \tau_2^{1/2}\,\mathrm{d}x_{678}^2 + \tau_2^{5/2}\,\mathrm{d}x_9^2\,,$$

$$\mathrm{e}^{-2\tilde{\phi}} = \tau_2^{-1}\,,\qquad \beta^{3\cdots 8} = m\,y^{\mathrm{D}}_{345}\,. \qquad \big(\,\tau_2 = h + m\,|x^9|\,\big)$$

winding coordinate of D3-brane

This background has a constant R-flux:

$$R_{(5)}^{3\cdots 8,345} = \partial_{\mathrm{D}}^{345}\,\beta^{3\cdots 8} = m\,.$$

$$\mathrm{e}^{2(1-5)\,\tilde{\phi}}\,\tilde{g}_{m_1n_1}\cdots\tilde{g}_{m_6n_6}\tilde{g}_{p_1q_1}\tilde{g}_{p_2q_2}\tilde{g}_{p_3q_3}\,\tilde{*}_{10}R_{(5)}^{m_1\cdots m_6,\,p_1p_2p_3}$$

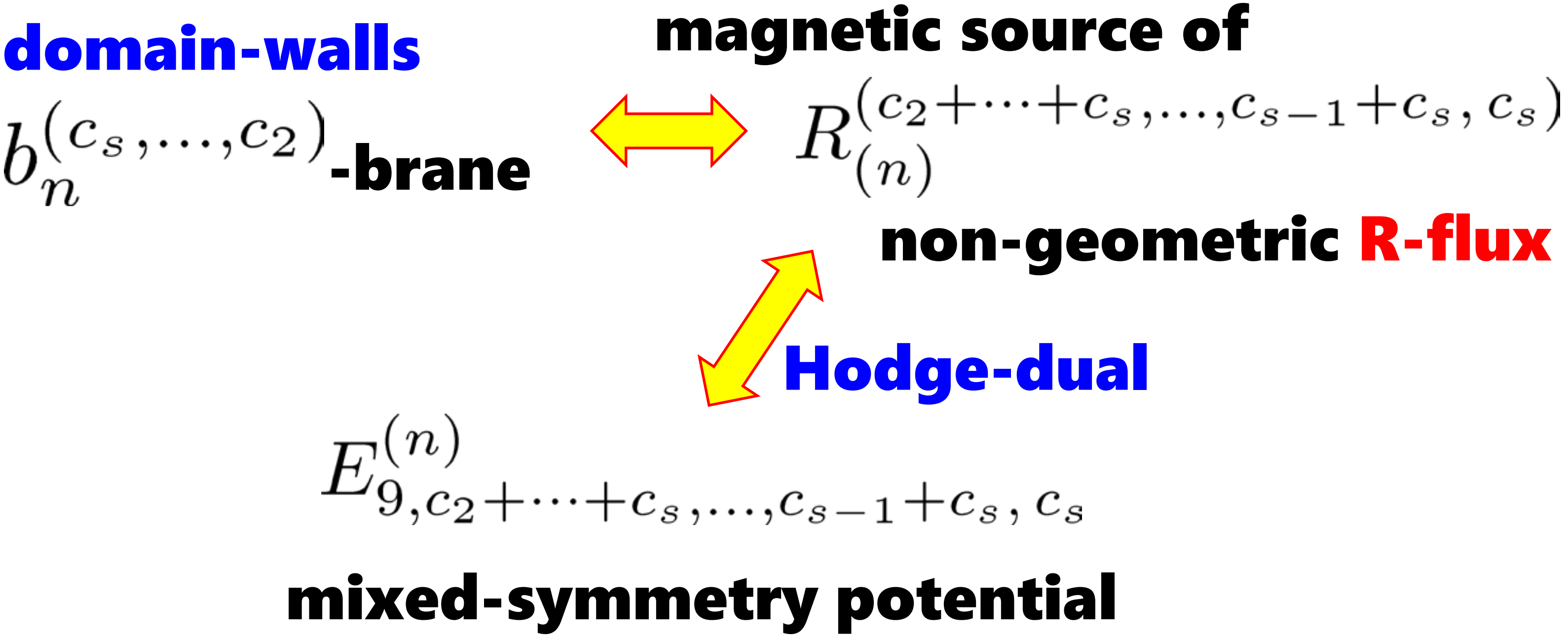
$$\equiv \mathrm{d}E^{(5)}_{9,n_1\cdots n_6,\,q_1q_2q_3}$$

mixed-symmetry potential $G_{9,6,3}$

$$\boxed{E^{(5)}_{0\cdots 8,\,3\cdots 8,345} = -m\,\tau_2^{-1}\,.}$$

[Bergshoeff, Riccioni arXiv:1710.00642]

Short summary



domain-walls

$b_n^{(c_s,\ldots,c_2)}$-brane

magnetic source of

$R_{(n)}^{(c_2+\cdots+c_s,\ldots,c_{s-1}+c_s,\,c_s)}$

non-geometric R-flux

Hodge-dual

$E^{(n)}_{9,c_2+\cdots+c_s,\ldots,c_{s-1}+c_s,\,c_s}$

mixed-symmetry potential

We constructed domain-wall solutions in EFT

The EFT solutions include
a linear winding-coordinate dependence.

Relation to deformed SUGRAs

D8-brane

[Hohm, Kwak, arXiv:1108.4937]

In the D8 solution, R-R 1-form has

a linear winding-coordinate dependence.

$$C_1(x) = \hat{C}_1(x^i) + m\,\tilde{x}_8\,\mathrm{d}x^8\,,$$

ansatz

$$F_0 = \tilde{\partial}^8 C_8 = m$$

E.O.M. of DFT

converted

Type IIA SUGRA E.O.M. with a mass deformation

(E.O.M. of massive type IIA SUGRA).

5^3_2-brane

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots 5} + \tau_2^{-1}\,\mathrm{d}x^2_{678} + \tau_2\,\mathrm{d}x^2_9\,, \quad \mathrm{e}^{-2\tilde{\phi}} = \tau_2^2\,, \quad \underbrace{\beta^{67} = m\,\tilde{x}_8\,.}$$

This is a solution of DFT/EFT.

Ansatz:

$$\mathcal{H}_{MN} = \bigl(U^{\mathrm{T}}\,\hat{\mathcal{H}}(x)\,U\bigr)_{MN}\,, \qquad U^M{}_N \equiv \begin{pmatrix} \delta^m_n & -\beta^{mn} \\ 0 & \delta^n_m \end{pmatrix}.$$

$\Downarrow$

constant R-flux $R^{678} = m$

$$\mathcal{L}_{\mathrm{DFT}} = \mathcal{L}_{\mathrm{SUGRA}} \underbrace{- \mathrm{e}^{-2d}\Bigl(3\,m\,\hat{\mathcal{H}}_{[6}{}^{n}\,\hat{\mathcal{H}}_{7|}{}^{M}\,\partial_n\hat{\mathcal{H}}_{|8]M} + \frac{m^2}{12}\,\det\hat{\mathcal{H}}_{ij}\Bigr)}_{\textbf{deformation}}$$

$(i, j = 6, 7, 8)$

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots 5} + \tau_2^{-1}\,\mathrm{d}x^2_{678} + \tau_2\,\mathrm{d}x^2_9\,, \quad \mathrm{e}^{-2\tilde{\phi}} = \tau_2^2\,.$$

Without β-field, it is a solution of the deformed SUGRA.

5^3_2-brane

$$\mathrm{d}\tilde{s}^2 = \mathrm{d}x^2_{01\cdots5} + \tau_2^{-1}\,\mathrm{d}x^2_{678} + \tau_2\,\mathrm{d}x^2_9\,, \quad \mathrm{e}^{-2\tilde{\phi}} = \tau_2^2\,, \quad \beta^{67} = m\,\tilde{x}_8\,.$$

According to the section condition,
if there is a winding-coordinate dependence,
fields cannot depend on all of the physical coordinates.

Consistency condition: $R^{mnp}\,\partial_p = 0$

$$(R^{678} = m) \quad \Rightarrow \quad (x^0, \cdots, x^5, \cancel{x}^6, \cancel{x}^7, \cancel{x}^8, x^9)$$

The deformed SUGRA is effectively 7-dimensional.

$7_3^{(1,0)}$-brane

$7_3^{(1,0)}$-brane solution (IIA):

$$\mathrm{d}\tilde{s}^2 = \tau_2^{1/2}\,(\mathrm{d}x_{01\cdots 7}^2 + \tau_2\,\mathrm{d}x_9^2) + \tau_2^{-3/2}\,\mathrm{d}x_8^2\,,$$

$$\mathrm{e}^{-2\tilde{\phi}} = \tau_2^{-1/2}\,, \qquad \gamma^8 = m\,\tilde{x}_8\,.$$

Ansatz: $\mathcal{M}_{IJ} = \left(U^{\mathrm{T}}\,\hat{\mathcal{M}}\,U\right)_{IJ}\,, \quad U \equiv \mathrm{e}^{-m\,\tilde{x}_8\,K_8{}^{\mathrm{M}}}\,,$

generator of E$_n$

$$\Downarrow$$

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SUGRA}} + \text{(deformation)}$$

$$R^{8,8} = m \quad \Longrightarrow \quad \partial_8 = 0$$

This deformed SUGRA is effectively 9-dimensional.

$$\mathrm{d}s^2 = \tau_2^{1/2}\,(\mathrm{d}x_{01\cdots 7}^2 + \tau_2\,\mathrm{d}x_9^2) + \tau_2^{-3/2}\,\mathrm{d}x_8^2\,, \qquad \mathrm{e}^{-2\Phi} = \tau_2^{-1/2}\,.$$

KK8A solution of **[Meessen, Ortin, hep-th/9806120]**

$6_3^{(1,1)}$-brane

$6_3^{(1,1)}$-brane solution (IIB):

$$\mathrm{d}\tilde{s}^2 = \tau_2^{1/2}\,(\mathrm{d}x_{01\cdots 6}^2 + \tau_2\,\mathrm{d}x_9^2) + \tau_2^{-1/2}\,\mathrm{d}x_7^2 + \tau_2^{-3/2}\,\mathrm{d}x_8^2\,,$$

$$\mathrm{e}^{-2\tilde{\phi}} = 1\,, \qquad \gamma^{78} = m\,\tilde{x}_8\,.$$

Ansatz:

$$\mathcal{M}_{IJ} = \big(U^{\mathrm{T}}\,\hat{\mathcal{M}}\,U\big)_{IJ}\,, \quad U \equiv \mathrm{e}^{-m\,\tilde{x}_8\,R^2_{78}}\,,$$

generator of E$_n$

$\Downarrow$

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SUGRA}} + \text{(deformation)}$$

An effectively 8-dimensional deformed SUGRA.

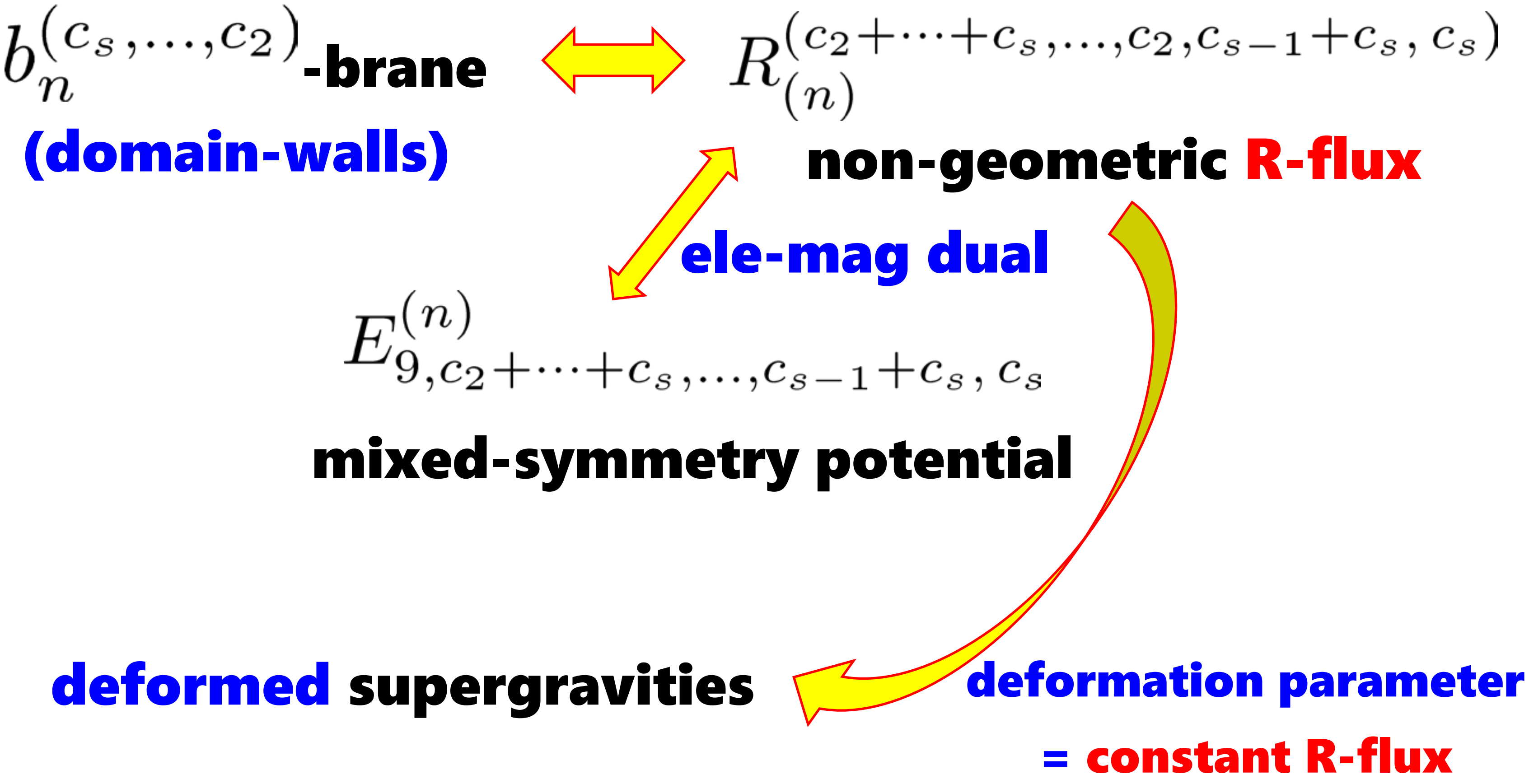
$$\mathrm{d}s^2 = \tau_2^{1/2}\,(\mathrm{d}x_{01\cdots 6}^2 + \tau_2\,\mathrm{d}x_9^2) + \tau_2^{-1/2}\,\mathrm{d}x_7^2 + \tau_2^{-3/2}\,\mathrm{d}x_8^2\,.$$

"Unknown brane (6,2,1)" **[Meessen, Ortin, hep-th/9806120]**

Summary

We constructed the domain-wall solutions in EFT

or deformed SUGRAs.



$b_n^{(c_s,\dots,c_2)}$**-brane**

(domain-walls)

$R_{(n)}^{(c_2+\cdots+c_s,\dots,c_2,c_{s-1}+c_s,\,c_s)}$

non-geometric R-flux

ele-mag dual

$E^{(n)}_{9,c_2+\cdots+c_s,\dots,c_{s-1}+c_s,\,c_s}$

mixed-symmetry potential

deformed supergravities

deformation parameter

= constant R-flux

Summary

Exceptional Field Theory is a useful framework

to describe SUGRA backgrounds
of **the domain-wall branes** in string/M-theory.

to systematically reproduce **deformed SUGRAs**
(usually non-covariant)
that contains **a domain-wall brane** as the solution.

$b_n^{(c_s,\dots,c_2)}$ -branes

Killing (isometry) directions

Yoshida-san's talk

Generalized SUGRA is also a similar **deformed SUGRA**.