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Yang-Baxter deformations and generalized supergravity

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0. Introduction

String Theory

A promising candidate of the unified theory of 4 forces in nature.

However, String Theory has not been completed yet!

In particular, String Theory is defined only perturbatively, and there are various approaches towards the non-perturbative formulation of String Theory.

EX String Field Theory, Matrix Model, Tensor Model etc.

Question

Is there anything to consider for perturbative string theory? YES!

3 well-known formulations of perturbative string theory

1. NS-R formulation (world-sheet fermions)
2. Green-Schwarz formulation (space-time fermions + kappa symmetry)
3. Pure Spinor formulation (space-time fermion + pure spinor condition)

The Green-Schwarz (GS) formulation of type IIB superstring

Space-time fermions contain 32 components (= 2 x 16 comps. of Majorana-Weyl spinor).

The on-shell condition reduces # of d.o.f. to 16, but # of physical comps. should be 8.

So it is necessary to impose an additional condition.

→ Kappa-symmetry (a fermionic gauge symmetry)

FACT: the on-shell condition of the standard type IIB SUGRA

→ kappa-invariant GS string theory

[Grisaru-Howe-Mezincescu -Nilsson-Townsend, 1985]

The inverse was conjectured, but not true.

New result: kappa-invariant GS string theory on an arbitrary background

→ the generalized type IIB SUGRA [Tseytlin-Wulf, 1605.04884]

This issue has been resolved after more than 30 years from the old work.

This is the recent fundamental progress in String Theory!

What does this result indicate?

Low Energy Effective Theory emerging from String Theory may be more general than the well-known SUGRA!

Generalized SUGRA = SUGRA + an extra vector field

We may have missed an important ingredient in String Phenomenology for more than 30 years.

It is really significant to study the generalized SUGRA in more detail.

It may be possible to get a nice idea to solve the long-standing problems such as cosmological constant problem and stable de Sitter vacuum.

The plan of this talk

1. What is the generalized type IIB SUGRA? (15 mins.)
   a) Its definition -- the appearance of an extra vector field *I* .
   b) Relation to Yang-Baxter (YB) deformations

2. What is the physical interpretation of *I* ? (15 mins.)
   a) Divergence formula -- a relation to NC-geometry and beta-field
   b) *I* is a non-geometric flux

3. Summary and discussion (1 slide)

1. What is the generalized type IIB SUGRA?

The generalized eqns of type IIB SUGRA

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

[Tseytlin-Wulf, 1605.04884]

$$R_{MN} - \frac{1}{4}H_{MKL}H_N{}^{KL} - T_{MN} + D_M X_N + D_N X_M = 0\,,$$

$$\frac{1}{2}D^K H_{KMN} + \frac{1}{2}F^K F_{KMN} + \frac{1}{12}F_{MNKLP}F^{KLP} = X^K H_{KMN} + D_M X_N - D_N X_M$$

$$R - \frac{1}{12}H^2 + 4D_M X^M - 4X_M X^M = 0\,,$$

$$D^M \mathcal{F}_M - Z^M \mathcal{F}_M - \frac{1}{6}H^{MNK}\mathcal{F}_{MNK} = 0\,, \qquad I^M \mathcal{F}_M = 0\,, \qquad \mathcal{F}_{n_1 n_2 \ldots} = e^{\Phi} F_{n_1 n_2 \ldots}$$

$$D^K \mathcal{F}_{KMN} - Z^K \mathcal{F}_{KMN} - \frac{1}{6}H^{KPQ}\mathcal{F}_{KPQMN} - (I \wedge \mathcal{F}_1)_{MN} = 0\,,$$

$$D^K \mathcal{F}_{KMNPQ} - Z^K \mathcal{F}_{KMNPQ} + \frac{1}{36}\epsilon_{MNPQRSTUVW}H^{RST}\mathcal{F}^{UVW} - (I \wedge \mathcal{F}_3)_{MNPQ} = 0$$

$$T_{MN} \equiv \frac{1}{2}\mathcal{F}_M\mathcal{F}_N + \frac{1}{4}\mathcal{F}_{MKL}\mathcal{F}_N{}^{KL} + \frac{1}{4\times 4!}\mathcal{F}_{MPQRS}\mathcal{F}_N{}^{PQRS} - \frac{1}{4}G_{MN}\left(\mathcal{F}_K\mathcal{F}^K + \frac{1}{6}\mathcal{F}_{PQR}\mathcal{F}^{PQR}\right)$$

Modified Bianchi identities

$$(d\mathcal{F}_1 - Z \wedge \mathcal{F}_1)_{MN} - I^K \mathcal{F}_{MNK} = 0\,,$$

$$(d\mathcal{F}_3 - Z \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{MNPQ} - I^K \mathcal{F}_{MNPQK} = 0\,,$$

$$(d\mathcal{F}_5 - Z \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{MNPQRS} + \frac{1}{6}\epsilon_{MNPQRSTUVW}I^T \mathcal{F}^{UVW} = 0$$

New ingredients: $X, \quad I, \quad Z$ 3 vector fields

But $X_M \equiv I_M + Z_M$, so two of them are independent.

Then I & Z satisfy the following relations:

$$D_M I_N + D_N I_M = 0\,, \quad D_M Z_N - D_N Z_M + I^K H_{KMN} = 0\,, \quad I^M Z_M = 0$$

Assuming that I is chosen such that the Lie derivative

$$(\mathcal{L}_I B)_{MN} = I^K \partial_K B_{MN} + B_{KN} \partial_M I^K - B_{KM} \partial_N I^K$$

vanishes, the 2nd equation above can be solved by

$$Z_M = \partial_M \Phi - B_{MN} I^N \quad .$$

Thus only I is independent after all.

Note When $I = 0$, the usual type IIB SUGRA is reproduced.

Some comments:

In the original AFHRT and TW papers, the classical action has not been constructed.

It has been shown that the generalized type II SUGRAs can be reproduced from the classical action of Double Field Theory (DFT) or Exceptional Field theory (EFT) by taking a slightly different section condition.

[Sakatani-Uehara-KY, 1611.05856] [Baguet-Magro-Samtleben, 1612.07210] [Sakamoto-Sakatani-KY, 1703.09213]

Simultaneously, the generalized type IIA supergravity has also been constructed.

[Sakamoto-Sakatani-KY, 1703.09213]

NOTE: The generalized type IIB SUGRA was originally derived in another context, in the study of integrable deformations of the $AdS_5 x S^5$ superstring.

||

Yang-Baxter (YB) deformations

In the following, let me briefly introduce what is Yang-Baxter deformation.

Yang-Baxter deformations

[Klimcik, 2002, 2008]

Integrable deformation!

An example

G-principal chiral model

Yang-Baxter sigma model

$$S = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}(J_\mu J_\nu) \quad \longrightarrow \quad S^{(\eta)} = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}\left(J_\mu \frac{1}{1-\eta R} J_\nu\right)$$

$$J_\mu = g^{-1}\partial_\mu g, \quad g \in G$$

η : a const. parameter

What is *R*?

$$R \ : \ \mathfrak{g} \longrightarrow \mathfrak{g}$$

a linear op.

a classical r-matrix satisfying

the modified classical Yang-Baxter eq. (mCYBE)

An integrable deformation can be specified by a classical r-matrix.

Strong advantage

Given a classical r-matrix, a Lax pair follows automatically.

No need to construct Lax pair in an intuitive manner case by case

Relation between R-operator and classical r-matrix

A linear R-operator ⟷ A skew-symmetric classical r-matrix

$R \ : \ \mathfrak{g} \longrightarrow \mathfrak{g}$ $\qquad$ $r \in \mathfrak{g} \otimes \mathfrak{g}$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

Two sources of classical r-matrices

1) modified classical Yang-Baxter eq. (mCYBE) ⟵ the original work by Klimcik

$$[R(X), R(Y)] - R\left([R(X), Y] + [X, R(Y)]\right) = \underline{-c^2 [X, Y]} \qquad (c \in \mathbb{C})$$

2) classical Yang-Baxter eq. (CYBE) $(c = 0)$ ⟵ a possible generalization

The list of generalizations of Yang-Baxter deformations (2 classes)

(i) modified CYBE (trigonometric class)

a) Principal chiral model [Klimcik, hep-th/0210095, 0802.3518]

b) Symmetric coset sigma model [Delduc-Magro-Vicedo, 1308.3581]

1) c) The $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, 1309.5850]

(ii) CYBE (rational class)

a) Principal chiral model [Matsumoto-KY, 1501.03665]

b) Symmetric coset sigma model [Matsumoto-KY, 1501.03665]

2) c) The $AdS_5 \times S^5$ superstring [Kawaguchi-Matsumoto-KY, 1401.4855]

NOTE bi-Yang-Baxter deformation [Klimcik, 0802.3518, 1402.2105]

(applicable only for principal chiral models)

Yang-Baxter deformations of the AdS_5 x S^5 superstring

$$S = -\frac{1}{2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \; P_-^{\alpha\beta} \mathrm{Str}\left[A_\alpha d \circ \frac{1}{1-\eta\,[R]_g \circ d}(A_\beta) \right]$$

Integrable deformations are specified by inserting classical r-matrices here.

There are two sources for classical r-matrices:

1) modified classical Yang-Baxter eq. (mCYBE) [Delduc-Magro-Vicedo, 1309.5850]
2) homogeneous classical Yang-Baxter eq. (CYBE) [Kawaguchi-Matsumoto-KY, 1401.4855]

- Kappa invariance : a consistency as string theory at classical level
- Lax pair is constructed : classical integrability

The undeformed limit: $\eta \to 0$ ⟶ the Metsaev-Tseytlin action

[Metsaev-Tseytlin, hep-th/9805028]

An outline of supercoset construction

[Arutyunov-Borsato-Frolov, 1507.04239]
[Kyono-KY, 1605.02519]

By taking a representation of the group element and expanding w.r.t. the fermions, the deformed action can be rewritten into the canonical form:

$$S = -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \left[\gamma^{ab} G_{MN} \partial_a X^M \partial_b X^N - \epsilon^{ab} B_{MN} \partial_a X^M \partial_b X^N\right]$$
$$-\frac{\sqrt{\lambda_c}}{2} i\bar{\Theta}_I(\gamma^{ab}\delta^{IJ} - \epsilon^{ab}\sigma_3^{IJ})\, e_a^m \Gamma_m D_b^{JK} \Theta_K + \mathcal{O}(\theta^4)$$

In general, the covariant derivative D is given by

[Cvetic-Lu-Pope-Stelle, hep-th/9907202]

$$D_a^{IJ} \equiv \delta^{IJ}\left(\partial_a - \frac{1}{4}\omega_a^{mn}\Gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ} e_a^m H_{mnp}\Gamma^{np}$$
$$-\frac{1}{8}e^{\Phi}\left[\epsilon^{IJ}\Gamma^p F_p + \frac{1}{3!}\sigma_1^{IJ}\Gamma^{pqr} F_{pqr} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\Gamma^{pqrst} F_{pqrst}\right] e_a^m \Gamma_m$$

From this expression, one can read off all of the fields of type IIB SUGRA.

Summary of the resulting backgrounds

1) The mCYBE case [Delduc-Magro-Vicedo, 1309.5850]

η-deformation or standard q-deformation [Arutyunov-Borsato-Frolov, 1312.3542]

The background is not a sol. of the usual type IIB SUGRA,

but satisfies the generalized type IIB SUGRA. [Arutyunov-Borsato-Frolov, 1507.04239]

(the original derivation!) [Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

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2) The CYBE case [Kawaguchi-Matsumoto-KY, 1401.4855]

A certain class of classical r-matrices satisfying

The unimodularity condition [Borsato-Wulff, 1608.03570]

$r^{ij}[b_i, b_j] = 0$ for a classical r-matrix $r = r^{ij} b_i \wedge b_j$

→ Sols. of the standard type IIB SUGRA

EX Lunin-Maldacena, Maldacena-Russo backgrounds [Matsumoto-KY, 1404.1838 ,1404.3657] [Kyono-KY, 1605.02519]

Otherwise, the backgrounds become sols. of the generalized type IIB SUGRA.

i) Unimodular example: gravity duals for SYM on non-commutative space

c.f. Seiberg-Witten, 1999

Abelian Jordanian r-matrix: $r = \frac{1}{2}\, p_2 \wedge p_3$

[Matsumoto-KY, 1404.3657]

where $p_\mu \equiv \frac{1}{2}\gamma_\mu - m_{\mu 5}\,, \quad m_{\mu 5} = \frac{1}{4}\left[\gamma_\mu, \gamma_5\right]\,, \quad \gamma_\mu$: a basis of $\mathfrak{su}(2,2)$

Metric: $ds^2 = \frac{1}{z^2}(-dx_0^2 + dx_1^2) + \frac{z^2}{z^4+\eta^2}(dx_2^2 + dx_3^2) + \frac{dz^2}{z^2} + d\Omega_5^2$

B-field: $B_2 = \frac{\eta}{z^4+\eta^2}\, dx^2 \wedge dx^3\,,$ dilaton: $\Phi = \frac{1}{2}\log\left(\frac{z^4}{z^4+\eta^2}\right)$

R-R : $F_3 = \frac{4\eta}{z^5}\, dx^0 \wedge dx^1 \wedge dz\,,$ $F_5 = 4\left[\mathrm{e}^{2\Phi}\omega_{AdS_5} + \omega_{\mathrm{S}^5}\right]\,.$

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

Note This solution can also be reproduced as a special limit of η-deformed AdS_5 .

[Arutyunov-Borsaro-Frolov, 1507.04239] [Kameyama-Kyono-Sakamoto-KY, 1509.00173]

ii) non-unimodular example: a solution of the generalized SUGRA

$$
\begin{aligned}
r &= E_{24} \wedge (c_1 E_{22} - c_2 E_{44}) \\
&= (p_0 - p_3) \wedge \left[a_1 \left(\frac{1}{2}\gamma_5 - n_{03} \right) - a_2 \left(n_{12} - \frac{i}{2}\mathbf{1}_4 \right) \right]
\end{aligned}
$$

$$
a_1 \equiv \frac{c_1 + c_2}{2} = \mathrm{Re}(c_1)\,, \qquad a_2 \equiv \frac{c_1 - c_2}{2\,i} = \mathrm{Im}(c_1)
$$

The resulting background: [Kyono-KY, 1605.02519]

$$
\begin{aligned}
ds^2 &= \frac{-2dx^+dx^- + d\rho^2 + \rho^2 d\phi^2 + dz^2}{z^2} - 4\eta^2 \left[(a_1^2 + a_2^2)\frac{\rho^2}{z^6} + \frac{a_1^2}{z^4} \right] (dx^+)^2 + ds^2_{\mathrm{S}^5}\,, \\
B_2 &= 8\eta \left[\frac{a_1 x^1 + a_2 x^2}{z^4} dx^+ \wedge dx^1 + \frac{a_1 x^2 - a_2 x^1}{z^4} dx^+ \wedge dx^2 + a_1 \frac{1}{z^3} dx^+ \wedge dz \right]\,, \\
F_3 &= 8\eta \left[\frac{a_2 x^1 - a_1 x^2}{z^5} dx^+ \wedge dx^1 \wedge dz + \frac{a_1 x^1 + a_2 x^2}{z^5} dx^+ \wedge dx^2 \wedge dz + \frac{a_1}{z^4} dx^+ \wedge dx^1 \wedge dx^2 \right]\,, \\
F_5 &= \text{undeformed}\,, \qquad \Phi = \text{const}
\end{aligned}
$$

$$
I = -\frac{2\eta a_1}{z^2}\, dx^+\,, \qquad Z = 0
$$

c.f, the a1=0 case corresponds to the usual SUGRA solution [Hubeney-Rangamani-Ross, hep-th/0504034]

This is a solution of the generalized SUGRA!.

2. What is the interpretation of I ?

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

[Sakamoto-Sakatani-KY, 1703.09213, 1705.07116]

[Fernandez Melgarejo-Sakamoto-Sakatani-KY, 1710.06849]

The open string picture ?

So far, we have considered the closed string picture with (g_{MN}, B_{MN}, g_s).

But it is also interesting to consider the open string picture with $(G_{MN}, \beta^{MN}, G_s)$.

The relations

$$G_{MN} = (g - Bg^{-1}B)_{MN} \qquad G_s = g_s \left(\frac{\det(g+B)}{\det g} \right)^{1/2}$$

$$\beta^{MN} = -((g+B)^{-1}B(g-B)^{-1})^{MN}$$

The open string picture of YB deformations of AdS_5 with homogeneous CYBE:

G_{MN} : the undeformed $AdS_5 \times S^5$ $\qquad$ G_s : const.

Only the non-commutative parameter β^{MN} depends on the deformation.

→ Classical r-matrices determine non-commutativities

[van Tongeren, 1506.01023, 1610.05677]

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

The relation between SUGRA and noncommutativity

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

The on-shell condition of type IIB SUGRA (= the unimodularity condition)

$$\Rightarrow \quad \nabla_M \beta^{MN} = 0$$

This condition is necessary for the cyclic property of star product.

For the generalized type IIB SUGRA,

$$\Rightarrow \quad \nabla_M \beta^{MN} = I^N$$

The extra vector field *I* has been related to the noncommutativity!

This is the first result that relates *I* to a physical quantity like non-commutativity.

What is the implication of the divergence formula?

This formula was derived by considering YB deformations of $AdS_5 \times S^5$, but this may be much more general.

NOTE: The transformation to the open string picture appeared in a different context when considering duality transformations. [Duff, NPB335 (1990) 610]

Then the non-commutativity is called the beta field.

Then, by using the beta field, a certain flux, called Q-flux, can be defined as

$$Q_p{}^{mn} \equiv \partial_p \beta^{mn}$$

[Grana-Minasian-Petrini-Waldram, 0807.4527]

For a constant shift for a direction $x \to x+1$, one can introduce

The monodromy

$$\beta^{mn}(x+1) - \beta^{mn}(x) = \int_x^{x+1} dx'^p \, \partial_p \beta^{mn} = \int_x^{x+1} dx'^p \, Q_p{}^{mn}(x')$$

If this monodromy is non-trivial along the x-direction, this flux is non-geometric.

Our proposal [Sakamoto-Sakatani-KY, 1705.07116]

$$I^m \equiv D_n \beta^{mn}$$ = trace of Q-flux + Chrsitoffel symbols

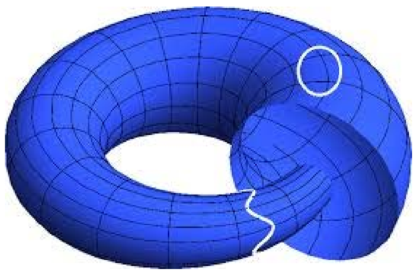
(divergence formula)

When the extra vector field $I \neq 0$, solutions may be non-geometric.

Hence we have checked some YB-deformed backgrounds with $I \neq 0$ and obtained non-trivial monodromies. [Fernandez Melgarejo-Sakamoto-Sakatani-KY, 1710.06849]

⟹ (At least some) YB-deformed backgrounds with $I \neq 0$ are T-folds.

NOTE: A T-fold is a generalized notion of a manifold.
It is locally a Riemannian manifold, but the patches are glued with diffeomorphism and T-duality.



[Blumenhagen, et al., 1510.04059]

In general, solutions of the generalized SUGRA are non-geometric!

What happens to the string world-sheet theory?

Pathology? [Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

When the background is a solution of the generalized SUGRA, scale invariance is ensured, but Weyl invariance is not.

To resolve this issue, the DFT picture is very useful.

By allowing the dilaton to depend on the dual coordinates, the appropriate counter-term can be constructed.

For the bosonic string case, see [Sakamoto-Sakatani-KY, 1703.09213]

→ String theory on the generalized background may be Weyl invariant!

NOTE In the case of superstring, the analysis should be very complicated, but the essential part of the proof of Weyl invariance has been resolved.

3. Summary and Discussion

Summary & discussion

I have given a review of recent progress on

generalized SUGRA and Yang-Baxter deformations

Kappa-symmetry of GS superstring → generalized SUGRA

Summary

- YB-deformation can be used to generate solutions of the generalized SUGRA
- Solutions of the generalized SUGRA are non-geometric in general.
- Superstring theory on the generalized background may be Weyl invariant.

Future directions

- Applications to phenomenology -- cosmological constant problem, stabilization of de Sitter vacuum
- Black Hole solution and its entropy?
- More fundamental formulation of superstring theory?

Thank you!

Back up

Definitions of the quantities

Maurer-Cartan 1-form

$$A_\alpha \equiv g^{-1}\partial_\alpha g\,, \quad g \in SU(2,2|4) \quad ,$$

Projection on the group manifold

$$d \equiv P_1 + 2P_2 - P_3$$

Projection on the world-sheet

$$P_\pm^{\alpha\beta} \equiv \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$$

$$\left[\begin{array}{l} \gamma^{\alpha\beta} = \mathrm{diag}(-1,1) \\ \epsilon^{\alpha\beta} : \ \text{anti-symm. tensor} \end{array}\right.$$

A chain of operations

$$R_g(X) \equiv g^{-1}R(gXg^{-1})g\,, \qquad \forall X \in \mathfrak{su}(2,2|4)$$

A group element: $g = g_{\rm b}\, g_{\rm f} \quad \in SU(2,2|4)$

$$g_{\rm b} = {g_{\rm b}}^{\rm AdS_5}\, {g_{\rm b}}^{\rm S^5}\,,$$

[For a big review, Arutyunov-Frolov, 0901.4937]

$$g_{\rm f} = \exp(\mathbf{Q}^I \theta_I)\,, \quad \mathbf{Q}^I \theta_I \equiv (\mathbf{Q}^{\check{\alpha}\hat{\alpha}})^I\, (\theta_{\check{\alpha}\hat{\alpha}})_I \quad (I = 1,2;\ \check{\alpha},\hat{\alpha} = 1,\ldots,4)$$

When we take a parametrization like

$$\begin{aligned} {g_{\rm b}}^{\rm AdS_5} &= \exp\Big[x^0\, P_0 + x^1\, P_1 + x^2\, P_2 + x^3\, P_3\Big]\, \exp\Big[(\log\, z)\, D\Big]\,, \\ {g_{\rm b}}^{\rm S^5} &= \exp\Big[\frac{i}{2}(\phi_1\, h_1 + \phi_2\, h_2 + \phi_3\, h_3)\Big]\, \exp\Big[\xi\, \mathbf{J}_{68}\Big]\, \exp\Big[-i\, r\, \mathbf{P}_6\Big]\,, \end{aligned}$$

the metric of AdS_5 x S^5 is given by

(the undeformed case)

$$\begin{aligned} ds^2 &= ds^2_{\rm AdS_5} + ds^2_{\rm S^5}\,, \\ ds^2_{\rm AdS_5} &= \frac{-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + \frac{dz^2}{z^2}\,, \\ ds^2_{\rm S^5} &= dr^2 + \sin^2 r\, d\xi^2 + \cos^2\xi \sin^2 r\, d\phi_1^2 + \sin^2 r \sin^2\xi\, d\phi_2^2 + \cos^2 r\, d\phi_3^2 \end{aligned}$$

The AdS/CFT correspondence

type IIB string on $AdS_5 \times S^5$ ⟷ 4D $\mathcal{N}=4$ SU(N) SYM $(N \to \infty)$

Recent progress: the discovery of integrability [For a big review, Beisert et al., 1012.3982]

Integrability is so powerful!

The integrability enables us to compute exactly physical quantities even at finite coupling, without relying on supersummetries.

EX anomalous dimensions, amplitudes etc.

Indeed, there are many directions of study with this integrability.

Here, among them, we are concerned with

the classical integrability on the string-theory side.

→ The existence of Lax pair (kinematical integrability)

The classical integrability of the AdS_5 x S^5 superstring

The coset structure of AdS_5 x S^5 is closely related to the integrability.

$$\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}$$

: symmetric coset

Z_2-grading → classical integrability

Including fermions

$$\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

: super coset

[Metsaev-Tseytlin, 1998]

Z_4-grading → classical integrability

elucidated by
[Bena-Polchinski-Roiban, 2003]

This fact is the starting point of our later argument.

The next issue

Integrable deformations of the $AdS_5 \times S^5$ superstring

Integrable deformations → Deformed $AdS_5 \times S^5$ geometries

(as a 2D non-linear sigma model)

Questions

Do the integrable deformations lead to solutions of type IIB SUGRA?

or, do they break the on-shell condition of type IIB SUGRA?

The main subject of my talk is to answer these questions

for a specific class of integrable deformations called

Yang-Baxter deformations

Yang-Baxter deformations

What is Yang-Baxter deformation?

Recent progress on this issue

Yang-Baxter deformations

[Klimcik, 2002, 2008]

Integrable deformation!

An example

G-principal chiral model

$$S = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}(J_\mu J_\nu)$$

$$J_\mu = g^{-1}\partial_\mu g, \quad g \in G$$

$\longrightarrow$

Yang-Baxter sigma model

$$S^{(\eta)} = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}\left(J_\mu \frac{1}{1-\eta R} J_\nu\right)$$

η : a const. parameter

What is *R*?

$$R \;:\; \mathfrak{g} \longrightarrow \mathfrak{g}$$

a linear op.

a classical r-matrix satisfying

the modified classical Yang-Baxter eq. (mCYBE)

An integrable deformation can be specified by a classical r-matrix.

Strong advantage

Given a classical r-matrix, a Lax pair follows automatically.

No need to construct Lax pair in an intuitive manner case by case

Relation between R-operator and classical r-matrix

A linear R-operator ⟷ A skew-symmetric classical r-matrix

$R \; : \; \mathfrak{g} \longrightarrow \mathfrak{g}$ $\qquad$ $r \in \mathfrak{g} \otimes \mathfrak{g}$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

Two sources of classical r-matrices

1) modified classical Yang-Baxter eq. (mCYBE) ⟵ the original work by Klimcik

$$[R(X), R(Y)] - R\left([R(X), Y] + [X, R(Y)]\right) = \underline{-c^2[X, Y]} \qquad (c \in \mathbb{C})$$

2) classical Yang-Baxter eq. (CYBE) $(c = 0)$ ⟵ a possible generalization

The list of generalizations of Yang-Baxter deformations (2 classes)

(i) modified CYBE (trigonometric class)

a) Principal chiral model [Klimcik, hep-th/0210095, 0802.3518]

b) Symmetric coset sigma model [Delduc-Magro-Vicedo, 1308.3581]

1) c) The $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, 1309.5850]

(ii) CYBE (rational class)

a) Principal chiral model [Matsumoto-KY, 1501.03665]

b) Symmetric coset sigma model [Matsumoto-KY, 1501.03665]

2) c) The $AdS_5 \times S^5$ superstring [Kawaguchi-Matsumoto-KY, 1401.4855]

NOTE bi-Yang-Baxter deformation [Klimcik, 0802.3518, 1402.2105]

(applicable only for principal chiral models)

i) gamma-deformations of S^5

c.f. Leigh-Strassler deformation

[Matsumoto-KY, 1404.1838]

Abelian classical r-matrix: $$r = \frac{1}{8}(\mu_3\, h_1 \wedge h_2 + \mu_1\, h_2 \wedge h_3 + \mu_2\, h_3 \wedge h_1)$$

where μ_i and h_i $(i = 1, 2, 3)$ are deformation parameters and the Cartan generators of $\mathfrak{su}(4)$.

Metric: $$ds^2 = ds^2_{\mathrm{AdS}_5} + \sum_{i=1}^{3}(d\rho_i^2 + G\rho_i^2 d\phi_i^2) + \eta^2 G \rho_1^2\rho_2^2\rho_3^2 \left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right)^2 ,$$

B-field: $$B_2 = \eta\, G\left(\mu_3\, \rho_1^2\rho_2^2\, d\phi_1 \wedge d\phi_2 + \mu_1\, \rho_2^2\rho_3^2\, d\phi_2 \wedge d\phi_3 + \mu_2\, \rho_3^2\rho_1^2\, d\phi_3 \wedge d\phi_1\right) ,$$

dilaton: $$\Phi = \frac{1}{2}\log G\,, \quad G^{-1} \equiv 1 + \eta^2(1 + \mu_3^2\rho_1^2\rho_2^2 + \mu_1^2\rho_2^2\rho_3^2 + \mu_2^2\rho_3^2\rho_1^2)\,, \qquad \textstyle\sum_{i=1}^{3}\rho_i^2 = 1$$

R-R: $$F_3 = -4\eta \sin^3\alpha\, \cos\alpha\, \sin\theta\, \cos\theta \left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right) \wedge d\alpha \wedge d\theta\,,$$

$$F_5 = 4\left[\omega_{\mathrm{AdS}_5} + G\,\omega_{\mathrm{S}^5}\right] .$$

$$\rho_1 = \sin\alpha\cos\theta\,, \quad \rho_2 = \sin\alpha\sin\theta\,, \quad \rho_3 = \cos\alpha\,.$$

[Lunin-Maldacena, Frolov, 2005]

ii) Gravity duals for SYM on non-commutative space

c.f. Seiberg-Witten, 1999

Abelian Jordanian r-matrix: $r = \frac{1}{2}\, p_2 \wedge p_3$

[Matsumoto-KY, 1404.3657]

where $p_\mu \equiv \frac{1}{2}\gamma_\mu - m_{\mu 5}\,, \quad m_{\mu 5} = \frac{1}{4}\,[\gamma_\mu, \gamma_5]\,, \quad \gamma_\mu$: a basis of $\mathfrak{su}(2,2)$

Metric: $$ds^2 = \frac{1}{z^2}(-dx_0^2 + dx_1^2) + \frac{z^2}{z^4+\eta^2}(dx_2^2 + dx_3^2) + \frac{dz^2}{z^2} + d\Omega_5^2$$

B-field: $$B_2 = \frac{\eta}{z^4+\eta^2}\, dx^2 \wedge dx^3\,,$$ dilaton: $$\Phi = \frac{1}{2}\log\left(\frac{z^4}{z^4+\eta^2}\right)$$

R-R : $$F_3 = \frac{4\eta}{z^5}\, dx^0 \wedge dx^1 \wedge dz\,, \qquad F_5 = 4\left[\mathrm{e}^{2\Phi}\omega_{AdS_5} + \omega_{\mathrm{S}^5}\right]\,.$$

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

Note This solution can also be reproduced as a special limit of η-deformed AdS_5 .

[Arutyunov-Borsaro-Frolov, 1507.04239] [Kameyama-Kyono-Sakamoto-KY, 1509.00173]

iii) Schrödinger spacetimes

c.f. [Son, 0804.3972], [Balasubramanian-McGreevy, 0804.4053]

Mixed r-matrix: $r = -\frac{i}{4}\, p_- \wedge (h_4 + h_5 + h_6)$

[Matsumoto-KY, 1502.00740]

Metric: $$ds^2 = \frac{-2dx^+dx^- + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} - \eta^2 \frac{(dx^+)^2}{z^4} + ds^2_{S^5}$$

B-field: $$B_2 = \frac{\eta}{z^2} dx^+ \wedge (d\chi + \omega)\,,$$

dilaton: $$\Phi = \text{const.}$$

The R-R sector is the same as $AdS_5 \times S^5$.

[Herzog-Rangamani-Ross, 0807.1099]
[Maldacena-Martelli-Tachikawa, 0807.1100]
[Adams-Balasubramanian-McGreevy, 0807.1111]

S^5-coordinates: $$ds^2_{S^5} = (d\chi + \omega)^2 + ds^2_{\mathbb{C}P^2}\,,$$
$$ds^2_{\mathbb{C}P^2} = d\mu^2 + \sin^2\mu \left(\Sigma_1^2 + \Sigma_2^2 + \cos^2\mu\, \Sigma_3^2\right)$$

NOTE the dilaton and R-R sector have not been deformed.

In the middle of computation, the fermionic sector becomes really messy and quite complicated. So the cancellation of the deformation effect seems miraculous.

Some comments

- Special case

The a1=0 case is special. The classical r-matrix becomes unimodular.

The background is a solution of type IIB SUGRA. [Hubeney-Rangamani-Ross, hep-th/0504034]

- General case

The resulting background is not a solution of type IIB SUGRA,

but still satisfies the generalized equations with

It is more interesting to perform ``generalized T-dualities'' for this solution

(i.e., a generalized Buscher rule) ➡ a solution of the usual type IIB SUGRA.

Furthermore, this ``T-dualized'' background is locally equivalent to

the undeformed $AdS_5 \times S^5$! [Orlando-Reffert-Sakamoto-KY, 1607.00795]

What is the physical interpretation of this result?

3. An argument for the Weyl invariance

[Sakamoto-Sakatani-KY, 1703.09213]

Weyl invariance of the bosonic string theory (D=26)

<u>The classical action</u>

$a, b = \tau, \sigma, \quad \varepsilon^{\tau\sigma} = 1/\sqrt{-\gamma}, \quad \varepsilon_{\tau\sigma} = -\sqrt{-\gamma}$

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{-\gamma} \left[G_{mn}\gamma^{ab} - B_{mn}\varepsilon^{ab} \right] \partial_a X^m \partial_b X^n$$

At classical level,

$$T^a{}_a \equiv \frac{4\pi}{\sqrt{-\gamma}} \gamma^{ab} \frac{\delta S}{\delta \gamma^{ab}} = 0$$

➡ Weyl invariant

But at quantum level, the trace anomaly appears [Callan-Friedan-Martinec-Perry, '85]

$$2\alpha' \langle T^a{}_a \rangle = \left(\beta^G_{mn} \gamma^{ab} - \beta^B_{mn} \varepsilon^{ab} \right) \partial_a X^m \partial_b X^n$$

where

$$\beta^G_{mn} = \alpha' \left(R_{mn} - \frac{1}{4} H_{mpq} H_n{}^{pq} \right), \quad \beta^B_{mn} = \alpha' \left(-\frac{1}{2} D^k H_{kmn} \right)$$

Quantum scale invariance

[Hull-Townsend, '86]

If the beta functions take the following forms:

$$\beta^{G}_{mn} = -2\alpha' D_{(m}Z_{n)}, \quad \beta^{B}_{mn} = -2\alpha'(Z^k H_{kmn} + 2D_{[m}I_{n]}),$$

then scale invariance is preserved at quantum level.

Here *Z* and *I* are arbitrary vector fields, but of course these are nothing but the *Z* and *I* have already appeared in the generalized supergravity!

➡ The origin of the generalized supergravity

NOTE: In fact, the trace anomaly can be rewritten into a total derivative form:

$$\langle T^{a}{}_{a} \rangle = -\mathcal{D}_a \left[(Z_n \gamma^{ab} - I_n \varepsilon^{ab}) \partial_b X^n \right]$$

where the eom of X has been utilized.

Quantum Weyl invariance

As a special case of Hull and Townsend, one may take

$$Z_m = \partial_m \Phi, \qquad I_m = 0 \quad .$$

Then the trace anomaly is given by

$$\langle T^a{}_a \rangle = -\mathcal{D}^a \partial_a \Phi \ .$$

This anomaly can be cancelled out by adding the Fradkin-Tseytlin (FT) term:

$$S_{\mathrm{FT}} \equiv \frac{1}{4\pi} \int \mathrm{d}^2\sigma \sqrt{-\gamma}\, R^{(\gamma)} \Phi$$

← alpha' is not contained!

because

$$\langle T^a{}_a \rangle_{\mathrm{FT}} = \frac{4\pi}{\sqrt{-\gamma}} \gamma^{ab} \frac{\delta S_{\mathrm{FT}}}{\delta \gamma^{ab}} = \mathcal{D}^a \partial_a \Phi \, .$$

Note: the FT term itself should be regarded as quantum contribution.

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STRINGS IN BACKGROUND FIELDS

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Comment on the FT term

The closed string has one other massless excitation, namely the dilaton, and we should be able to give it a background expectation value as well. How to do this is a bit mysterious since all the renormalizable and Weyl-invariant sigma model terms have been used up! Fradkin and Tseytlin [9] have suggested that one should add to S_{nlsm} the renormalizable, but not Weyl invariant, term

$$S_{\mathrm{dil}} = \frac{1}{4\pi}\int \mathrm{d}^2\xi\,\sqrt{\gamma}\,R^{(2)}\Phi(X)\,,$$

where $R^{(2)}$ is the scalar curvature of the two-dimensional manifold and $\Phi(X)$ is the background dilaton field in the spacetime $\mathfrak{M}$. Since Weyl invariance is so crucial to the consistency of string theory, it seems mad to introduce terms which explicitly break it. Nevertheless, we shall show that, properly treated, S_{dil} does the right thing.

Regarding the FT term as one-loop quantum contributions

Here $H_{\mu\nu\lambda} = 3\nabla_{[\mu}B_{\nu\lambda]}$ is the antisymmetric tensor field strength, and $R_{\mu\nu}$ is the Ricci tensor. The leading term in β^{Φ} was discovered by Polyakov [8] (the 26 arises from the conformal gauge Faddeev-Popov determinant), the $R_{\mu\nu}$ term was discussed by Friedan and others [4], and the inclusion of the H-field torsion has been considered by Witten [10] and Curtright and Zachos [11]. It is important to note that since the coefficient of $R^{(2)}\Phi$ is smaller by a factor α' (the loop-expansion parameter) than the other couplings, its *classical* contribution is of the same order as the one-loop *quantum contributions* of the $G_{\mu\nu}$ and $B_{\mu\nu}$ couplings. This is because $R^{(2)}\Phi$ is scale non-invariant at the classical level while the other couplings only lose scale invariance at the quantum level.

A generalization of the FT term

Question: Can one generalize the FT term for the case with $I_m \neq 0$?

Generalized FT term: [Sakamoto-Sakatani-KY, 1703.09213]

Dual coordinates

$$S^{(*)}_{\mathrm{FT}} = \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{-\gamma}\,R^{(\gamma)}\Phi_*\,, \qquad \Phi_* = \Phi + I^i\widetilde{Y}_i$$

$$\langle T^a{}_a\rangle^{(*)}_{\mathrm{FT}} = \frac{4\pi}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S^{(*)}_{\mathrm{FT}}}{\delta\gamma^{ab}} = +\mathcal{D}_a\left[\left(Z_n\gamma^{ab} - I_n\varepsilon^{ab}\right)\partial_b X^n\right]$$

Exactly cancels out Hull-Townsend's trace anomaly!

Here we have used the eom of Hull's double sigma model,

$$\partial_a\widetilde{Y}_i - G_{in}\varepsilon^b{}_a\partial_b X^n - B_{in}\partial_a X^n = 0$$

This implies the generalized FT term would be non-local.

What does this non-locality mean?

POINT The (modified) FT term should be regarded as a quantum correction, and this means that it should be derived by integrating out fluctuations in more fundamental theory.

If this postulated fundamental theory should be local, then there is no problem. Of course, String Theory is incomplete and we have not understood what is the fundamental theory.

Anyway, the cancellation of the anomaly is very impressive. It is important to study carefully the world-sheet theory with T-fold and confirm the world-sheet picture in the case of the generalized SUGRA.

Of course, this non-locality should be related to the non-geometricity of I.

Implications and significance of the Weyl invariance

What does this Weyl invariance mean?

If the Weyl invariance is really broken, everyone claims that the Weyl invariance must be respected and hence there is no room for the generalized SUGRAs.

However, now the Weyl invariance is definitely positive and this indicates that the generalized SUGRAs may appear as low-energy effective theories of String Theory

(A democratic point of view)

Take-home message:

The proof of the Weyl invariance opens up a new arena to build up new phenomenological models based on the generalized SUGRA.

(In particular, the extra vector field may play an important role)