BRST covariant Yang-Mills squared

A gauge-independent map of equations of motion

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Reminder

- The basic building block is a *x*-space convolution

$$F(x) = (f^I \star \phi_{II'} \star g^{I'})(x) \equiv (f \circ g)(x)$$

which allows to derive the action of certain operators on gravitational fields from that of the analogous operators on the gauge side:

$$\mathcal{O}F = \hat{\mathcal{O}}(f \circ g) \qquad \text{given} \qquad \hat{\mathcal{O}}f^I, \hat{\mathcal{O}}\phi_{II'}, \hat{\mathcal{O}}g^{I'}$$

- E.g., for symmetries $\delta F = \hat{\delta}(f \circ g) = \hat{\delta}f \circ g + f \circ \hat{\delta}g$

Motivation

- BCJ double copy: (super)gravity amplitudes from (S)YM
- Classical solutions: substitution rules, case-by-case basis, gauge-dependent!
- In spacetime approach, relation between equations on motion restricted Lorenz gauge.

Goal

To extend the *field-theoretic* dictionary, to obtain content, symmetries and equations of motion of (super)gravity from those of the underlying (two copies of) Yang-Mills, *without restricting to a specific choice of gauge fixing*.

Yang-Mills, ghosts and BRST symmetry

$$\mathcal{L}_{A_\mu} = \mathrm{tr}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + b\partial^\mu A_\mu - \frac{\xi}{2}b^2 - \bar{c}\Box c\right)$$

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- The dynamical equations of motion are

$$\partial^\mu F^I_{\mu\nu} = \partial_\nu b^I$$
$$\Box c^{Ia} = 0$$

while the Lautrup-Nakanishi auxiliary field is constrained

$$b^I = \frac{1}{\xi}\partial^\mu A_\mu$$

and free (due to current conservation)

$$\Box b^I = 0$$

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Yang-Mills, ghosts and BRST symmetry

$$\mathcal{L}_{A_\mu} = \mathrm{tr}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2\xi}(\partial^\mu A_\mu)^2 - \bar{c}\Box c\right)$$

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$$\Box A^I_\mu - \xi' \partial_\mu \partial A^I = 0 \qquad \xi' \equiv \frac{\xi+1}{\xi}$$

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- The theory is invariant under the following (on-shell nilpotent) BRST transformations, which encode the late gauge invariance (fixed):

$$QA^I_\mu = \partial_\mu c^I, \qquad Qc^I = 0, \qquad Q\bar{c}^I = \frac{1}{\xi}\partial^\mu A^I_\mu$$

- Simplifying features for certain choices of "gauge"

$$\xi = 0 \qquad\qquad \text{(Landau gauge)}$$
$$\xi = -1 \quad (\xi' = 0) \qquad\qquad \text{(Feynman-'t Hooft gauge)}$$

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Keep it general!

Gauge x gauge = ?

	$\tilde{A}_\nu \ (0)$
$A_\mu \ (0)$	$A_{(\mu} \circ \tilde{A}_{\nu)} - \frac{\eta_{\mu\nu}}{D} A^\rho \circ \tilde{A}_\rho$ $A_{[\mu} \circ \tilde{A}_{\nu]}$ $A^\rho \circ \tilde{A}_\rho$

Gauge x gauge = ?

	$\tilde{A}_\nu$ (0)	$\tilde{c}^b$ (± 1)
A_μ (0)	$A_{(\mu} \circ \tilde{A}_{\nu)} - \frac{\eta_{\mu\nu}}{D} A^\rho \circ \tilde{A}_\rho$ $A_{[\mu} \circ \tilde{A}_{\nu]}$ $A^\rho \circ \tilde{A}_\rho$	$A_\mu \circ \tilde{c}^b$
c^a (± 1)	$c^a \circ \tilde{A}_\mu$	$c^{(a} \circ \tilde{c}^{b)}$ $c^a \circ \tilde{c}_a$

$$gh(f \circ \tilde{g}) = gh(f) + gh(\tilde{g})$$
$$\epsilon(f \circ \tilde{g}) = \epsilon(f) + \epsilon(\tilde{g}) \quad (\text{mod } 2)$$

Gauge x gauge = ?

	$\tilde{A}_\nu\ (0)$	$\tilde{c}^b\ (\pm 1)$
$A_\mu\ (0)$	$A_{(\mu} \circ \tilde{A}_{\nu)} - \frac{\eta_{\mu\nu}}{D} A^\rho \circ \tilde{A}_\rho$ $A_{[\mu} \circ \tilde{A}_{\nu]}$ $A^\rho \circ \tilde{A}_\rho$	$A_\mu \circ \tilde{c}^b$
$c^a\ (\pm 1)$	$c^a \circ \tilde{A}_\mu$	$c^{(a} \circ \tilde{c}^{b)}$ $c^a \circ \tilde{c}_a$

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$\underline{1}$: $$c^a \circ \tilde{c}_a = \varepsilon_{ab} c^a \circ \tilde{c}^b = c \circ \bar{\tilde{c}} - \bar{c} \circ \tilde{c} \qquad (0)$$

$\underline{3}$: $$c^{(a} \circ \tilde{c}^{b)} = \begin{cases} c \circ \tilde{c} & (2) \\ c \circ \bar{\tilde{c}} + \bar{c} \circ \tilde{c} & (0) \\ \bar{c} \circ \bar{\tilde{c}} & (-2) \end{cases}$$

Gauge x gauge = ?

	$\tilde{A}_\nu$ (0)	$\tilde{c}^b$ (± 1)
A_μ (0)	$h^t_{\mu\nu}$ $B_{\mu\nu}$ (0) φ, h	$A_\mu \circ \tilde{c}^b$
c^a (± 1)	$c^a \circ \tilde{A}_\mu$	$c^{(a} \circ \tilde{c}^{b)}$ $c^a \circ \tilde{c}_a$

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3: $$c^{(a} \circ \tilde{c}^{b)} = \begin{cases} c \circ \tilde{c} & (2) \\ c \circ \bar{\tilde{c}} + \bar{c} \circ \tilde{c} & (0) \\ \bar{c} \circ \bar{\tilde{c}} & (-2) \end{cases}$$

Gravitational theory: BRST

$$\mathcal{L}_{h_{\mu\nu}} = -\frac{1}{4} h^{\mu\nu} E_{\mu\nu} + \frac{1}{2\xi_{(h)}} \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h \right)^2 - \bar{c}^\mu \Box c_\mu - \frac{1}{4} \partial^\mu \varphi \partial_\mu \varphi$$

- Invariance under the set of BRST variations

$$Qh_{\mu\nu} = 2\partial_{(\mu} c_{\nu)} \qquad Qc_\mu = 0$$

$$Q\bar{c}_\mu = \frac{1}{\xi_{(h)}} \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h \right) \qquad Q\varphi = 0$$

Field	Ghost number	Mass dimension
$h_{\mu\nu}$	0	1
c_μ	1	0
$\bar{c}_\mu$	-1	2

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c_μ	1	0
$\bar{c}_\mu$	-1	2

$$\mathcal{L}_{B_{\mu\nu}} = -\frac{1}{24}H^{\mu\nu\rho}H_{\mu\nu\rho} + \frac{1}{2\xi_{(B)}}\left(\partial_\mu B^{\mu\nu} + l_{(B)}\partial^\nu \eta\right)^2 - \bar{d}_\nu \Box d^\nu + \frac{\xi_{(d)} - m_{(d)}}{\xi_{(d)}}\bar{d}_\mu \partial_\mu \partial^\nu d_\nu + m_{(d)}\bar{d}\Box d$$

$$QB_{\mu\nu} = 2\partial_{[\mu}d_{\nu]} \qquad Qd_\mu = \partial_\mu d$$

$$Q\bar{d}_\mu = \frac{1}{\xi_{(B)}}\left(\partial^\nu B_{\mu\nu} + l_{(B)}\partial_\mu \eta\right) \qquad Qd = 0$$

$$Q\bar{d} = \frac{1}{\xi_{(d)}}\partial^\mu \bar{d}_\mu \qquad Q\eta = \frac{m_{(d)}}{l_{(B)}\xi_{(d)}}\partial^\mu d_\mu$$

Field	Ghost number	Mass dimension
$B_{\mu\nu}$	0	1
d_μ	1	0
$\bar{d}_\mu$	-1	2
d	2	-1
$\bar{d}$	-2	3
η	0	1

	$\tilde{A}_\nu$ (0)	$\tilde{c}^b$ (± 1)
A_μ (0)	$h^t_{\mu\nu}$ $B_{\mu\nu}$ (0) φ, h	$A_\mu \circ \tilde{c}^b$
c^a (± 1)	$c^a \circ \tilde{A}_\mu$	$c^{(a} \circ \tilde{c}^{b)}$ $c^a \circ \tilde{c}_a$

$\underline{1}$: $$c^a \circ \tilde{c}_a = \varepsilon_{ab} c^a \circ \tilde{c}^b = c \circ \bar{\tilde{c}} - \bar{c} \circ \tilde{c} \qquad (0)$$

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$$\underline{1}: \quad c^a \circ \tilde{c}_a = \varepsilon_{ab} c^a \circ \tilde{c}^b = \varphi,\ h \qquad (0)$$

$$\underline{3}: \quad c^{(a} \circ \tilde{c}^{b)} = \begin{cases} d & (2) \\ \eta & (0) \\ \overline{d} & (-2) \end{cases}$$

Not enough..

- Consider the obvious choice (of all possible local scalar terms)

$$\varphi = A^\rho \circ \tilde{A}_\rho + \alpha\, c^a \circ \tilde{c}_a$$

Indeed, it is not enough to map both BRST symmetry and e.o.m in a gauge-independent fashion: while it accommodates BRST, it fails to map the e.o.m in an arbitrary gauge,

$$\begin{aligned} \Box A^I_\mu - \xi' \partial_\mu \partial A^I &= 0 \\ \Box c^{Ia} &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \Box\varphi &= \Box(A^\rho \circ \tilde{A}_\rho) + \alpha\, \Box(c^a \circ \tilde{c}_a) \\ &= \xi'(\xi' - 2)(\partial A \circ \partial \tilde{A}) \\ &\overset{!}{=} 0 \end{aligned}$$

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$\longrightarrow$

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$\searrow$

$$\xi' = 0, 2$$

Convolution and boundary conditions

- The functions require nice fall-offs at infinity in order for the convolution integral to converge and to obey the *derivative rule*:

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

- Encode boundary conditions in effective sources, $\Box f = j$ and define Green's function operator

$$G \star j \equiv \frac{1}{\Box} j$$

Need for Green's function

- Instead, add all non-local terms consistent with mass dimension, ghost number, parity:

$$\varphi = A^\rho \circ \tilde{A}_\rho + \alpha_1 c^a \circ \tilde{c}_a + \frac{\alpha_2}{\Box} \partial A \circ \partial \tilde{A}$$

- The YM equations are

$$\Box A^I_\mu - \xi' \partial_\mu \partial A^I = j^I_\mu(A)$$
$$\Box c^{Ia} = j^{Ia}(c)$$

$$\Box(A^\rho \circ \tilde{A}_\rho) = \frac{1}{\Box} \Box A^\rho \circ \Box \tilde{A}_\rho$$
$$= \xi'(\xi' - 2) \partial A \circ \partial \tilde{A} + \frac{1}{\Box} j^\rho \circ \tilde{j}_\rho$$
$$\Box(c^a \circ c_a) = \frac{1}{\Box} j^a(c) \circ \tilde{j}_a(c)$$
$$\Box(\Box^{-1} \partial A \circ \partial \tilde{A}) = \partial A \circ \partial \tilde{A}$$

- This dictionary reproduces BRST and e.o.m.

$$\Box \varphi = j(\varphi)$$
$$Q\varphi = 0$$

$$\varphi = A^\rho \circ \tilde{A}_\rho + (\xi' - 1) c^a \circ \tilde{c}_a + \xi'(\xi' - 2) \frac{1}{\Box} \partial A \circ \partial \tilde{A}$$

$$j(\varphi) = \frac{1}{\Box} j^\rho \circ \tilde{j}_\rho + \frac{\alpha_1}{\Box} j^a \circ \tilde{j}_a$$

Dictionary

Physical sector:

$$h_{\mu\nu} = 2A_{(\mu} \circ \tilde{A}_{\nu)} + a_1 \frac{\partial_\mu \partial_\nu}{\Box} A \circ \tilde{A} + a_2 \frac{\partial_\mu \partial_\nu}{\Box} c^a \circ \tilde{c}_a + \frac{a_3}{\Box} \left(\partial A \circ \partial_{(\mu} \tilde{A}_{\nu)} + \partial_{(\mu} A_{\nu)} \circ \partial \tilde{A} \right)$$
$$+ \eta_{\mu\nu} \left(b_1 A \circ \tilde{A} + b_2 c^a \circ \tilde{c}_a + \frac{b_3}{\Box} \partial A \circ \partial \tilde{A} \right)$$

$$B_{\mu\nu} = 2A_{[\mu} \circ \tilde{A}_{\nu]} + \left(\frac{\xi' - 3}{1 - \xi'} \right) \frac{1}{\Box} \left(\partial A \circ \partial_{[\mu} \tilde{A}_{\nu]} - \partial_{[\mu} A_{\nu]} \circ \partial \tilde{A} \right)$$

$$\varphi = A^\rho \circ \tilde{A}_\rho + (\xi' - 1) c^a \circ \tilde{c}_a + \xi'(\xi' - 2) \frac{1}{\Box} \partial A \circ \partial \tilde{A}$$

$$a_1 = 2\frac{\xi' - 1}{\xi' - 2}, \qquad a_2 = \frac{\xi'}{\xi' - 2}, \qquad a_3 = -1$$
$$b_1 = \frac{2}{(2 - D)(2 - \xi')}, \qquad b_2 = (\xi' - 1) b_1, \qquad b_3 = \xi'(\xi' - 2) b_1$$

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$$QA^I_\mu = \partial_\mu c^I, \qquad Qc^I = 0, \qquad Q\bar{c}^I = \frac{1}{\xi} \partial^\mu A^I_\mu$$

↓

$$Qh_{\mu\nu} = 2\partial_{(\mu} c_{\nu)} \qquad Qc_\mu = 0$$
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$$+ \eta_{\mu\nu} \Big(b_1 A \circ \tilde{A} + b_2 c^a \circ \tilde{c}_a + \frac{b_3}{\Box} \partial A \circ \partial \tilde{A} \ldots$$

$$B_{\mu\nu} = 2A_{[\mu} \circ \tilde{A}_{\nu]} + \left(\frac{\xi' - 3}{1 - \xi'} \right) \frac{1}{\Box} \left(\partial A \circ \partial_{[\mu} \tilde{A}_{\nu]} - \ldots \right)$$

$$\varphi = A^\rho \circ \tilde{A}_\rho + (\xi' - 1) c^a \circ \tilde{c}_a + \xi'(\xi' - 2) \frac{1}{\Box} \partial A \circ \partial \tilde{A}$$

$$c_\mu = \frac{1}{2} \left(c \circ \tilde{A}_\mu + A_\mu \circ \tilde{c} \right) - \frac{\xi'}{2} \frac{\partial_\mu}{\Box} \left(c \circ \partial \tilde{A} + \partial A \circ \tilde{c} \right)$$

$$d_\mu = \frac{\xi' + 1}{2(\xi' - 1)} \left(c \circ \tilde{A}_\mu - A_\mu \circ \tilde{c} \right) - \frac{\xi'(\xi' + 1)}{2(\xi' - 1)} \frac{\partial_\mu}{\Box} \left(c \circ \partial \tilde{A} - \partial A \circ \tilde{c} \right)$$

$$d = \left(1 - \xi'^2 \right) c \circ \tilde{c}$$

$$\eta = \frac{(\xi' - 2)(\xi' + 1)}{2(1 - \xi')} \left(c \circ \tilde{\bar{c}} + \bar{c} \circ \tilde{c} \right)$$

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$$\downarrow$$

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$$+ \eta_{\mu\nu} \left(b_1 A \circ \tilde{A} + b_2 c^a \circ \tilde{c}_a + \frac{b_3}{\Box} \partial A \circ \partial \tilde{A} \right)$$
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$$\Box A^I_\mu - \xi' \partial_\mu \partial A^I = j^I_\mu(A)$$
$$\Box c^{Ia} = j^{Ia}(c)$$

$$\xi'$$

↓

$$\xi = \xi_{(h)} = \xi_{(B)} = \xi_{(d)}$$

$$\Box h_{\mu\nu} - 2\xi'_{(h)} \partial^\rho \partial_{(\mu} h_{\nu)\rho} + \xi'_{(h)} \partial_\mu \partial_\nu h = j_{\mu\nu}(h)$$
$$\Box B_{\mu\nu} - \xi'_{(B)} \partial^\rho \partial_{[\mu} B_{\nu]\rho} = j_{\mu\nu}(B)$$
$$\Box \varphi = j(\varphi)$$

$$\xi'_{(h)}, \ \xi'_{(B)}, \ \xi'_{(d)}$$

Conclusions

- The tensor product of two Yang-Mills theories written in the BRST basis (A, 2c) yields exactly the field content of graviton, Kalb-Ramond 2-form and dilaton, together with their respective ghosts (first and second generation).

- Dictionary constructs the correct action of the gravitational BRST operator on gravity fields from that of the YM BRST.

- The gravity fields derived have the correct dynamics, owing to the underlying YM equations of motion. This holds for any gauge-fixing parameter and, as a by-product, map between different gauges.

- Formalism is automatically anti-BRST covariant as well (both YM and gravity sides). This is a general property due to linearity of gauge-fixing functional. Importantly, anti-BRST is anti-commuting with BRST on all fields.

- Tensoring two YM over (A, 2c, b) induces spurious Weyl scaling invariance?

Thank you!