

AdS₄/CFT₃ holography from massive IIA

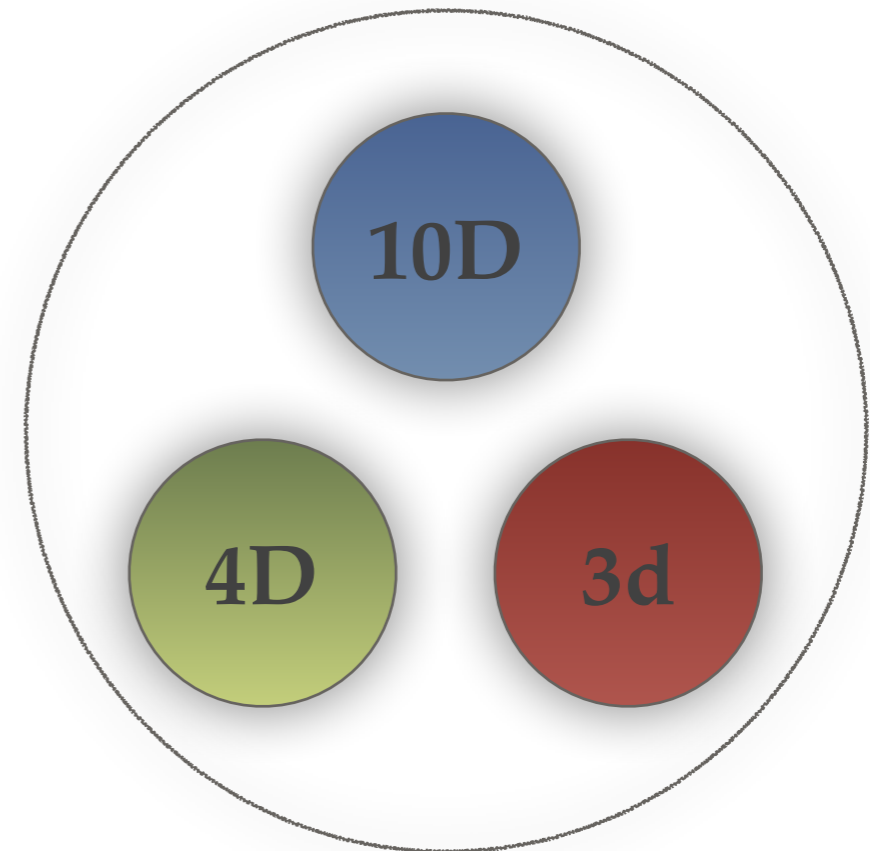
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With [D. Jafferis](#), [J. Tarrío](#) and [O. Varela](#) :

[arXiv:1504.08009](#) , [arXiv:1508.04432](#) , [arXiv:1509.02526](#)

[arXiv:1605.09254](#) , [arXiv:1703.10833](#) , [arXiv:1706.01823](#)

[arXiv:1712.09549](#)



Outlook

- Electric-magnetic duality in N=8 supergravity
- Massive IIA on S^6 / SYM-CS duality
- Holographic RG flows: domain-walls & black holes



Electric-magnetic duality in $N=8$ supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars
 (s = 2) (s = 3/2) (s = 1) (s = 1/2) (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$ [Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

- Reduction of M-theory on a *sphere* S^7 down to 4D produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]
- Reduction of M-theory on S^1 (Type IIA) and subsequently on S^6 down to 4D produces $N = 8$ supergravity with $G = ISO(7) = SO(7) \times \mathbb{R}^7$ [Hull '84]

* These gauged supergravities believed to be **unique** for 30 years...

Electric-magnetic deformations

- Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $\text{AdS}_5 \times S^5$ (D3-brane \sim N=4 SYM in 4d) [Maldacena '97]

M-theory : $\text{AdS}_4 \times S^7$ (M2-brane \sim ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

- N=8 supergravity in 4D admits a **deformation parameter** c yielding **inequivalent** theories. It is an **electric/magnetic** deformation

$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

g = 4D gauge coupling
 c = **deformation param.**

[Dall'Agata, Inverso, Trigiante '12]

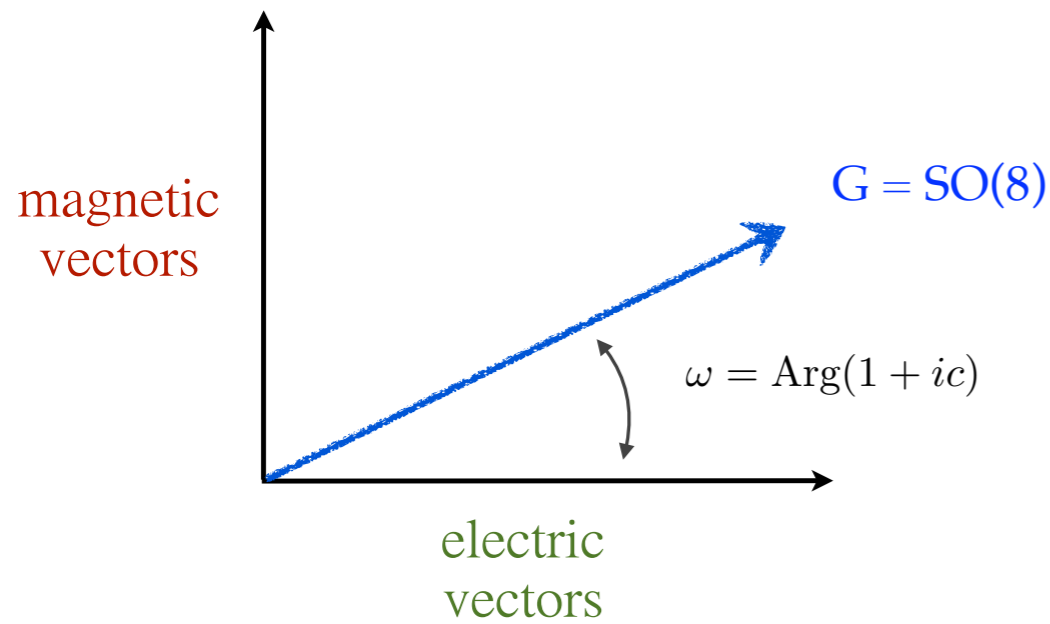
- There are two generic situations :

1) Family of $\text{SO}(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous param [similar for $\text{SO}(p,q)_c$]

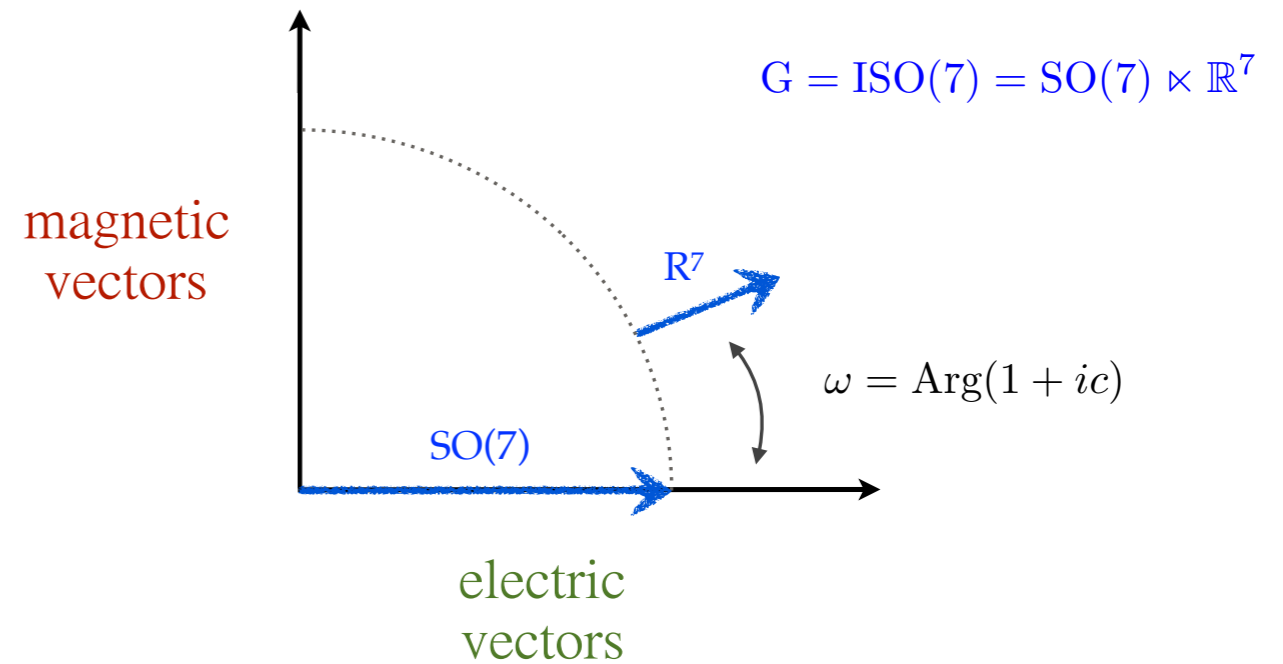
2) Family of $\text{ISO}(7)_c$ theories : $c = 0 \text{ or } 1$ is an (on/off) param [same for $\text{ISO}(p,q)_c$]

[Dall'Agata, Inverso, Marrani '14]

SO(8)_c vs ISO(7)_c



$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$



$$D = \partial - g A_{\text{SO}(7)}^{\text{elec}} - g (A_{\mathbb{R}^7}^{\text{elec}} - c \tilde{A}_{\mathbb{R}^7 \text{ mag}})$$

Higher-dimensional origin?

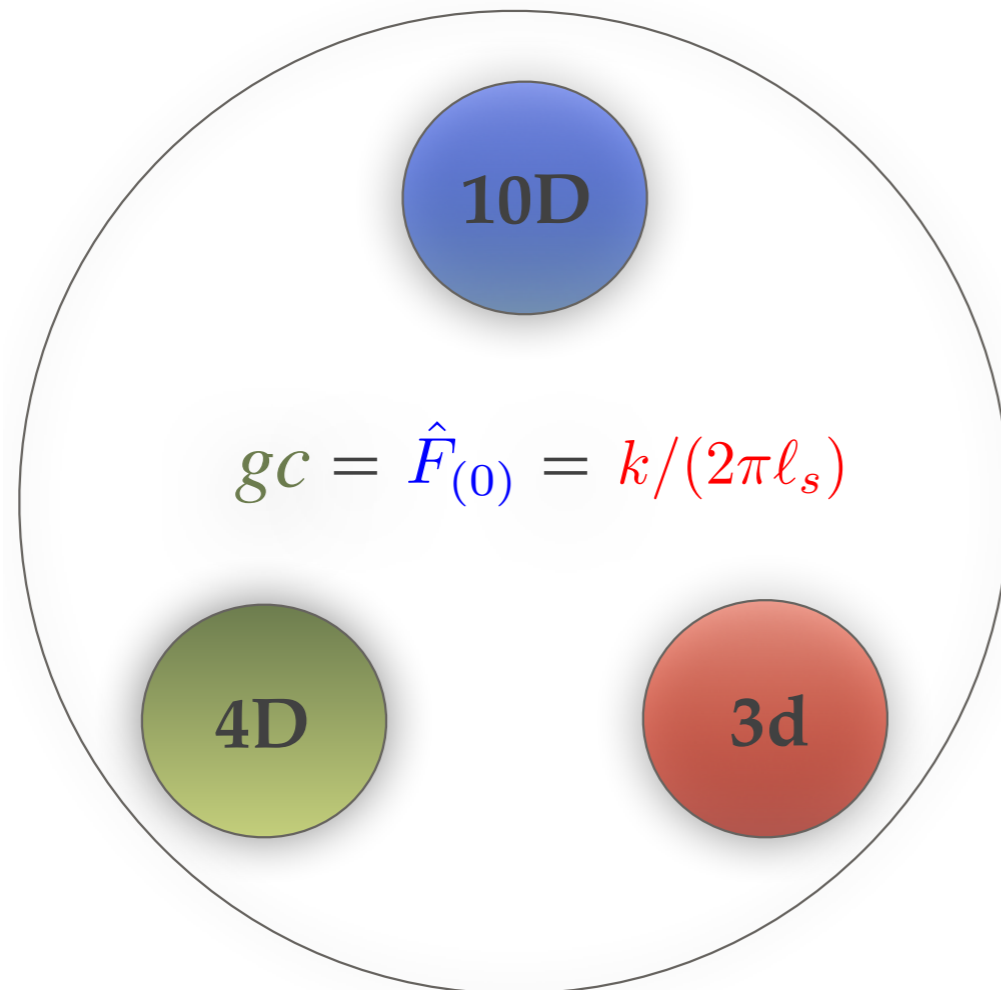
Obstruction for SO(8)_c, cf. [de Wit, Nicolai '13]

[Lee, Strickland-Constable, Waldram '15]

Holographic dual?

A new 10D/4D/3d correspondence

massive IIA on S^6 « $ISO(7)_c$ -gauged sugra » $SU(N)_k$ SYM-CS theory



gc = elec/mag deformation in 4D

$\hat{F}_{(0)}$ = Romans mass in 10D

k = Chern-Simons level in 3d

[AG, Jafferis, Varela '15]

[AG, Varela '15]

Well-established and independent dualities :

Type IIB on S^5 / $N=4$ SYM — M-theory on S^7 / ABJM — **mIIA on S^6 / SYM-CS**



Massive IIA on S^6 / SYM-CS duality

4D : ISO(7)_c Lagrangian

$$M = 1, \dots, 56$$

$$\Lambda = 1, \dots, 28$$

$$I = 1, \dots, 7$$

$$\begin{aligned} \mathcal{L}_{\text{bos}} = & (R - V) \text{vol}_4 - \frac{1}{48} D\mathcal{M}_{\text{MIN}} \wedge *D\mathcal{M}^{\text{MIN}} + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge *\mathcal{H}_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge \mathcal{H}_{(2)}^\Sigma \\ & + g \, c \left[\mathcal{B}^I \wedge (\tilde{\mathcal{H}}_{(2)I} - \frac{g}{2} \delta_{IJ} \mathcal{B}^J) - \frac{1}{4} \tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J \wedge (d\mathcal{A}^{IJ} + \frac{g}{2} \delta_{KL} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL}) \right] \end{aligned}$$

◆ Setting $c = 0$, all the magnetic pieces in the Lagrangian disappear.

* Ingredients :

- Electric vectors (21 + 7) : $\mathcal{A}^{IJ} = \mathcal{A}^{[IJ]}$ [SO(7)] and \mathcal{A}^I [R⁷] with $\mathcal{H}_{(2)}^\Lambda = (\mathcal{H}_{(2)}^{IJ}, \mathcal{H}_{(2)}^I)$
- Auxiliary magnetic vectors (7) : $\tilde{\mathcal{A}}_I$ [R⁷] with $\tilde{\mathcal{H}}_{(2)I}$ field strength
- E₇/SU(8) scalars : \mathcal{M}_{MIN}
- Auxiliary two-forms (7) : \mathcal{B}^I [R⁷]
- Topological term : $g \, c [\dots]$
- Scalar potential : $V(\mathcal{M}) = \frac{g^2}{672} X_{\text{MN}}^{\text{R}} X_{\text{PQ}}^{\text{S}} \mathcal{M}^{\text{MP}} (\mathcal{M}^{\text{NQ}} \mathcal{M}_{\text{RS}} + 7 \delta_{\text{R}}^{\text{Q}} \delta_{\text{S}}^{\text{N}})$

AdS₄ solutions

[AG, Varela '15]

\mathcal{N}	G_0	$c^{-1/3} \chi$	$c^{-1/3} e^{-\varphi}$	$c^{-1/3} \rho$	$c^{-1/3} e^{-\phi}$	$\frac{1}{4} g^{-2} c^{1/3} V_0$	$M^2 L^2$
$\mathcal{N} = 1$	G_2	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{2^{22/3} 3^{1/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, -\frac{1}{6}(11 \pm \sqrt{6})$
$\mathcal{N} = 2$	$U(3)$	$-\frac{1}{2}$	$\frac{3^{1/2}}{2}$	0	$\frac{1}{2^{1/2}}$	$-3^{3/2}$	$3 \pm \sqrt{17}, 2, 2$
$\mathcal{N} = 1$	$SU(3)$	$\frac{1}{2^2}$	$\frac{3^{1/2} 5^{1/2}}{2^2}$	$-\frac{3^{1/2}}{2^2}$	$\frac{5^{1/2}}{2^2}$	$-\frac{2^6 3^{3/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, 4 \pm \sqrt{6}$
$\mathcal{N} = 0$	$SO(6)_+$	0	$2^{1/6}$	0	$\frac{1}{2^{5/6}}$	$-3 2^{5/6}$	$6, 6, -\frac{3}{4}, 0$
$\mathcal{N} = 0$	$SO(7)_+$	0	$\frac{1}{5^{1/6}}$	0	$\frac{1}{5^{1/6}}$	$-\frac{3 5^{7/6}}{2^2}$	$6, -\frac{12}{5}, -\frac{6}{5}, -\frac{6}{5}$
$\mathcal{N} = 0$	G_2	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$-\frac{2^{10/3}}{3^{1/2}}$	$6, 6, -1, -1$
$\mathcal{N} = 0$	$SU(3)$	0.455	0.838	0.335	0.601	-5.864	6.214, 5.925, 1.145, -1.284
$\mathcal{N} = 0$	$SU(3)$	0.270	0.733	0.491	0.662	-5.853	6.230, 5.905, 1.130, -1.264

◆ $\mathcal{N} = 2$ solution will play a central role in holography !!

10D : ISO(7)_c into type IIA supergravity

[AG, Varela '15]

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} Dy^m Dy^n ,$$

$$\begin{aligned} \hat{A}_{(3)} = & \mu_I \mu_J (C^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \frac{1}{6} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \frac{1}{6} \mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K) \\ & + g^{-1} (\mathcal{B}_J^I + \frac{1}{2} \mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \frac{1}{2} \mathcal{A}^I \wedge \tilde{\mathcal{A}}_J) \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ & - \frac{1}{2} \mu_I B_{mn} \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \frac{1}{6} A_{mnp} Dy^m \wedge Dy^n \wedge Dy^p , \end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I (\mathcal{B}^I + \frac{1}{2} \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J) - g^{-1} \tilde{\mathcal{A}}_I \wedge D\mu^I + \frac{1}{2} B_{mn} Dy^m \wedge Dy^n ,$$

$$\hat{A}_{(1)} = -\mu_I \mathcal{A}^I + A_m Dy^m .$$

where we have defined : $Dy^m \equiv dy^m + \frac{1}{2} g K_{IJ}^m \mathcal{A}^{IJ}$, $D\mu^I \equiv d\mu^I - g \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$\begin{aligned} g^{mn} &= \frac{1}{4} g^2 \Delta \mathcal{M}^{IJKL} K_{IJ}^m K_{KL}^n , & B_{mn} &= -\frac{1}{2} \Delta g_{mp} K_{IJ}^p \partial_n \mu^K \mathcal{M}^{IJ}{}_{K8} , \\ A_m &= \frac{1}{2} g \Delta g_{mn} K_{IJ}^n \mu_K \mathcal{M}^{IJK8} , & A_{mnp} &= \frac{1}{8} g \Delta g_{mq} K_{IJ}^q K_{np}^{KL} \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} . \end{aligned}$$

N=2 solution of massive type IIA

- N=2 & U(3) AdS₄ point of the ISO(7)_c theory

$$d\hat{s}_{10}^2 = L^2 \frac{(3 + \cos 2\alpha)^{\frac{1}{2}}}{(5 + \cos 2\alpha)^{-\frac{1}{8}}} \left[ds^2(\text{AdS}_4) + \frac{3}{2} d\alpha^2 + \frac{6 \sin^2 \alpha}{3 + \cos 2\alpha} ds^2(\mathbb{CP}^2) + \frac{9 \sin^2 \alpha}{5 + \cos 2\alpha} \boldsymbol{\eta}^2 \right],$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{(5 + \cos 2\alpha)^{3/4}}{3 + \cos 2\alpha}, \quad \hat{H}_{(3)} = 24\sqrt{2} L^2 e^{\frac{1}{2}\phi_0} \frac{\sin^3 \alpha}{(3 + \cos 2\alpha)^2} \mathbf{J} \wedge d\alpha,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6} \frac{\sin^2 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} - 3\sqrt{6} \frac{(3 - \cos 2\alpha)}{(5 + \cos 2\alpha)^2} \sin \alpha d\alpha \wedge \boldsymbol{\eta},$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6 \text{vol}_4 + 12\sqrt{3} \frac{7 + 3 \cos 2\alpha}{(3 + \cos 2\alpha)^2} \sin^4 \alpha \text{vol}_{\mathbb{CP}^2} + 18\sqrt{3} \frac{(9 + \cos 2\alpha) \sin^3 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} \wedge d\alpha \wedge \boldsymbol{\eta},$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{5}{8}} 3^{-1} g^{-2} c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}} c^{-\frac{5}{6}}$

- ◆ The angle $0 \leq \alpha \leq \pi$ locally foliates S₆ with S₅ regarded as Hopf fibrations over \mathbb{CP}^2

3D : CFT₃ dual & matching of free energies

[Schwarz '04]

[Gaiotto, Tomasiello '09]

- 3d SYM + (N=2) Chern-Simons with **simple** group SU(N) , level k , three adjoint matter and a cubic superpotential $W = \text{Tr}(X[Y,Z])$

- The 3d free energy $F = -\text{Log}(Z)$, where Z is the partition function of the CFT on a Euclidean S^3 , can be computed via localisation over supersymmetric configurations $N \gg k$

$$F = \frac{3^{13/6} \pi}{40} \left(\frac{32}{27} \right)^{2/3} k^{1/3} N^{5/3}$$

[Pestun '07] [Kapustin, Willett, Yaakov '09]

[Jafferis '10] [Jafferis, Klebanov, Pufu, Safdi '11]

[Closset, Dumitrescu, Festuccia, Komargodski '12 '13]

- The gravitational free energy can be computed from the warp factor in the N=2 massive IIA solution. Using the charge quantisation condition $N = -(2\pi\ell_s)^{-5} \int_{S^6} e^{\frac{1}{2}\phi} \hat{*}\hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6} \hat{F}_{(0)} \hat{B}_{(2)}^3$ for the D2-brane, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} k^{1/3} N^{5/3} \quad \text{provided}$$

$$g c = \hat{F}_{(0)} = k/(2\pi\ell_s)$$

[Emparan, Johnson, Myers '99]



Holographic RG flows: domain-walls & black holes

Holographic description of RG flows

[Boonstra, Skenderis, Townsend '98]

- RG flows are described holographically as non-AdS₄ solutions in gravity
- RG flows on M2-brane : SO(8)-gauged sugra from M-theory on S⁷

[Ahn, Paeng '00] [Ahn, Itoh '01]

[Bobev, Halmagyi, Pilch, Warner '09]

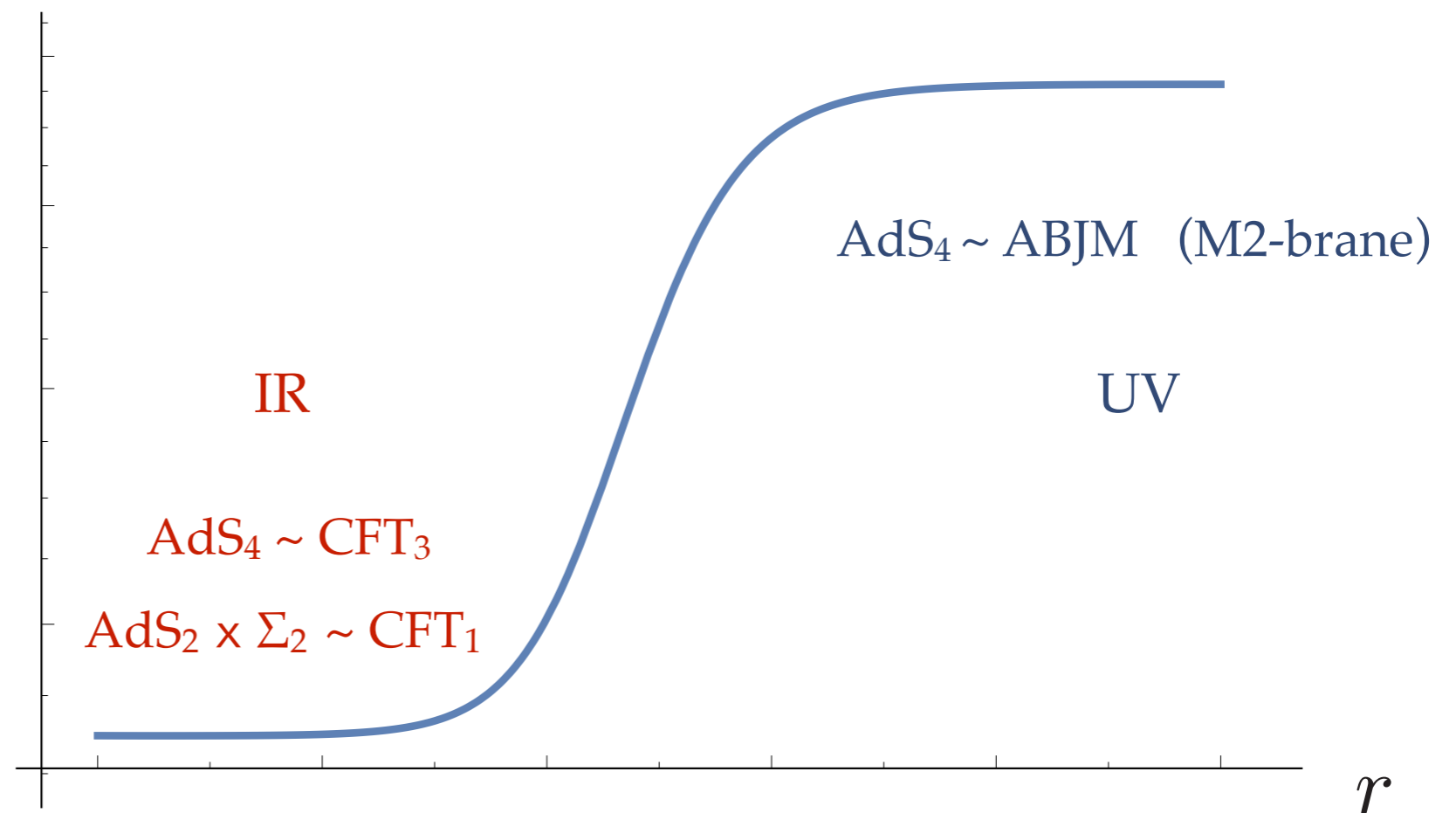
[Cacciatori, Klemm '09]

[Halmagyi, Petrini, Zaffaroni '13]

[Chimento, Klemm, Petri '15]

[Benini, Hristov, Zaffaroni '15 '16]

AdS₄ in IR : domain-wall
AdS₂ × Σ₂ in IR : black hole



- RG flows on D3-brane : SO(6)-gauged sugra from type IIB on S⁵ and N=4 SYM in 4D

[Freedman, Gubser, Pilch, Warner '99]

[Pilch, Warner '00] [Benini, Bobev '12, '13]

Holographic RG flows on the D2-brane

- D2-brane :

$$d\hat{s}_{10}^2 = e^{\frac{3}{4}\phi} \left(-e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(\psi-U)} ds_{\Sigma_2}^2 \right) + g^{-2} e^{-\frac{1}{4}\phi} ds_{S^6}^2$$

$$e^{\hat{\Phi}} = e^{\frac{5}{2}\phi}$$

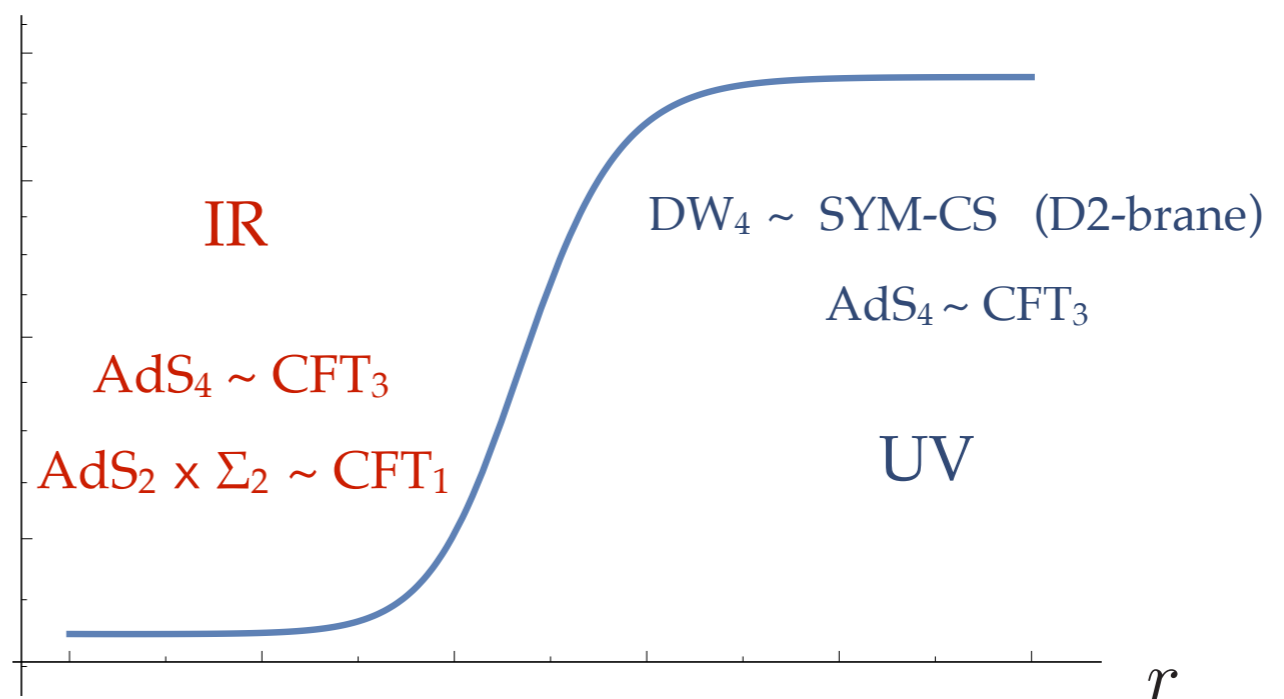
$$\hat{F}_{(4)} = 5g e^{\phi} e^{2(\psi-U)} dt \wedge dr \wedge d\Sigma_2$$

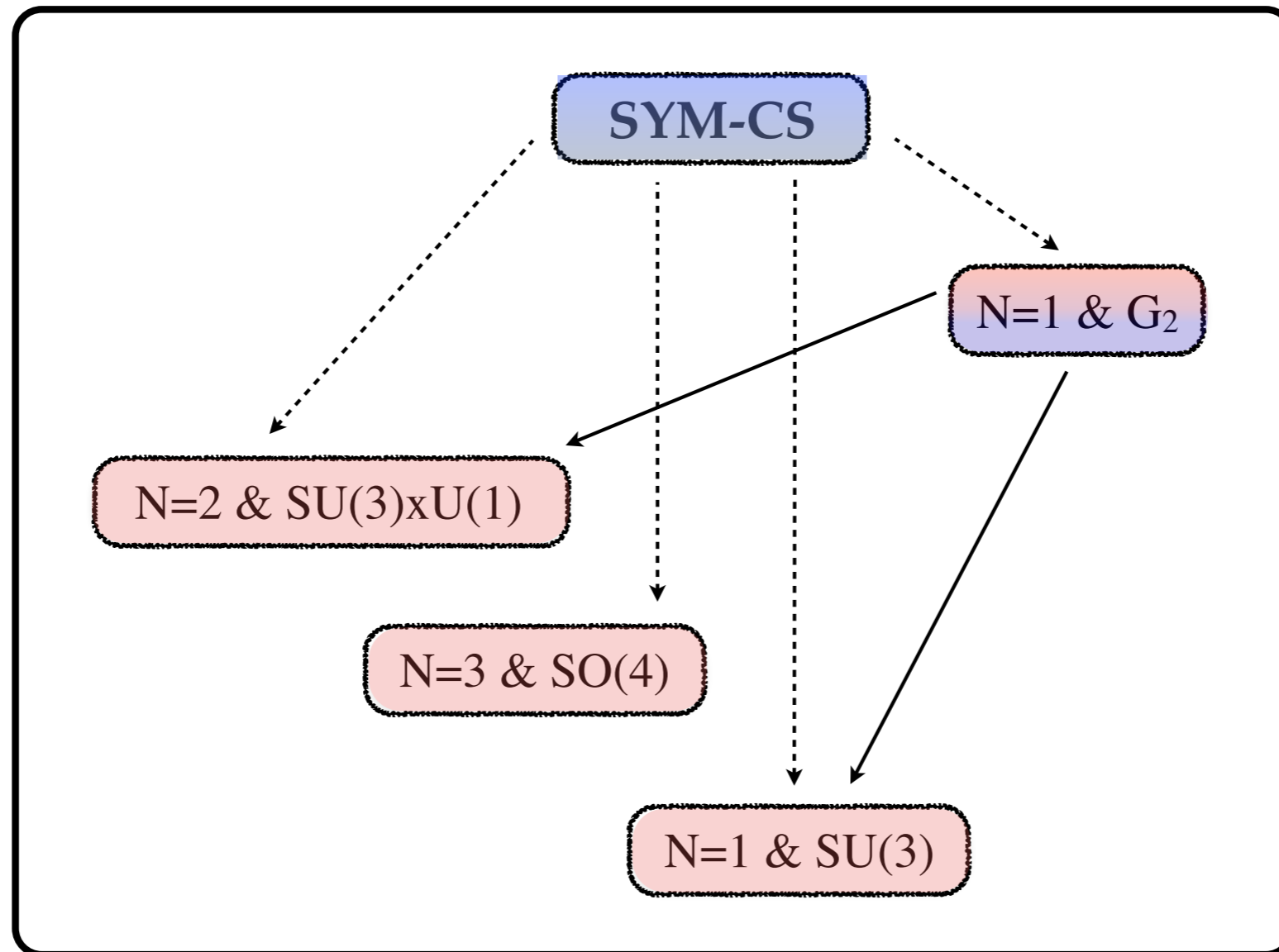
with $e^{2U} \sim r^{\frac{7}{4}}$, $e^{2(\psi-U)} \sim r^{\frac{7}{4}}$ and $e^{\phi} \sim r^{-\frac{1}{4}}$ \rightarrow

DW₄
domain-wall
(SYM-CS)

- RG flows on D2-brane : ISO(7)_c-gauged sugra from mIIA on S⁶

AdS₄ in IR : domain-wall
AdS₂ × Σ₂ in IR : black hole





- RG flows from **SYM-CS** (dotted lines) and between **CFT's** (solid lines) dual to BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity

Black holes (I)

$$\Lambda = 0, 1$$

- N=2 model with 1 vector + 1 hyper (universal)

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} dr^2 + e^{2(\psi(r)-U(r))} \left(d\theta^2 + \left(\frac{\sin \sqrt{\kappa} \theta}{\sqrt{\kappa}} \right)^2 d\phi^2 \right)$$

- Black hole Ansatz :

$$\begin{aligned} \mathcal{A}^\Lambda &= \mathcal{A}_t^\Lambda(r) dt - p^\Lambda \frac{\cos \sqrt{\kappa} \theta}{\kappa} d\phi \\ \tilde{\mathcal{A}}_\Lambda &= \tilde{\mathcal{A}}_{t\Lambda}(r) dt - e_\Lambda \frac{\cos \sqrt{\kappa} \theta}{\kappa} d\phi \end{aligned} \quad \mathcal{B}^0 = b_0(r) \frac{\sin \sqrt{\kappa} \theta}{\sqrt{\kappa}} d\theta \wedge d\phi$$

- Attractor equations :

$$\begin{aligned} Q &= \kappa L_{\Sigma_2}^2 \Omega \mathcal{M} Q^x \mathcal{P}^x - 4 \text{Im}(\bar{\mathcal{Z}} \mathcal{V}) , \\ \frac{L_{\Sigma_2}^2}{L_{\text{AdS}_2}} &= -2 \mathcal{Z} e^{-i\beta} , \\ \langle \mathcal{K}^u, \mathcal{V} \rangle &= 0 , \end{aligned}$$

[Dall'Agata, Gneccchi '10]
[Klemm, Petri, Rabbiosi '16]

- Unique AdS₂ × H² :

(N=2 & U(3) AdS₄ vev's)

$$\begin{aligned} e^{\varphi_h} &= \frac{2}{\sqrt{3}} \left(\frac{g}{m} \right)^{\frac{1}{3}} , \quad \chi_h = -\frac{1}{2} \left(\frac{g}{m} \right)^{-\frac{1}{3}} , \quad e^{\phi_h} = \sqrt{2} \left(\frac{g}{m} \right)^{\frac{1}{3}} , \quad a_h = \zeta_h = \tilde{\zeta}_h = 0 , \\ p^0 + \frac{1}{2} m b_0^h &= \pm \frac{1}{6} m^{\frac{2}{3}} g^{-\frac{5}{3}} , \quad e_0 + \frac{1}{2} g b_0^h = \pm \frac{1}{6} m^{-\frac{1}{3}} g^{-\frac{2}{3}} , \\ p^1 &= \mp \frac{1}{3} g^{-1} , \quad e_1 = \pm \frac{1}{2} m^{\frac{1}{3}} g^{-\frac{4}{3}} , \\ L_{\text{AdS}_2}^2 &= \frac{1}{4\sqrt{3}} m^{\frac{1}{3}} g^{-\frac{7}{3}} , \quad L_{\text{H}^2}^2 = \frac{1}{2\sqrt{3}} m^{\frac{1}{3}} g^{-\frac{7}{3}} . \end{aligned}$$

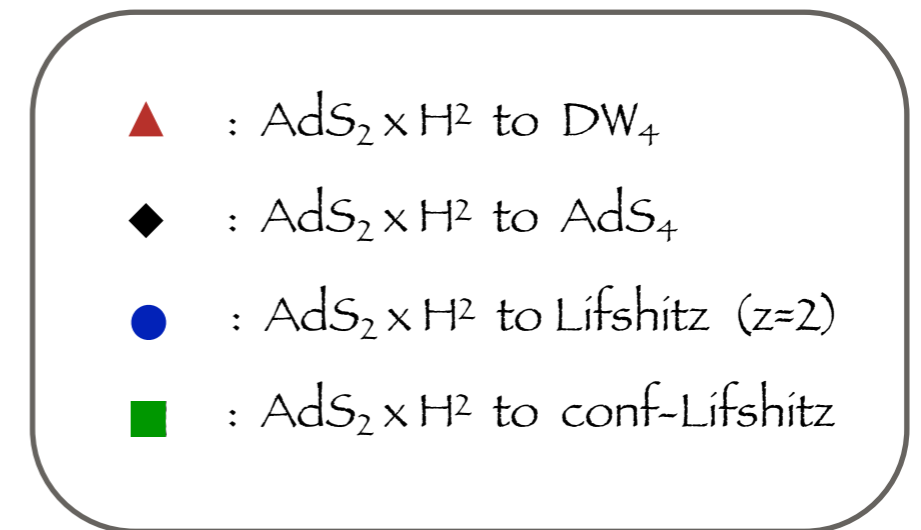
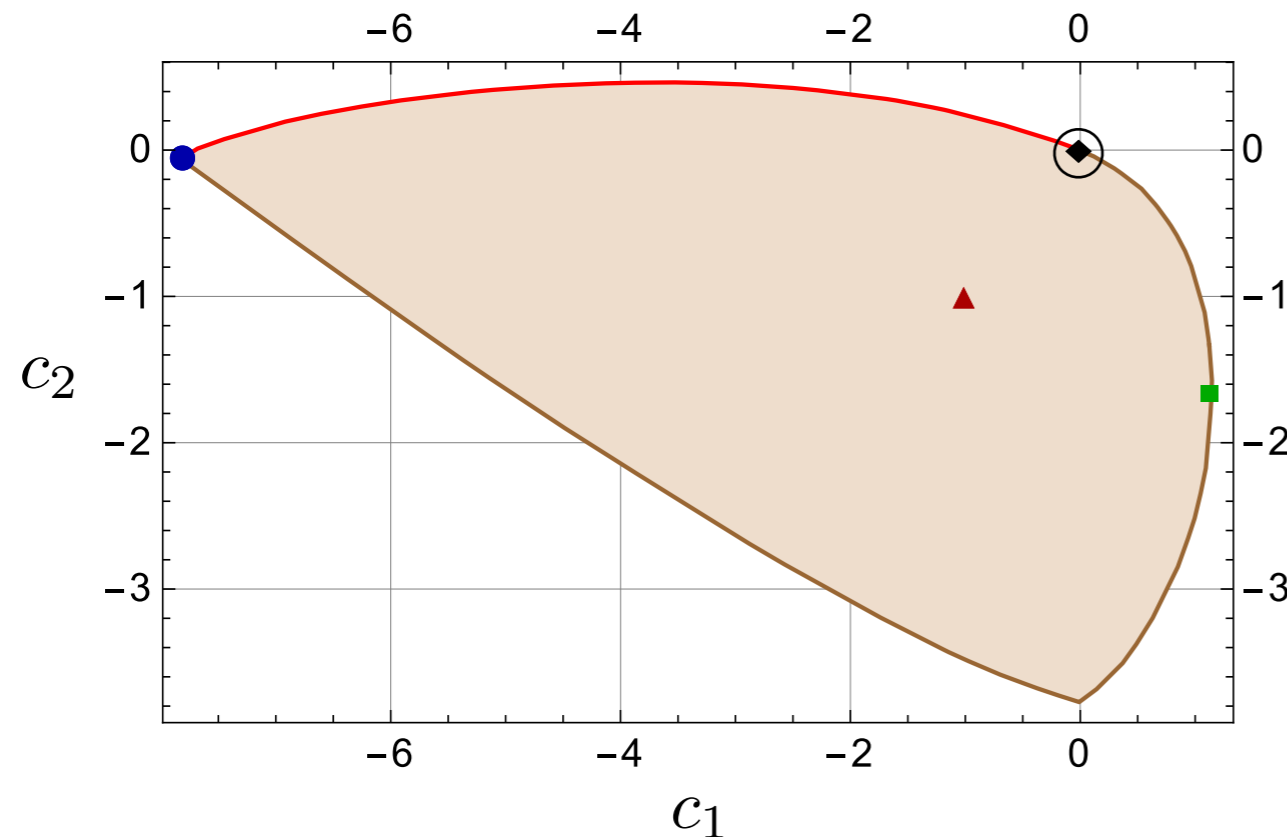
[AG, Tarrío '17]

Black holes (II)

[Dall'Agata, Gneccchi '10]

[Klemm, Petri, Rabbiosi '16]

- Two irrelevant modes (c_1, c_2) when perturbing around the $AdS_2 \times H^2$ solution in the IR



[AG, Tarrío '17]

- RG flows across dimension from **SYM-CS or CFT_3 or non-relativistic** to **CFT_1**

- Universal (constant scalars) RG flow (◆) **CFT_3** to **CFT_1**

[Caldarelli, Klemm '98]

- $AdS_2 \times \Sigma_g$ horizons for mIIA on $H^{(p,q)}$: STU-models with 3 vectors + 1 hyper

[AG '17]

Summary

- Dyonic $N = 8$ supergravity with $ISO(7)_c$ gauging connected to massive IIA reductions on S^6 .
- Any 4D configuration (AdS, DW, BH) is embedded into 10D via the uplifting formulas.
Example : $AdS_4 \times S^6$ solution of massive IIA based on an $N = 2$ & $U(3)$ AdS_4 vacuum.
- CFT_3 dual for the $N = 2$ $AdS_4 \times S^6$ solution of mIIA based on the D2-brane field theory (SYM-CS).
- Holographic study of RG flows on D2-brane :
DW solutions (CFT_3 / CFT_3 & SYM-CS / CFT_3)
BH solutions (CFT_3 / CFT_1 & SYM-CS / CFT_1)
- Generalisation & further tests/conjectures on the duality (semiclassical observables, level-rank duality, ...)
[Fluder, Sparks '15] [Passias, Prins, Tomasiello '18]
[Araujo, Nastase '16] [Araujo, Itsios, Nastase, Ó Colgáin '17]
- Recent progress in the holographic counting of BH microstates
[Benini, Hristov, Zaffaroni '16]
[Azzurli, Bobev, Crichigno, Min, Zaffaroni '17]
[Hosseini, Hristov, Passias '17] [Benini, Khachatryan, Milan '17]
- $SO(8)_c$ theories? [work in progress...]

¡ Muchas gracias !