α'-corrected solutions of the Heterotic Superstring

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Plan of the talk:

1.- Intro: Superstring effective actions and solutions.

2.- The Heterotic superstring effective action.

3.- A "black-hole ansatz"

4.- The rôle of non-Abelian fields in solving the $\mathcal{O}(\alpha')$ e.o.m.

5.- α'-corrected T-duality

1.- The Superstring effective action & its solutions

$$S_{eff}[g_{\mu\nu}, \phi, B_{\mu\nu}, \ldots] \sim \sum_{n=0}^{\infty} \alpha'^{n} \sum_{g=0}^{\infty} e^{-2(g+1)\phi_\infty} S_{n,g}$$

$$\left.\begin{array}{l} \alpha' = l_s^2 \\ e^{-\phi_\infty} = g_s \end{array}\right\} \Rightarrow \quad S_{n,g} \sim \frac{1}{16\pi G_N^{(d)}} \int d^dx \sqrt{|g|}\, e^{-2(g+1)\phi} R^{n+1}$$

and other field strengths.

Very little is known about $S_{n,g}$ for $n > 1$, $g \geq 1$

Focus on $S_{1,0}$

First stringy effects!

Solutions ($g=0$)

$$g_{\mu\nu}(x) = g_{\mu\nu}(x) + \alpha' \delta g_{\mu\nu}(x) + O(\alpha'^2)$$
$$\phi(x) = \phi(x) + \alpha' \delta\phi(x) + O(\alpha'^2)$$
$$\vdots$$

1.- If $\alpha'\delta g, \alpha'\delta\phi \ldots \gtrsim g, \phi, \ldots$ the zeroth-order solutions are not good solutions of S_{eff}: the corrections cannot be ignored.

2.- The $O(\alpha'^2)$ corrections can be ignored if they are much smaller than $O(1)$ and $O(\alpha')$

3.- Curvature invariants $= 0$
$\Longrightarrow$ corrections $= 0$.

4.- Finding $\mathcal{O}(\alpha')$ corrections is an interesting but difficult problem.

- Only some regions (near-H or vacua)
- Exact solutions ("anomaly cancellation")

non-Abelian fields

5.- $\mathcal{O}(\alpha')$-corrections are not well known in $d=4,5$

$d=10$ Heterotic superstring.

$$\downarrow T^4$$

$d=5$ gauged SUGRA

$$\downarrow S^1$$

$d=4$ gauged SUGRA

The Heterotic Superstring Effective Action

$B = \frac{1}{2} B_{\mu\nu}\, dx^\mu \wedge dx^\nu \;;\quad e^a = e^a{}_\mu\, dx^\mu \;;\quad \phi \;;\quad A^A \;;$

$H^{(0)} \equiv dB \qquad \omega^a{}_b(e)\;; \qquad d\phi\;;$

$$\Omega^{(0)\,a}_{(\pm)\;\;b} = \omega^a{}_b \pm \tfrac{1}{2} H^{(0)a}_{\mu\;\;\;b} dx^\mu$$

$$R^{(0)\,a}_{(\pm)\;\;b} = d\Omega^{(0)\,a}_{(\pm)\;\;b} - \Omega^{(0)\,a}_{(\pm)\;\;c} \wedge \Omega^{(0)\,c}_{(\pm)\;\;b}\,,$$

$$\omega^{\mathrm{L}\,(0)}_{(\pm)} = d\Omega^{(0)\,a}_{(\pm)\;\;b} \wedge \Omega^{(0)\,b}_{(\pm)\;\;a} - \tfrac{2}{3}\Omega^{(0)\,a}_{(\pm)\;\;b} \wedge \Omega^{(0)\,b}_{(\pm)\;\;c} \wedge \Omega^{(0)\,c}_{(\pm)\;\;a}$$

$$F^A = dA^A + \tfrac{1}{2}\epsilon^{ABC} A^B \wedge A^C\,,$$

$$\omega^{\mathrm{YM}} = dA^A \wedge A^A + \tfrac{1}{3}\epsilon^{ABC} A^A \wedge A^B \wedge A^C$$

$$\boxed{H^{(1)} = dB + 2\alpha'\left(\omega^{\mathrm{YM}} + \omega^{\mathrm{L}\,(0)}_{(-)}\right)} \qquad \left(\longrightarrow \Omega^{(1)\,a}_{(\pm)\;\;b} \;\ldots\right)$$

T- tensors

They codify the first α' corrections

$R^{(0)}_{(-)}$

$$T^{(4)} \equiv 6\alpha' \left[F^A \wedge F^A + R_{(-)}{}^a{}_b \wedge R_{(-)}{}^b{}_a \right] , \rightarrow \text{Bianchi of } H$$

$$T^{(2)}{}_{\mu\nu} \equiv 2\alpha' \left[F^A{}_{\mu\rho} F^A{}_\nu{}^\rho + R_{(-)\,\mu\rho}{}^a{}_b R_{(-)\,\nu}{}^{\rho\, b}{}_a \right] , \rightarrow \text{Einstein eq.}$$

$$T^{(0)} \equiv T^{(2)\,\mu}{}_\mu . \rightarrow \text{Dilaton eq.}$$

$\mathcal{O}(\alpha')$ effective action:

$$\boxed{S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 - \frac{1}{2} T^{(0)} \right\}}$$

(Bergshoeff & de Roo 1989)

→ Very complicated e.o.m. but

$$\begin{aligned}
\delta S &= \frac{\delta S}{\delta g_{\mu\nu}} \delta g_{\mu\nu} + \frac{\delta S}{\delta B_{\mu\nu}} \delta B_{\mu\nu} + \frac{\delta S}{\delta A^{A_i}{}_\mu} \delta A^{A_i}{}_\mu + \frac{\delta S}{\delta \phi} \delta\phi \\
&= \left.\frac{\delta S}{\delta g_{\mu\nu}}\right|_{\text{exp.}} \delta g_{\mu\nu} + \left.\frac{\delta S}{\delta B_{\mu\nu}}\right|_{\text{exp.}} \delta B_{\mu\nu} + \left.\frac{\delta S}{\delta A^{A_i}{}_\mu}\right|_{\text{exp.}} \delta A^{A_i}{}_\mu + \frac{\delta S}{\delta \phi} \delta\phi \\
&\quad + \frac{\delta S}{\delta \Omega_{(-)}{}^a{}_b} \left(\frac{\delta \Omega_{(-)}{}^a{}_b}{\delta g_{\mu\nu}} + \frac{\delta \Omega_{(-)}{}^a{}_b}{\delta B_{\mu\nu}} \delta B_{\mu\nu} + \frac{\delta \Omega_{(-)}{}^a{}_b}{\delta A^{A_i}{}_\mu} \delta A^{A_i}{}_\mu \right) .
\end{aligned}$$

$$\frac{\delta S}{\delta \Omega_{(-)}{}^a{}_b} \sim \alpha' \frac{\delta S^{(0)}}{\delta (g_{\mu\nu}, B_{\mu\nu}, \phi)}$$

(Bergshoeff & de Roo 1989)

→ If a configuration solves the $\mathcal{O}(1)$ e.o.m. up to terms of $\mathcal{O}(\alpha')$, the e.o.m. that need to be checked are $\left.\frac{\delta S}{\delta \text{fields}}\right|_{\underline{\text{exp.}}} = 0$

$$
\begin{aligned}
R_{\mu\nu} - 2\nabla_\mu \partial_\nu \phi + \tfrac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma} - T^{(2)}{}_{\mu\nu} &= 0\,, \\
(\partial\phi)^2 - \tfrac{1}{2}\nabla^2\phi - \tfrac{1}{4\cdot 3!} H^2 + \tfrac{1}{8} T^{(0)} &= 0\,, \\
d\left(e^{-2\phi} \star H\right) &= 0\,, \\
\alpha' e^{2\phi} \mathfrak{D}_{(+)} \left(e^{-2\phi} \star F^{A_i}\right) &= 0\,,
\end{aligned}
$$

$i = 1,2$
$SU(2) \times SU(2)$

If the configuration is described in terms of H, add

$$
dH - \tfrac{1}{3} T^{(4)} = 0\,,
$$

REMARKS:

1.- Ignoring all the terms in $\Omega_{(-)} \Rightarrow$ exact SUGRA

+ compactification on $T^{5,6}$ + truncation

$\longrightarrow$ gauged $\mathcal{N}=2$, $d=5,4$ SUGRA coupled to vector multiplets.

+ solution-generating techniques

$\longrightarrow$ black holes, rings; global monopoles, instantons; multi-center solutions — with non-Abelian hair in fully analytic form.

2.- $\Omega_{(-)}$ occurs as another gauge field

$\longrightarrow$ its presence should modify the $\mathcal{O}(1)$ solutions

$\boxed{\text{just as } A^A \text{ does.}}$

The Ansatz

$i = 2,3,4,5$

1.- Metric:

$$ds^2 = \frac{2}{\mathcal{Z}_-} du \left[dv - \tfrac{1}{2}\mathcal{Z}_+ du \right] - \mathcal{Z}_0 d\sigma^2 - dy^i dy^i \,,$$

Fundamental strings

pp - waves

Solitonic 5- branes

$$d\sigma^2 = h_{\underline{mn}} dx^m dx^n \,, \qquad m = \sharp, 1, 2, 3$$

hyper-Kähler 4-manifold (KK monopoles)

	u, v: 0	1	y^i: 2	3	4	5	x^m: $\sharp$	1	2	3
F1	×	×								
W	×	×								
S5	×	×	×	×	×	×				
KK	×	×	×	×	×	×	O			

$$\mathcal{Z}_{0,+,-} = \mathcal{Z}_{0,+,-}(x)$$

2.- KALB-RAMOND 3-FORM:

$$H = d\mathcal{Z}_{-}^{-1} \wedge du \wedge dv + \star_{(4)} d\mathcal{Z}_0 \,,$$

B ?

Fundamental strings

Solitonic 5-branes

HK

3.- Dilaton:

$$e^{-2\phi} = \underbrace{e^{-2\phi_\infty}}_{1/g_s^2} \frac{\mathcal{Z}_-}{\mathcal{Z}_0} \,,$$

This configuration is an exact, supersymmetric solution of the $O(1)$ theory if

$$\boxed{\nabla^2_{(4)} \mathcal{Z}_{0,+,-} = 0}$$

HK

$T^5 \longrightarrow d=5$ BHs

$T^6 \longrightarrow d=4$ BHs

(HK $\longrightarrow$ Gibbons-Hawking)

4.— Yang-Mills fields:

$$F^{A_{1,2}} = + \star_{(4)} F^{A_{1,2}}$$

Same selfduality as HK: $R^{mn} = * R^{mn}$

HK

(Papadopoulos 0809:1156)

We are going to need an explicit construction of $A^{A_{1,2}}$ because we want to compute the T-tensors explicitely: Typically the Biandi identity is solved via the "anomaly-cancellation mechanism" (Green-Schwarz)

$$T^{(4)} = 6\alpha' \left[F^{A_1} \wedge F^{A_1} + F^{A_2} \wedge F^{A_2} + R_{(-)}{}^{a}{}_{b} \wedge R_{(-)}{}^{b}{}_{a} \right] = 0; \Rightarrow dH = 0;$$

Relations between SU(2) bundles

We want independent Yang-Mills fields.

Solving the Bianchi identity of H

For our Ansatz

$$dH = d\star_{(4)}\, d\mathcal{Z}_0 = -\nabla^2_{(4)}\mathcal{Z}_0 |v| d^4x\,.$$

$$\left(h_{\underline{mn}} = v^p{}_{\underline{m}} v^p{}_{\underline{n}}\,. \qquad \det(v^p{}_{\underline{m}}) = |v| \right)$$

$$\Rightarrow \nabla^2_{(2)} \mathcal{Z}_0 |v| d^4x + 2\alpha' \left[F^{Ai} \wedge F^{Ai} + R_{(-)}{}^a{}_b \wedge R_{(-)}{}^b{}_a \right] = 0$$

We know from gauged $N=2$, $d=5,4$ SUGRA how $F\wedge F$ contributes to $\mathcal{Z}_0$! $\Rightarrow F^{Ai} \wedge F^{Ai} \sim \nabla^2_{(4)}(\text{Something}) |v| d^4x$

We have found two constructions of selfdual SU(2) bundles on HK spaces with this property:

1.- 't Hooft ansatz

2.- Kronheimer-Bogomol'nyi-Protogenov

't Hooft Ansatz

Based on $so(4) \simeq su^+(2) \times su^-(2)$

$$\{(\mathbb{M}_{mn})^{pq} \equiv 2\delta_{mn}{}^{pq}\},$$ generate $so(4)$

$$\{(\mathbb{M}^{\pm}_{mn})^{pq} = \delta_{mn}{}^{pq} \pm \tfrac{1}{2}\varepsilon_{mn}{}^{pq} = (\mathbb{M}^{\pm}_{pq})^{mn}\,,\ \tfrac{1}{2}\varepsilon_{mn}{}^{pq}\mathbb{M}^{\pm}{}_{pq} = \pm\mathbb{M}^{\pm}{}_{mn}\},$$ generate $su^{\pm}(2)$

$$\left([\mathbb{M}^{\pm}_{\sharp i}, \mathbb{M}^{\pm}_{\sharp j}] = \mp\varepsilon_{ijk}\mathbb{M}^{\pm}_{\sharp k}\,. \quad su^{\pm}(2)\right)$$

$$J^i_{mn} \equiv 2(\mathbb{M}^-_{\sharp i})^{mn}$$ 't Hooft symbols

$su^-(2)$ generators in 4 rep.

hypercomplex structure

in $so(4)$ indices

HK $\Longleftrightarrow$ $\nabla_m J^i{}_{np} = 0\,,$ $\longrightarrow$ $[\omega, J^i] = 0\,, \quad \Rightarrow \quad \omega = \omega^+\,,$

$$[\nabla_m, \nabla_n] J^i{}_{pq} = 0\,, \qquad [R, J^i] = 0\,, \quad \Rightarrow \quad R = R^+\,,$$

Bianchi $\longrightarrow$ as 2-form

$\Rightarrow$ Ricci flat.

Consider A^A $SU(2)$ connection on HK

$$A^A \longrightarrow A^{\# i} \longrightarrow A^{mn} = -\frac{1}{2}\varepsilon^{mnpq}A^{pq} \, ;$$

$$F^A = dA^A + \tfrac{1}{2}\epsilon^{ABC}A^B \wedge A^C \, , \qquad \omega^{\rm YM} = dA^A \wedge A^A + \tfrac{1}{3}\epsilon^{ABC}A^A \wedge A^B \wedge A^C \, .$$

$$F^{mn} = dA^{mn} + A^{mp} \wedge A^{pn} \, , \qquad \omega^{\rm YM} \equiv - (dA^{mn} \wedge A^{nm} + \tfrac{2}{3}A^{mn} \wedge A^{np} \wedge A^{pm}) \, ,$$

↪ relevant sign!

In this notation, the 't Hooft Ansatz reads

$$A = \mathbb{M}^-_{mp} V^p v^m \, .$$

(2 $\bar{su}(2) \subset so(4)$ indices not written)

$\mathbb{M}^-_{mp}$ → $J^i{}_{mn}$

v^m → HK

$\nabla_{(4)m} M^-{}_{pq} = 0$

$$F = + \star_{(4)} F \iff V_p = \partial_p \log P \, ; \quad \nabla^2_{(4)} P = 0$$

Then :

$$\omega^{\rm YM} = -\star_{(4)} dV^2 = -\star_{(4)} d(\partial \log P)^2 ,$$

$$F^A \wedge F^A = d\omega^{\rm YM} = -d\star_{(4)} d(\partial \log P)^2 = \nabla^2_{(4)} \left[(\partial \log P)^2\right] |v| d^4x ,$$

For this Ansatz, it just happens that

$$\Omega_{(-)+-m} = \Omega_{(-)m+-} = \mathcal{Z}_0^{-1/2}\partial_m \log \mathcal{Z}_- , \qquad \Omega_{(-)++m} = \tfrac{1}{2}\mathcal{Z}_-\mathcal{Z}_0^{-1/2}\partial_m \mathcal{Z}_+ ,$$

$$\Omega_{(-)mnp} = \mathcal{Z}_0^{-1/2}\left[\underset{\substack{\downarrow\\ \text{HK}\\ \omega^+}}{\varpi_{mnp}} + \underset{\substack{\downarrow\\ \text{'t Hooft-like}}}{(\mathbb{M}^-_{mq})_{np}\partial_q \log \mathcal{Z}_0}\right] ,$$

$$\omega^{\rm L}_{(-)} \equiv d\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_a - \tfrac{2}{3}\Omega_{(-)}{}^a{}_b \wedge \Omega_{(-)}{}^b{}_c \wedge \Omega_{(-)}{}^c{}_a$$

$$= d\Omega_{(-)mn} \wedge \Omega_{(-)nm} + \tfrac{2}{3}\Omega_{(-)mn} \wedge \Omega_{(-)np} \wedge \Omega_{(-)pm} ,$$

$$= \omega^{\rm LHK} + \omega^{\rm LSS} ;$$

$$\omega^{\rm LSS} = \star_{(4)} d(\log \mathcal{Z}_0)^2 ;$$

How about $\omega^{\rm LHK}$?

"Twisted" 't Hooft Ansatz

If the HK space is a Gibbons-Hawking space

$$d\sigma^2 = H^{-1}(d\eta + \chi)^2 + H dx^x dx^x\,, \quad \partial_{\underline{x}} H = \varepsilon_{xyz}\partial_{\underline{y}}\chi_{\underline{z}}\,.$$

Simplest frame

$$\left\{\begin{aligned} v^\sharp &= H^{-\frac{1}{2}}[d\eta + \chi_{\underline{x}} dx^x]\,, \\ v^x &= H^{\frac{1}{2}} dx^x\,, \end{aligned}\right. \longrightarrow \omega_{mn} = (\mathbb{N}^+_{mn})_{pq}\partial_q \log H v^p \,\vdots$$

where we are using the twisted su(2) generators

$$(\mathbb{N}^\pm_{mn})^{pq} \equiv \eta_{mr}\eta_{ns}(\mathbb{M}^\mp_{rs})^{pq} \qquad \Rightarrow \qquad (\mathbb{N}^\pm_{mn})^{pq} = (\mathbb{N}^\mp_{pq})^{mn}\,,$$

$\eta = \mathrm{diag}(-+++)$

Then a calculation similar to that of the Yang-Mills one

$$\omega^{\mathrm{LHK}} = \star_{(4)} d(\partial \log H)^2\,,$$

$$R^{mn}\wedge R^{nm} = d\omega^{\mathrm{LHK}} = d\star_{(4)} d(\partial\log H)^2 = -\nabla^2\left[(\partial \log H)^2\right] |v| d^4x\,.$$

Then, using the 't Hooft Ansatz for the SU(2) gauge fields and using a Gibbons-Hawking HK space, the Bianchi identity takes the form

$$\nabla^2_{(4)} \left\{ \mathcal{Z}_0 + 2\alpha' \left[(\partial \log P_1)^2 + (\partial \log P_2)^2 - (\partial \log \mathcal{Z}_0)^2 - (\partial \log H)^2 \right] \right\} |v| d^4x = \mathcal{O}(\alpha'^2),$$

and is solved to this order by

$$\mathcal{Z}_0 = \mathcal{Z}_0^{(0)} - 2\alpha' \Big[\underbrace{(\partial \log P_1)^2}_{SU(2)} + \underbrace{(\partial \log P_2)^2}_{SU(2)} - \underbrace{(\partial \log \mathcal{Z}_0^{(0)})^2}_{S5} - \underbrace{(\partial \log H)^2}_{HK-GH} \Big] + \mathcal{O}(\alpha'^2),$$

with

$$\nabla^2_{(4)} \mathcal{Z}_0^{(0)} = 0. \qquad \left(\text{Same for } P_{1,2} \text{ but } \nabla^2_{\mathbb{R}^3} H = 0 \right)$$

Before we study the e.o.m. are there other ways of getting Laplacians? KBP for Yang-Mills. →

Kronheimer – Bogomol'nyi – Protogenov

Kronheimer (1985) : $\boxed{\hat{F} = +*_{(4)}\hat{F}}$ on Gibbons - Hawking $(\hat{\ })$

instantons

$\updownarrow$

monopoles

Define $\begin{cases} \Phi \equiv -H\hat{A}_{\#} \\ A_r \equiv \hat{A}_r - \alpha_r \hat{A}_{\#} ; \quad r=1,2,3 \end{cases}$

$\Longrightarrow \boxed{D_r \Phi = \frac{1}{2}\varepsilon_{rst}F_{st}}$ on $\mathbb{R}^3$

Bogomol'nyi equation (Bogomol'nyi 1976)

In this case: $\hat{F}^A \wedge \hat{F}^A = \hat{F}^A \wedge * \hat{F}^A = \cdots = 2\,\hat{F}^A_{\# r}\hat{F}^A_{\# r}\,|v|\,d^4x =$

$= 2\,H^{-2}\left[D_r\Phi^A - \Phi^A\partial_r \log H\right]^2 |v|\,d^4x$

$= \cdots = \frac{1}{H}\partial_r\partial_r\left(\Phi^A\Phi^A/H\right)|v|\,d^4x = \nabla^2_{(4)}\left(\frac{|\Phi|^2}{H}\right)|v|\,d^4x$

$D^2\Phi^A = 0 \;;\quad \partial_r\partial_r H = 0 \;;$

Solutions to the SU(2) Bogomol'nyi Eqs.

a) Spherically symmetric (Protogenov 1977)

General form
$$\begin{cases} \breve{A}^A = -h(r)\, \varepsilon^A{}_{rs}\, x^r\, dx^s ; \\ \breve{\Phi}^A = -f(r)\, \delta^A{}_r\, x^r ; \end{cases}$$

BPS 't Hooft-Polyakov magnetic monopole

$$f = -\frac{1}{g r^2}\left[1 - \mu r \coth(\mu r + s)\right];$$
$$h = \frac{1}{g r^2}\left[\frac{\mu r}{\sinh(\mu r + s)} - 1\right];$$

Protogenov's hair parameter

Coloured monopoles → BPST instantons $\left(H = \frac{1}{2}\right)$

$$f = -\frac{1}{g r^2 (1 + \lambda^2 r)} ; \quad h = -f ;$$

f) Multicenter solutions

Ramirez's multimonopole solution 2015

$$\Phi^A = -\delta^{Ar}\frac{1}{gP}\partial_{\underline{r}}P \; ; \qquad \breve{A}^A{}_{\underline{r}} = -\varepsilon^A{}_{rs}\frac{1}{gP}\partial_{\underline{s}}P \; ;$$

$$\partial_{\underline{r}}\partial_{\underline{r}}P = 0 \quad ; \qquad P = \lambda^2 + \frac{1}{r}$$ → Coloured monopole

No more simple solutions known

Summarizing:

$$\omega^{YM} = - *_{(4)}\, d \left\{ \begin{matrix} (\partial \log P_1)^2 + (\partial \log P_2)^2 \\ |\Phi_1|^2/H + |\Phi_2|^2/H \end{matrix} \right\}$$

Gibbons-Hawking only

$$\omega^{L}_{(-)} = *_{(4)}\, d \left[(\partial \log Z_0)^2 + (\partial \log H)^2 \right] ;$$

$$T^{(4)} = 6\alpha' \nabla^2_{(4)} \left[(\partial \log P_1)^2 + (\partial \log P_2)^2 - (\partial \log Z_0^{(0)})^2 + (\partial \log H)^2 \right]$$

$$Z_0 = Z_0^{(0)} - 2\alpha' \left[(\partial \log P_1)^2 + (\partial \log P_2)^2 - (\partial \log Z_0^{(0)})^2 + (\partial \log H)^2 \right]$$

$$H = d\frac{1}{Z_-} \wedge du \wedge dv + *_{(4)} dZ_0$$

$$= dB - 2\alpha' *_{(4)} d \left[(\partial \log P_1)^2 + (\partial \log P_2)^2 - (\partial \log Z_0^{(0)})^2 + (\partial \log H)^2 \right]$$

$$\Rightarrow \quad dB = \underbrace{d\frac{1}{Z_-} \wedge du \wedge dv + *_{(4)} dZ_0^{(0)}}_{}$$

$O(1) \rightarrow$ no α' corrections to B

The Yang-Mills equations

→ Automatically solved for this Ansatz.

The Kalb-Ramond equations

→ Solved for any Z_- $\boxed{\nabla^2_{(4)} Z_- = 0}$

The dilaton equations

→ Automatically solved for this Ansatz

SUSY

The Einstein equations

→ All automatically solved for this Ansatz except for the ++ component

$$T^{(2)}_{++} = -2\alpha' R_{(-)\,+abc} R_{(-)\,+}{}^{abc} = -2\alpha' \frac{\mathcal{Z}_-}{\mathcal{Z}_0} \nabla^2_{(4)} \left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-} \right) + \mathcal{O}(\alpha'^2)\,.$$

(This component does not contribute to $T^{(0)}$ or any other invariants!)

$$\Rightarrow \quad \mathcal{Z}_+ = \mathcal{Z}_+^{(0)} - 4\,\alpha' \left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-} \right) + \mathcal{O}(\alpha'^2)\,.$$

Harmonic in HK

It can be solved but it is not clear why

Exact $\mathcal{O}(\alpha')$ solution:

$$\begin{aligned}
\mathcal{Z}_+ &= \mathcal{Z}_+^{(0)} - 4\,\alpha' \left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-^{(0)}}{\mathcal{Z}_0^{(0)}\,\mathcal{Z}_-} \right) + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_- &= \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_0 &= \mathcal{Z}_0^{(0)} \\
&\quad -2\alpha' \left[(\partial \log P_1^{(0)})^2 + (\partial \log P_2^{(0)})^2 - (\partial \log \mathcal{Z}_0^{(0)})^2 - (\partial \log H^{(0)})^2 \right] \\
&\quad +\mathcal{O}(\alpha'^2)\,, \\
H &= H^{(0)} + \mathcal{O}(\alpha'^2)\,, \\
P_{1,2} &= P_{1,2}^{(0)} + \mathcal{O}(\alpha'^2)\,.
\end{aligned}$$

EXACT $\mathcal{O}(\alpha')$ SOLUTION:

$$
\begin{aligned}
\mathcal{Z}_+ &= \mathcal{Z}_+^{(0)} - 4\,\alpha' \left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-^{(0)}}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-}\right) + \mathcal{O}(\alpha'^2)\,,\\
\mathcal{Z}_- &= \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2)\,,\\
\mathcal{Z}_0 &= \mathcal{Z}_0^{(0)}\\
&\quad -2\alpha' \left[(\partial \log P_1^{(0)})^2 + (\partial \log P_2^{(0)})^2 - (\partial \log \mathcal{Z}_0^{(0)})^2 - (\partial \log H^{(0)})^2\right]\\
&\quad +\mathcal{O}(\alpha'^2)\,,\\
H &= H^{(0)} + \mathcal{O}(\alpha'^2)\,,\\
P_{1,2} &= P_{1,2}^{(0)} + \mathcal{O}(\alpha'^2)\,.
\end{aligned}
$$

$d=5$ option: $H^{(0)} = 1$; $P_2^{(0)} = 1$; harmonic functions in $\mathbb{R}^4$

EXACT $\mathcal{O}(\alpha')$ SOLUTION:

$$
\begin{aligned}
\mathcal{Z}_+ &= \mathcal{Z}_+^{(0)} + 4\,\alpha' \left(\frac{\partial_n \mathcal{Z}_+^{(0)} \partial_n \mathcal{Z}_-^{(0)}}{\mathcal{Z}_0^{(0)} \mathcal{Z}_-} \right) + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_- &= \mathcal{Z}_-^{(0)} + \mathcal{O}(\alpha'^2)\,, \\
\mathcal{Z}_0 &= \mathcal{Z}_0^{(0)} \\
&\quad -2\alpha' \left[(\partial \log P_1^{(0)})^2 + (\partial \log P_2^{(0)})^2 + (\partial \log \mathcal{Z}_0^{(0)})^2 - (\partial \log H^{(0)})^2 \right] \\
&\quad +\mathcal{O}(\alpha'^2)\,, \\
H &= H^{(0)} + \mathcal{O}(\alpha'^2)\,, \\
P_{1,2} &= P_{1,2}^{(0)} + \mathcal{O}(\alpha'^2)\,.
\end{aligned}
$$

"SUGRA option": only α' correction in $\mathcal{Z}^{(0)}$ due to $P_{1,2}$

OBSERVE:

1.- The solution is exact to $\mathcal{O}(\alpha')$ with no use of the anomaly-cancellation mechanism (now $\mathcal{P}_1^{(0)} = \mathcal{Z}_0^{(0)}$; $\mathcal{P}_2^{(0)} = \mathcal{H}^{(0)}$)

2.- For BHs the corrected solution covers near-horizon and asymptotic regions.

3.- $\mathcal{Z}_+^{(0)} = \mathcal{Z}_-^{(0)} = \mathcal{H}^{(0)} = \mathcal{P}_2^{(0)} = 1$; $\mathcal{Z}_0^{(0)} = \mathcal{P}_1^{(0)}$ "symmetric 5-brane" (Callan, Harvey, Strominger (1991))

4.- Singular gauge → spurious singularities ⇔ shift in harmonic functions ⇔ change in the number of branes ⇒ important for entropy calculations
(Removable singularity theorem Uhlenbeck (1982))

α'-corrected T-duality

These solutions have 2 non-trivial isometries:

1.- u: the direction of propagation of the wave (string)

2.- z: the triholomorphic isometry of the Gibbons-Hawking space

At zeroth order in α' T-duality along

u: interchanges waves and strings (momentum and winding) $\qquad \mathcal{Z}_+^{(0)} \rightleftarrows \mathcal{Z}_-^{(0)}$

z: interchanges S5-branes and KK monopoles

$$\mathcal{Z}_0^{(0)} \rightleftarrows H^{(0)}$$

α'-corrected

Now, using the α'- corrected Buscher T- duality rules (Bergshoeff, Janssen, O. (1995)) (never before used due to lack of α'- corrected solns)

$$
\begin{aligned}
g'_{\mu\nu} &= g_{\mu\nu} + \left[g_{\underline{xx}} G_{\underline{x}\mu} G_{\underline{x}\nu} - 2 G_{\underline{xx}} G_{\underline{x}(\mu} g_{\nu)\underline{x}} \right] / G^2_{\underline{xx}}\,, \qquad (\mu, \nu \neq x) \\
B'_{\mu\nu} &= B_{\mu\nu} - G_{\underline{x}[\mu} G_{\nu]\underline{x}} / G_{\underline{xx}}\,, \\
g'_{\underline{x}\mu} &= -g_{\underline{x}\mu} / G_{\underline{xx}} + g_{\underline{xx}} G_{\underline{x}\mu} / G^2_{\underline{xx}}\,, \qquad B'_{\underline{x}\mu} = -B_{\underline{x}\mu} / G_{\underline{xx}} - G_{\underline{x}\mu} / G_{\underline{xx}}\,, \\
g'_{\underline{xx}} &= g_{\underline{xx}} / G^2_{\underline{xx}}\,, \qquad e^{-2\phi'} = e^{-2\phi} |G_{\underline{xx}}|\,, \\
A'^{A}_{\underline{x}} &= -A^{A}_{\underline{x}} / G_{\underline{xx}}\,, \qquad A'^{A}_{\mu} = A^{A}_{\mu} - A^{A}_{\underline{x}} G_{\underline{x}\mu} / G_{\underline{xx}}\,,
\end{aligned}
$$

where

$$
G_{\mu\nu} \equiv g_{\mu\nu} - B_{\mu\nu} - 2\alpha' \left\{ A^{A}_{\mu} A^{A}{}_{\nu} + \Omega_{(-)\,\mu}{}^{a}{}_{b} \Omega_{(-)\,\nu}{}^{b}{}_{a} \right\} .
$$

$$
\boxed{Z^{(0)}_{0} \rightleftarrows H^{(0)} \qquad Z^{(0)}_{+} \rightleftarrows Z^{(0)}_{-}}
$$

Highly non-trivial test passed!

CONCLUSIONS:

1.- It is possible to compute α' corrections to physically interesting backgrounds (complete BHs)

2.- It is possible to apply Wald's entropy formula directly in $d=10$ avoiding unwarranted assumptions, incomplete actions etc.

3.- non-Abelian solutions are an essential ingredient (but we do not know them w/o SUSY)

4.- $d=4$ BHs include a KK monopole (in progress)

Thanks!