On gauged maximal d=8 supergravities

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Geometry, Duality and Strings
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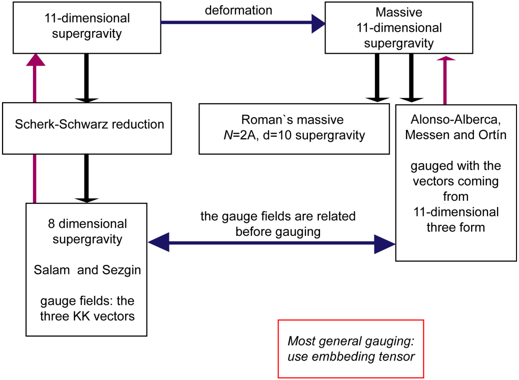
Overview

1. General Overview.
2. 8-dimensional field theories.
3. Non-Abelian and massive deformations: the tensor hierarchy.
   - The embedding tensor formalism.
   - Gauging the global symmetries using the embedding tensor.
4. The 8-dimensional supergravities with SO(3) gaugings
   - Supergravity in 11-dimensions.
   - Supergravity in 8-dimensions

Overview

- The gauging of maximal 8-dimensional supergravity: Salam and Sezgin obtained this theory with an $\mathrm{SO}(3) \subset \mathrm{SL}(3,\mathbb{R})$ gauging (Other 3-dimensional groups can be obtained by the same procedure)
- The gauge fields of this theory are the three Kaluza-Klein vectors. However, the theory has another $\mathrm{SL}(3,\mathbb{R})$ triplet of vectors that can be used as gauge fields: the vectors that come from the 11-dimensional 3-form.
- It is not known how to obtain this theory from the conventional 11-dimensional supergravity.
- It is believed that it should be possible to obtain this second SO(3)-gauged theory by an SL(2) rotation of the Salam-Sezgin one.
- From the 8-dimensional point of view, these two theories should be equivalent.
- It is hard to say whether these two theories are equivalent from the 11-dimensional point of view.

From 11-dimensional to 8-dimensional supergravity



Salam and Sezgin, (1985),
Alonso-Alberca,Messen, Ortín, (2001),
Alonso-Alberca, Bergshoeff, Gran, Linares, Ortín, Roest, (2003),
Puigdomènech, de Roo, (2008).

- We want to show that the gauged theory obtained from the compactification of *massive 11-dimensional supergravity* (The AAMO theory. Alonso-Alberca,Messen, Ortín (2000)) is indeed one of the SO(3)-gauged maximal supergravities that can be obtained using the embedding tensor method.

Goals

- We want to show that the gauged theory obtained from the compactification of *massive 11-dimensional supergravity* (The AAMO theory. Alonso-Alberca,Messen, Ortín (2000)) is indeed one of the SO(3)-gauged maximal supergravities that can be obtained using the embedding tensor method.
- We want to show that, from the 8-dimensional point of view, it is related to the Salam-Sezgin one (The SS theory, Salam and Sezgin, (1985)) by an $SL(2,\mathbb{R})$ transformation. We will achieve both goals by constructing a 1-parameter family of SL(2)-related SO(3)-gauged supergravities that interpolates between the SS and AAMO theories.

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- the metric $g_{\mu\nu}$,
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- 1-form fields $A^I = A^I{}_\mu dx^\mu$,
- 2-form fields $B_m = \frac{1}{2} B_{m\mu\nu} dx^\mu \wedge dx^\nu$ and
- 3-form fields $C^a = \frac{1}{3!} C^a{}_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$.

The way

What is the simplest theory one can construct with these fields?.

The simplest field strengths are the exterior derivatives:

$$F^I \equiv dA^I\,, \qquad H_m \equiv dB_m\,, \qquad G^a \equiv dC^a\,.$$

Ungauged $d = 8$ theories

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The most general gauge-invariant action is:

$$\begin{aligned} S \;=\; & \int \Big\{ \star R + \tfrac{1}{2} \mathscr{G}_{xy} d\phi^x \wedge \star d\phi^y - \tfrac{1}{2} \mathscr{M}_{IJ} F^I \wedge \star F^J + \tfrac{1}{2} \mathscr{M}^{mn} H_m \wedge \star H_n \\ & - \tfrac{1}{2} \Im\mathfrak{m} \mathscr{N}_{ab} G^a \wedge \star G^b - \tfrac{1}{2} \Re\mathfrak{e} \mathscr{N}_{ab} G^a \wedge G^b \Big\} , \end{aligned}$$

where the kinetic matrices $\mathscr{G}_{xy}, \mathscr{M}_{IJ}, \mathscr{M}^{mn}, \Im\mathfrak{m}\mathscr{N}_{ab}$ as well as the matrix $\Re\mathfrak{e}\mathscr{N}_{ab}$ are scalar-dependent.

Gauging the global symmetries of the theory

The most general possibilities can be explored using the embedding tensor formalism

Cordaro, Fré,Gualtieri, Termonia and Trigiante (1998). Nicolai and Samtleben (2001).De Wit and Samtleben (2001). De Wit, Samteblen and Trigiante (2003)

Bonus

The tensor hierarchy

De Wit and Samtleben (2005). De Wit, Nicolai and Samteblen (2008). Bergshoeff, Hartong, Hohm,Hubscher and Ortín (2009). De Wit and Zalk (2009)

- It turns out that all couplings that deform an ungauged supergravity into a gauged one, can be given in terms of the embedding tensor.
- Gauged supergravities are classified by the embedding tensor, subject to a number of algebraic or group-theoretical constraints.
- The embedding tensor ${\Theta_M}^\alpha$ pairs the generators t_α of the group G with the vector fields ${A_\mu}^M$ used for the gauging.

$$ {A_\mu}^M {\Theta_M}^\alpha , $$

- Which combinations of group generators

$$ X_M = {\Theta_M}^\alpha \, t_\alpha $$

can be seen as the generators of the gauge group?.

The gauging of the global symmetry

We promote the global parameters α^A to local ones $\alpha^A(x)$ and we make the identifications:

The embedding tensor and the global parameters

$$\alpha^A \equiv \sigma^I \vartheta_I{}^A .$$

The embedding tensor and the $1-forms$

The gauge fields for these symmetries are given by

$$A^A \equiv A^I \vartheta_I{}^A .$$

The derivatives transform covariantly under gauge transformations $\delta_\sigma = \sigma^I \vartheta_I{}^A \delta_A$ provided that the embedding tensor is gauge-invariant

$$\delta_\sigma \vartheta_I{}^A = 0\,,$$

and provided that the 1-forms transform as

$$\delta_\sigma A^I = \mathscr{D}\sigma^I + \Delta A^I\,, \quad \text{where} \quad \left\{ \begin{array}{rcl} \Delta A^I \vartheta_I{}^A & = & 0\,, \\ \mathscr{D}\sigma^I & = & d\sigma^I - A^J X_J{}^I{}_K \sigma^K\,, \end{array} \right.$$

The gauge invariance of the embedding tensor leads to the so-called *quadratic constraint*

$$\vartheta_J{}^B \left[T_B{}^K{}_I \vartheta_K{}^A - f_{BC}{}^A \vartheta_I{}^C \right] = 0\,.$$

To determine ΔA^I we have to construct the gauge-covariant 2-form field strengths F^I.

$$
\begin{aligned}
F^I &= dA^I - \tfrac{1}{2} X_J{}^I{}_K A^{JK} + Z^{I\,m} B_m \,, \\
H_m &= \mathscr{D} B_m - d_{mIJ} dA^I A^J + \tfrac{1}{3} X_J{}^M{}_K A^{IJK} + Z_{im} C^i \,, \\
G^i &= \mathscr{D} C^i + d^i{}_I{}^n \left[F^I B_n - \tfrac{1}{2} Z^{I\,p} B_n B_p + \tfrac{1}{3} d_{nJK} dA^J A^{KI} + \tfrac{1}{12} d_{mMJ} X_K{}^M{}_L A^{IJKL} \right] - Z_{im} \tilde{H}^m \\
\tilde{H}^m &= \mathscr{D} \tilde{B}^m - d_{iI}{}^m F^I C^i + d^{mnp} B_n \left(H_p + \Delta H_p - 2 Z_{ip} C^i \right) \\
&+ d^m{}_{IJK} dA^I dA^J A^K \\
&+ \left(\tfrac{1}{12} d_{iJ}{}^m d^j{}_K{}^n d_{nIL} - \tfrac{3}{4} d^m{}_{IJM} X_K{}^M{}_L \right) dA^I A^{JKL} \\
&+ \left(\tfrac{3}{20} d^m{}_{NPM} X_I{}^N{}_J - \tfrac{1}{60} d_{iM}{}^m d^{i}_{I}{}^n d_{nPJ} \right) X_K{}^P{}_L A^{IJKLM} \\
&+ Z^{Im} \tilde{A}_I \,,
\end{aligned}
$$

- See OLA, Ortín (2016), The Tensor Hierarchies of 8-dimensional Theories
- The simplest mechanical procedure to obtain them from 11-dimensional supergravity would be to perform the standard Scherk-Schwarz reduction that gives an 8-dimensional SO(3)-gauged maximal supergravity in which the 3 Kaluza-Klein vectors play the role of gauge fields and then perform the $SL(2,\mathbb{R})$ duality transformations. Salam and Sezgin (1985).
- The 8-dimensional $SL(2,\mathbb{R})$ duality transformations have no clear 11-dimensional counterpart, though, and the SO(3)-gauged maximal supergravities in which the triplet of gauge fields are not the first component of the $SL(2,\mathbb{R})$ doublet obtained in this way cannot be uplifted to 11 dimensions.

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- Maybe find more general non-covariant deformations of 11-dimensional supergravity leading to the rest of SO(3)-gauged 8-dimensional maximal supergravities.

Cremmer and Julia (1978)

The bosonic fields of $N=1, d=11$ supergravity are:

$$\left\{ \hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}}, \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} \right\}.$$

The field strength of the 3-form is

$$\hat{\hat{G}} = 4\partial\,\hat{\hat{C}},$$

and is obviously invariant under the gauge transformations

$$\delta\,\hat{\hat{C}} = 3\partial\,\hat{\hat{\chi}},$$

where $\hat{\hat{\chi}}$ is a 2-form.

The action for these bosonic fields is

$$\hat{\hat{S}} = \int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|}\left[\hat{\hat{R}} - \tfrac{1}{2\cdot 4!}\,\hat{\hat{G}}^2 - \tfrac{1}{6^4}\,\frac{1}{\sqrt{|\hat{\hat{g}}|}}\,\hat{\hat{\varepsilon}}\partial\,\hat{\hat{C}}\partial\,\hat{\hat{C}}\,\hat{\hat{C}}\right].$$

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$\mathcal{N} = 2,\ d = 8$ supergravity

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- The scalars of the theory parametrize the coset spaces $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ and $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$ and the U-duality group of the theory is $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(3,\mathbb{R})$ and its fields are either invariant or transform in the fundamental representations of both groups.

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- We use the indices $i, j, k = 1, 2$ for $\mathrm{SL}(2,\mathbb{R})$ doublets and $m, n, p = 1, 2, 3$ for $\mathrm{SL}(3,\mathbb{R})$ triplets.

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- The bosonic fields are

$$g_{\mu\nu}, C, B_m, A^{i\,m}, a, \varphi, \mathcal{M}_{mn},$$

where C is a 3-form, B_m a triplet of 2-forms, $A^{i\,m}$, a doublet of triplets of 1-forms (six in total), a and φ are the axion and dilaton fields which can be combined into the axidilaton field

$$\tau \equiv a + ie^{-\varphi}\,,$$

or into the $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ symmetric matrix

The bosonic action is:

$$
\begin{aligned}
S \quad = \quad & \int \Big\{ -\star R + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{M}\mathscr{M}^{-1}\wedge\star d\mathscr{M}\mathscr{M}^{-1}\right) + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{W}\mathscr{W}^{-1}\wedge\star d\mathscr{W}\mathscr{W}^{-1}\right) \\
& + \tfrac{1}{2}\mathscr{W}_{ij}\mathscr{M}_{mn}F^{im}\wedge\star F^{jn} + \tfrac{1}{2}\mathscr{M}^{mn}H_m\wedge\star H_n + \tfrac{1}{2}e^{-\varphi}G^1\wedge\star G^1 - \tfrac{1}{2}aG^1G^1 \\
& + \tfrac{1}{3}G^1\left[H_mA^{2m} - B_mF^{2m} + \tfrac{1}{2}\varepsilon_{mnp}F^{2m}A^{1n}A^{2p}\right] \\
& + \tfrac{1}{3}H_mF^{2m}\left[C^1 + \tfrac{1}{6}\varepsilon_{mnp}A^{1m}A^{1n}A^{2p}\right] \\
& + \tfrac{1}{3!}\varepsilon^{mnp}H_mH_n\left(B_p - \tfrac{1}{2}\varepsilon_{pqr}A^{1q}A^{2r}\right)\Big\}\,.
\end{aligned}
$$

and the field strengths

$$
\begin{aligned}
F^{im} \quad &= \quad dA^{im}\,, \\
H_m \quad &= \quad dB_m + \tfrac{1}{2}\varepsilon_{ij}\varepsilon_{mnp}F^{in}A^{jp}\,, \\
G^1 \quad &= \quad dC^1 + F^{1m}B_m + \tfrac{1}{6}\varepsilon_{ij}\varepsilon_{mnp}A^{1m}F^{in}A^{jp}\,.
\end{aligned}
$$

- The only structure constants that we need to know explicitly are those of the $SO(3)$ subgroup

$$[T_m, T_n] = f_{mn}{}^{p} T_p = -\varepsilon_{mn}{}^{p} T_p \,,$$

- The indices $I, J, \dots$ must be replaced by composite indices im, jn etc. where $i, j, \dots = 1, 2$ and $m, n, \dots = 1, 2, 3$ are indices in the fundamental representations of $\mathrm{SL}(2, \mathbb{R})$ and $\mathrm{SL}(3, \mathbb{R})$, respectively.

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- In the electric 3-forms the lower index 1 is equivalent to an upper index 2: $C_1 = \varepsilon_{12} C^2 = C^2$ and, therefore $(C^i) = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1 \\ C^2 \end{pmatrix}$. On the other hand, $C_i \equiv \varepsilon_{ij} C^j$.

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- Comparing the field strengths of this theory with those of the generic ungauged theory we get that the d-tensors can be constructed entirely in terms of the U-duality invariant tensors $\delta^i{}_j, \varepsilon_{ij}, \delta^m{}_n, \varepsilon_{mnp}$:

$$d_{mIJ} \quad \rightarrow \quad d_{m\,in\,jp} = -\tfrac{1}{2}\varepsilon_{mnp}\varepsilon_{ij} \,,$$

$$d^i{}_I{}^m \quad \rightarrow \quad d^i{}_{jn}{}^m = \delta^i{}_j \delta^m{}_n \,.$$

  Moreover

$$d^i{}_{(I|}{}^m d_{i|J)}{}^n = -2 d^{mnp} d_{pIJ} \,, \quad \Rightarrow \quad d^{mnp} = +\tfrac{1}{2}\varepsilon^{mnp} \,.$$

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- From the 8-dimensional supergravity point of view, one could use any other $SL(2,\mathbb{R})$ transformed of the A^{1m} triplet as gauge fields.The corresponding embedding tensor has the form

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- The SO(3) gauge fields are combinations of the two triplets of vector fields

$$\vartheta_{in}{}^{m}A^{in} = v_iA^{im}\,,$$

  and include, as limiting cases, the SS and the AAMO theories.

We have found a set of deformation parameters which are a solution for all constraints

$$\vartheta_{im}{}^{n} = v_i \delta_m{}^{n}\,, \quad Z^{imn} = v^i \delta^{mn}\,, \quad Z_{im} = 0$$

- The kinetic terms in the action

$$S^{(0)} = \int \{ -\star R + \tfrac{1}{4}\mathrm{Tr}\left(\mathscr{D}\mathscr{M}\mathscr{M}^{-1} \wedge \star \mathscr{D}\mathscr{M}\mathscr{M}^{-1}\right) + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{W}\mathscr{W}^{-1} \wedge \star d\mathscr{W}\mathscr{W}^{-1}\right)$$
$$+\tfrac{1}{2}\mathscr{W}_{ij}\mathscr{M}_{mn}F^{im} \wedge \star F^{jn} + \tfrac{1}{2}\mathscr{M}^{mn}H_m \wedge \star H_n + \tfrac{1}{2}e^{-\varphi}G \wedge \star G - \tfrac{1}{2}aG \wedge G - V \}$$

- We add

$$S^{(1)} = \int \{ -dC^1 \Delta G^2 - \tfrac{1}{2}\Delta G^1 \Delta G^2 - \tfrac{1}{12}\varepsilon^{mnp}B_m \mathscr{D}B_n \mathscr{D}B_p + \tfrac{1}{4}\varepsilon^{mnp}B_m H_n H_p$$
$$-\tfrac{1}{24}\varepsilon_{ij}A^{im}A^{jn}\Delta H_m \mathscr{D}B_n \} ,$$

- Another correction

$$S^{(2)} = \int \{ -\tfrac{1}{12}v_i(F^{im} - v^i B_m)B_m B_n B_n + \tfrac{1}{4}\varepsilon^{mnp}B_m \Delta H_n \Delta H_p - \tfrac{1}{2}\varepsilon_{ij}\Box G^i \Box F^{jm} B_m$$
$$+\tfrac{1}{24}\varepsilon_{ij}A^{im}A^{in}\mathscr{D}B_m \Delta H_n \} .$$

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp},$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

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- They can be written in terms of the *dressed structure constants* of the gauge group.

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp},$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

- No general rules available in the literature to construct the fermion shifts of any gauged supergravity. Castellani, Ceresole, Ferrara, D´Auria, Fré and Maina,(1986), (1985). Bandos and Ortín (2016).
- They can be written in terms of the *dressed structure constants* of the gauge group.

The scalar potential

$$V = -\tfrac{1}{4} S_{IJ} S^{*\,IJ} + \tfrac{1}{8}\delta^{\mathbf{mn}} N_{\mathbf{m}}{}^I{}_J N^*_{\mathbf{n}\,I}{}^J = -\tfrac{1}{2}\mathscr{W}^{ij} v_i v_j \left[\mathrm{Tr}(\mathscr{M})^2 - 2\mathrm{Tr}(\mathscr{M}^2)\right],$$

where $\mathscr{W}^{ij}$ is the $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ symmetric matrix, and where we have used

$$\mathscr{M}_{mn} \equiv L_m{}^{\mathbf{p}} L_n{}^{\mathbf{p}}, \text{ so that } T = \mathrm{Tr}(\mathscr{M}), \text{ and } T^{\mathbf{mn}} T^{\mathbf{mn}} = \mathrm{Tr}(\mathscr{M}^2).$$


$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$
$\sum_{x=0}^{\infty} P(x) = 1$
$\sigma_{xy} = \frac{\sum_{i=1}^{n}(x_i - \langle x \rangle)(y_i - \langle y \rangle)}{n-1}$
$\Gamma(y) = \int_0^\infty e^{-x} x^{y-1} dx$
$\Gamma^{(n)}(y) = \int_0^\infty e^{-x} x^{y-1} \ln^n(x) dx$
$\int_{-\infty}^{\infty} F(x)\delta(x)dx = F(0)$
$\frac{p(x)}{((x-b)^2+c^2)^n} = \sum_{k=1}^{n} \frac{A_k x + B}{((x-b)^2+c^2)^k}$
$\delta(x) = \lim_{\varepsilon \to 0} P(\varepsilon, x)$