Double-Higgs boson production at NLO in the high-energy limit

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JHEP 1803 (2018) 048 [arXiv:1801.09696]
&work in progress

Karlsruhe Institute of Technology

Institute for Theoretical Particle Physics

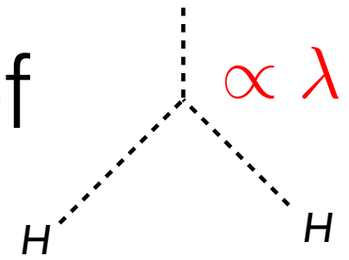
One of the subjects for HL-LHC

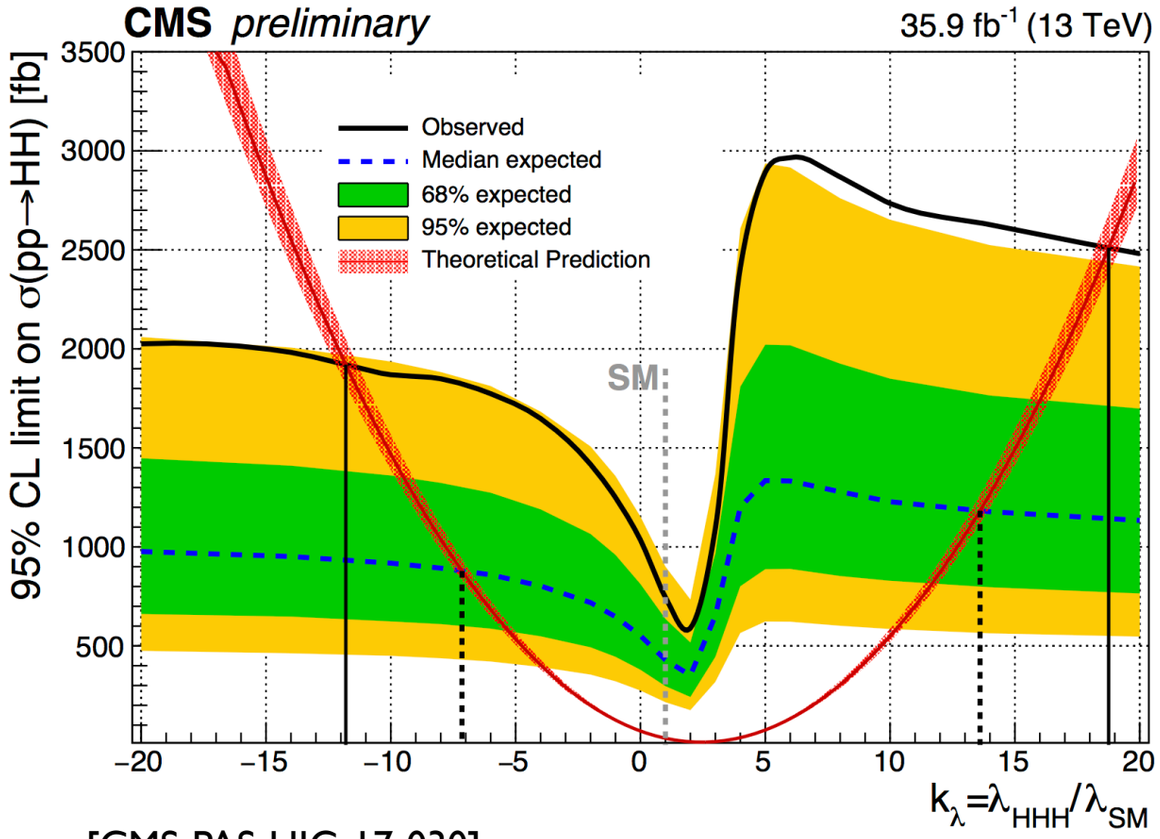
the talk by Mia Liu yesterday

Fermilab M.LIU

Di-higgs

Interpretation in terms of





[CMS-PAS-HIG-17-030]

- **CMS** Di-Higgs combination used to constraint the trilinear coupling modifier $k_\lambda = \lambda/\lambda_{SM}$

$$-11.8 < k_\lambda < 18.8$$

$(-7.1 < k_\lambda < 13.6$ exp.)

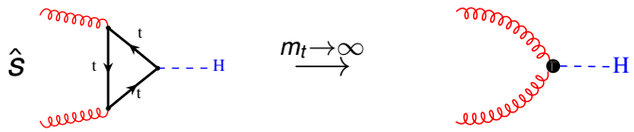
@ 95% CL

$gg \to hh$: previous works

exact analytic@LO
[Eboli, Marques, Novaes, Natale, '87, Glover, van der Bij '88]

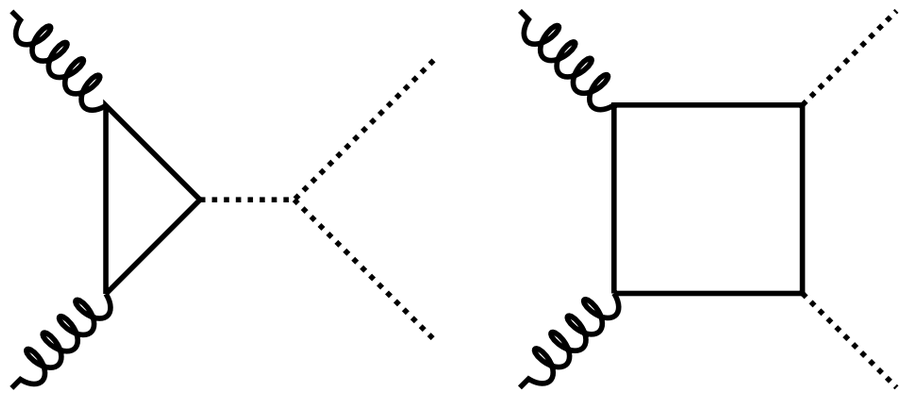
Born-improved HEFT@NLO
[Dawson, Dittmaier Spira, '98]

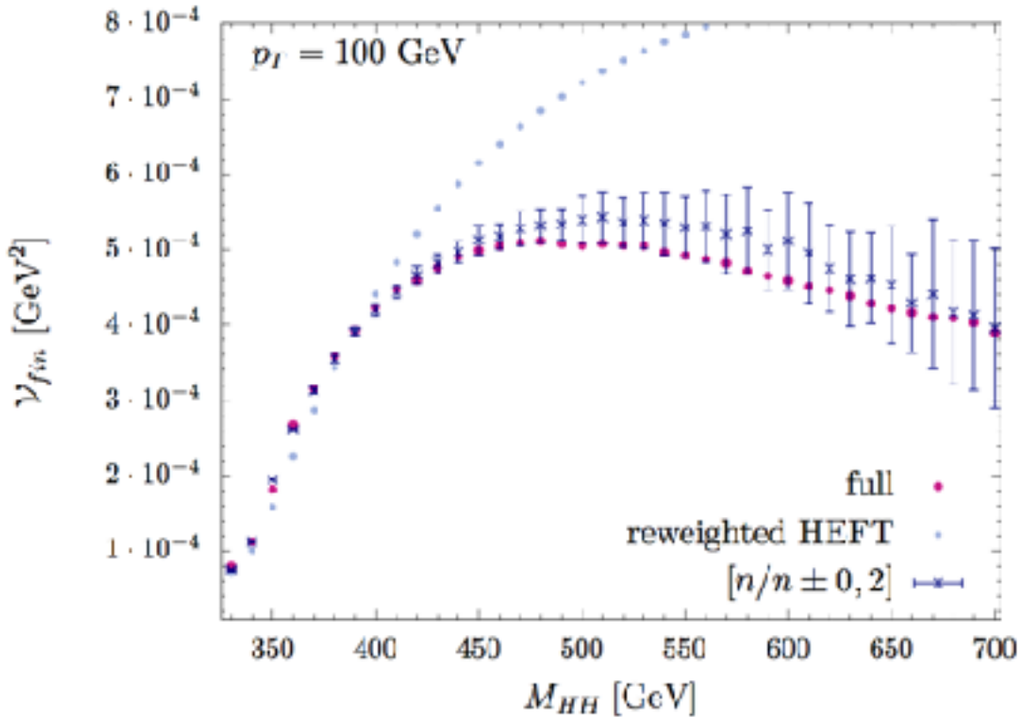
FT$_{\text{approx}}$, FT'$_{\text{approx}}$
[Maltoni, Vryonidou, Zaro, '14]



HEFT@NNLO with 1/mt corr.
[Grigo, Hoff, Melnikov, Steinhauser, '13,
Grigo, Melnikov, Steinhauser, '14,
Grigo, Hoff, Steinhauser, '15, Degrassi, Giardino, Gröber, '16]

exact numerical@NLO (14TeV, 100TeV)
[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zicke, '16]





Padé approximation using the large top-mass and the threshold expansion@NLO
[Gröber, Maier Rauh, '17]

Expansion in a small Higgs transverse momentum $p_T^2 + m_h^2 \leq \frac{s}{4}$
[Bonciani, Degrassi, Giardino, Gröber, '18]

We supplement these works by providing the high energy expansion@NLO.

Complementarity of our approximation and others

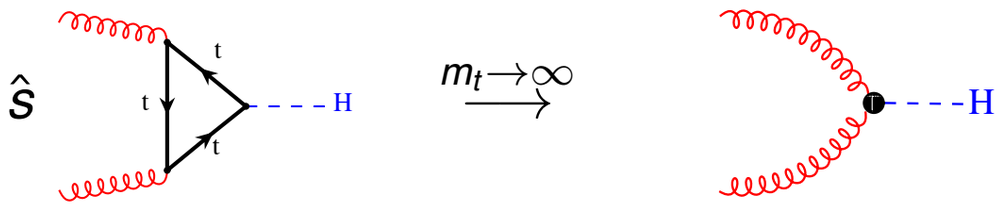
large pT

our high energy approximation

$$\frac{x}{X} \ll 1$$

$$x = \{m_H^2, m_t^2\}$$

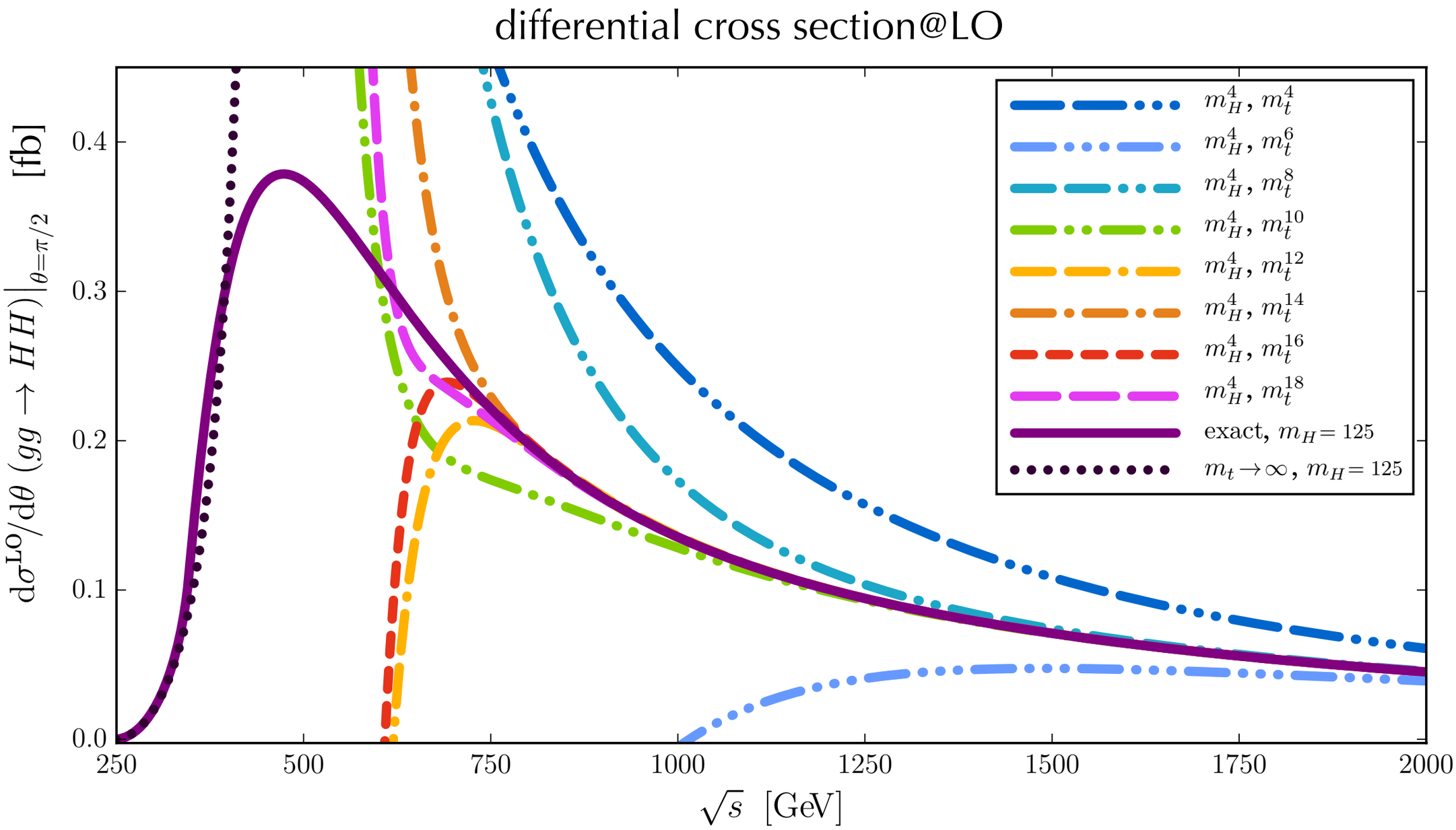
$$X = \{S, |T|, |U|\}$$



HEFT, large mt, Higgs pT expansion

large center-of-mass energy

$gg \to hh$: our aim is to obtain the high energy expansion@NLO.



Setup to calculate the two-loop amplitude

qgraf **[Nogueira, '93]** : generate amplitudes

q2e/exp **[Harlander, Seidensticker, Steinhauser, '98, Seidensticker, '99]** : rewrite output to FORM notation

TFORM 4.2 **[Rujil, Ueda, Vermaseren '17]** : projection to the form factors

LiteRed **[Lee, '13]** : mH expansion

FIRE **[Smirnov, '14]** (with LiteRed rules **[Lee, '13]**) : reduction to master integrals

tsort **[Smirnov, Pak]** : minimization of master integrals

Up to this point, we retain the full top mass dependence.

Feynman Diagrams: 8 (LO) + 118 (NLO)

Scalar Integrals: 26K (+120K mH expansion)

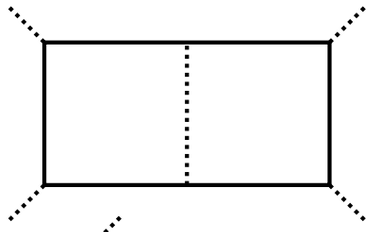
Master Integrals: 10 (LO) + 221 (NLO)

Minimal Master Integrals: 10(LO) + 161 (NLO)

Master integrals at 2 loop

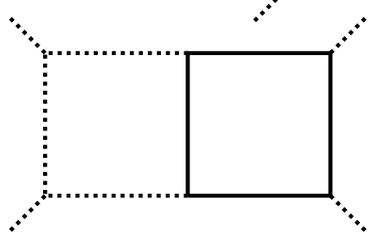
161 = 131 (planar&crossing) + 30 (nonplanar&crossing)

= 28 +(20+15+19+11+9)



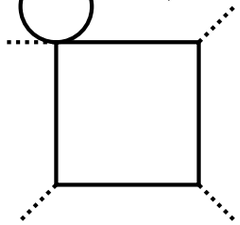
+crossing

+15 +(8+3)



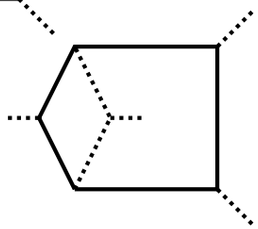
+crossing

+1 +(2)



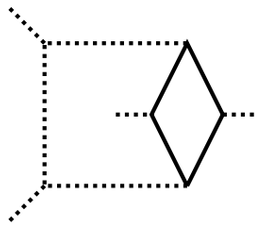
+crossing

+11 +(6+6)



+crossing

+7



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[arXiv:1801.09696]

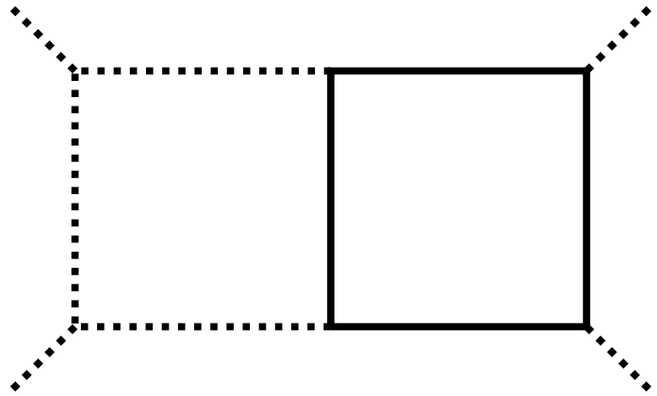
coming soon

The exact analytic result is not available yet.

planar integrals:

We still have unsolved 2-fold integrals in the half-massive double box sector.

[Becchetti, Bonciani '17]



The 2-fold integrals converge only in an Euclidian kinematical configuration, therefore cannot be used in the physical calculation so far.

non-planar integrals:

- None of the non-planar 7line integrals are solved analytically.
- The 6line three point functions available for the physical kinematics. **[Manteuffel, Tancredi, 17]**

Method of region (asymptotic expansion of Feynman integral)

[Smirnov `90, Beneke, Smirnov '97, Smirnov `02, Jantzen `11]

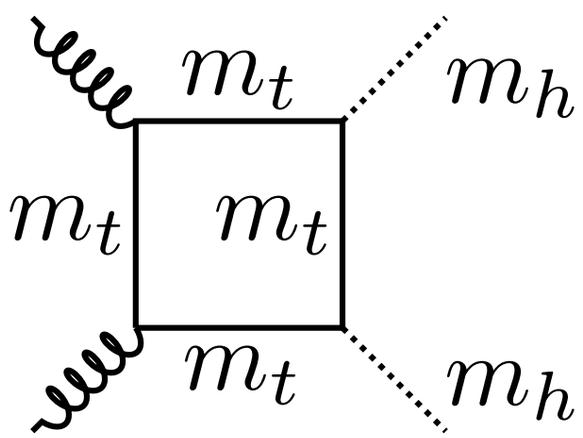
is useful when (i) the integral is hard to solve due to multi-scale complexity
(ii) certain hierarchy in dimensionful parameters makes sense

In our case, we assume $m_h^2 < m_t^2 \ll |S| \sim |T| \sim |U|$

In the practical calculation,

(1) expand in m_h

(2) expand in m_t



We use "asy.m" to perform the asymptotic expansion.
[Pak, Smirnov '10, Jantzen, Smirnov, Smirnov '12]

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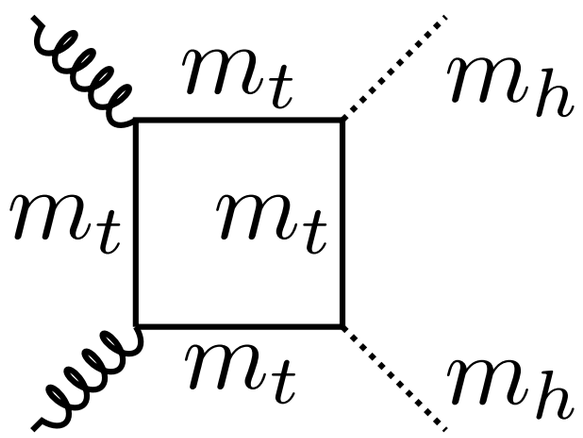
a subsets of the mt-expansion

covers the essential ingredients of the method of region



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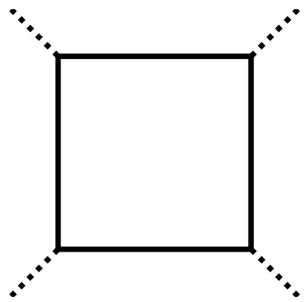
Expansion in m_t



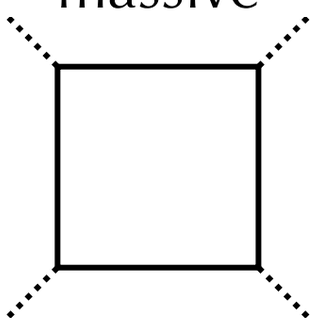
$$= \int Dk \frac{1}{k^2 - m_t^2} \frac{1}{(k+p_1)^2 - m_t^2} \frac{1}{(k+p_1+p_2)^2 - m_t^2} \frac{1}{(k+p_3)^2 - m_t^2}$$

$$= \sum_{n=0}^{\infty} (m_t^2)^n f_n(S, T, \log m_t)$$

Naive expansion of the integrand like

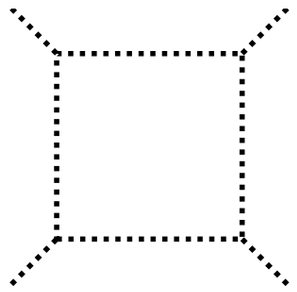
$$\frac{1}{k^2 - m_t^2} = \frac{1}{k^2} + \frac{m_t^2}{(k^2)^2} + \cdots$$ gives wrong result.

massive



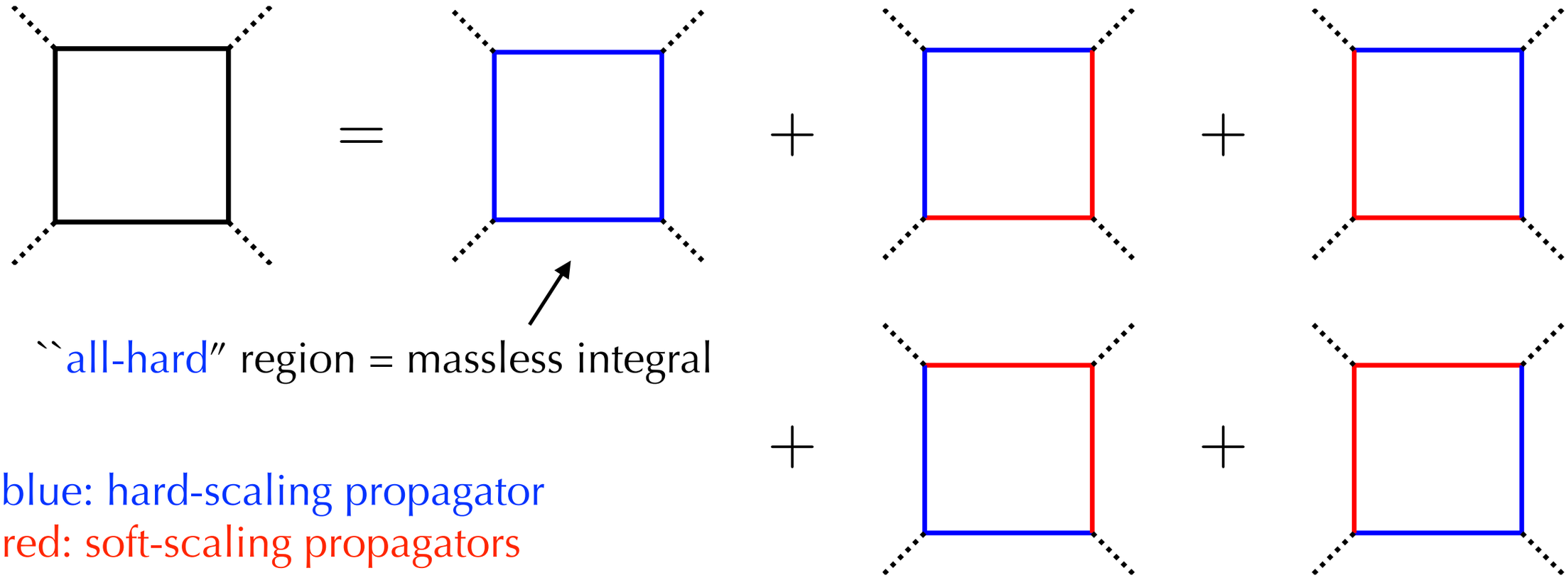
is finite.

massless



$$= \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right) + \mathcal{O}(\epsilon)$$

Method of region [Beneke, Smirnov '97, Smirnov `02, Jantzen `11]



``all-hard'' region = massless integral

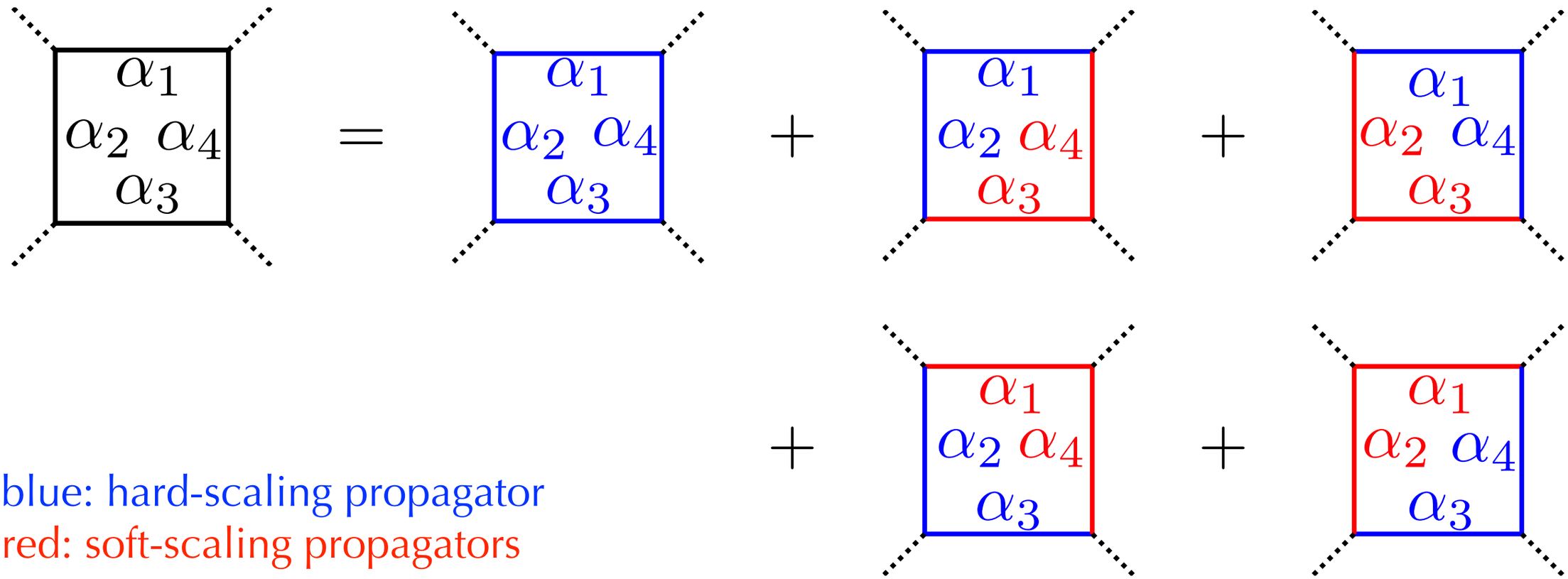
blue: hard-scaling propagator
red: soft-scaling propagators

the scaling of propagators in terms of alpha-parameter representation

$$\int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n\right) \alpha_{1234}^{-d/2}\, \mathrm{e}^{-m^2\alpha_{1234}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{1234}}$$

$$\alpha_{1234} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

Method of region [Beneke, Smirnov '97, Smirnov `02, Jantzen `11]



blue: hard-scaling propagator
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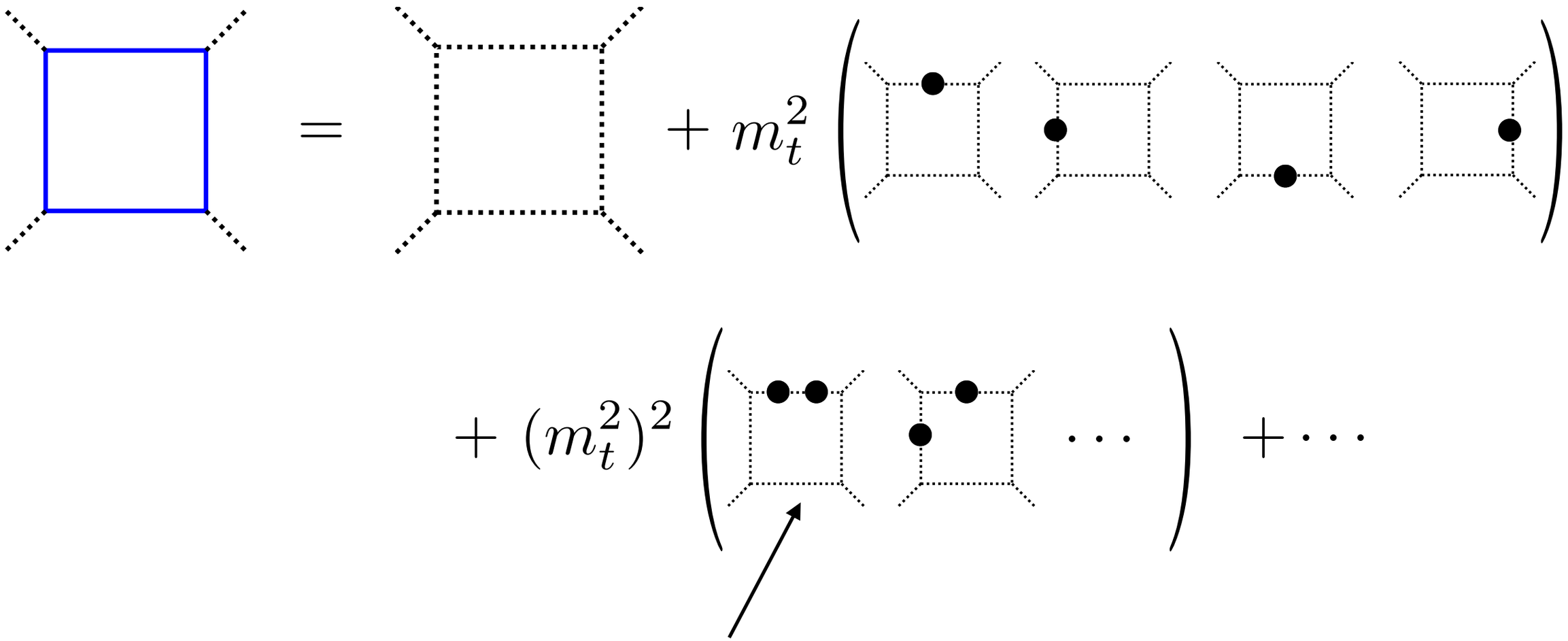
$$\alpha_{1234} = \alpha_1+\alpha_2+\alpha_3+\alpha_4$$

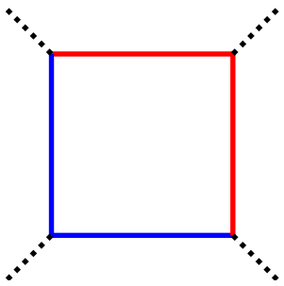
Method of region: ``all-hard'' region

In our case, the expansion in this region corresponds to the **naive Taylor expansion**.
The right hand side consists of massless diagrams with dots.



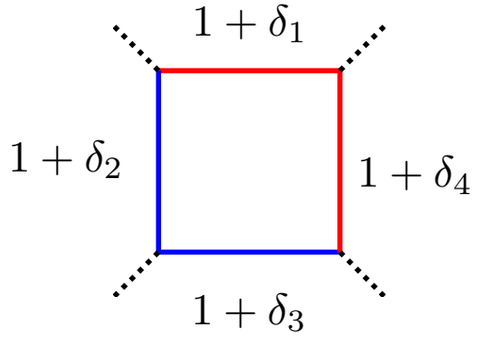
We can apply the integration by parts (IBP) reduction.

Method of region: soft-collinear regions



$$= \int_0^\infty \left(\prod_{n=1}^{4} d\alpha_n\right) \alpha_{12}^{-d/2}\, \mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}}$$
$$- \alpha_{12}^{-d/2-2}(\alpha_3+\alpha_4)((d/2)\alpha_{12}+m^2(\alpha_{12})^2-s\alpha_1\alpha_3-t\alpha_2\alpha_4)$$
$$\times \mathrm{e}^{-m^2\alpha_{12}-(s\alpha_1\alpha_3+t\alpha_2\alpha_4)/\alpha_{12}}$$
$$+ \cdots$$

Usual momentum representation is not always possible…

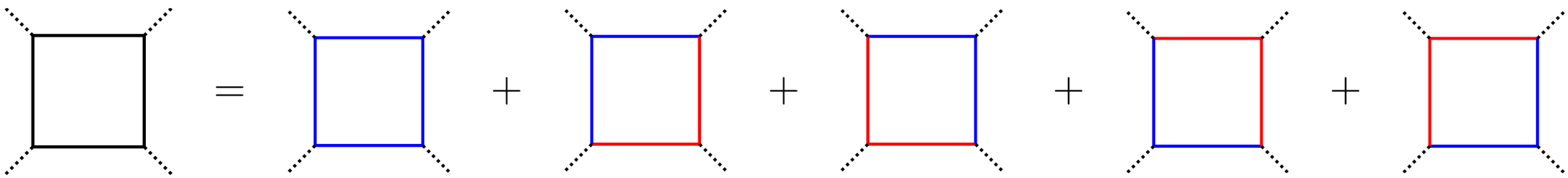


The integrals are ill-defined, so we have to introduce analytic regularization of the exponent of propagators.

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log st\right)\right]$$
$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}-\frac{1}{\delta_4}+\log t/m^2\right)+\frac{\pi^2}{12}\right]$$
$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3}+\frac{1}{\delta_4}+\log s/m^2\right)+\frac{\pi^2}{12}\right]$$
$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}-2\log m^2\right)+\frac{\pi^2}{6}\right]$$

Cancellation of auxiliary parameters between soft regions occurs.

Method of region: total



$$f_0^{(1)} = \frac{1}{st}\left(\frac{4}{\varepsilon^2} - \frac{2\log st}{\varepsilon} + 2\log s \log t - \frac{4\pi^2}{3}\right)$$

$$f_0^{(2)} = \frac{(m^2)^{-\varepsilon}}{st}\left[\frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log st\right)\right]$$

$$f_0^{(3)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} - \frac{1}{\delta_4} + \log t/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(4)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{\delta_3} + \frac{1}{\delta_4} + \log s/m^2\right) + \frac{\pi^2}{12}\right]$$

$$f_0^{(5)} = \frac{(m^2)^{-\varepsilon}}{st}\left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{1}{\delta_3} + \frac{1}{\delta_4} - 2\log m^2\right) + \frac{\pi^2}{6}\right]$$

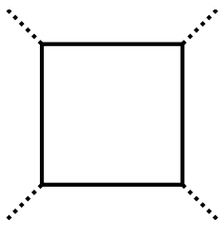
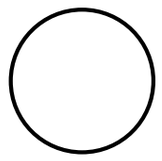
$$I = \sum_{n=0}^{\infty}(m^2)^n f_n$$

$$f_0 = \frac{1}{st}(2\log\frac{s}{m^2}\log\frac{t}{m^2} - \pi^2)$$

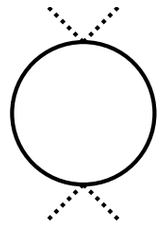
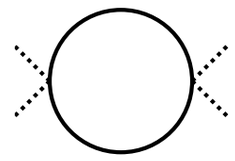
Cancellation of auxiliary parameters between soft regions occurs.

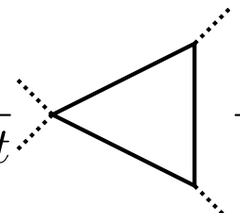
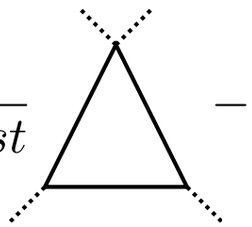
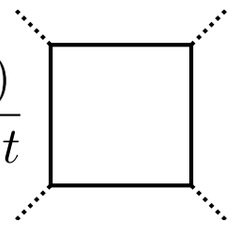
Expansion in m_t : using differential equation

[Kotikov '91]

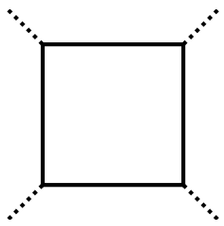
$$\frac{\partial}{\partial(m_t^2)}$$  $$= -\frac{2(d-2)\left(2m^2(s+t)+st\right)}{m^4\left(4m^2+s\right)\left(4m^2+t\right)\left(4m^2(s+t)+st\right)}$$ 

We used LiteRed [Lee '13] for obtaining the diff.-eq.

$$-\frac{2(d-3)t}{m^2\left(4m^2+t\right)\left(4m^2(s+t)+st\right)}$$  $$-\frac{2(d-3)s}{m^2\left(4m^2+s\right)\left(4m^2(s+t)+st\right)}$$ 

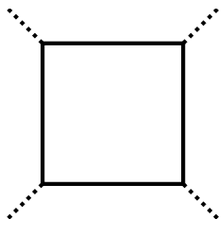
$$-\frac{(d-4)s}{4m^4(s+t)+m^2st}$$  $$-\frac{(d-4)t}{4m^4(s+t)+m^2st}$$  $$-\frac{2(d-5)(s+t)}{4m^2(s+t)+st}$$ 

Substituting the form,

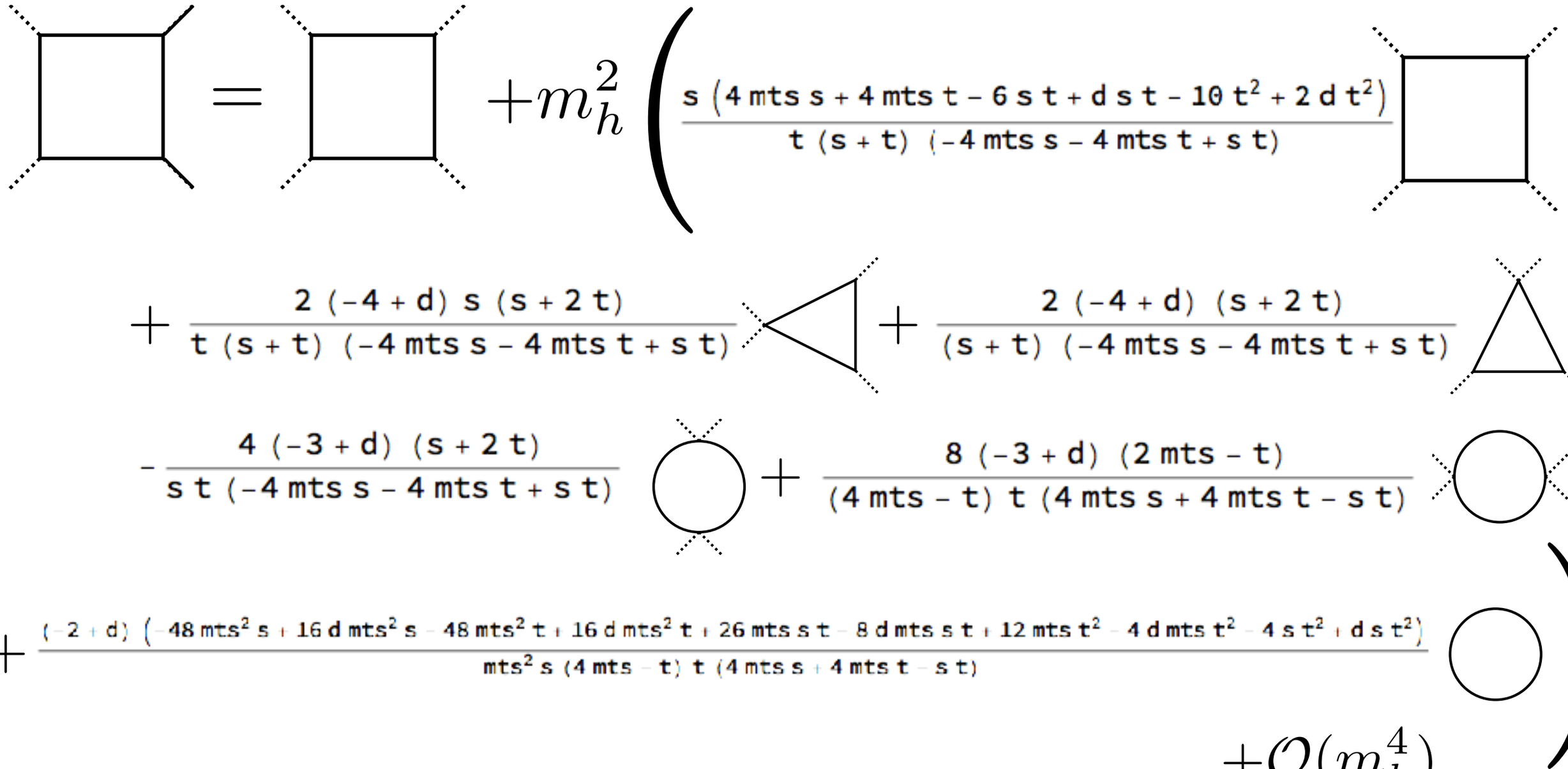
 $$= \sum_{n_1,n_2} c_{n_1,n_2}(m_t^2)^{n_1}(\log m_t)^{n_2}$$

we obtain recursive relations of C_n's.

See also
[Melnikov, Tancredi, Wever '16]

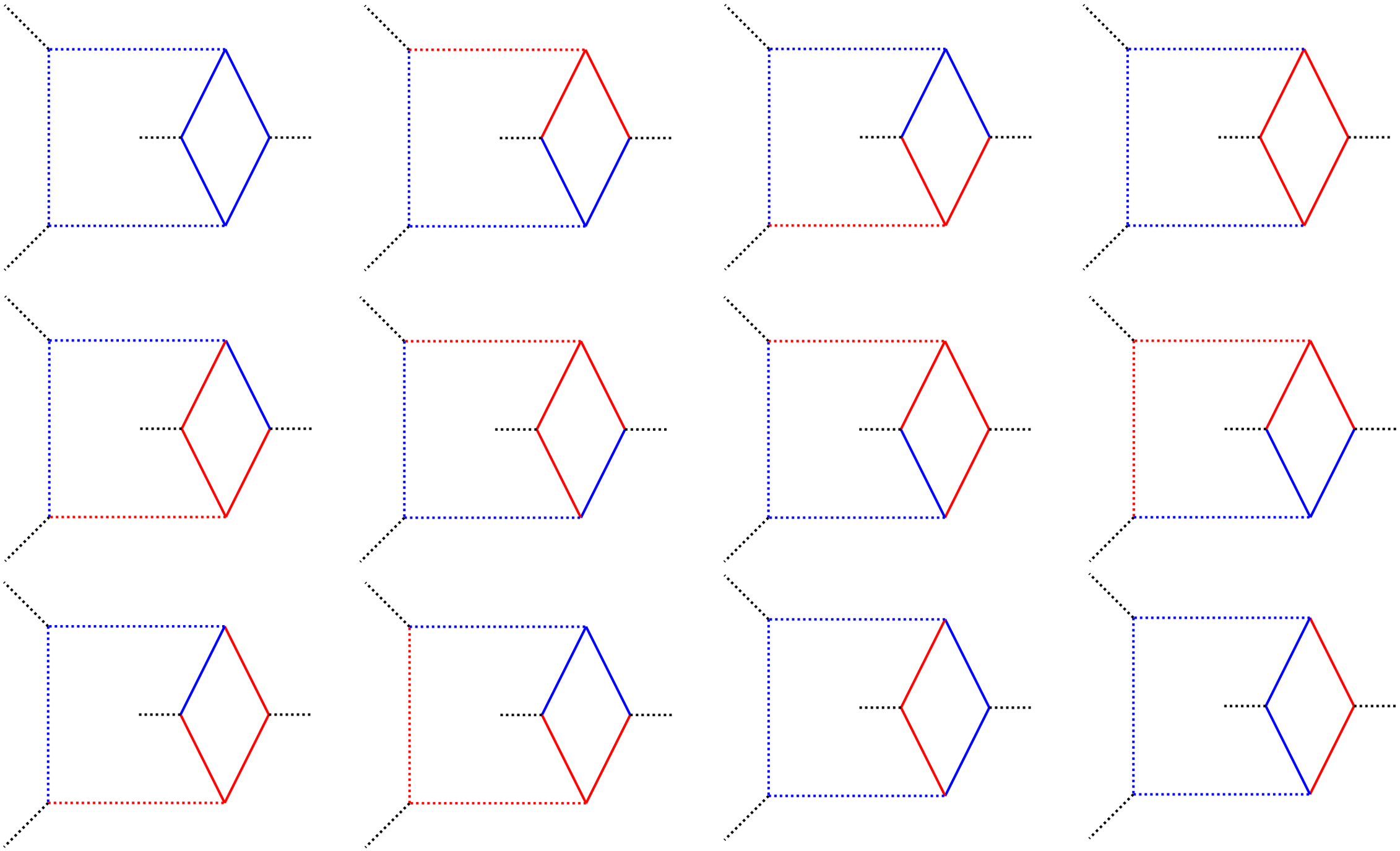
 $$= (m_t^2)^0 f_0 + (m_t^2)^1 f_1 + (m_t^2)^2 f_2 + \cdots$$

Expansion in m_h: only the all-hard region



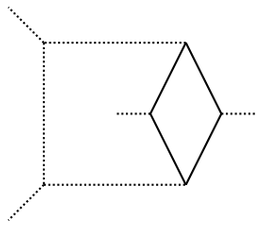
The massive-Higgs diagram can be expressed as an infinite sum of the massless-Higgs diagrams.

Method of region: non-planar diagrams

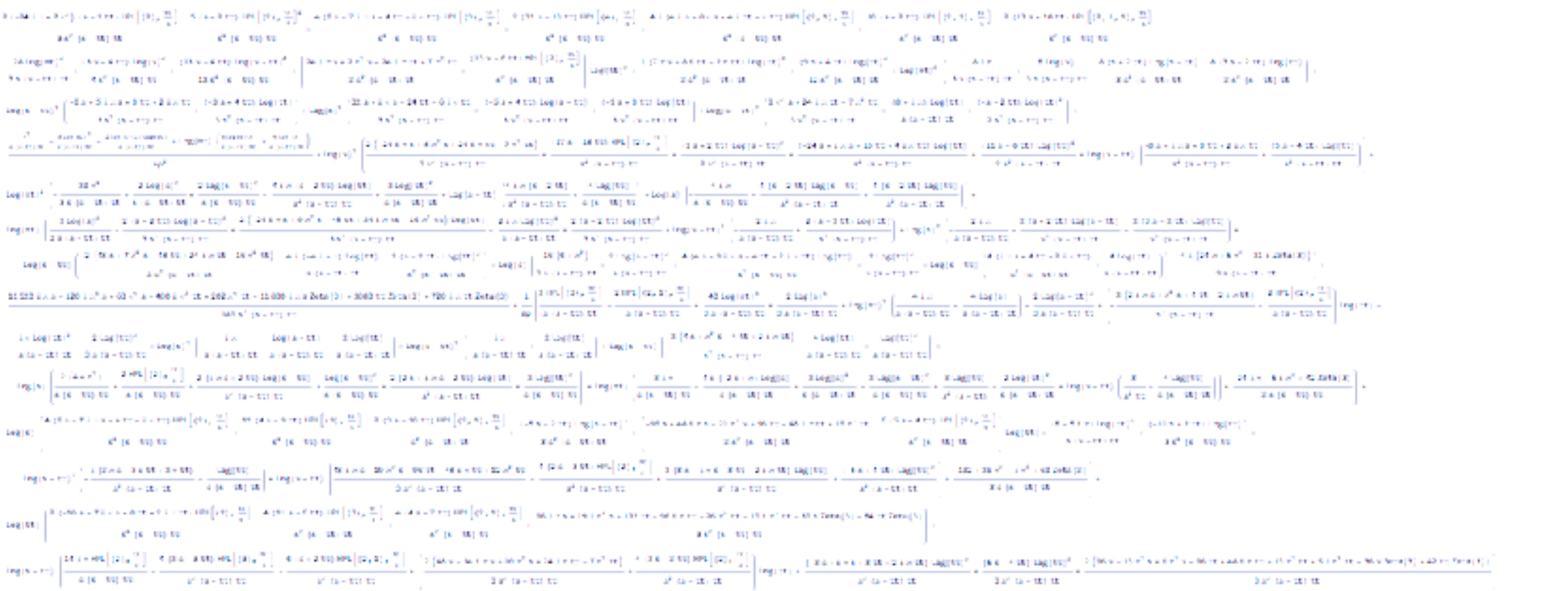


blue: hard-scaling propagator, red: soft-scaling propagators

Method of region: non-planar diagrams



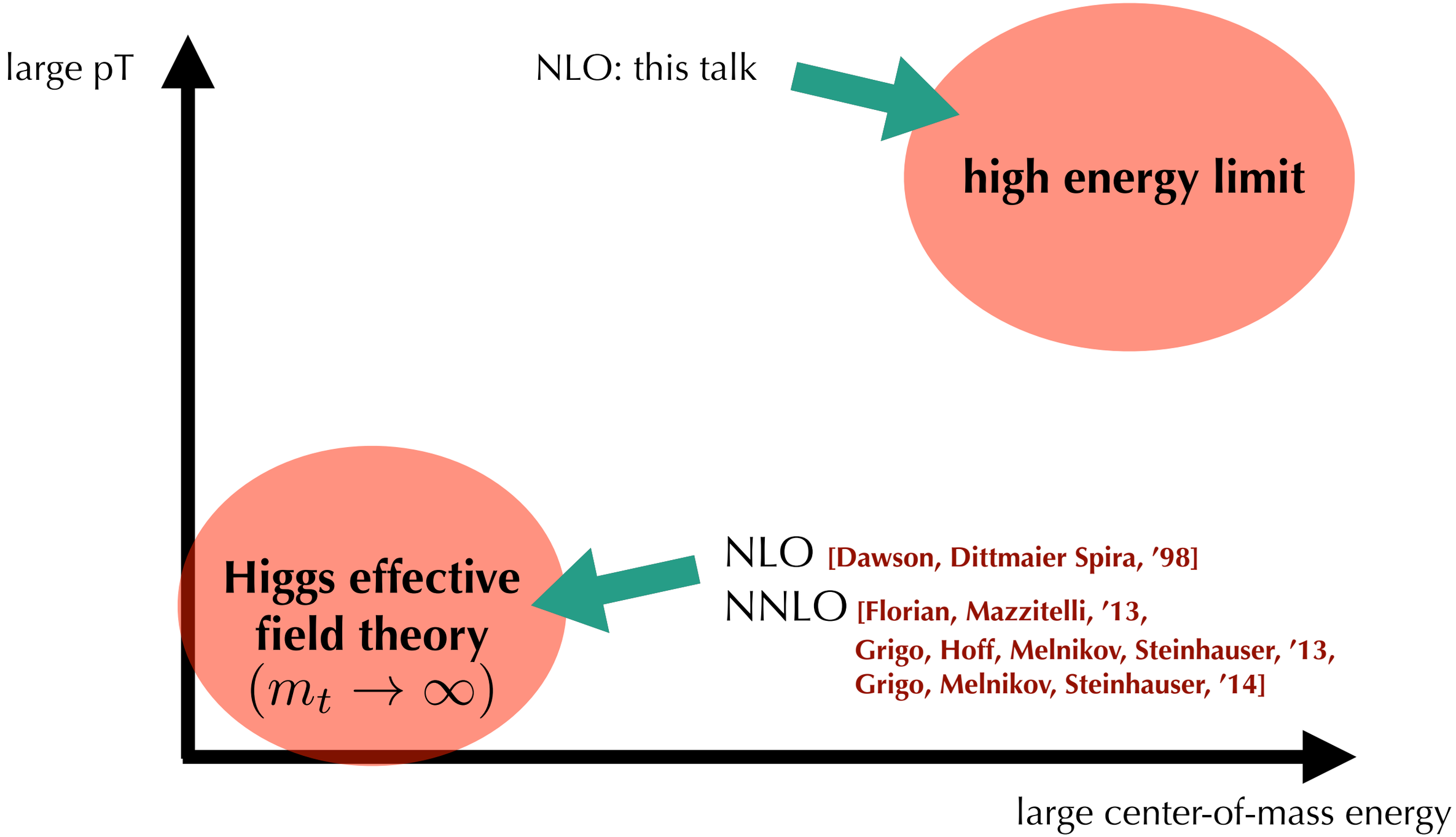
$=$



$+\mathcal{O}(\epsilon, m_t)$

- used as the boundary condition of m_t -differential equation
- expressed in terms of HPL's
- suited for the evaluation at the physical kinematical configuration (s>0, t<0)
- satisfy the t-differential equation (independent check)
- consistent with the Higgs+jet result **[Kudashkin, Melnikov, Wever, '17]**

Prospect for the effective theory



Prospect for the effective theory

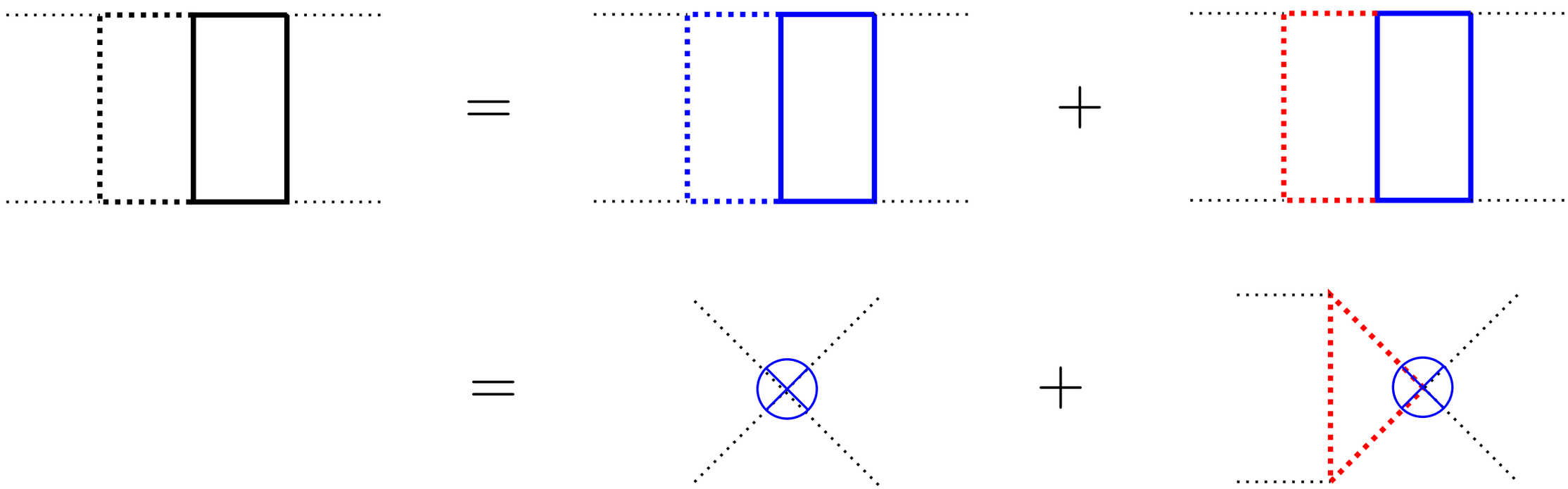
	Lagrangian	diagram generation	IBP reduction	evaluating the MIs
HEFT	expanded in 1/mt	expanded in 1/mt	expanded in 1/mt	expanded in 1/mt
high energy limit	**exact in mt**	**exact in mt**	**exact in mt**	expanded in mt
exact	**exact in mt**	**exact in mt**	**exact in mt**	**exact in mt**
desirable high energy EFT (not known)	expanded in mt	expanded in mt	expanded in mt	expanded in mt

our method so far

Method of region may give us hints towards the EFT

There is a explicit one-to-one correspondence between
the method of region and the EFT in the case of $m_t \to \infty$.

example



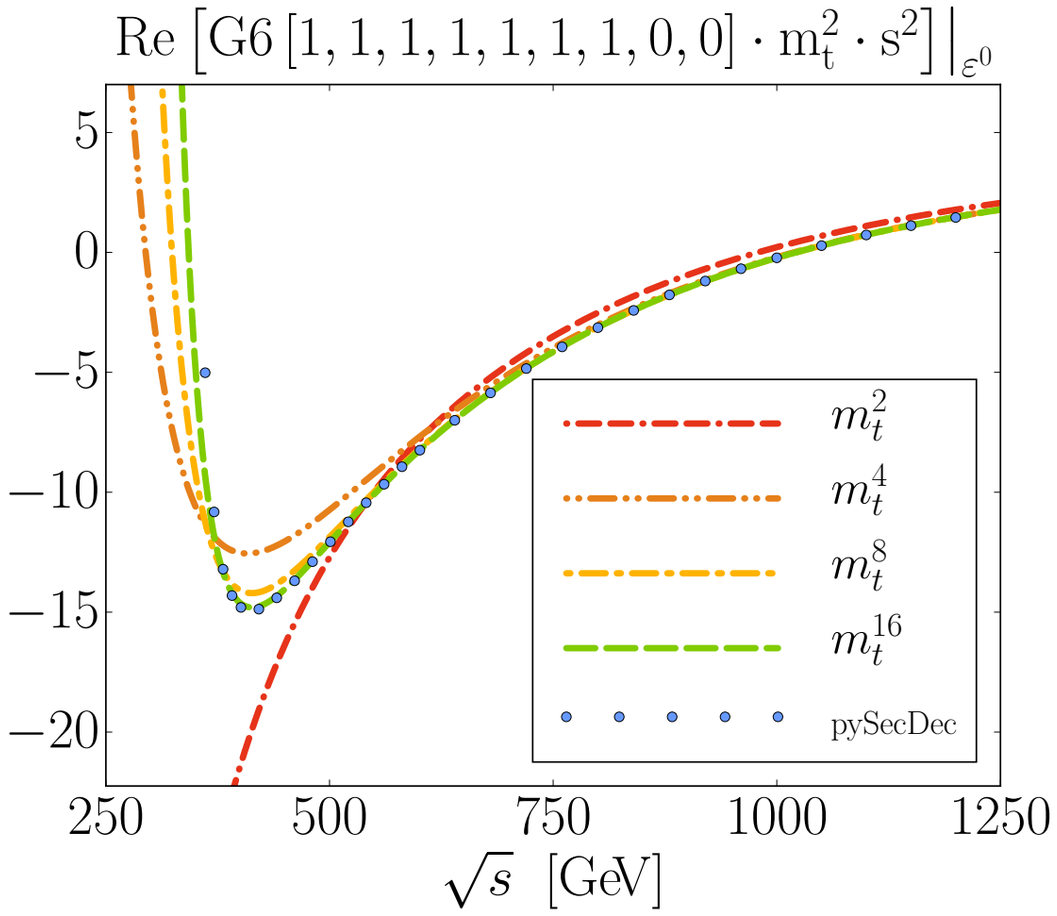
Wilson coefficient@2loop
EFT matrix element@tree

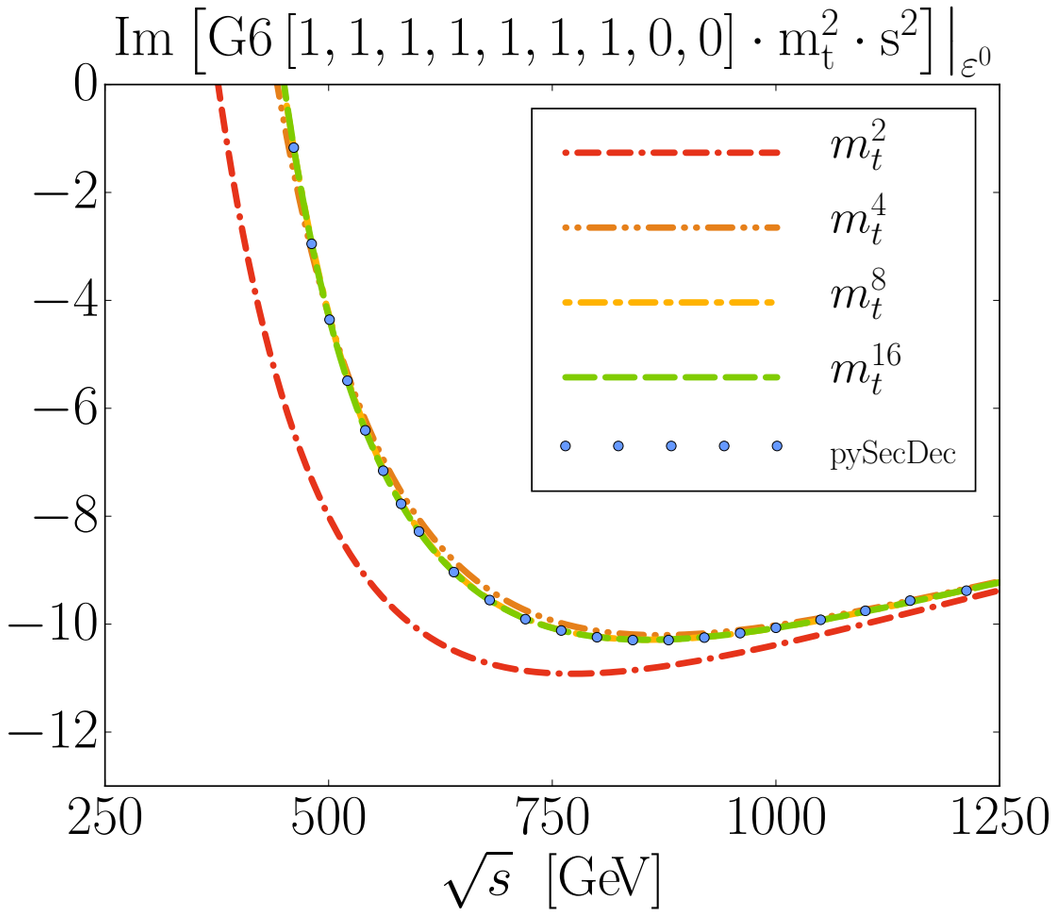
Wilson coefficient@1loop
EFT matrix element@1loop

Summary

- We are calculating the two-loop gg->HH amplitude in the high energy approximation.
- Reduction to the master integrals is done.
- The 131 planar integrals are calculated and published.
- Most of the non-planar integrals are calculated.
- The method of region may give us hints towards the desirable EFT, which could let the NNLO calculation possible.

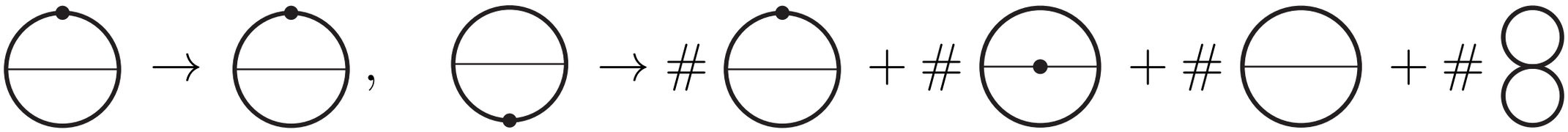
good agreement with the exact numerical result



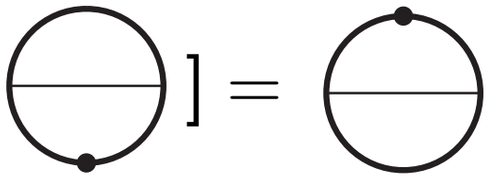


Minimization of the Master Integrals

FIRE yields **221**^{NLO} two-loop masters: this is not a minimal set! E.g.,



Make use of FIRE command FindRules. Finds equivalent integrals:



For any integral I we should have that, after applying reduction rules,

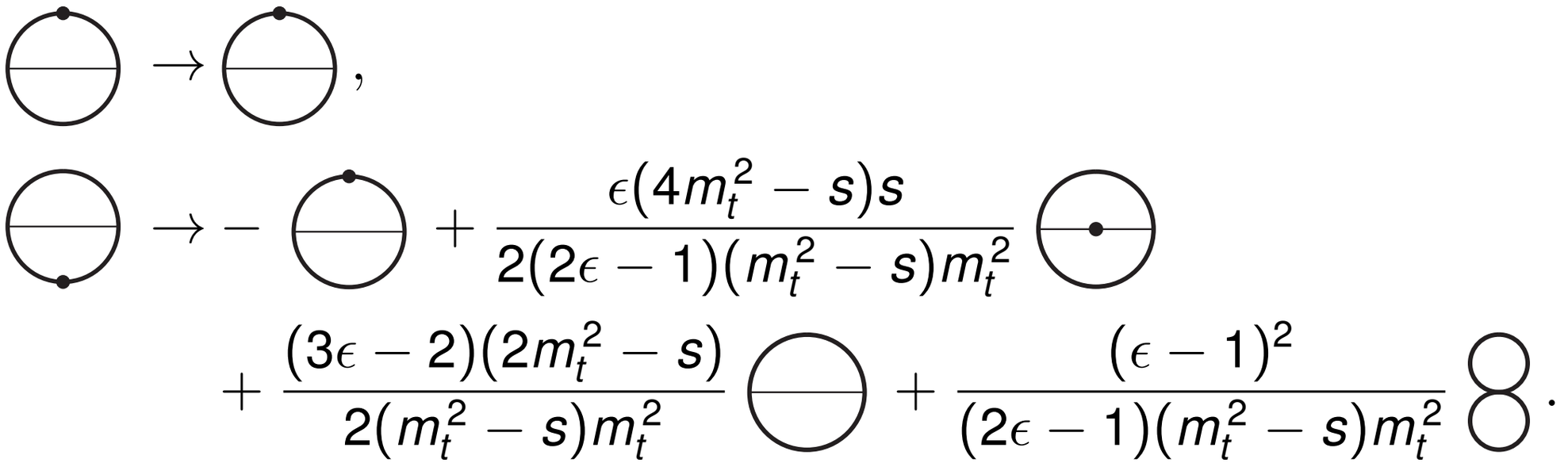
$$\texttt{FindRules}[I] = I$$

Method: build such equations for many integrals

- use to derive extra reduction relations
- final result: **161**^{NLO} master integrals

Minimization of the Master Integrals: Example

Reduction rules:

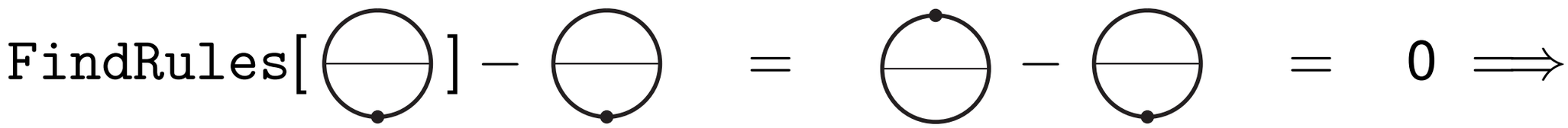


$$\to \frac{\epsilon(4m_t^2-s)s}{2(2\epsilon-1)(m_t^2-s)m_t^2}$$

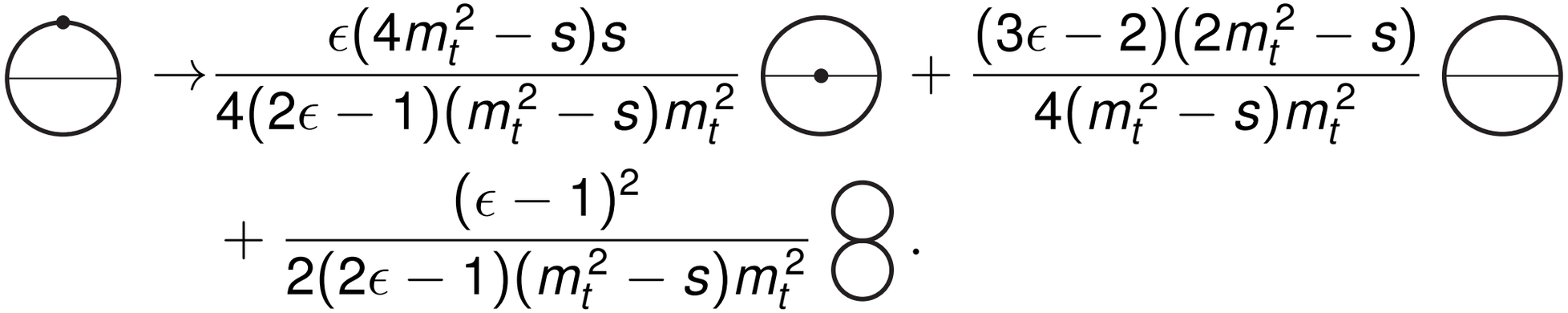
$$+\frac{(3\epsilon-2)(2m_t^2-s)}{2(m_t^2-s)m_t^2}$$

$$+\frac{(\epsilon-1)^2}{(2\epsilon-1)(m_t^2-s)m_t^2}\,.$$

Then,



$$\texttt{FindRules}[\quad]-\quad=\quad-\quad=\quad 0\Longrightarrow$$



$$\to\frac{\epsilon(4m_t^2-s)s}{4(2\epsilon-1)(m_t^2-s)m_t^2}$$

$$+\frac{(3\epsilon-2)(2m_t^2-s)}{4(m_t^2-s)m_t^2}$$

$$+\frac{(\epsilon-1)^2}{2(2\epsilon-1)(m_t^2-s)m_t^2}\,.$$