N3LL+NNLO resummation for color singlet production at the LHC

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LoopFest XVII
Michigan State University, 19 July 2018

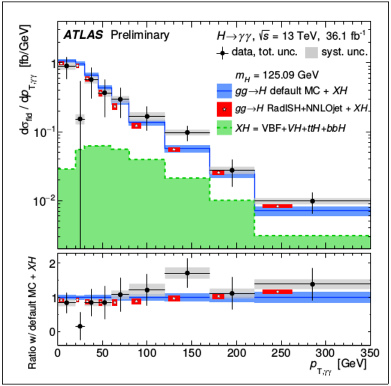
outline

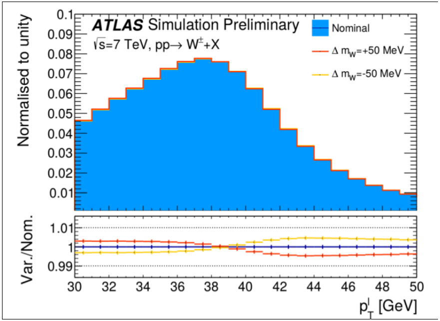
- Introduction
- resummation for transverse observables in momentum-space
- Predictions at N3LL+NNLO for color-singlet production at the LHC
  - Higgs production [$p_{\mathrm{T,H}}$]
  - Drell-Yan production [$p_{\mathrm{T,V}}$, ϕ^*]
- Conclusions

based on:

Monni,ER,Torrielli '16
Bizon,Monni,ER,Rottoli,Torrielli '17
Bizon,Chen,Gehrmann-De Ridder,Gehrmann,Glover,Huss,Monni,ER,Rottoli,Torrielli '18

Color-singlet production at the LHC





- $p_{\mathrm{T,H}}$ is one of the more important observables for current Higgs studies at the LHC:
  - "easy" to measure
  - possible to probe deviation from SM, in tail, but also at medium-small $p_{\mathrm{T,H}}$
- $p_{\mathrm{T,V}}$ and ϕ^* data are extremely precise:
  - SM measurement: test QCD at higher orders, extract PDFs, ...
  - W-mass extraction: modelling of $p_{\mathrm{T,W}}$ and $p_{\mathrm{T,Z}}$ is crucial

Transverse observables in color-singlet production

Transverse and inclusive observables in color-singlet production offer a clean experimental and theoretical environment for precision physics.

$$V(\{\tilde{p}\},k) = d_\ell g_\ell(\phi)\left(\frac{k_t}{M}\right)^a$$

$$V(\{\tilde{p}\},k_1,...,k_n) = V(\{\tilde{p}\},k_1+...+k_n)$$

- direct probe the kinematics of the color-singlet
- sensitivity to non-perturbative effects (hadronisation, intrinsic kt) only through transverse recoil
- very limited/no sensitivity to multi-parton interaction

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1. soft/collinear limit: $v \to 0 \Rightarrow$ resummation of large logarithms $L = \log(1/v)$.
   Logarithmic accuracy usually defined at the level of the logarithm of the cumulative cross section Σ

$$\frac{\Sigma(v)}{\sigma_{LO}} = \int_0^v dv' \frac{1}{\sigma_{LO}}\frac{d\sigma}{dv'} \sim \exp\left\{\sum_n \left[\underbrace{\mathcal{O}(\alpha_S^n L^{n+1})}_{LL} + \underbrace{\mathcal{O}(\alpha_S^n L^n)}_{NLL} + \underbrace{\mathcal{O}(\alpha_S^n L^{n-1})}_{NNLL} + ...\right]\right\}$$

2. matching to accurate (NNLO) fixed order results is also needed

V/H at small transverse momentum

Aside from the obvious need to resum large logarithms, the $p_t \to 0$ limit is also theoretically interesting:

$$V(\{\tilde{p}\}, k_1 \dots k_n) = |\vec{k}_{t1} + ... + \vec{k}_{tn}| := p_t$$

- p_t is a vectorial quantity $\to$ it absorbs the recoil of all emissions $k_{t,i}$
- when $p_t \to 0$, two mechanism compete:
  - Sudakov (exponential) suppression when $k_{ti} \sim p_t$
  - azimuthal cancellations when $k_{ti} \gg p_t$

- the latter mechanism is dominant when $p_t \to 0$: $\Sigma(p_t) \sim p_t^2$ [Parisi,Petronzio '79]

resummation in momentum-space

- hierarchy in $\log(M/p_t)$ doesn't work, as neglected effects actually dominate the limit. It's impossible to recover power behaviour at any given order in L.
- Moreover, at any log order in $L = \log(M/p_t)$, resummation in direct space cannot be, at the same time, free of subleading terms and of spurious singularities at finite p_t

[Frixione,Nason,Ridolfi '98]

- when going in b-space, the vectorial nature of azimuthal cancellations is taken care by a Fourier transform [Parisi,Petronzio '79, CSS '85, Bozzi et al. '05, Becher et al. '10-'12]

$$\delta^{(2)}(\vec{p_t} - (\vec{k}_{t1} + ... + \vec{k}_{tn})) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^{n} e^{-i\vec{b}\cdot\vec{k}_{ti}}$$

$$\frac{d^2\Sigma(v)}{d\Phi_B\, dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1 c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\mathrm{CSS}}(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$

$$\times \exp\left\{ -\sum_{\ell=1}^{2} \int_{b_0/b}^{M} \frac{dk_t}{k_t} \mathbf{R}'_{\mathrm{CSS},\ell}(k_t) \right\}$$

C/H: [Catani,Grazzini '11-'12, Gehrmann et al. '14]

anom. dimensions: [Davies,Stirling '84, De Florian,Grazzini '01, Becher,Neubert '10, Li,Zhu / Vladimirov '16]

$\Rightarrow$ resummation directly in momentum space now possible

[Monni,ER,Torrielli '16, Bizon,Monni,et al. '17]
[Ebert,Tackmann '16]
[Kang,Lee,Vaidya '17]

small p_t resummation in momentum space (I)

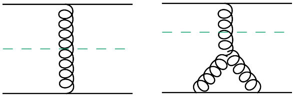
- Write all-order cross-section for $v = p_t$ $(V(\{\tilde{p}\}, k_1 \dots k_n) = |\vec{k}_{t1} + \dots + \vec{k}_{tn}|)$

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 \, \Theta\left(v - V(\{\tilde{p}\}, k_1 \dots k_n)\right)$$

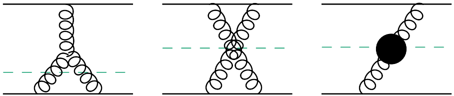
$\mathcal{V}(\Phi_B)$: all-order form factor $\qquad$ $|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2$: real emissions

- re-organize multiple-emission squared amplitudes into (iterations of) "n-particle-correlated blocks"

$$|\tilde{M}(k)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2}$$



$$|\tilde{M}(k_a, k_b)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a, k_b)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} - \frac{1}{2!} |\tilde{M}(k_a)|^2 |\tilde{M}(k_b)|^2$$



- each $|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2$ can be rewritten as a sum of products of $|\tilde{M}|^2$.
- the rIRC safety of the observable guarantees an hierarchy between the different blocks (n-particle $\rightarrow$ one higher log-order than $n-1$-particle):
  LL : $|\tilde{M}^{(0)}(k_i)|^2$; NLL : $|\tilde{M}^{(0)}(k_a, k_b)|^2$, $|\tilde{M}^{(1)}(k_i)|^2$; ...

small p_t resummation in momentum space (II)

- for *inclusive observables*, radiation within each block can be integrated before evaluating the observable (keeping a fixed k_t and rapidity).

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \ldots, k_n)|^2 \longrightarrow |M_B(\tilde{p}_1, \tilde{p}_2)|^2$$

$$\times \frac{1}{n!}\left\{\prod_{i=1}^{n}\left(|\tilde{M}(k_i)|^2 + \int [dk_a][dk_b]|\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti})\delta(Y_{ab} - Y_i)\right.\right.$$

$$\left.\left. + \int [dk_a][dk_b][dk_c]|\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti})\delta(Y_{abc} - Y_i) + \ldots\right)\right\}$$

- now we need to cancel (at all orders) the IRC poles between $\mathcal{V}$ and real emissions
  - introduce a resolution scale ϵk_{t1} (not $\epsilon\, p_t$)
  - emissions with $k_{ti} < \epsilon k_{t1}$ are unresolved. They don't contribute to the observable, hence they exponentiate $\rightarrow$ regularize virtual corrections and leave a Sudakov factor:

$$\mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(\ldots\right) \Theta(\epsilon k_{t1} - k_{ti}) \sim \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(\epsilon k_{t1})} R'(k_{t1})$$

  where

$$R(\epsilon k_{t1}) = \sum_{\ell=1}^{2} \int_{\epsilon k_{t1}}^{M} \frac{dk_t}{k_t} R'_\ell(k_t) = \sum_{\ell=1}^{2} \int_{\epsilon k_{t1}}^{M} \frac{dk_t}{k_t} \left(A_\ell(\alpha_S(k_t)) \ln \frac{M^2}{k_t^2} + B_\ell(\alpha_S(k_t))\right)$$

  - A and B as in CSS, with differences starting at N3LL.
  - resolved blocks contribute to the observable: treated exclusively in 4 dimensions, parametrized as derivatives of the Sudakov ($R'(k_{ti})$), generated as MC events

small p_t resummation in momentum space (III)

- final result:

$$\hat{\Sigma}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0))H(\mu_R)\mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right]\int_0^M \frac{dk_{t1}}{k_{t1}}\int_0^{2\pi}\frac{d\phi_1}{2\pi}$$
$$\times e^{-\mathbf{R}(\epsilon k_{t1})}\exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\frac{\alpha_s(k_t)}{\pi}\mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0}\frac{dk_t}{k_t}\mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$
$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi}\mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$
$$\times\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=2}^{n+1}\int_\epsilon^1\frac{d\zeta_i}{\zeta_i}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi}\mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$
$$\times\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_{n+1})\right)$$

$$\left[\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1}\frac{dN_1}{2\pi i}\int_{\mathcal{C}_2}\frac{dN_2}{2\pi i}x_1^{-N_1}x_2^{-N_2}\sum_{c_1,c_2}\frac{d|M_B|^2_{c_1c_2}}{d\Phi_B}\mathbf{f}^T_{N_1}(\mu_0)\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v)\mathbf{f}_{N_2}(\mu_0)\right]$$

- $\zeta_i = k_{ti}/k_{t1}$ $\quad\mathbf{\Gamma}$ and $\mathbf{\Gamma}^{(C)}$ anomalous dimensions of PDFs and coeff. function
- ϵ dependence in the resolved emissions cancels against the one in the Sudakov, leaving ϵ^p effects
- at any logarithmic order only a finite number of DGLAP-evolution steps necessary: can do the Mellin inversion and have only quantities in momentum space (with convolutions)

comments I

- role of subleading terms
  - logarithmic counting is defined in terms of $\log(M/k_{ti})$.
  - in the Sudakov limit, the hierarchy in $\log(M/p_t)$ makes sense, one has $k_{ti} \sim p_t \sim 0$.
    - same as resummation of $\log(M/p_t)$, i.e. log accuracy in $\log(M/k_{ti})$ translates into the same accuracy in $\log(M/p_t)$, plus subleading terms.
  - similar conclusions were found by Ebert,Tackmann '16
- resolved k_{ti} are of the same order of k_{t1}
  - expand k_{ti} around k_{t1} in the resolved radiation at the desired logarithmic accuracy.
  - higher-order corrections to the NLL resolved reals: one correction at a time [one at NNLL, two at N3LL,...]

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\mathrm{N^3LL}}(k_{t1}) \right) \int d\mathcal{Z}[\{R', k_i\}] \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$
$$+ (1 \text{ correction})$$
$$+ (2 \text{ corrections})$$

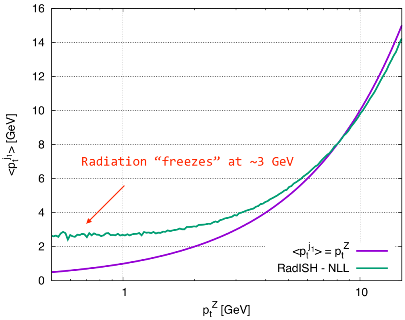
  - NLL real emissions $d\mathcal{Z}[\{R', k_i\}]$ generated as a parton shower
  - H and C absorbed in $\mathcal{L}$
- we did many checks; among them, we reproduced the CSS result (and, by doing this, extracted the correct A and B)

comments II

- azimuthal cancellations [at NLL, with $\mathcal{L} = 1$ for simplicity]

$$\frac{d^2\Sigma(p_t)}{d^2p_t d\Phi_B} = \sigma^{(0)}(\Phi_B)\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi} e^{-R(k_{t1})} R'(k_{t1}) \int d\mathcal{Z}[\{R', k_i\}]\delta^{(2)}(\vec{p_t} - \vec{k}_{t1} - \ldots - \vec{k}_{t,n+1})$$

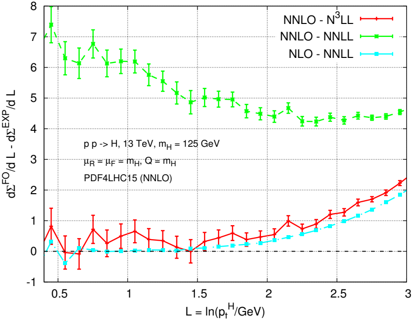
Sudakov freezes at $k_{t1} \gg p_t$, random azimuthal orientation given by $d\mathcal{Z}[\{R', k_i\}]$

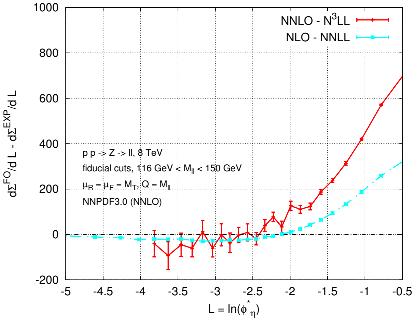


matching to fixed order I

- all the above has been implemented in a MC code (`RadISH`)
- we have obtained resummed+matched results (at N3LL+NNLO) for Higgs and Drell-Yan production
  - N3LO (for Higgs) — from Anastasiou et al., '15
  - $pp \to Hj$ at NNLO: first paper: — Boughezal, Caola, et al., '15
  - results presented here [$pp \to Hj$ and $pp \to Zj$] — `NNLOJET`, Chen et al., '16
  - anomalous dimension — Li, Zhu '16, Vladimirov '16 (except 4-loop cusp)

- expansion of resummation vs. fixed-order





matching to fixed order II

$$\Sigma(p_t, \Phi_B) = \int_0^{p_t} dp_t' \frac{d\sigma}{dp_t' d\Phi_B} \qquad \begin{cases} \to \Sigma_{\rm res} & \text{if } p_t \ll M_B \\ \to \Sigma_{\rm F.O.} & \text{if } p_t \gtrsim M_B \end{cases}$$

additive matching

$$\Sigma^{add}_{\rm matched}(v) = \Sigma_{\rm res}(v) + \Sigma_{\rm F.O.}(v) - \Sigma_{\rm res,exp}(v)$$

multiplicative matching

$$\Sigma^{mult}_{\rm matched}(v) = \Sigma_{\rm res}(v) \frac{\Sigma_{\rm F.O.}(v)}{\Sigma_{\rm res,exp}(v)}$$

- there's no rigorous theory argument to favour a prescription over the other

- additive: probably the more natural choice, simpler and clear
- numerically delicate when $p_t \to 0$ (F.O. result needs to be extremely stable)

- multiplicative: numerically more stable, as physical suppression at small v fixes potentially unstable F.O. results
- allows to include constant terms from F.O.

- to avoid contamination of hard region from resummation, we use modified logs:

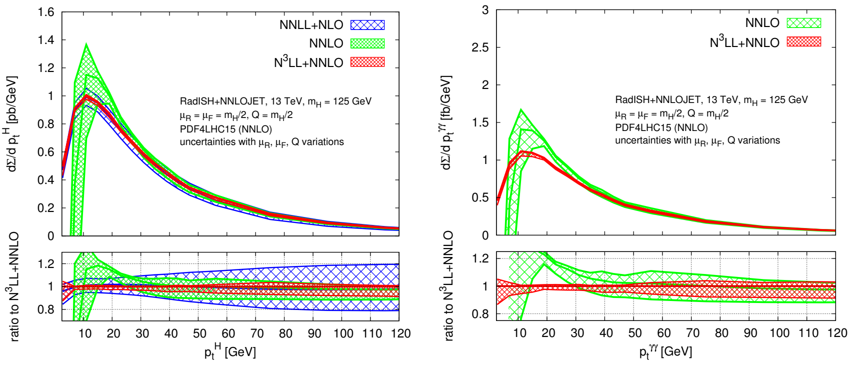
$$\ln(Q/k_{t1}) \to \frac{1}{p} \ln\left(1 + \left(\frac{Q}{k_{t1}}\right)^p\right)$$

- improved multiplicative matching (to avoid spurious $(1 + \mathcal{O}(\alpha_S^4))$ terms)

$$\Sigma^{mult}_{\rm matched}(v) = \frac{\Sigma_{\rm res}(v)}{\Sigma_{\rm res,asym}} \left[\Sigma_{\rm res,asym} \frac{\Sigma_{\rm F.O.}(v)}{\Sigma_{\rm res}(v)}\right]_{\rm exp} \qquad \Sigma_{\rm res,asym} = \int_{\text{with cuts}} d\Phi_B \left(\lim_{L\to 0} \mathcal{L}_{\rm N^kLL}\right)$$

Higgs p_t

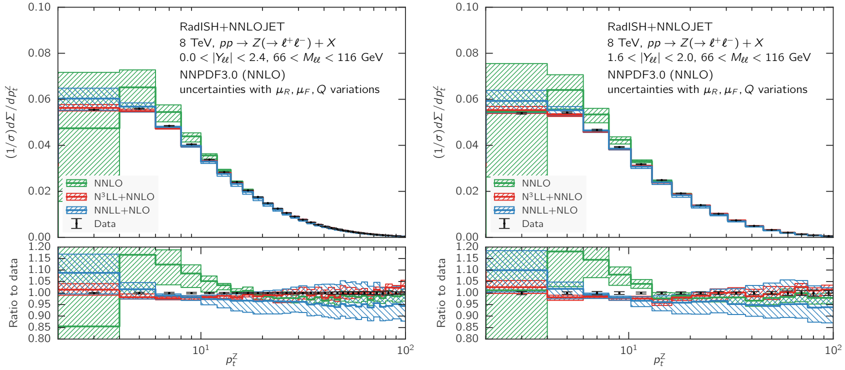
[Bizon,Chen,Gehrmann-De Ridder,Gehrmann,Glover,Huss,Monni,ER,Rottoli,Torrielli '18]



- on the right: fiducial distribution (we didn't have it last year)
- good convergence pattern
- N3LL correction amount to few %
- with $\mu = m_H/2$, important cancellations (with $\mu = m_H$, bands are larger)

Drell-Yan p_t

[Bizon,Chen,Gehrmann-De Ridder,Gehrmann,Glover,Huss,Monni,ER,Rottoli,Torrielli '18]



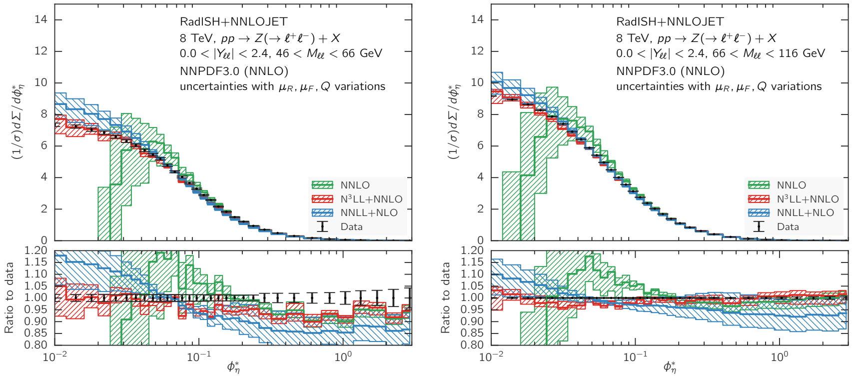
. $\mu = \sqrt{M^2 + p_t^2}, Q = M$

- good convergence and good agreement with data
- for $p_{\mathrm{T,Z}} > 20$ GeV, NNLO seems perfectly adequate to describe data
- leftover uncertainty at small p_t is at the few-percent level

Drell-Yan ϕ^*

$$\phi^* = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \sin\theta^*$$

- θ^*: angle between electron and beam axis, in Z boson rest frame
- ATLAS uses slightly different definition: $\cos\theta^* = \tanh((y_{l^-} - y_{l^+})/2)$



- TH convergence pattern is good
- for $\phi^* < 0.2$ resummation is relevant
- in the low invariant-mass region, disagreement with data at medium-large values, already observed previously

conclusions

- higher-order resummation can be formulated directly in momentum space (without the need for a factorisation for the considered observable)
- shown results for Higgs and Drell-Yan
- can be easily extended to other color-singlet processes (for sure W)
- possible developments
  - closer connections to a parton-shower formalism
  - might allow joint resummation (the k_{ti} are not integrated over), although accuracy needs to be carefully addressed
  - we haven't included any assessment of NP effects, quark-mass corrections, QED, theory uncertainties in PDFs...(probably need to study them for W-mass extraction)

Thanks for your attention