Stochastic Sampling Color Parton Shower

Joshua Isaacson

LoopFest XVII

19 July 2018

Outline

- Parton Shower Basics
- Color Flow Basis
- Algorithm
- Matching to Hadronization
- Validation
- Results

[Isaacson, Prestel: 2018 (arxiv:1806.10102)]

Parton Shower Basics

$$\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \left[P_{ab}(z)\right]_+ f_b\left(\frac{x}{z}, t\right)$$

- Solve using Markovian Monte-Carlo algorithms
- Treat P_{ab} as a probability
- Virtual corrections defined at kinematic endpoints by $+$ prescription

Catani-Seymour Dipole Subtraction

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle\mathcal{M}_n| \frac{\mathbf{T}_{\tilde{k}}\cdot\mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \quad \mathcal{V}_{\tilde{ij},\tilde{k}} \quad |\mathcal{M}_n\rangle,$$

Catani-Seymour Dipole Subtraction

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle \mathcal{M}_n | \frac{\mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \quad \mathcal{V}_{\tilde{ij},\tilde{k}} \quad | \mathcal{M}_n \rangle,$$

- Color factor

Catani-Seymour Dipole Subtraction

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle \mathcal{M}_n | \frac{\mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \mathcal{V}_{\tilde{ij},\tilde{k}} | \mathcal{M}_n \rangle,$$

- Color factor
- Kinematic factor

Catani-Seymour Dipole Subtraction

$$|\mathcal{M}_{n+1}|^2 \simeq -\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}}^{n} \langle \mathcal{M}_n | \frac{\mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}}}{\mathbf{T}^2_{\tilde{ij}}} \mathcal{V}_{\tilde{ij},\tilde{k}} | \mathcal{M}_n \rangle,$$

- Color factor
- Kinematic factor

Conservation of Color:

$$\sum_{\tilde{ij},\tilde{k}\neq\tilde{ij}} \mathbf{T}_{\tilde{k}} \cdot \mathbf{T}_{\tilde{ij}} = -\mathbf{T}^2_{\tilde{ij}}$$

Dipole Shower: Splitting Kernels

DIRE Parton Shower Kernels:

$$P_{qq}\left(z,\kappa^2\right) = C_F\left[\frac{2\left(1-z\right)}{\left(1-z\right)^2+\kappa^2} - \left(1+z\right)\right],$$

$$P_{gg}\left(z,\kappa^2\right) = \frac{C_A}{2}\left[\frac{2\left(1-z\right)}{\left(1-z\right)^2+\kappa^2} - 2 + z\left(1+z\right)\right],$$

$$P_{gq}\left(z,\kappa^2\right) = \frac{T_R}{2}\left(1-2z\left(1-z\right)\right)$$

$$\kappa^2 = \frac{t}{Q^2}$$

Leading Color

Gluon Propagator can be written as:

$$\langle (\mathcal{A}_\mu)^{i_1}_{j_1} (\mathcal{A}_\mu)^{i_2}_{j_2} \rangle \propto \delta^{i_1}_{j_2}\delta^{i_2}_{j_1} - \frac{1}{N_C}\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}.$$

Taking $N_C \to \infty$ limit reduces the Catani-Seymour Dipoles to:

$$|\mathcal{M}_{n+1}|^2 \approx \sum_{\tilde{ij},\tilde{k}\in \mathsf{LC}}^{n} \langle \mathcal{M}_n | \mathcal{V}_{\tilde{ij},\tilde{k}} | \mathcal{M}_n \rangle$$

Can capture dominate subleading color in parton showers by leaving $C_F = \frac{4}{3}$.

Algorithm: Color Flow

Separate gluon into "nonet" $(\delta^{i1}_{j2}\delta^{i_2}_{j_1})$ and "singlet" $(\delta^{i1}_{j1}\delta^{i_1}_{j_1})$ components

Non-orthogonal and overcomplete basis

Easy to compute numerically with

Represented by color-anticolor pairs, denoting flow of color from one leg to another

	λ_i	$\overline{\lambda}_i$
Quark	$\sqrt{T_R}$	0
Antiquark	0	$\sqrt{T_R}$
Gluon	$\sqrt{T_R}$	$\sqrt{T_R}$

Decompose color operators in color flow basis

$$|\sigma\rangle = \left| \begin{array}{cccc} 1 & 2 & \dots & n \\ \overline{\sigma}(1) & \overline{\sigma}(2) & \dots & \overline{\sigma}(n) \end{array} \right\rangle = \delta^{c_1}_{\overline{c}_{\sigma(1)}} \delta^{c_2}_{\overline{c}_{\sigma(2)}} \dots \delta^{c_n}_{\overline{c}_{\sigma(n)}}$$

$$\mathbf{T}_i = \lambda_i \mathbf{t}_{c_i} - \overline{\lambda}_i \overline{\mathbf{t}}_{\overline{c}_i} - \frac{1}{N}\left(\lambda_i - \overline{\lambda}_i\right)\mathbf{s},$$

[Ángeles Martínez, De Angelis, Forshaw, Plätzer, Seymour: 2018 (arxiv:1802.08531)]

Challenges

- Color Flow basis grows like: $(n_q + n_g)!$
- Color Flow basis is not orthogonal
- Matching $N_C = 3$ PS to an $N_C \to \infty$ Hadronization model

Algorithm: Overview

- **Goal**: Exponentiate Catani-Seymour exactly:
  $$-\sum_{k,ij}\langle\mathcal{M}'|\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}\mathcal{V}_{ij,k}|\mathcal{M}\rangle$$
- First Accept/Reject is traditional LC Shower ($P^{\text{LC}}/P^{\text{over}}$)
- If below t_{FC}^{cut}, done
- Second Accept/Reject is Color factor accept reject:
  $$-\frac{\mathbf{T}_k\cdot\mathbf{T}_{ij}}{\mathbf{T}_{ij}^2}$$
- Randomly choose a specific color flow with probability:
  $$\frac{|t_k^\alpha\cdot t_{ij}^\beta|}{\sum_{\alpha,\beta}|t_k^\alpha\cdot t_{ij}^\beta|}$$
- Correct Accept/Reject with a weighting factor:
  $$\frac{t_k^\alpha\cdot t_{ij}^\beta}{|t_k^\alpha\cdot t_{ij}^\beta|}\frac{\sum_{\alpha,\beta}|t_k^\alpha\cdot t_{ij}^\beta|}{\sum_{\alpha,\beta}t_k^\alpha\cdot t_{ij}^\beta}$$

Algorithm: Splitting Kernels

- Work in explicit $N_C = 3$
- Break splitting kernels up based on color and splitting type
- Increases number of splitting kernels by ≈ 6
- P_{gq} is tricky:
  - Determined by color conservation
  - Tricky to determine spectator

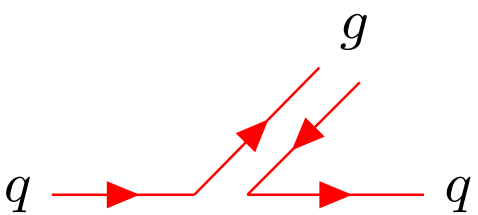
$$P_{qq}^{(9)}(x) = \lambda_i \mathbf{t}_{c_i} \frac{P_{qq}(x)}{C_F},$$

$$P_{qq}^{(1)}(x) = -\frac{\lambda_i}{N_C} \mathbf{s} \frac{P_{qq}(x)}{C_F},$$

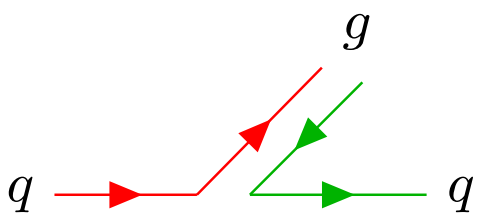
$$P_{gg}^{(+)} = \lambda_i \mathbf{t}_{c_i} \frac{P_{gg}(x)}{C_A},$$

$$P_{gg}^{(-)} = -\overline{\lambda}_i \overline{\mathbf{t}}_{\bar{c}_i} \frac{P_{gg}(x)}{C_A}$$

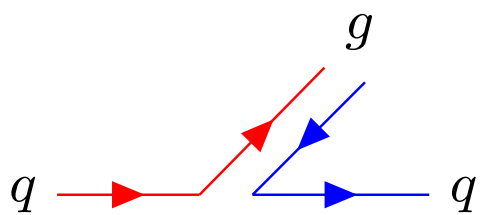
Example Splitting Kernels: Graphically



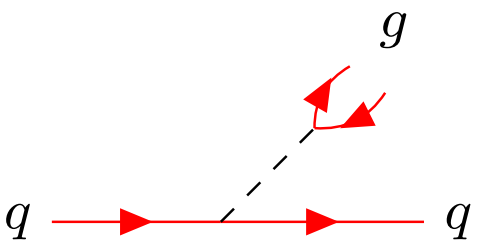
(a) $P_{rr}^{(9)}$



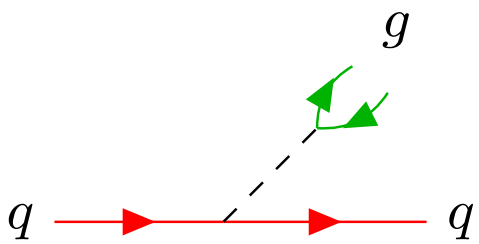
(b) $P_{rg}^{(9)}$



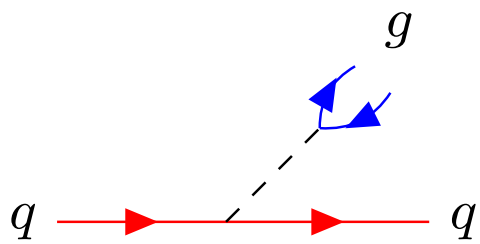
(c) $P_{rb}^{(9)}$



(d) $P_{rr}^{(1)}$

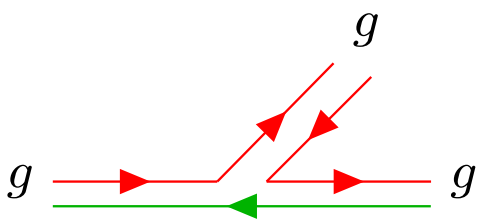


(e) $P_{rg}^{(1)}$

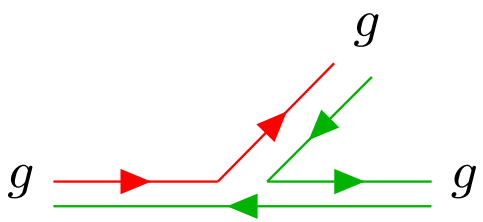


(f) $P_{rb}^{(1)}$

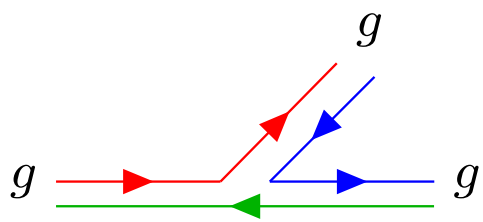
Example Splitting Kernels: Graphically



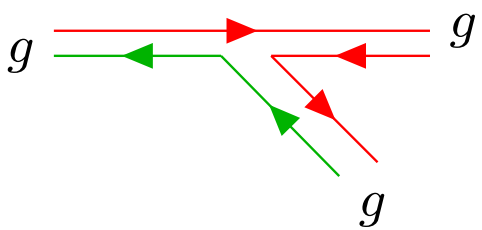
(g) $P_{rr}^{(+)}$



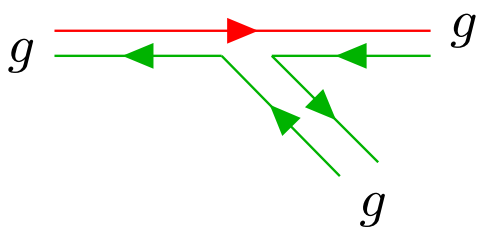
(h) $P_{rg}^{(+)}$



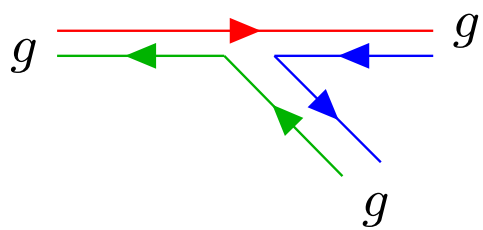
(i) $P_{rb}^{(+)}$



(j) $P_{gr}^{(-)}$

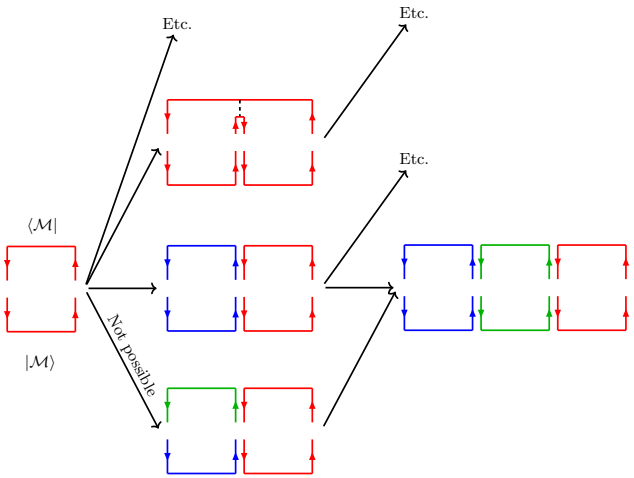


(k) $P_{gg}^{(-)}$



(l) $P_{gb}^{(-)}$

Color Flow Sampling



- 36 possible configurations at each emission
- "color weight" for each configuration given by:

$$P_{\alpha\beta} = |\langle \mathcal{M}' | \mathbf{t}_k^\alpha \cdot \mathbf{t}_{ij}^\beta | \mathcal{M} \rangle|$$

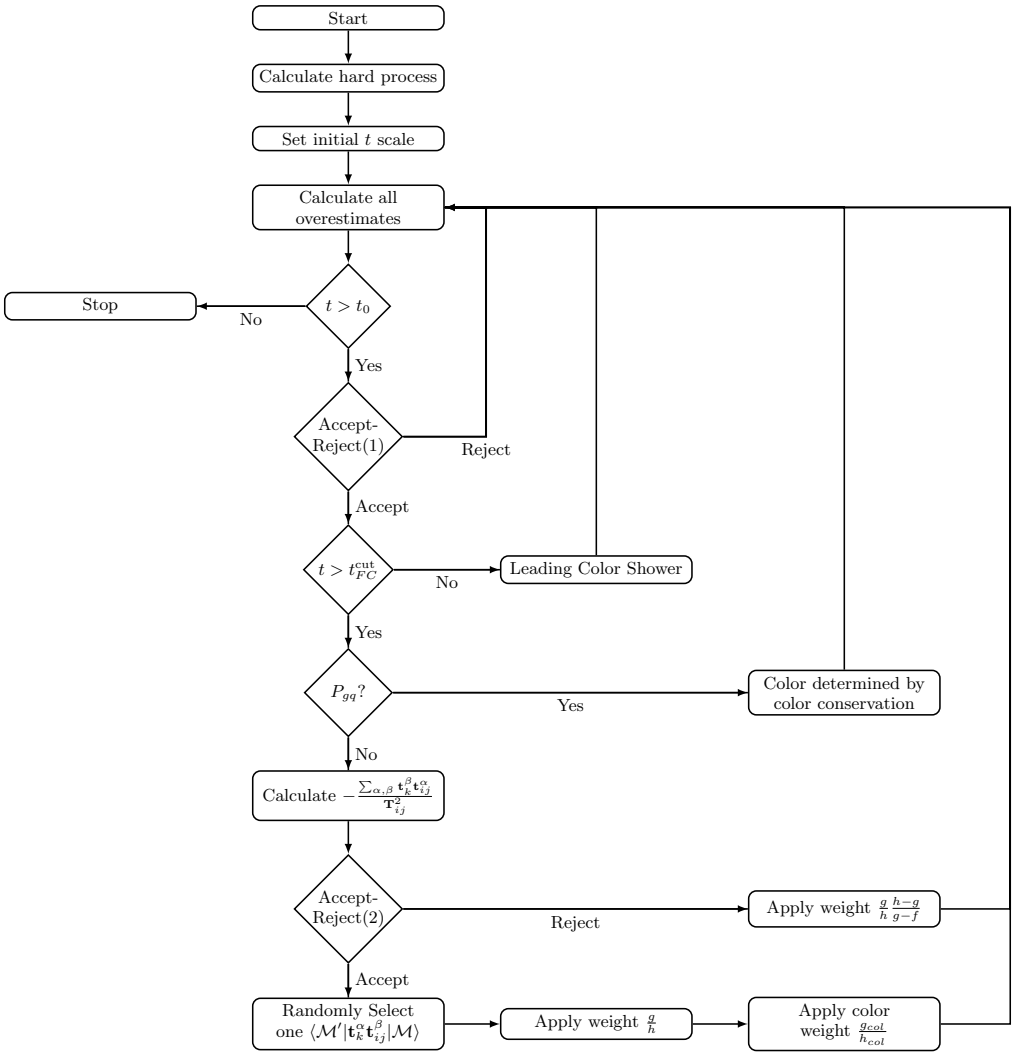
- Choose one color structure with probability:

$$P = \frac{P_{\alpha\beta}}{\sum_{\alpha,\beta} P_{\alpha\beta}}$$

- Probabilities from top down: $\frac{1}{12}, \frac{1}{4}, 0$
- Correct overall sign:

$$\frac{g_{col}}{h_{col}} = \frac{\mathbf{t}_k^\alpha \cdot \mathbf{t}_{ij}^\beta}{|\mathbf{t}_k^\alpha \cdot \mathbf{t}_{ij}^\beta|} \frac{\sum_{\alpha,\beta} |\mathbf{t}_k^\alpha \cdot \mathbf{t}_{ij}^\beta|}{\sum_{\alpha,\beta} \mathbf{t}_k^\alpha \cdot \mathbf{t}_{ij}^\beta}.$$

Flow Chart


Start
Calculate hard process
Set initial t scale
Calculate all overestimates
$t > t_0$
No
Stop
Yes
Accept-Reject(1)
Reject
Accept
$t > t_{FC}^{\text{cut}}$
No
Leading Color Shower
Yes
P_{gq}?
Yes
Color determined by color conservation
No
Calculate $-\frac{\sum_{\alpha,\beta} \mathbf{t}_k^\beta \mathbf{t}_{ij}^\alpha}{\mathbf{T}_{ij}^2}$
Accept-Reject(2)
Reject
Apply weight $\frac{g}{h}\frac{h-g}{g-f}$
Accept
Randomly Select one $\langle \mathcal{M}' | \mathbf{t}_k^\alpha \mathbf{t}_{ij}^\beta | \mathcal{M} \rangle$
Apply weight $\frac{g}{h}$
Apply color weight $\frac{g_{col}}{h_{col}}$

Matching to Leading Color/Hadronization

- Use Lund String Model ($N_C \to \infty$)
- Bookkeep 2 "LC" shower evolution in background
  - One "LC" event uses $N_C \to \infty$ but includes singlet gluons
  - One "LC" event replaces singlet gluons by nonet gluons
- String Fragmentation: gluons interpreted as links in a chain of color dipoles
- Links only induce transverse "kinks" on the string
- Second event ensures that hadronization does not spoil gauge invariance

Results

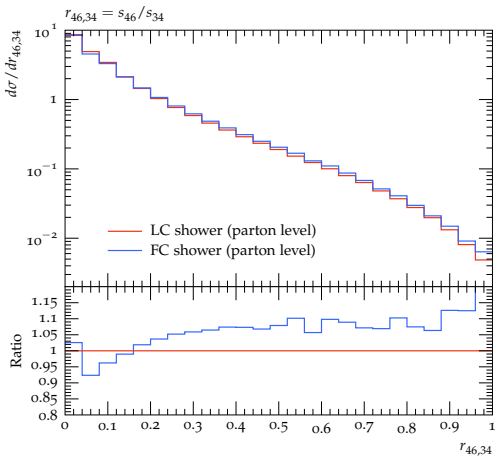
Parameters

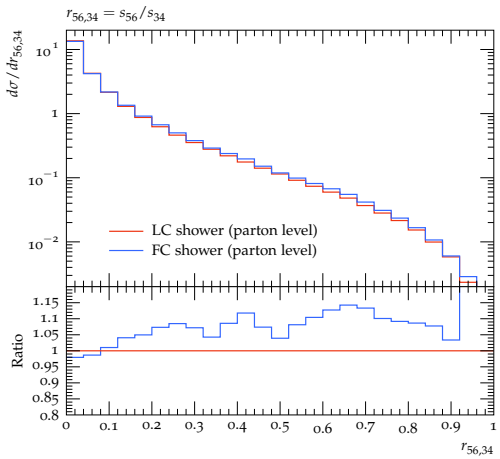
- No inclusion of Γ_2
- DIRE PYTHIA hadronization tune
- $\alpha_s(M_Z) = 0.125$

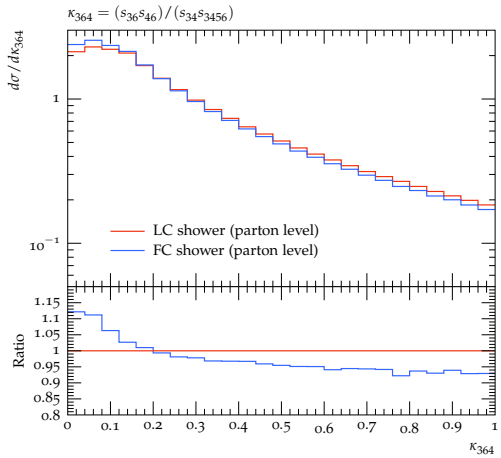
Validation Invariants:

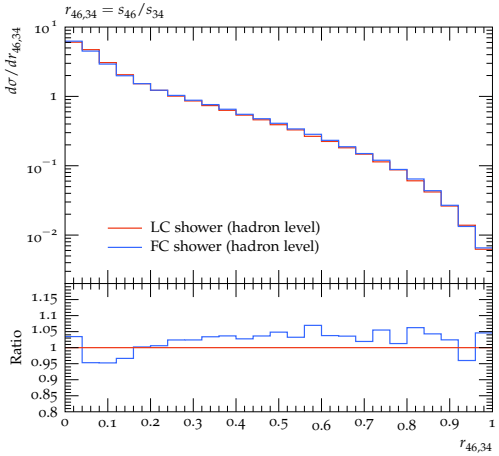
- $\frac{s_{46}}{s_{34}}$
- $\frac{s_{56}}{s_{34}}$
- $\kappa_{364} = \frac{s_{36}s_{64}}{s_{34}s_{3456}}$

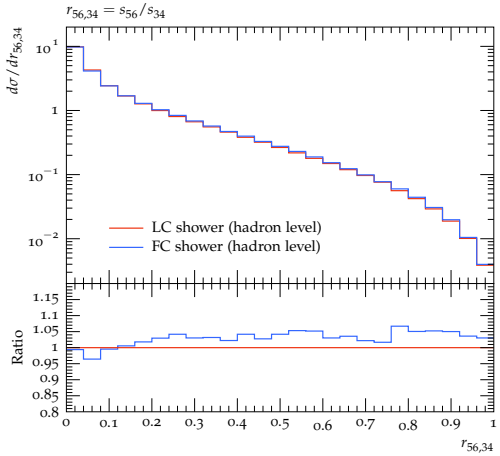
Validation

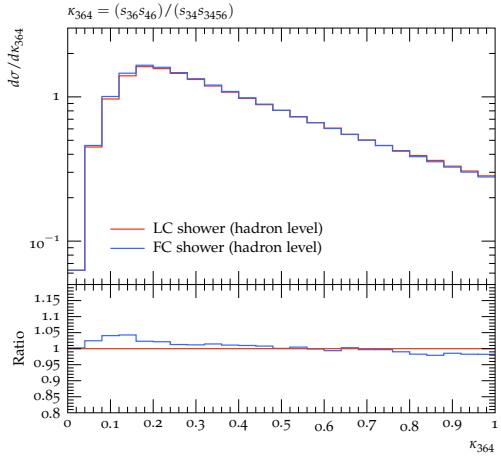

$r_{46,34} = s_{46}/s_{34}$
$d\sigma/dr_{46,34}$
LC shower (parton level)
FC shower (parton level)
Ratio
$r_{46,34}$


$r_{56,34} = s_{56}/s_{34}$
$d\sigma/dr_{56,34}$
LC shower (parton level)
FC shower (parton level)
Ratio
$r_{56,34}$

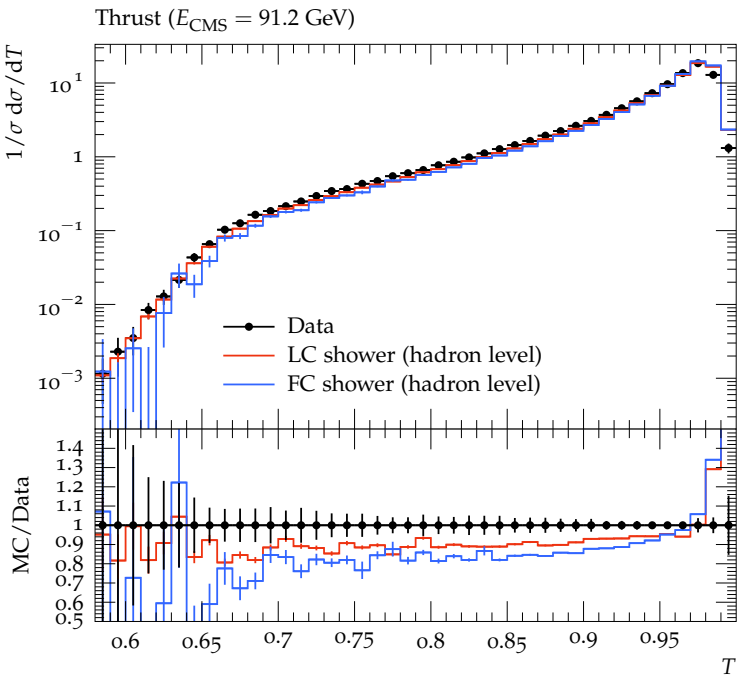

$\kappa_{364} = (s_{36}s_{46})/(s_{34}s_{3456})$
$d\sigma/d\kappa_{364}$
LC shower (parton level)
FC shower (parton level)
Ratio
κ_{364}


$r_{46,34} = s_{46}/s_{34}$
$d\sigma/dr_{46,34}$
LC shower (hadron level)
FC shower (hadron level)
Ratio
$r_{46,34}$

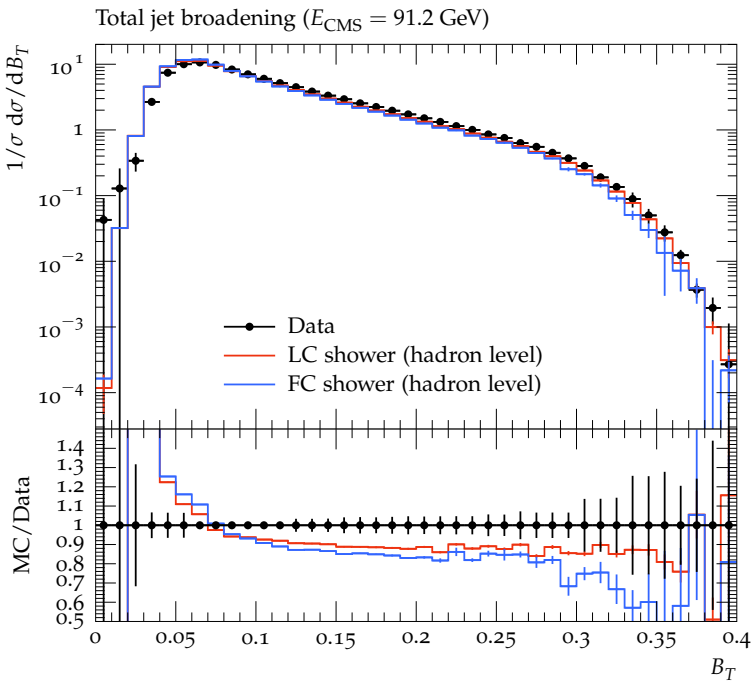

$r_{56,34} = s_{56}/s_{34}$
$d\sigma/dr_{56,34}$
LC shower (hadron level)
FC shower (hadron level)
Ratio
$r_{56,34}$


$\kappa_{364} = (s_{36}s_{46})/(s_{34}s_{3456})$
$d\sigma/d\kappa_{364}$
LC shower (hadron level)
FC shower (hadron level)
Ratio
κ_{364}

Results:

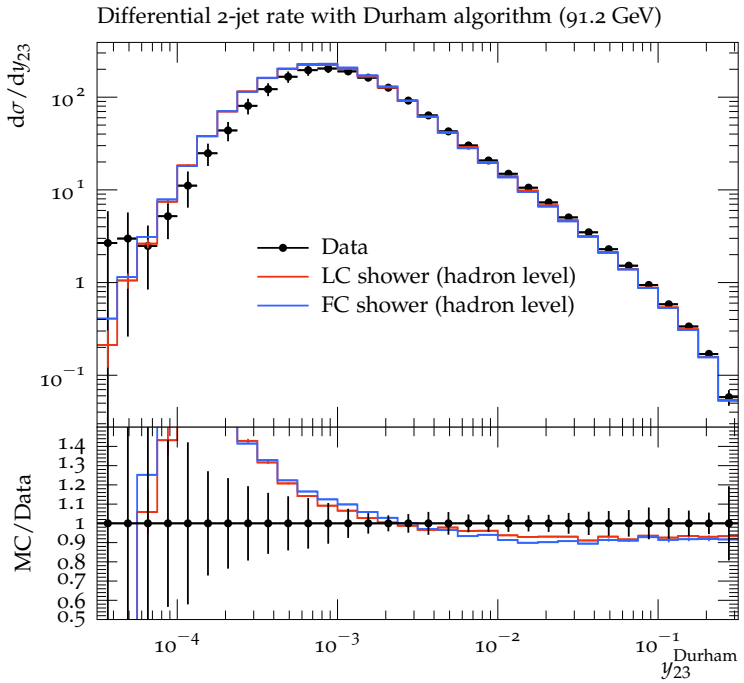


Thrust as measured by ALEPH

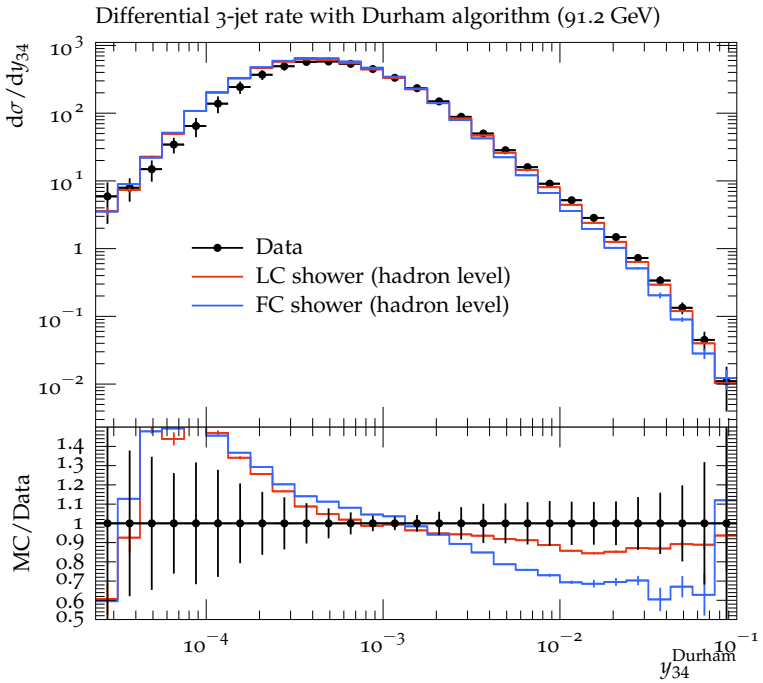


Total jet broadening as measured by ALEPH

Results:



Jet separation between two- and three-jet configurations in the Durham algorithm, as measured by OPAL



Jet separation between three- and four-jet configurations in the Durham algorithm, as measured by OPAL

Conclusions

Conclusions:

- Steps toward a Full Color Parton Shower
- Able to handle any number of emissions (additional cutoff for stability)
- Matching to Hadronization when including "singlet" gluons

Future Steps:

- Match to $\mathcal{O}\left(\alpha_s^2\right)$ corrections to recover Γ_2
- Extend to initial-state splittings
- Study interplay of sub-leading color, kinematics, spin, and higher-order corrections

Backup