UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Automated calculations of two-loop soft functions in Soft-Collinear Effective Theory

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based on work with Guido Bell, Bahman Dehnadi, Tobias Mohrmann (Siegen) and Jim Talbert (DESY)

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Outline

1. ***Resummation in SCET***
    (a) Large logarithms
    (b) SCET and Factorisation
    (c) Renormalisation group resummation
2. ***Universal soft functions - dijet***
    (d) Generic statements
    (e) NLO application
    (f) NNLO application
    (g) `SoftSERVE`
3. ***N-jet soft functions***

Large logarithms

- Perturbative calculations involving widely separated scales exhibit large *Sudakov logarithms* spoiling convergence

- Examples:
  - Event shapes
  - p_T spectra
  - …

$$\alpha_s^n \ln^{2n} \tau$$

(Thrust)

$$\alpha_s^n \ln^{2n} \left(\frac{m_Z^2}{p_T^2}\right)$$

(Z @ small p_T)

- Origin: soft and collinear radiation, constrained real emission

- Analytic resummation:
  - "direct QCD": coherent branching [Catani, Trentadue, Turnock, Webber, '93]
  - "Factorisation": RG evolution [Collins, Soper, Sterman, '85] SCET

SCET: Introduction

[Bauer, Fleming, Pirjol, Stewart, '00]
[Beneke, Chapovsky, Diehl, Feldmann, 02]

- SCET is an effective theory for soft and collinear QCD field modes
- It is highly observable dependent (field modes, relative scaling,…)
- Formalises the expansion of QCD near the problematic limit
- Allows us to derive *factorisation theorems*

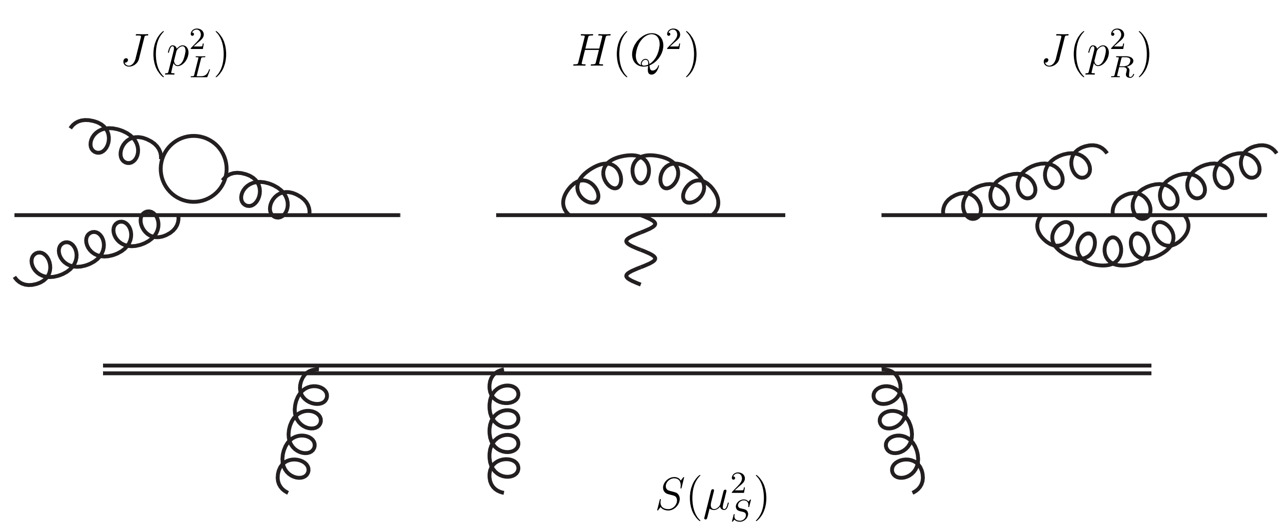
SCET: Factorisation

- If the observable is compatible, SCET matrix elements factorise:

$$|C_V|^2 \sum_X |\langle 0|\mathcal{O}_{n\bar{n}}|X\rangle|^2$$
$$= |C_V|^2 \,\langle 0| \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right] \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right]^\dagger |0\rangle \,\langle 0| \left[W^0_n \zeta^0_n\right] \left[W^0_n \zeta^0_n\right]^\dagger |0\rangle \,\langle 0| \left[S^\dagger_{\bar{n}} S_n\right] \left[S^\dagger_{\bar{n}} S_n\right]^\dagger |0\rangle$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \int dp_R^2 \;\; J(p_L^2,\mu) \;\; J(p_R^2,\mu) \;\; S(\tau Q - \frac{p_L^2 + p_R^2}{Q}, \mu)$$

(for dijet thrust)



SCET: Resummation

- Functions in factorisation theorem only know one scale each
  e.g. for Thrust:

$$\mu_h = Q,\ \mu_c = \sqrt{\tau}Q,\ \mu_s = \tau Q$$

- On their own they also require regularisation

- Resummation in SCET:
  - Renormalise
  - Run from the natural scales to a common scale
  - RG running exponentiates logarithms

- We need anomalous dimensions and renormalised functions

- Added difficulty: SCET_2 type observables show rapidity divergences
  - ➡ Additional analytic regulator required
  - ➡ Resummation via *collinear anomaly* or *rapidity renormalisation group*

[Becher, Bell, '11]
[Becher, Neubert, '10]
[Chiu, Jain, Neill, Rothstein, '11]

Motivation for Automation

- For NNLL resummation, some two-loop ingredients are required, we look at soft functions
- So far we proceed observable by observable individually, e.g.:
  - Thrust
    [Becher, Schwartz, '08]
  - C-Parameter
    [Hoang, Kolodrubetz, Mateu, Stewart, '14]
  - Angularities
    [Bell, Hornig, Lee, Talbert, WIP]
  - …
  - Threshold Drell-Yan
    [Becher, Neubert, Xu, ’07]
  - W/Z/H @ large p_T
    [Becher, Bell, Lorentzen, Marti, '13,'14]
  - Jet veto
    [Becher et al. ’13, Stewart et al., ’13]
  - …
- Can this be made more systematic? It’s possible in full QCD…

Automation in QCD: CAESAR/ARES: [Banfi, Salam, Zanderighi, ’04], [Banfi, McAslan, Monni, Zanderighi, ’14]

Universal dijet soft functions

- The generic form of the dijet soft function we get from the factorisation:

$$S(\tau,\mu) = \frac{1}{N_c}\sum_X \mathcal{M}(\tau,k_i)\ \mathrm{Tr}\ |\langle 0|S^\dagger_{\bar{n}}(0)S_n(0)|X\rangle|^2 \qquad S_n(x) = Pexp(ig_s\int_{-\infty}^{0} n\cdot A_s(x+sn)ds)$$

- The **matrix element** is *independent of the observable* and is the source of divergences

- The **measurement function** (M) is *observable dependent* and harmless, e.g.

$$\mathcal{M}_{thrust}(\tau,\{k_i\}) = \exp\left(-\tau\textstyle\sum_i \min(k_i^+,k_i^-)\right)$$

(in Laplace space)

- **Idea**: isolate singularities at each order and calculate the associated coefficient numerically:

$$\mathcal{S}(\tau) \sim 1 + \alpha_s\{\frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon^1} + c_0\} + \mathcal{O}(\alpha_s^2)$$

Universal soft functions: NLO

- The virtual corrections are scaleless in dim reg, so the NLO soft function is:

$$S^{(1)}(\tau,\mu) = \frac{\mu^{2\varepsilon}}{(2\pi)^{d-1}} \int \delta(k^2)\,\theta(k^0)\,\frac{16\pi\alpha_s C_F}{k_+ k_-}\,\mathcal{M}(\tau,k)\,d^d k$$

- To disentangle the soft and collinear divergences we parametrise suitably:

$$k_- \to \frac{k_T}{\sqrt{y}} \qquad k_+ \to k_T\sqrt{y}$$

- We also must specify the measurement function *M*, and assume its form:

$$\mathcal{M}^{(1)}(\tau,k) = \exp\left(-\tau\, k_T\, y^{\frac{n}{2}}\, f\,(y,\vartheta)\right)$$

- The k_T integration can then be performed analytically, and yields the master formula:

$$S^{(1)}(\tau,\mu) \sim \Gamma(-2\epsilon)\int_0^\pi \mathrm{d}\vartheta \int_0^1 \mathrm{d}y\; y^{-1+n\epsilon} f(y,\vartheta)^{2\epsilon}$$

Measurement functions: NLO examples

$$\mathcal{M}^{(1)}(\tau, k) = \exp\left(-\tau\, k_T\, y^{\frac{n}{2}}\, f\,(y, \vartheta)\right)$$

Observable	n	$f(y,\vartheta)$
Thrust	1	1
Angularities	$1-A$	1
Recoil-free broadening	0	$1/2$
C-Parameter	1	$1/(1+y)$
Threshold Drell-Yan	-1	$1+y$
W @ large p_T	-1	$1+y-2\sqrt{y}\,\cos\theta$
e^+e^- transverse thrust	1	$\frac{1}{s\sqrt{y}}\left(\sqrt{\left(c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right)^2+1-\cos^2\theta}-\left\lvert c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right\rvert\right)$

- For transverse thrust, $s=\sin\theta_B$, $c=\cos\theta_B$, with $\theta_B = \angle$beam axis, thrust axis

Assumptions and classification: NLO

- *Assume*: Exponential function, motivated by Laplace space

$$\exp(-\tau\omega(\{k_i\})) = \int_0^\infty d\omega \exp(-\tau\omega)\,\delta(\omega - \omega(\{k_i\}))$$

- *Assume*: ω is *linear* in mass dimension

$$\mathcal{M} = \exp(-\tau k_T \hat{f}(y, \vartheta))$$

- *Classify*: How does the observable behave as y vanishes?

$$\mathcal{M} = \exp(-\tau k_T y^{\frac{n}{2}} f(y, \vartheta))$$

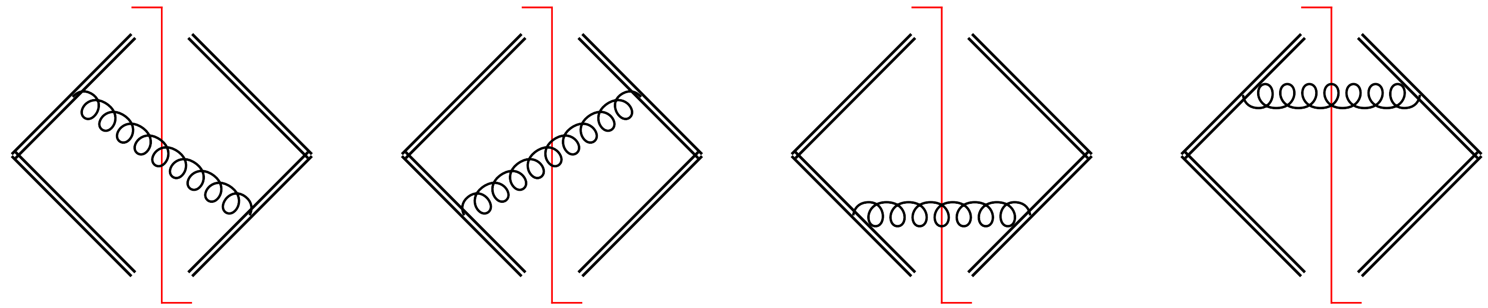
- *Assume*: f positive and non-vanishing over almost all of phase space

- This is enough to ensure the behaviour of the observable is under control in the critical limits:

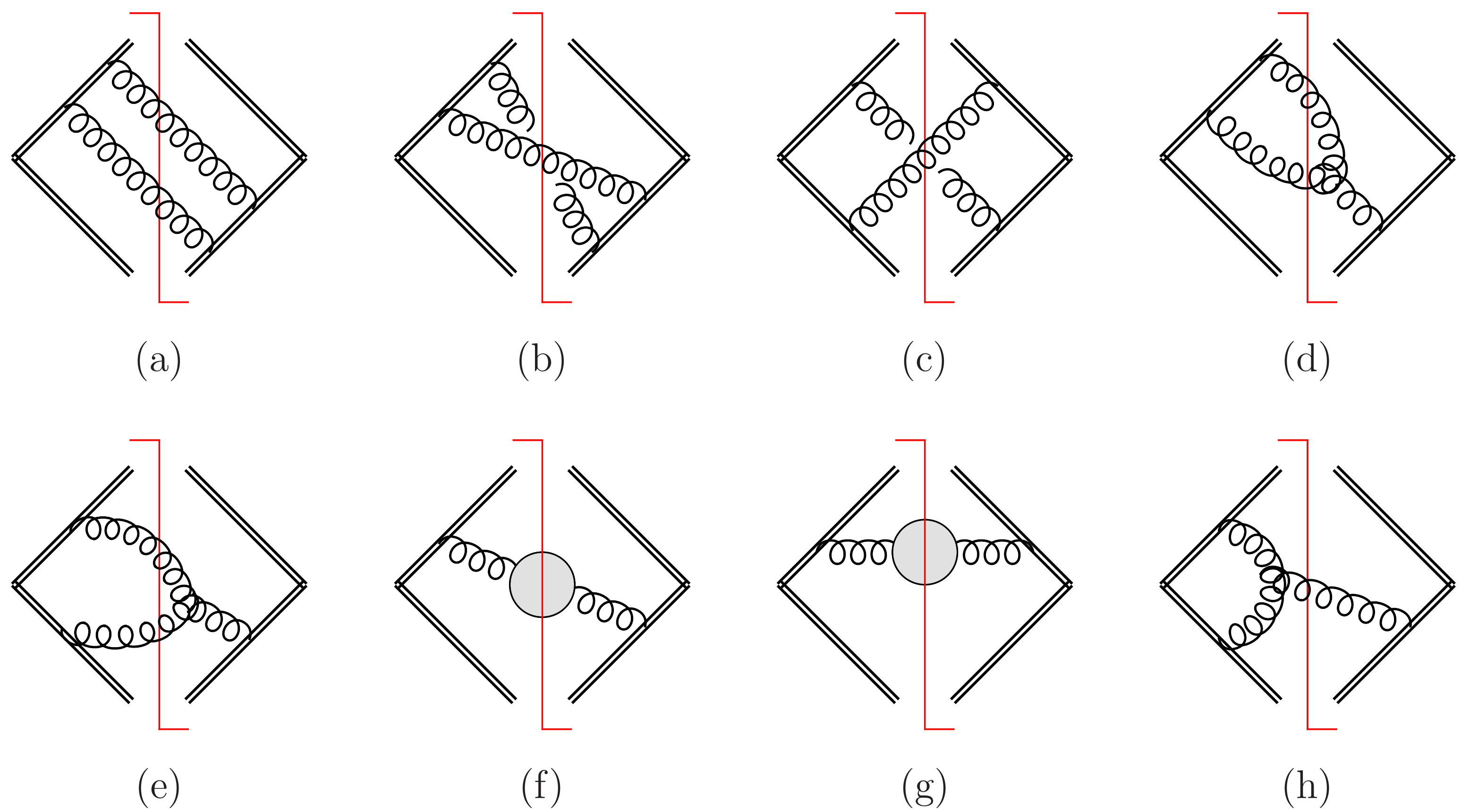
Soft	$(k_T \to 0)$	$\Rightarrow$	vanishes, fixed by mass dimension
Collinear	$(y \to 0)$	$\Rightarrow$	f finite

Universality: NLO vs. NNLO

NLO:



NNLO:



Universality: NLO vs. NNLO

- Consider the double real emission:

$$S^2_{RR}(\tau) = \frac{\mu^{4\epsilon}}{(2\pi)^{2d-2}} \int d^dk\ \delta(k^2)\ \theta(k^0) \int d^dl\ \delta(l^2)\ \theta(l^0)\ |\mathcal{A}(k,l)|^2\ \mathcal{M}(\tau,k,l)$$

- The matrix elements are no longer nice and easy, see e.g., the $C_F T_F n_f$ color structure:

$$|\mathcal{A}(k,l)|^2 = 128\pi^2 \alpha_s^2 C_F T_F n_f \frac{2k\cdot l(k_- + l_-)(k_+ + l_+) - (k_- l_+ - k_+ l_-)}{(k_- + l_-)^2 (k_+ + l_+)^2 (2k\cdot l)^2}$$

- The singularities are partially overlapping, not as easy to extract, but it's possible
- We then again assume the form of the measurement function:

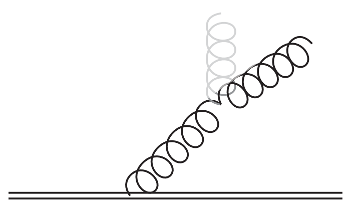
$$\mathcal{M}^{(2)}(\tau,k,l) = \exp\left(-\tau\, p_T\, y^{\frac{n}{2}}\ F\left(y,a,b,\vartheta,\vartheta_k,\vartheta_l\right)\right)$$

- Why is this enough?

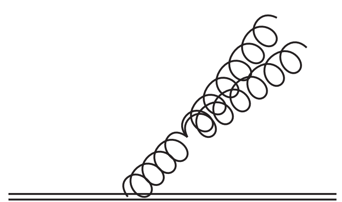
2-loop - Correlated emissions: $C_F C_A$, $C_F T_f n_f$

- Matrix element divergent in four critical limits: Behaviour

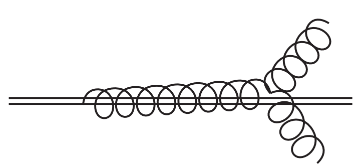
(Global soft) — fixed by mass dimension



(Individual soft) — fixed by IR safety



(emissions collinear) — fixed by collinear safety



(Global "jet-collinear") — unconstrained → Classify!

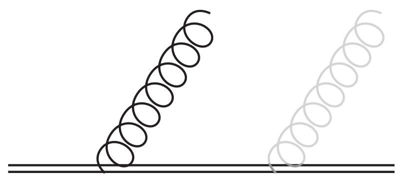
- Only one unconstrained variable
- Variable definition ensures commuting limits

$$\mathcal{M}^{(2)}(\tau,k,l) = \exp\left(-\tau\, p_T\, y^{\frac{n}{2}}\, F\left(y,a,b,\vartheta,\vartheta_k,\vartheta_l\right)\right)$$

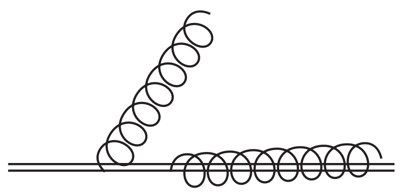
2-loop - Uncorrelated emissions: C_F^2

- 4 critical limits: Behaviour

(Global soft) fixed by mass dimension



(individual soft) fixed by IR safety

(

(one emission “jet-collinear”) unconstrained $)^2$

- Two “unconstrained” limits
- Worse: *Overlapping zeroes*:

$$\omega(k,l) = k_T\, y_k^{\frac{n}{2}}\, f(y_k) + l_T\, y_l^{\frac{n}{2}}\, f(y_l)$$

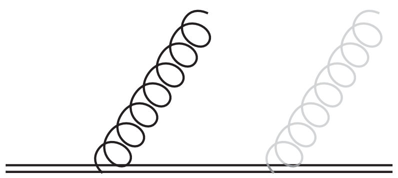
- Solution: adapt parametrisation for k_T, l_T:

$$k_T = q_T \frac{b}{1+b}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n,\ l_T = q_T \frac{1}{1+b}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n$$

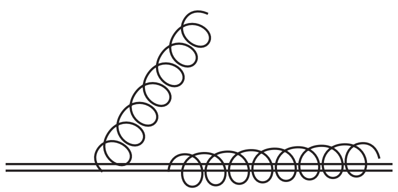
2-loop - Uncorrelated emissions: C_F^2

- 4 critical limits: Behaviour

(Global soft) — fixed by mass dimension



(individual soft) — fixed by IR safety

(

(one emission "jet-collinear") — unconstrained $)^2$

- Two "unconstrained" limits
- Worse: *Overlapping zeroes*:

$$\omega(k,l) = k_T\, y_k^{\frac{n}{2}}\, f(y_k) + l_T\, y_l^{\frac{n}{2}}\, f(y_l)$$

- Solution: adapt parametrisation for k_T, l_T:

$$k_T = q_T \frac{b}{1+b}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n, \; l_T = q_T \frac{1}{1+b}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n$$

2-loop - Uncorrelated emissions: C_F^2

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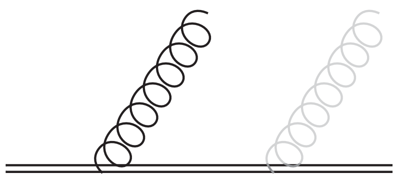

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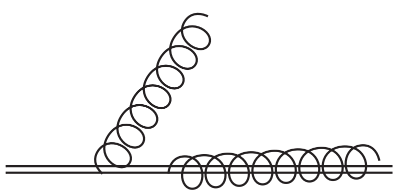
(

(one emission “jet-collinear”) unconstrained)^2

- Suitable parametrisation solves overlapping limit problem

$$\mathcal{M}^{(2)}(\tau; k, l) = \exp\left(-\tau\, q_T\, y_k^{\frac{n}{2}}\, y_l^{\frac{n}{2}}\, G(y_k, y_l, b, \vartheta, \vartheta_k, \vartheta_l)\right)$$

The completed framework

- Semi-Analytic expressions are available for anomalous dimensions

  [1805.12414]

- For finite parts we have an implementation using *pySecDec* and a dedicated C++ based program:

  [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke, 1703.09692]

(Soft function Simulation and Evaluation
for Real and Virtual Emissions)

- `SoftSERVE` uses the *Cuba* library's *Divonne* integrator, implements numerical improvements, and has multiprecision variable support

  [Hahn, hep-ph/0404043]

- Soon™ to be found on `HEPForge`, paper in preparation

SoftSERVE

- First release will be correlated emissions only
- User required input (C++ syntax):
  - correlated: two functions F (roughly: one for each hemisphere)
  - uncorrelated: three functions G
  - two parameters
  - optional: parameter values, integrator settings
- Two integrator settings pre-defined
- Scripts available for renormalisation in Laplace and momentum space, and for dealing with Fourier space observables

A few results

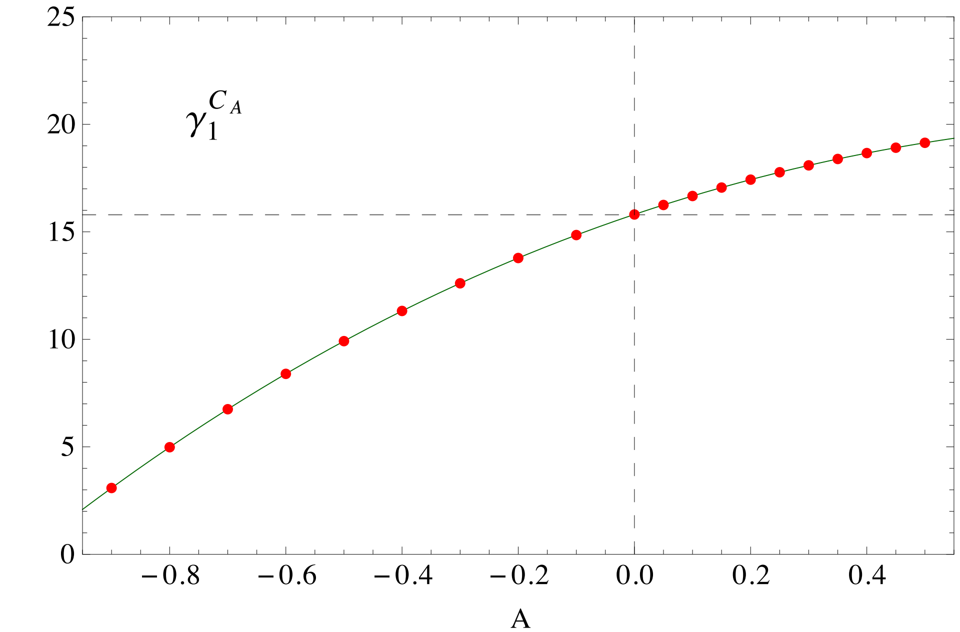
Soft function	$\gamma_1^{C_A}$	$\gamma_1^{n_f}$	$c_2^{C_A}$	$c_2^{n_f}$
Thrust [Kelley et al, '11] [Monni et al, '11]	15.7945 (15.7945)	3.90981 (3.90981)	-56.4992 (-56.4990)	43.3902 (43.3905)
C-Parameter [Hoang et al, '14]	15.7947 (15.7945)	3.90980 (3.90981)	-57.9754 [-58.16 ± 0.26]	43.8179 [43.74 ± 0.06]
Threshold Drell-Yan [Belitsky, '98]	15.7946 (15.7945)	3.90982 (3.90981)	6.81281 (6.81287)	-10.6857 (-10.6857)
W @ large p_T [Becher et al, '12]	15.7947 (15.7945)	3.90981 (3.90981)	-2.65034 (-2.65010)	-25.3073 (-25.3073)
Transverse Thrust [Becher, Garcia, Piclum, '15]	-158.278 [$-148\pm^{20}_{30}$]	19.3955 [$18\pm^{2}_{3}$]	—	—

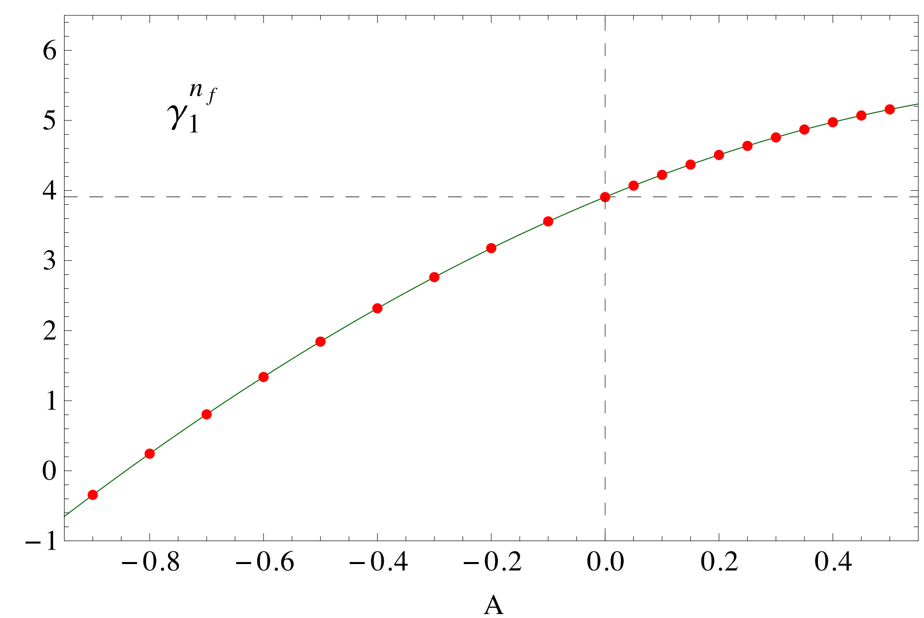
$$\gamma_1 = \gamma_1^{C_A} C_F C_A + \gamma_1^{N_f} C_F T_F n_f \qquad c_2 = c_2^{C_A} C_F C_A + c_2^{N_f} C_F T_F n_f + \frac{1}{2}(c_1)^2$$

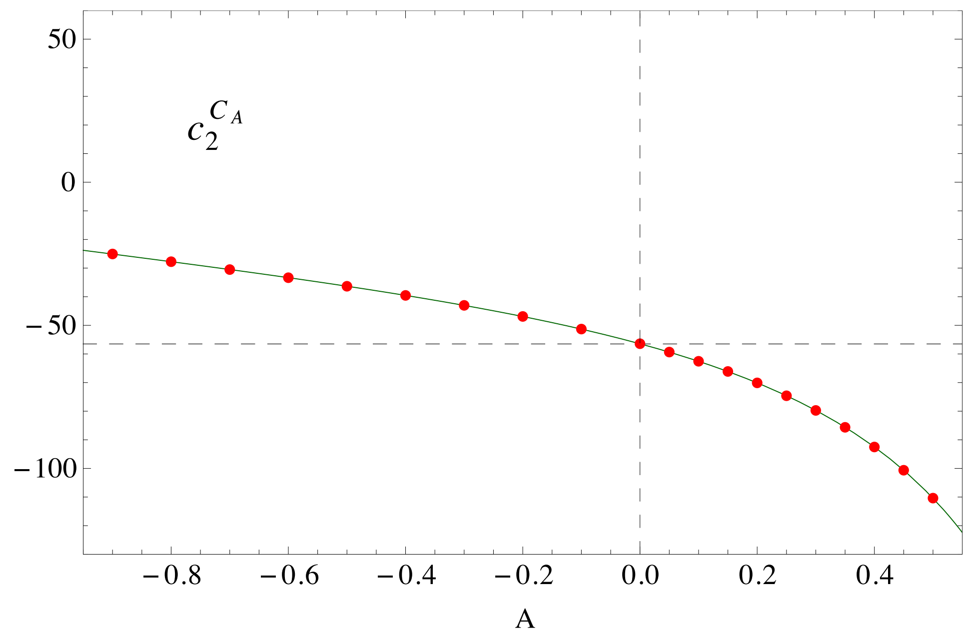
- Derived in few minutes to hours on an 8 core desktop machine
- Deviations from analytic results compatible with 1σ error estimate

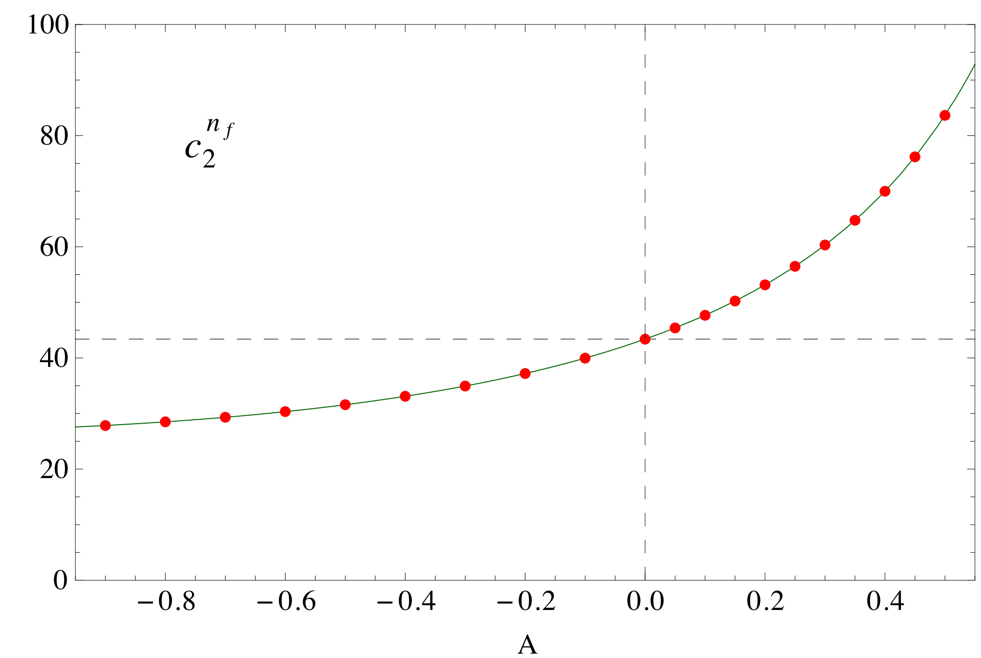
Results: Angularities

- Generalisation of thrust
- Obeys non-abelian observation
- New result, will feature in NNLL′ resummation paper soon™



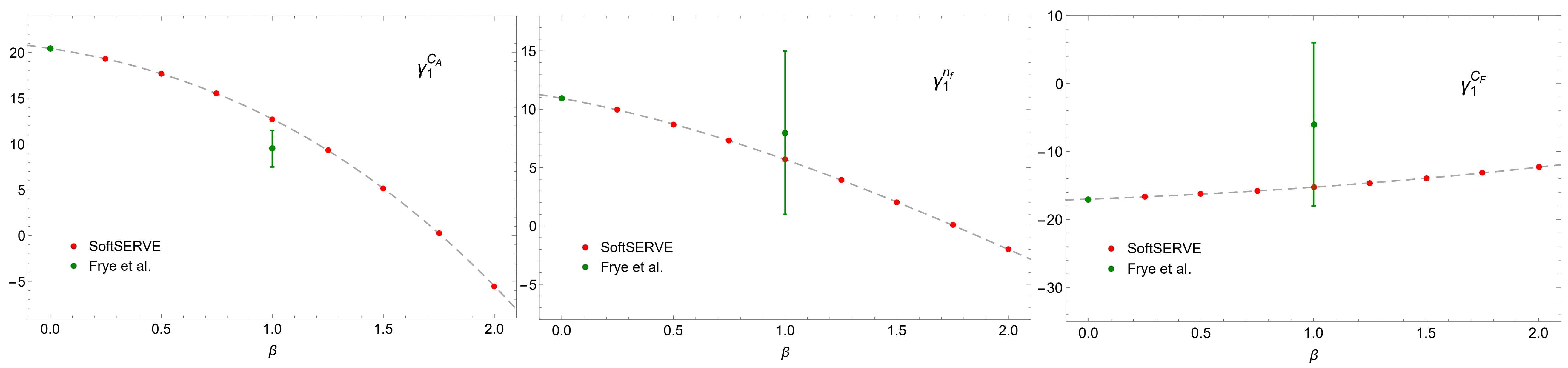






Results: Soft drop jet mass

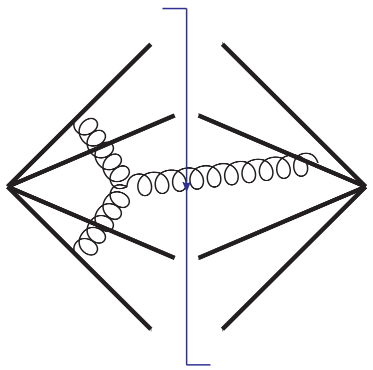
- Jet grooming procedures remove radiation from jets to reveal substructure
- For the *soft drop* groomer multiple observables have been proposed and factorised in [Frye et al, 1603.09338].
- For Cambridge / Aachen clustering and jet mass as the observable, EVENT2 fits were presented for the anomalous dimension
- Breaks non-abelian exponentiation



Extension to N jet directions

- There are now more jet/beam directions -> more Wilson lines:

$$S(\tau,\mu) = \sum_X \mathcal{M}(\tau, k_i)\, \langle 0|\, (S_{n_1} S_{n_2} S_{n_3} \ldots)^\dagger\, |X\rangle\langle X| S_{n_1} S_{n_2} S_{n_3} \ldots |0\rangle$$



  - Tripole and Quadrupole diagrams
    - ✓ Assume non-abelian exponentiation: only one tripole (RV)
  - Dipole directions are no longer back-to-back
    - ✓ Use boost-invariant parametrisation
    - ✓ Consequence: transverse space can acquire temporal direction

$$k = (k \cdot n, k \cdot \bar{n}, k_\perp)$$

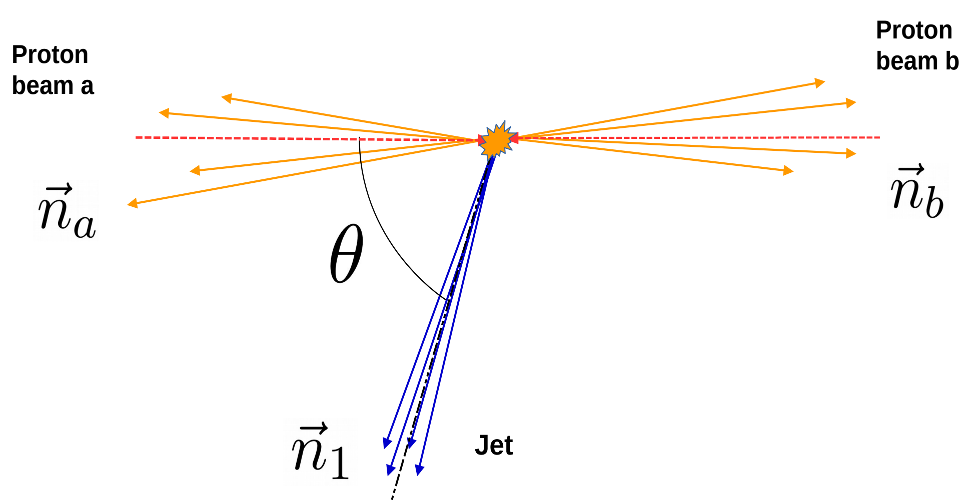
$$k = (k \cdot n_a, k \cdot n_b, k_\perp)$$

  - more complicated angular integrations
    - ✓ 5 angles instead of 3 at NNLO
  - external geometry must be sampled

1-Jettiness

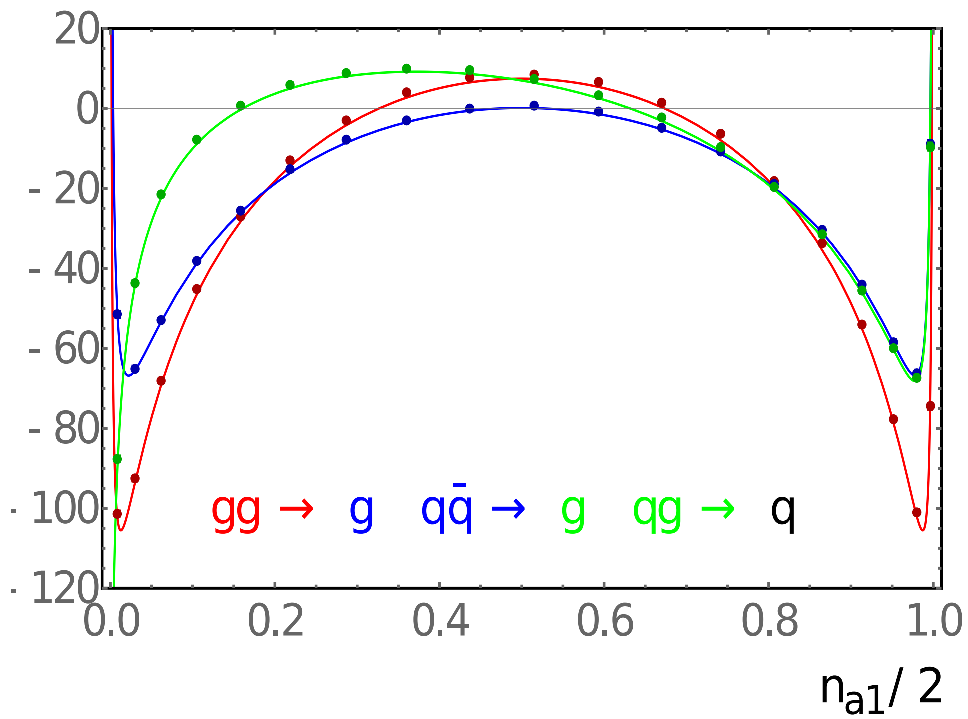
[Boughezal, Liu, Petriello, '15; Campbell, Ellis, Mondini, Williams, '17]

- Kinematics and Sampling



$$n_a \cdot n_b = 2$$
$$n_a \cdot n_1 = 1 - \cos\theta = n_{a1}$$
$$n_b \cdot n_1 = 1 + \cos\theta$$

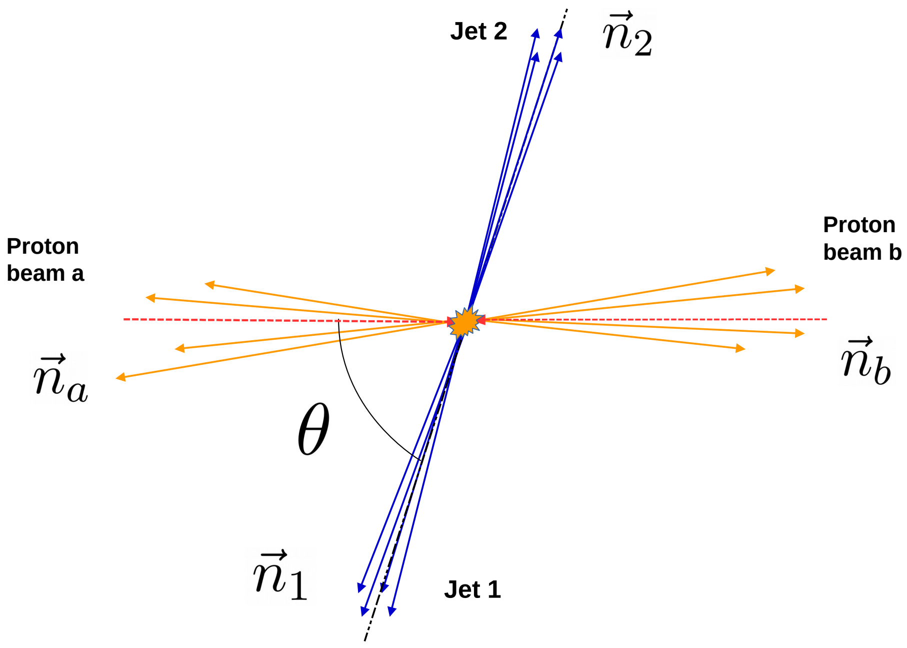
- Finite terms for different channels (preliminary)



Dots: [Bell, Dehnadi, Mohrmann, RR, in preparation]
Lines: [Campbell, Ellis, Mondini, Williams, '17]

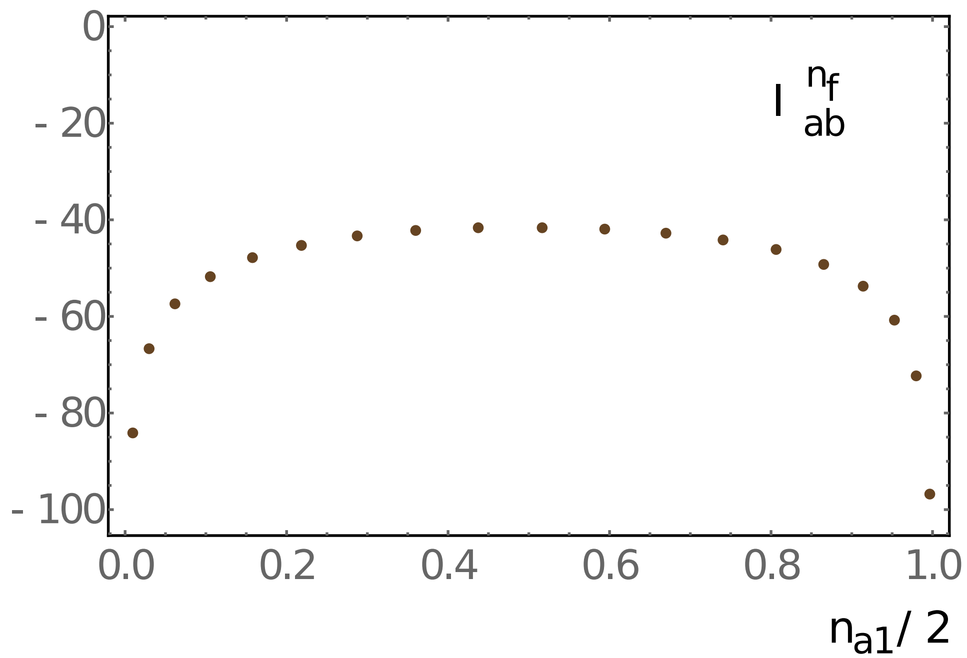
2-Jettiness

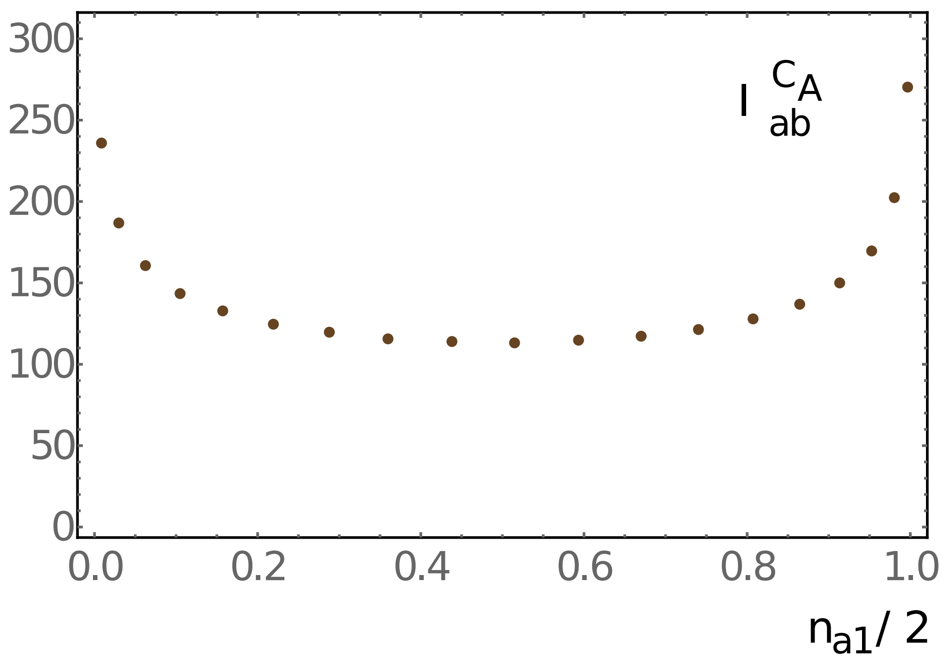
- Kinematics and Sampling



$$n_a \cdot n_b = n_1 \cdot n_2 = 2$$
$$n_a \cdot n_1 = n_b \cdot n_2 = 1 - \cos\theta = n_{a1}$$
$$n_a \cdot n_2 = n_b \cdot n_1 = 1 + \cos\theta$$

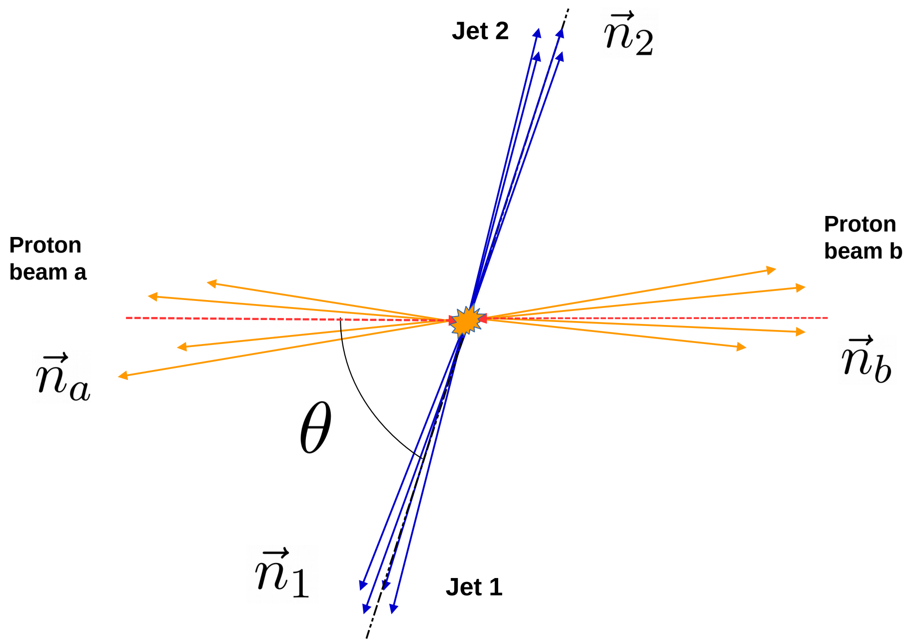
- Some preliminary results - dipoles





2-Jettiness

- Kinematics and Sampling



$$n_a \cdot n_b = n_1 \cdot n_2 = 2$$
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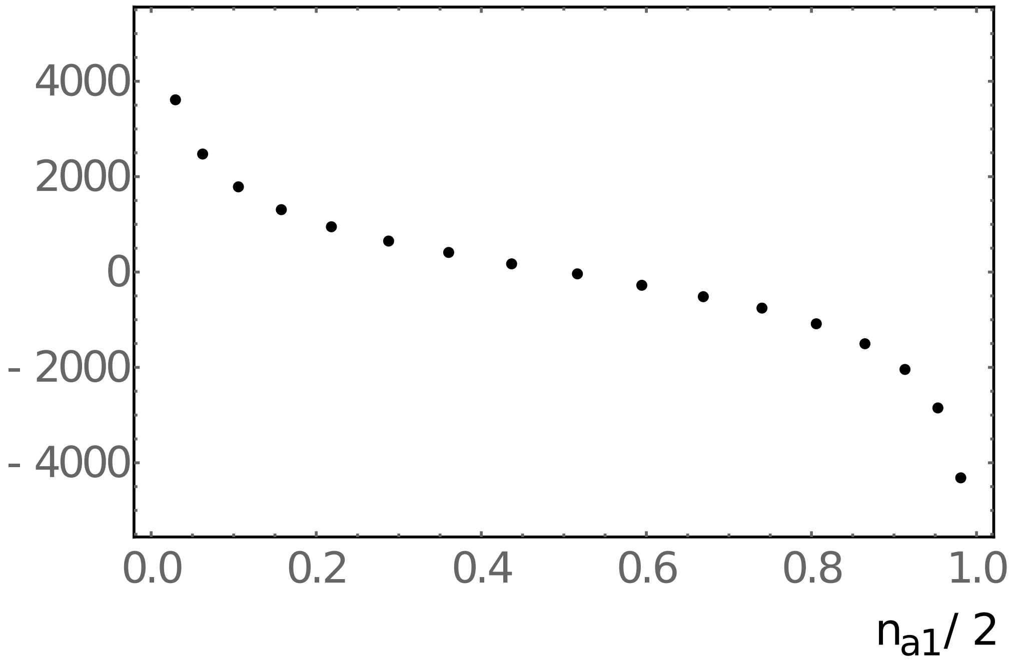
$$n_a \cdot n_2 = n_b \cdot n_1 = 1 + \cos\theta$$

- Some preliminary results - tripole



Conclusion

- SCET provides an efficient, analytic approach to high-order resummations necessary for precision collider physics.

- We have developed a framework to systematically compute generic *NNLO* dijet soft functions for wide ranges of observables at lepton and hadron colliders

- The program(s) based on this framework will soon™ be released into the wild

- An extension to N-jet observables seems possible, and we have already re-derived a few known results and are working on new ones

That's all folks!

Thank you!

Parametrisation uncorrelated

The parametrisation for the uncorrelated emissions

$$k_+ = q_T \frac{b}{1+b}\sqrt{y_k}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n \qquad k_- = q_T \frac{b}{1+b}\frac{1}{\sqrt{y_k}}\left(\frac{\sqrt{y_l}}{1+y_l}\right)^n$$

$$l_+ = q_T \frac{1}{1+b}\sqrt{y_l}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n \qquad l_- = q_T \frac{1}{1+b}\frac{1}{\sqrt{y_l}}\left(\frac{\sqrt{y_k}}{1+y_k}\right)^n$$

leads to divergences in b, y_k, y_l, q_T (analytic)

$$y_k = \frac{k_+}{k_-} \qquad b = \sqrt{\frac{k_+k_-}{l_+l_-}}^{\,1+n}\left(\frac{l_+ + l_-}{k_+ + k_-}\right)^n$$

$$y_l = \frac{l_+}{l_-} \qquad q_T = \sqrt{l_+l_-}\left(\frac{k_+ + k_-}{\sqrt{k_+k_-}}\right)^n + \sqrt{k_+k_-}\left(\frac{l_+ + l_-}{\sqrt{l_+l_-}}\right)^n$$

Parametrisation correlated

The parametrisation for the correlated emissions

$$k_+ = p_T \frac{b}{a+b}\sqrt{y} \qquad k_- = p_T \frac{ab}{1+ab}\frac{1}{\sqrt{y}}$$

$$l_+ = p_T \frac{a}{a+b}\sqrt{y} \qquad l_- = p_T \frac{1}{1+ab}\frac{1}{\sqrt{y}}$$

leads to divergences in y, *b*, p_T (analytic), and an overlapping divergence in $a \to 1$ (with transverse angle)

$$a = \sqrt{\frac{k_- l_+}{l_- k_+}} = \sqrt{\frac{y_l}{y_k}} \qquad b = \sqrt{\frac{k_+ k_-}{l_+ l_-}} = \frac{k_T}{l_T}$$

$$y = \frac{k_+ + l_+}{k_- + l_-} \qquad p_T = \sqrt{(k_+ + l_+)(k_- + l_-)}$$