Nested soft-collinear subtractions for color singlet production and decay

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Infrared singularities in QCD

IR singularities in higher order corrections from:

- **Virtual corrections – explicit** IR singularities amplitudes.
- **Real corrections:**
  - Unresolved phase space: emitted particle is soft and/or collinear.
  - IR singularities **after integration.**

To get **fully differential** results from **numerical** (Monte Carlo) **integration:**

Extract and cancel all singularities *prior* to integration.

- Solved at NLO (Catani-Seymour, Frixione-Kunszt-Signer,...).
  - **Essential precursor** to NLO revolution & automation of NLO calculations.
- **Highly non-trivial at NNLO:** multiple soft/collinear limits which may **overlap** – can approach a limit in different ways.

IR singularities at NNLO

– Slicing: $$\int |\mathcal{M}|^2 F_J d\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J d\phi_d\right]_{\text{s.c.}} + \int_\delta^\infty |\mathcal{M}_J|^2 F_J d\phi_4 + \mathcal{O}(\delta)$$

Born-like NLO+jet

- qT [Catani, Grazzini '07]
- N-jettiness [Gaunt *et al* '15; Boughezal *et al* '15]

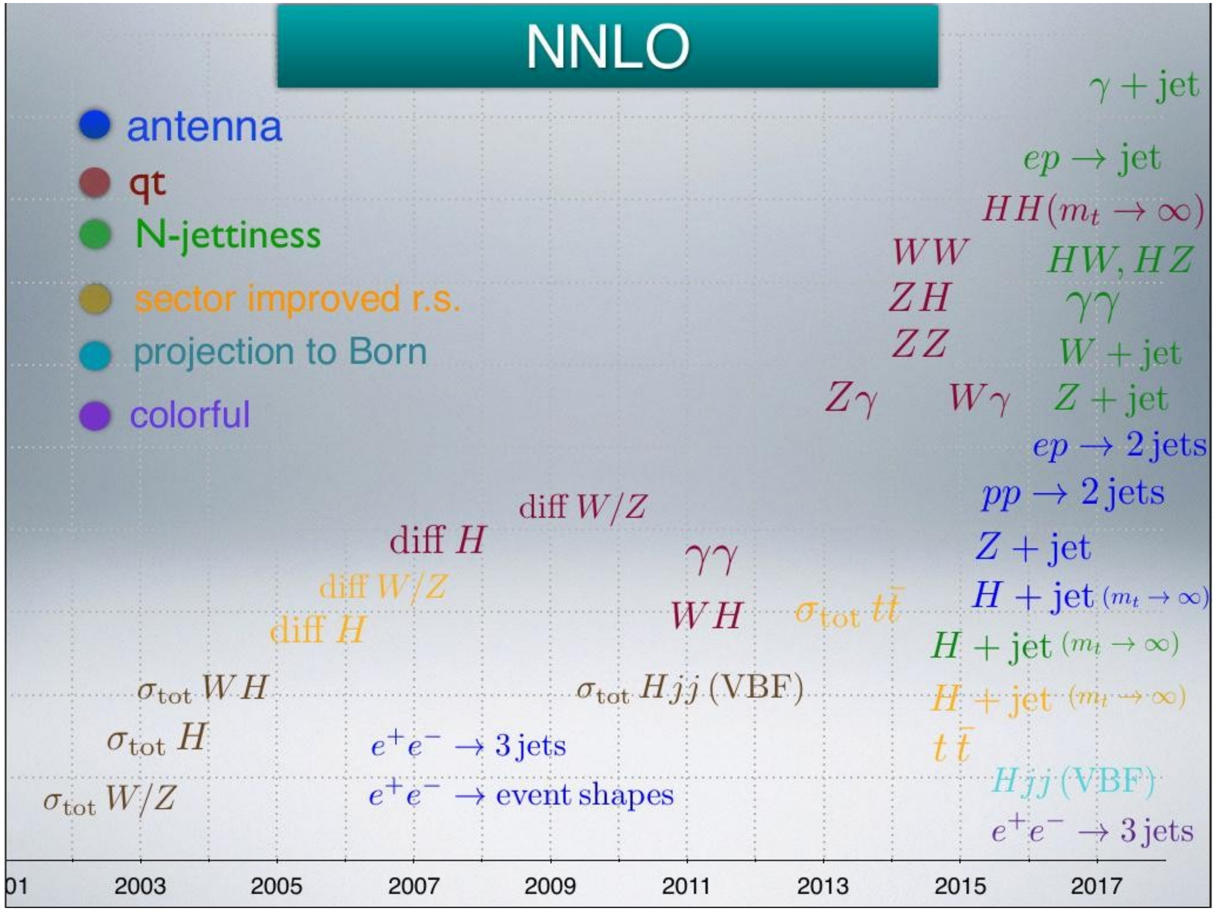
– Subtraction: $$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

- Antenna [Gehrmann-de Ridder, Gehrmann, Glover '05, …]
- STRIPPER [Czakon '10, '11]
- Projection-to-Born [Cacciari *et al* '15]
- CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca '05, ...]
- **Nested soft-collinear [Caola, Melnikov, R.R. '17]**
- Geometric [Herzog '18]
- Local analytic sector [Magnea *et al* '18]

The NNLO Revolution

Great progress in subtraction & slicing methods:

> All 2→2 processes and a few 2→3 processes (with special kinematics) known at **NNLO**.



Slide from Gudrun Heinrich, LHCP2017

The (NNLO) Revolution continues!

Problem solved, but solutions **not optimal** – room for improvement.

Current subtraction schemes:

- Are **complicated** – difficult to implement.
- Obscure the **physical origin of singularities** in intermediate steps.
- Are sometimes **process-dependent.**
- Require **large computational times** and **fast scaling:**
  - ~100 CPU hrs for V (differential)
  - ~100k CPU hrs for V+j (differential).
  - **2→3 processes, e.g.** ***H+2j*** **?**

The (NNLO) Revolution continues!

A "better" subtraction scheme should:

- Be fully local
  - *avoid large numerical cancellations in intermediate steps.*
- Have a minimal structure displaying a clear origin of physical singularities
  - *easier for others to implement.*
- Have explicit, analytic cancellation of poles
  - *control over singular structures.*
- Allow four-dimensional evaluation of amplitudes
  - *improved numerical efficiency.*
- Be process-independent.
- Be flexible
  - *allow freedom in phase-space parametrization/mapping.*

Nested soft-collinear subtraction

[Caola, Melnikov, R.R. '17]

- Extension of FKS subtraction to NNLO.
- Independent subtraction of soft and collinear divergences **(color coherence).**
- Use of sectors (as in STRIPPER) to separate overlapping ***collinear*** singularities.

[Czakon '10, '11]

  - Natural splitting by rapidity.

- Fully local. ✓
- Clear physical origin of singularities (soft & collinear). ✓
- **Recombination** of sectors leading to simplifications in integrated subtraction terms.
  - Final IR structure very transparent.
  - Explicit (not yet fully analytic) pole cancellation (independent of matrix element). ✓
- Allows four-dimensional evaluation of matrix elements. ✓
- Process-independent *in principle* – details only worked out for color singlet hadroproduction & color singlet decay. ✓
- Not tied to phase space parametrization (currently using STRIPPER parametrization of angular phase space). ✓

Current status and outline

- Color singlet production:
  - ✓ Corrections to $q\bar{q} \to V$ (e.g. DY, *VH*, *VV*,...)
  - ✓ Corrections to $gg \to V$ (e.g. *H*, *HH*, ...)
- Color singlet decay:
  - ✓ Corrections to $V \to q\bar{q}$ (e.g. $H \to b\bar{b}$)
  - ✓ Corrections to $V \to gg$
- Extension to initial & final states with color conceptually straightforward:
  - Corrections to DIS.
- Discuss corrections to $q\bar{q} \to V + ng$
  - Most complicated singular structure.

FKS subtraction at NLO: Notation

Consider color singlet production $q(p_1)\bar{q}(p_2) \to V + g(p_4)$:

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}g_4] F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4)\rangle.$$

$$F_{LM}(1,2,4) = \mathrm{dLips}_V \, |\mathcal{M}(1,2,4,V)|^2 \, \mathcal{F}_{\mathrm{kin}}(1,2,4,V) \qquad [\mathrm{d}g_4] = \frac{\mathrm{d}^{d-1}p_4}{(2\pi)^d 2E_4}\theta(E_{\max} - E_4)$$

- dLips_V: Lorentz-inv. Phase space for V **(incl. delta-fn)**
- $|\mathcal{M}(1,2,4,V)|^2$: Matrix-element sq.
- $\mathcal{F}_{\mathrm{kin}}$: IR-safe observable
- $[\mathrm{d}g_4]$: Integration in partonic CoM frame
- $E_{\max}$: Arbitrarily large energy parameter

Define soft and collinear operators:

$$S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A \qquad \rho_{ij} = 1 - \cos\theta_{ij}$$

FKS subtraction at NLO: Subtraction

Remove singular limits and add back as subtraction terms:

$$\begin{aligned}\langle F_{LM}(1,2,4)\rangle = & \langle (I - C_{41} - C_{42})(I - S_4) F_{LM}(1,2,4)\rangle + \\ & \langle S_4 F_{LM}(1,2,4)\rangle + \\ & \langle (C_{41} + C_{42})(I - S_4) F_{LM}(1,2,4)\rangle\end{aligned}$$

- First term: finite, can be integrated numerically in 4-dimensions.
- Second term: soft subtraction term – gluon decouples completely (need upper bound $E_{\rm max}$).
- Third term: collinear and soft+collinear subtraction terms – gluon decouples partially or completely.
- Singularities made explicit by integrating subtraction terms over unresolved gluon.

FKS Subtraction at NLO: Poles

After integrating:

$$\hat{O}_{\rm NLO} \equiv (I - C_{41} - C_{42})(I - S_4)$$

$$2s \cdot \mathrm{d}\sigma^{\rm R} = 2[\alpha_s] s^{-\epsilon} \left(\frac{C_F}{\epsilon^2} + \frac{3C_F}{2\epsilon} \right) \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \langle F_{LM}(1,2) \rangle + \langle \hat{O}_{\rm NLO} F_{LM}(1,2,4) \rangle$$

$$- \frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \int_0^1 \mathrm{d}z \mathcal{P}_{qq,R}(z) \left\langle \frac{F_{LM}(z\cdot 1, 2)}{z} + \frac{F_{LM}(1, z\cdot 2)}{z} \right\rangle .$$

- LO structures with and without boost, and regulated real emission:

$$\langle F_{LM}(1,2) \rangle \qquad \langle F_{LM}(z\cdot 1,2)/z \rangle \qquad \langle F_{LM}(1,z\cdot 2)/z \rangle \qquad \langle \hat{O}_{\rm NLO} F_{LM}(1,2,4) \rangle$$

- Remove soft limits of splitting functions from collinear emission → Altarelli-Parisi kernels

$$\mathcal{P}_{qq,R}(z) = \hat{P}^{(0)}_{qq}(z) + \epsilon \mathcal{P}^{(\epsilon)}_{qq,R}(z)$$

- Poles in first term cancel with virtual, poles in second term cancel with pdf renorm.
- Cancellation occurs ***within each structure!***

FKS subtraction at NLO: finite result

After cancelling poles, we can take the $\epsilon \to 0$ limit and compute everything in four dimensions.

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F F_{LM}(1,2)\right]\right\rangle + \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\rangle +$$
$$+ \frac{\alpha_s(\mu)}{2\pi}\int_0^1 \mathrm{d}z \left[\ln\frac{s}{\mu^2}\hat{P}_{qq}^{(0)}(z) - \mathcal{P}_{qq,R}^{(\epsilon)}(z)\right]\left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z}\right\rangle.$$

Sum of:

- **LO-like terms,** with or without convolutions with splitting functions.
- Real emission term, with singular configurations removed by iterated subtraction.
- Finite remainder of virtual corrections.

NNLO subtraction scheme

Aim to replicate NLO subtraction results *as much as possible:*

- Explicit (ideally analytical) cancellation of poles in each kinematic structure, *before* numerical implementation.
- Numerical implementation of finite result only: four-dimensional matrix elements.
- Finite result: (*relatively*) simple functions multiplying lower multiplicity structures – i.e. LO-like or NLO-like, with and without boosts – and regulated double-real term.

NNLO: Real-real Corrections

Real-real corrections – process $q\bar{q} \to V + gg$.

$$2s \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2!} \int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5).$$

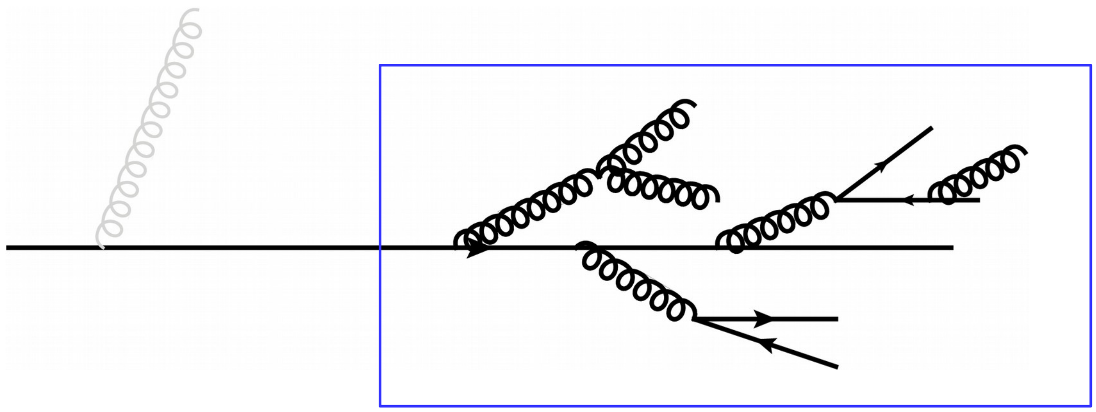
Singularity structure much more complicated:

- g_4 or g_5 → soft.
- g_4 or g_5 → collinear to initial state partons.
- g_4 or g_5 → collinear to each other.
- Combination of the above – can approach **each limit in different ways!**

Separating the singularities is the name of the game!

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Soft gluon cannot resolve details of later splittings; only sees total color charge.



→ Soft and collinear emissions can be treated **independently**:

- Regularize soft singularities first, then collinear singularities.
- No need for energy-angle ordering – energies and angles can be independently parametrized.

Treatment of real-real singularities

- **Step 1:** New limit operators.

$$\mathbb{S} A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$

$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i},$$

and recall $S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A.$

- **Step 2:** Order gluon energies $E_4 > E_5$.

$$2\,\mathrm{s} \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5] \theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4,5) \rangle.$$

- Gluon energies bounded by $E_{\max}$.
- Energies defined in CoM frame.
- Soft singularities: either double soft or g_5 soft.

Soft singularities

- **Step 3:** Regulate the soft singularities:

$$\left\langle F_{LM}(1,2,4,5)\right\rangle = \left\langle S\!\!\!\!S F_{LM}(1,2,4,5)\right\rangle + \left\langle S_5(I - S\!\!\!\!S)F_{LM}(1,2,4,5)\right\rangle + \left\langle (I-S_5)(I - S\!\!\!\!S)F_{LM}(1,2,4,5)\right\rangle.$$

- First term: both g_4 and g_5 soft.
- Second term: g_5 soft, soft singularities in g_4 are regulated.
- Third term: regulated against all soft singularities,
- All three terms contain **(overlapping)** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

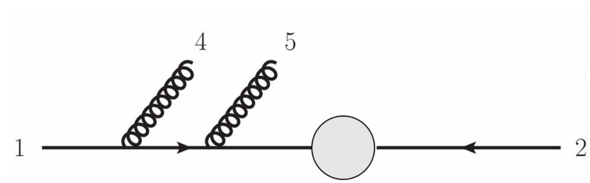
$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

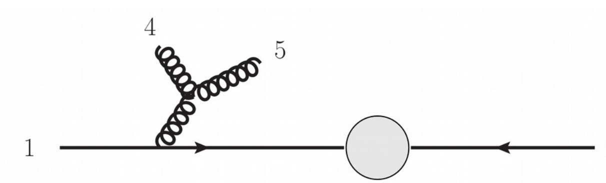
with

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0 \qquad\quad w^{24,25} \text{ contains } C_{42},\ C_{52},\ C_{45}$$

Triple collinear partition



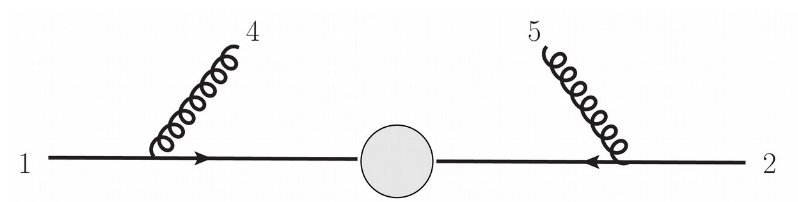


and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\ C_{52}$$

$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0 \qquad\quad w^{15,24} \text{ contains } C_{42},\ C_{51}$$

Double collinear partition



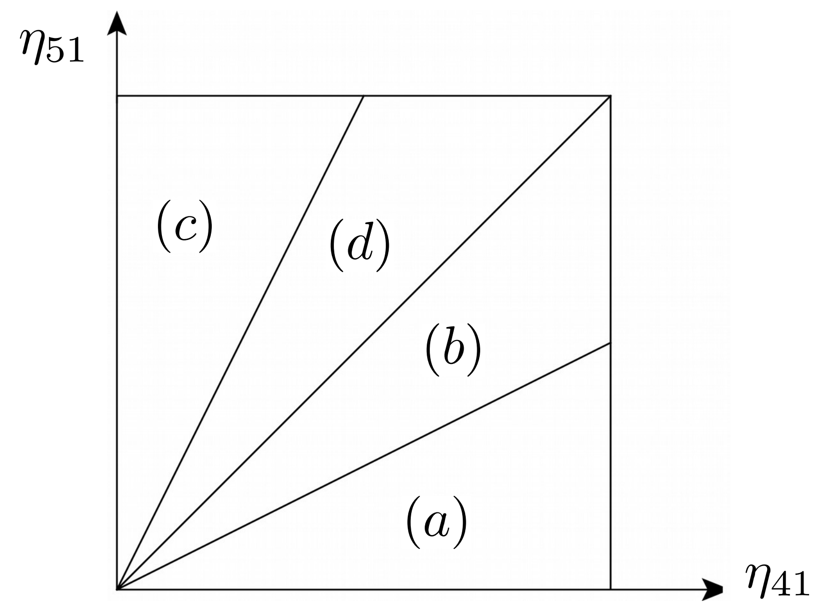
Sector Decomposition

- **Step 5: Sector decomposition:**

- Triple collinear sectors still have **overlapping** singularities.
- Define angular ordering to separate singularities.

$$\eta_{ij} = \rho_{ij}/2$$

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.
- Angular phase space parametrization [Czakon '10].

Removing collinear singularities

Then we can write soft-regulated term as

$$\big\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \big\rangle = \big\langle F_{LM}^{s_r c_s}(1,2,4,5) \big\rangle + \big\langle F_{LM}^{s_r c_t}(1,2,4,5) \big\rangle$$
$$+ \big\langle F_{LM}^{s_r c_r}(1,2,4,5) \big\rangle,$$

$$\langle F_{LM}^{s_r c_r}(1,2,4,5) \rangle$$

- All singularities removed through iterated subtractions – evaluated in 4-dimensions.
- Only term involving fully-resolved real-real matrix element.

$$\langle F_{LM}^{s_r c_{s,t}}(1,2,4,5) \rangle$$

- Contain (soft-regulated) single and triple collinear singularities.
- Matrix elements of lower multiplicity.
- Partitioning factors and sectors: one collinear singularity in each term.

Treating singular limits

We have four singular subtraction terms:

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5) \right\rangle \quad \left\langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z.*
- Significant analytic simplifications on recombining sectors after integration.
- Integration for first **three** subtraction terms done *analytically*, last one *numerically*.

(Double soft subtraction term computed recently [Caola, Delto, Frellesvig, Melnikov '18])

Treating singular limits

After integration: subtraction terms written as lower multiplicity terms:

- LO-like:

  $\langle F_{LM}(z\cdot 1, \bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1, 2)\rangle\,,\ \langle F_{LM}(1, z\cdot 2)\rangle\,,\ \langle F_{LM}(1,2)\rangle$

- NLO-real-like (regulated by iterative subtraction):

  $\langle \mathcal{O}_{NLO} F_{LM}(z\cdot 1, 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, z\cdot 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1,2,4)\rangle$

convoluted with splitting functions with explicit singularities

- Pole cancellation within each structure

  (to $1/\epsilon^2$ analytically, $1/\epsilon$ numerically).

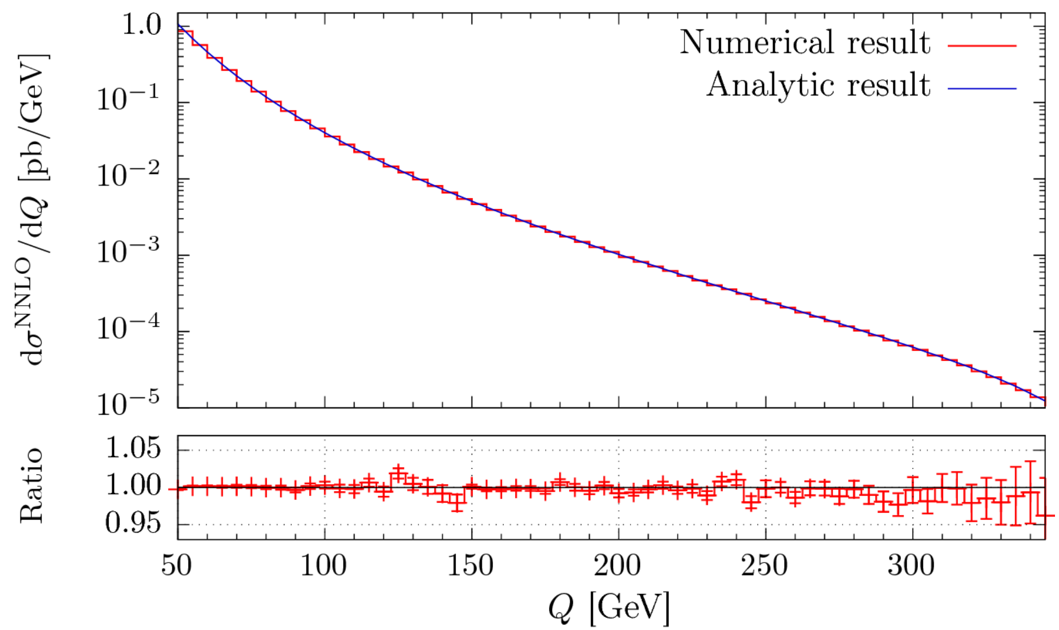
Finite remainders

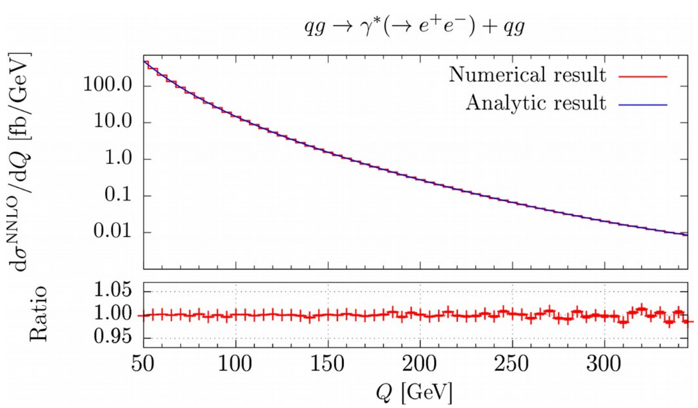
- **Relatively compact** expressions for finite remainders for each *lower-multiplicity structure.*
- Extension of NLO calculation to NNLO:
  - Boosted LO and NLO results multiplied by known functions.
  - Nested subtraction for real-real contribution.

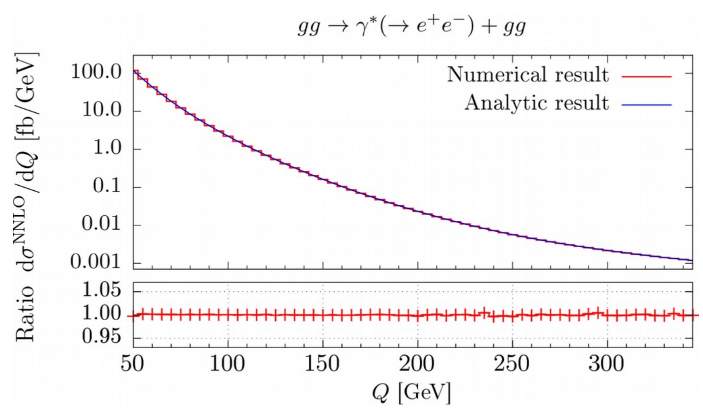
$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int_0^1 \mathrm{d}z \bigg\{ C_F^2 \bigg[8\tilde{\mathcal{D}}_3(z) + 4\tilde{\mathcal{D}}_1(z)(1+\ln 2) + 4\tilde{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2 \ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&\quad + 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\bigg[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\left[4\ln(1-z) - 6\right] + \ln^2 z \\
&\quad + \mathrm{Li}_2(1-z)\bigg] + (1+z)\bigg[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2\ln z - 2\ln 2\ln(1-z)\ln z \\
&\quad + 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \left[4\ln(1-z) - 2\ln 2\right]\mathrm{Li}_2(1-z)\bigg] \\
&\quad + \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\left[3\mathrm{Li}_2(1-z) - 2\ln^2 z\right] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&\quad + \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2\ln z\right]\bigg] \\
&\quad + C_A C_F\bigg[-\frac{22}{3}\tilde{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\tilde{\mathcal{D}}_1(z) + \bigg[-\frac{802}{27} + \frac{11}{18}\pi^2 \\
&\quad + (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\bigg]\tilde{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&\quad - \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&\quad - (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2\ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z)\times \\
&\quad \times\left[2\ln 2 + 3\ln(1-z)\right]\bigg] + R_+^{(c)}\mathcal{D}_0(z) + R^{(c)}(z)\bigg\}\left\langle \frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

Proof-of-principle

- Extensively tested in DY production against analytic results [Hamberg, Matsuura, van Neerven '91]:
  - All channels relevant for DY.
  - *NNLO corrections* to cross section agree at < 1 permille.
  - *NNLO corrections* show permille to percent agreement across 5 orders of magnitude in virtuality of vector boson Q.
  - Also in channels which are numerically negligible.







Color singlet decay

- NNLO corrections to $V \to q\bar{q}$ can be calculated with identical strategy.
- Integrated subtraction terms <u>much</u> simpler:

Consider collinear limit of $V \to q(p_1)\bar{q}(p_2)g(p_3)$:

$$C_{31}F_{LM}(1,2,3) = \frac{g_{s,b}^2}{E_1 E_3 \rho_{13}} P_{qq}\left(\frac{E_1}{E_1+E_3}\right) F_{LM}(1+3,2)$$

Integrate over the **full phase space** of all final state particles, so write energy integration as:

$$z = E_1/(E_1+E_3)$$

$$\int [dE_1][dE_3]C_{31}F_{LM}(1,2,3) = \left[\int dz(z(1-z))^{-2\epsilon}P_{qq}(z)\right] \times \left[\int [dE_{13}]E_{13}^{-2\epsilon}F_{LM}(1+3,2)\right]$$
$$= \text{const.} \times \langle F_{LM}(1,2)\rangle.$$

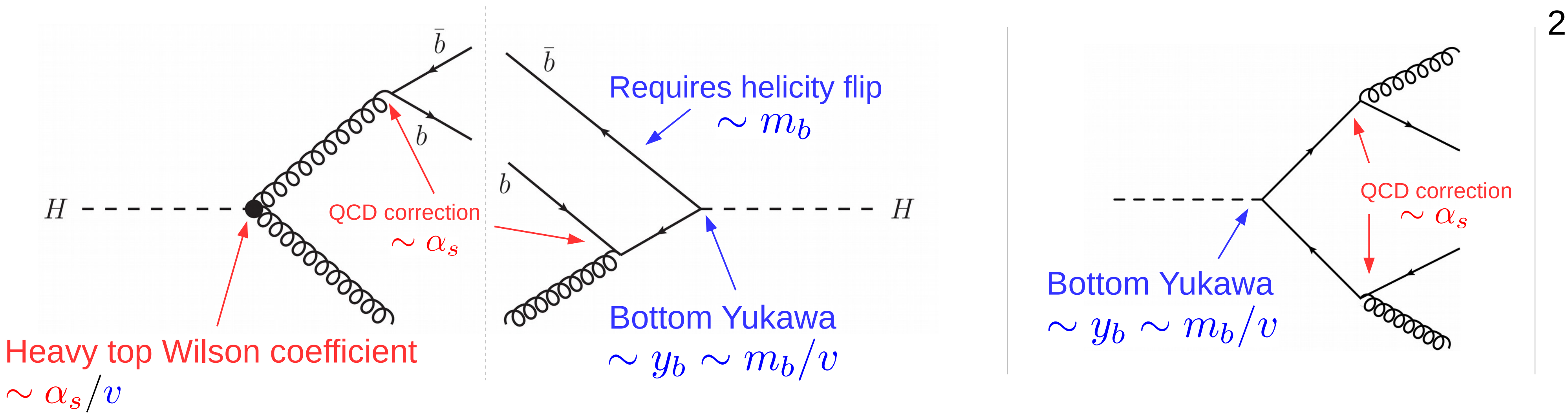
Lower multiplicity terms multiplied by constants rather than splitting functions.

Bottom mass effects in $H \rightarrow bb$

- In $H \rightarrow bb$ decay, want massless b-quarks but non-zero y_b

$$m_b \ll m_H \Rightarrow \mathrm{d}\sigma \sim y_b^2 (A + B\, m_b^2/m_H^2 + \ldots) = A y_b^2$$

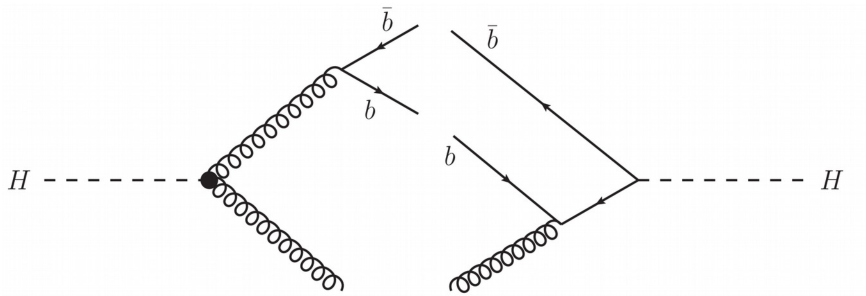
- Works at LO & NLO, but not at NNLO – **interference terms.**



Top-loop interference contribution $\sim \alpha_s^2 m_b^2 / v^2 \sim \alpha_s^2 y_b^2$	"Regular" contribution squared $\sim \alpha_s^2 m_b^2 / v^2 \sim \alpha_s^2 y_b^2$

Interference contribution has **identical parametric scaling** to other NNLO corrections.

Bottom mass interference



Obvious strategy: factor out one power of m_b and <u>then</u> take $m_b = 0$

BUT:

- Reduced matrix elements have unusual IR behaviour: *subleading power singularities*, e.g. **soft singularities from quarks!**
- $\log(m_b/m_H)$ **don't cancel** between real and virtual interference terms – cannot take massless limit!
- **Cannot** be regulated using flavor-kT algorithm (doesn't regulate soft quark singularity).
- **Cannot** define an inclusive cross section for *H* →*bb* at NNLO with massless *b*-quarks.
- Calculation in double-log approx: **~ 30%** of NNLO corrections to *H* → *bb* decay.
  - ➢ Effect on kinematic distributions?
- Different dependence on bottom Yukawa – **different behavior in BSM models.**

➡ NNLO calculation of *H*→*bb* to massive bottom quarks required.

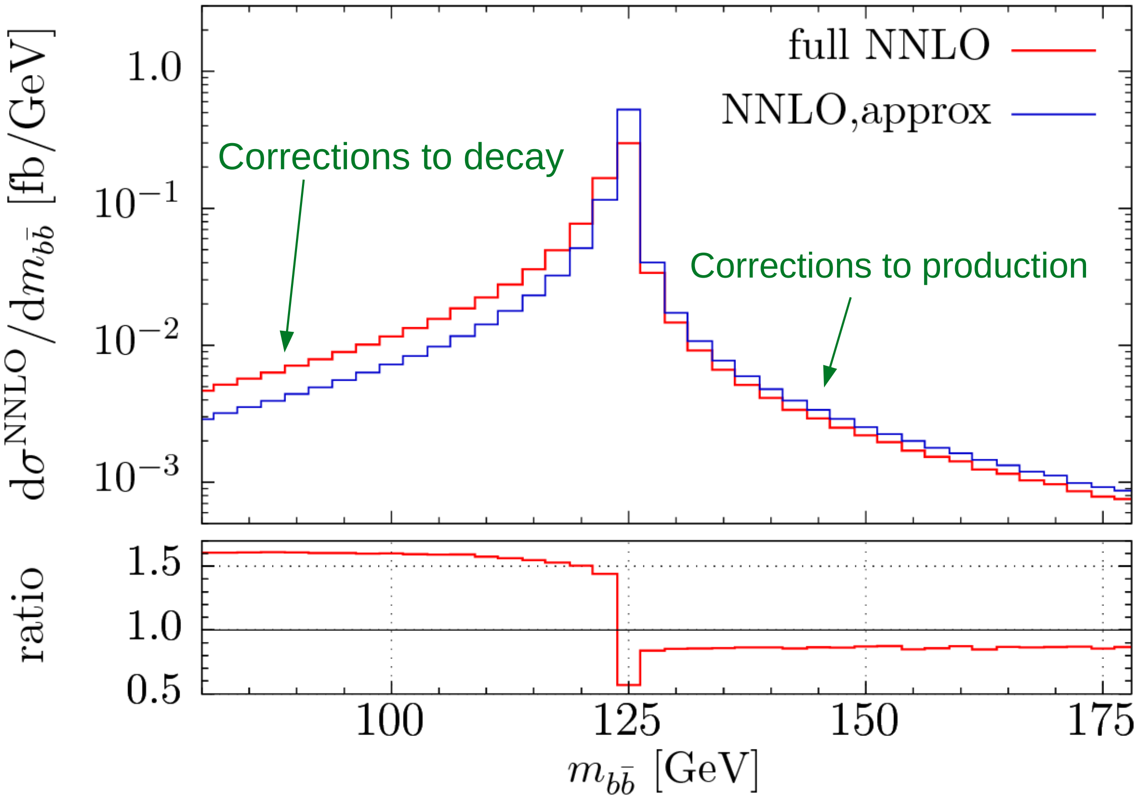
$VH(\to b\bar{b})$ to NNLO in production and decay

[Caola, Luisoni, Melnikov, R.R. ‘17]

NNLO corrections in production and decay in NWA.

Confirm results of [Ferrera, Somogyi, Tramantano ‘17]:

- Large (~60%) at low invariant mass.
- Sharp decrease at Higgs mass.
- ~ 15% depletion at high inv. mass.
- **Expected** as full NNLO includes corrections to decay – reduce inv. mass.
- Fairly well described by a parton shower.



Current work

- Corrections to $gg \to V; \;\; V \to gg$
  - Clear similarities with quark channels.

  ➡ Generic results for color singlet production and decay.

- Extension to colored final states: DIS
  - Double soft subtraction term originally computed **numerically.**
  - Major bottleneck for colored initial-final states: requires (numerical) integration of double soft eikonal function for different angles between hard partons (5-dim integration).
  - Double soft subtraction term now known **analytically** for arbitrary angles between hard partons. [Caola, Delto, Frellesvig, Melnikov '18]
  - Remaining subtraction terms ~ combination of previous results for color singlet production/decay.

Summary

- New method of handling NNLO subtraction, characterized by decoupling of soft and collinear limits.
- Developed iterative subtraction procedure:
  - Manifestly regulated finite term.
  - Integrated subtraction terms: convolutions of splitting function with explicit poles with lower multiplicity processes.
  - Transparent origin of IR poles.
  - Pole cancellation independent of matrix elements.
- Tested in DY and *W* production *for all partonic channels; H → bb* decay
  - Excellent agreement with analytic results in **all partonic channels.**
- Phenomenological application in $VH(\to b\bar{b})$.
- Ongoing work:
  - Remaining channels for color singlet production & color singlet decay.
  - Extension to colored initial-final state (DIS as first step).
  - Major obstacle removed: double soft subtraction term known analytically.

THANK YOU!

BACKUP SLIDES

Double-collinear partition

In single-collinear subtraction:

$$DC = \Big\langle [I - \mathbb{S}][I - S_5]\Big[(C_{41}[\mathrm{d}g_4] + C_{52}[\mathrm{d}g_5])\, w^{14,25} + (C_{42}[\mathrm{d}g_4] + C_{51}[\mathrm{d}g_5])\, w^{24,15} \Big] \times F_{LM}(1,2,4,5) \Big\rangle.$$

Collinear limit acts on phase space!

Consider fourth term:

$$\big\langle [I - \mathbb{S}][I - S_5] C_{51}[\mathrm{d}g_5] w^{24,15} F_{LM}(1,2,4,5) \big\rangle$$

$$= -\frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \int\limits_{z_{\min}(E_4)}^{1} \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}} \hat{\mathcal{P}}_{qq}^{(-)}(z) \big\langle \tilde{w}_{5||1}^{24,15} F_{LM}(z \cdot 1, 2, 4) \big\rangle.$$

$z_{\min}(E_4) = 1 - E_4/E_1$

ly: integral on [0:1]

Consider first term:

$$\big\langle [I - \mathbb{S}][I - S_5] C_{41}[\mathrm{d}g_4] w^{14,25} F_{LM}(1,2,4,5) \big\rangle$$

$$= -\frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \int\limits_{0}^{z_{\max}(E_5)} \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}} \mathcal{P}_{qq}(z) \big\langle \tilde{w}_{4||1}^{14,25} [I - S_5] F_{LM}(z \cdot 1, 2, 5) \big\rangle.$$

$z_{\max}(E_5) \equiv 1 - E_5/E_1$

Combining partitions

Rename the resolved gluon 4 in the first term and combine:

$$z_{\max}(E_4) \equiv 1 - E_4/E_1 = z_{\min}(E_4)$$

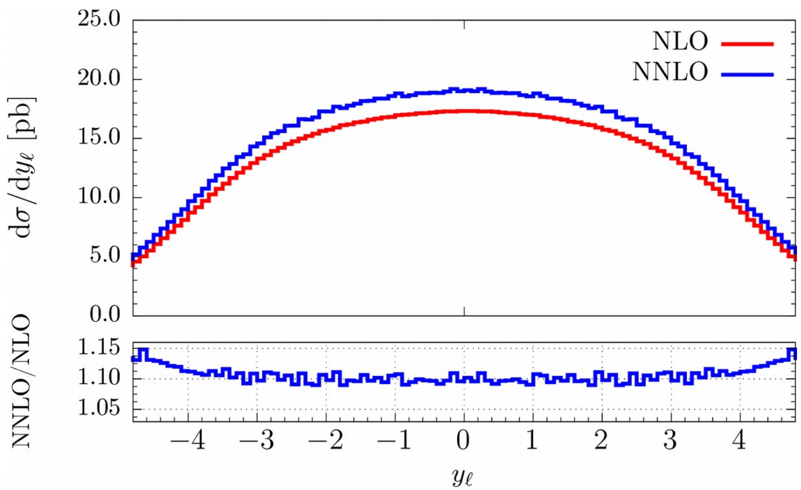
$$\Big\langle [I - \mathbb{S}]\,[I - S_5]\,\big[C_{41}[\mathrm{d}g_4]w^{14,25} + C_{51}[\mathrm{d}g_4]w^{15,24}F_{LM}(1,2,4,5)\big\rangle$$

$$= -\frac{[\alpha_s]s^{-\epsilon}}{\epsilon}\int\limits_0^1 \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}}\Big\langle \tilde{w}^{15,24}_{5||1}\Big(\hat{\mathcal{P}}^{(-)}_{qq}(z)\,[I - S_4]\,F_{LM}(z\cdot 1,2,4) +$$

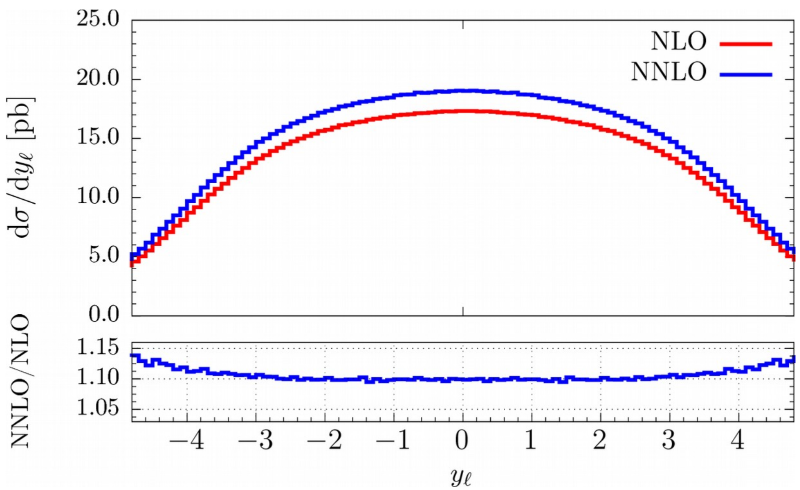
$$\theta(z_4 - z)2C_F\,[I - S_4]\,F_{LM}(1,2,4) + \theta(z_4 - z)\hat{\mathcal{P}}^{(-)}_{qq}(z)S_4F_{LM}(z\cdot 1,2,4)\Big)\Big\rangle.$$

- Simplifications after combining sectors.
- Different splitting functions in two terms → restrictions on *z.*
- Similar simplifications on combining terms from **double** & **triple** collinear partitions.

Differential distributions (I)



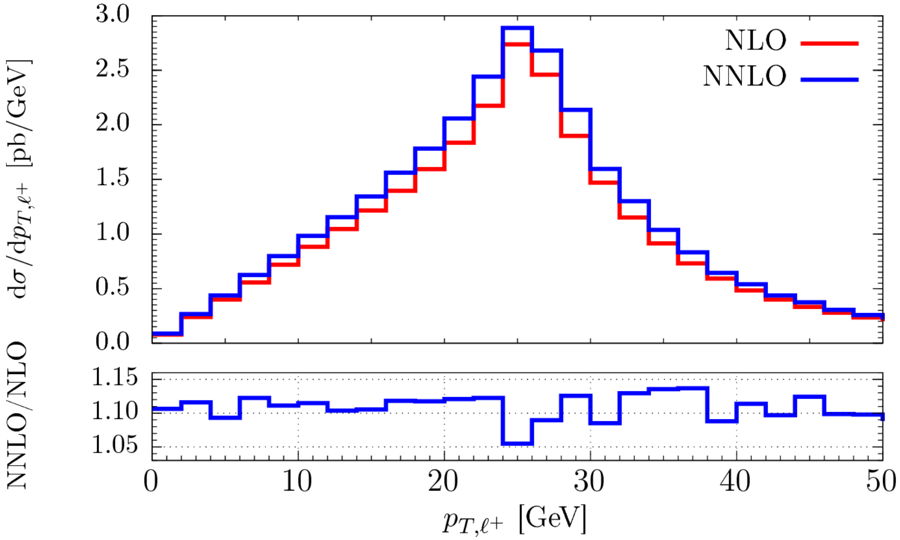
O(10 CPU hours) runtime

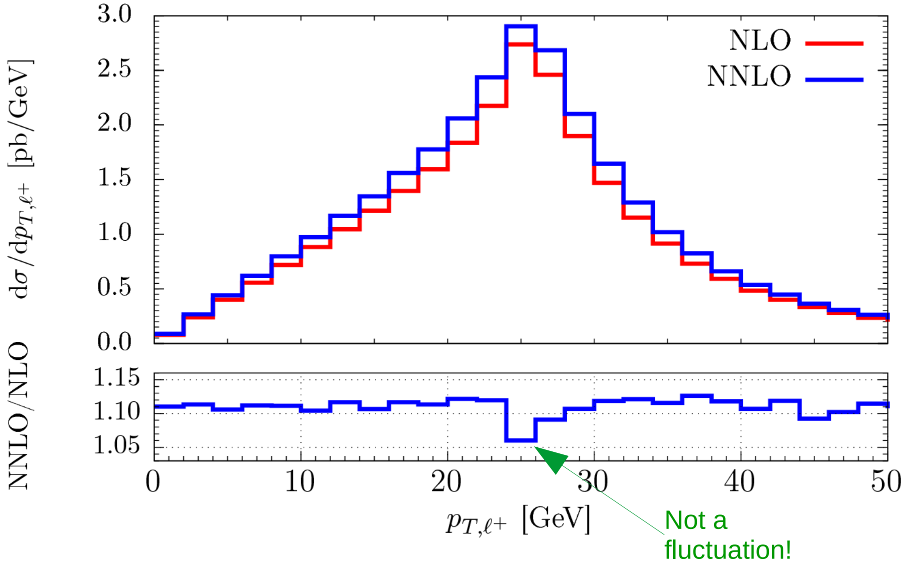


O(100 CPU hours) runtime

- Lepton rapidity.
- O(10 CPU hours): percent-level bin-to-bin fluctuations.
- O(100 CPU hours): per-mille bin-to-bin fluctuations.

Differential distributions (II)





O(10 CPU hours) runtime

O(100 CPU hours) runtime

- Lepton transverse momentum.
- O(100 CPU hours): percent-level bin-to-bin fluctuations.
- Delicate observable: receives contributions from large range of invariant masses.
  - Improves once introduce *Z* boson propagator.
  - Comparison with other NNLO codes?