

Two-Loop Five-Point Helicity Amplitudes in QCD via Integrand Reduction

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with S. Badger, C. Brønnum-Hansen and T. Peraro



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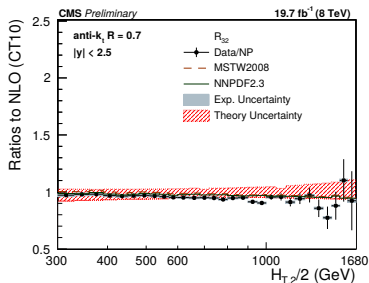


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NNLO frontier: 2 to 3 scattering

- ▶ $pp \rightarrow jjj$: $R_{3/2}$, $m_{jjj} \Rightarrow \alpha_s$ determination at multi-TeV range

$R_{3/2} \sim \alpha_s \rightarrow$ cancellation of uncertainties



$$\alpha_s(M_Z) = 01150 \pm 0.0010(\text{exp})$$

$$\pm 0.0013(\text{PDF})$$

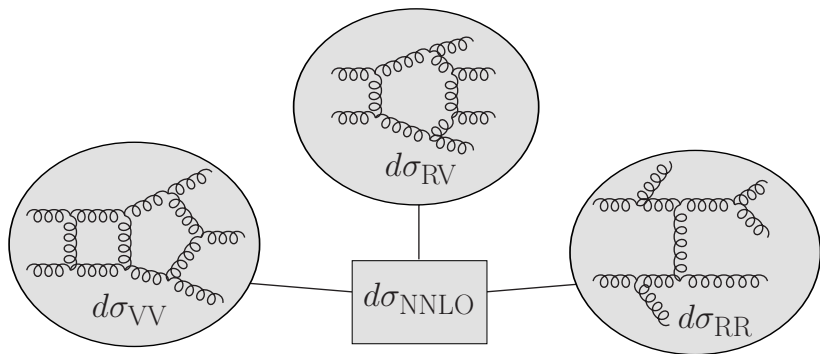
$$\pm 0.0015(\text{NP})$$

$$+0.0050$$

$$-0.0000(\text{scale})$$

[CMS-PAS-SMP-16-008]

- ▶ $pp \rightarrow \gamma\gamma j$: background to Higgs p_T , signal/background interference effects
- ▶ $pp \rightarrow Hjj$: Higgs p_T , background to VBF (probes Higgs coupling)
- ▶ $pp \rightarrow Vjj$: Vector boson p_T , W^+/W^- ratios, multiplicity scaling
- ▶ $pp \rightarrow VVj$: background for new physics



q_T subtraction

antenna subtraction

Nested Soft-Collinear Subtraction

N -jettiness subtraction

CoLoRFuNNLO

+ ...

projection to Born

STRIPPER

geometric subtraction

Integrand Reduction

[Ossola, Papadopoulos, Pittau, Mastrolia, Badger, Frellesvig, Zhang, Peraro, Mirabella, ...]
talks by U. Schubert, S. Abreu

Colour ordered amplitude:

$$\mathcal{A}^{(2)}(\{p\}) = \int \left(\prod_i^2 \frac{d^d k_i}{i\pi^{d/2} e^{-\epsilon\gamma_E}} \right) \frac{\mathcal{N}(\{k\}, \{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$

Integrand Reduction: construct irreducible numerators $\Delta_i(\{k\}, \{p\})$

$$\frac{\mathcal{N}(\{k\}, \{p\})}{\mathcal{D}_1 \cdots \mathcal{D}_n} = \sum_{i \in T} \frac{\Delta_i(\{k\}, \{p\})}{(\text{propagators})_i}$$

► $\mathcal{N}(\{k\}, \{p\})$: process dependent numerator function

⇒ generalized unitarity cuts → product of tree amplitudes

[Bern, Dixon, Dunbar, Kosower, 1994; Britto, Cachazo, Feng, 2004; Ellis, Giele, Kunszt, Melnikov, 2007-2008; ...]

⇒ Feynman diagram input (QGRAF, FeynArts, ...)

► fit $\Delta_i(\{k\}, \{p\})$ on the solutions of multiple cuts $\{\mathcal{D}_i = 0\}_{i \in T}$

Integrand Reduction

[Ossola, Papadopoulos, Pittau, Mastrolia, Badger, Frellesvig, Zhang, Peraro, Mirabella, ...]
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how can we parameterise the irreducible numerator?

Integrand Parameterization

Expand loop momenta to physical and transverse components [van Neerven, Vermaseren, 1983]

$$k_i^\mu = k_{\parallel,i}^\mu + k_{\perp,i}^\mu \quad k_{\perp,i} = k_{\perp,i}^{[4]} + k_{\perp,i}^{[-2\epsilon]}$$

Spanning basis for each topology:

$$k_{\parallel,i}^\mu = \sum_{j=1}^{d_{\parallel}} a_{ij} v_j^\mu, \quad k_{\perp,i}^{\mu,[4]} = \sum_{j=1}^{d_{\perp,[4]}} b_{ij} w_j^\mu, \quad \text{with } v_i \cdot w_j = 0$$

$$a_{ij}(k_i) \equiv a_{ij}(\{D\}, \{k \cdot q\}), \quad b_{ij}(k_i) \equiv b_{ij}(k_i \cdot w_j), \quad \mu_{ij} = -k_{\perp,i}^{[-2\epsilon]} \cdot k_{\perp,j}^{[-2\epsilon]}$$

$k_i \cdot q_j \rightarrow$ physical-space ISPs, $k_i \cdot w_j \rightarrow$ spurious ISPs, $\mu_{ij} \rightarrow$ extra-dim ISPs

Relations between monomials in the ISPs:

$$\mu_{ij} = k_i \cdot k_j - k_{\parallel,i} \cdot k_{\parallel,j} - k_{\perp,i}^{[4]} \cdot k_{\perp,j}^{[4]}$$

Any integrand:

$$\Delta_i(\text{ISP}) = \Delta_i(k_i \cdot q_j, k_i \cdot w_j, \mu_{ij})$$

Maximum rank of k in Δ is restricted by gauge theory power counting.

Constructing Integrand Basis

$$\begin{aligned}
 A^{(2),h} &= \int [dk_1][dk_2] \sum_{i \in T} \frac{\Delta_i^h(k_i \cdot q_j, k_i \cdot w_j, \mu_{ij})}{(\text{propagators})_i} \\
 &= \int [dk_1][dk_2] \sum_{i \in T} \frac{\sum_{\vec{\alpha}} c_{i,\vec{\alpha}}^h \cdot (\mathbf{m}_i(k_j))^{\vec{\alpha}}}{(\text{propagators})_i}
 \end{aligned}$$

$$\text{with } \mathbf{m}_i(k_j) = \{k_i \cdot q_j, k_i \cdot w_j, \mu_{ij}\}$$

- ▶ Write down an overcomplete set of monomials in $k_i \cdot q_j, k_i \cdot w_j, \mu_{ij}$ obeying power counting restrictions
- ▶ Order the monomials w.r.t. a set of criteria (e.g. prefer low rank over high rank, prefer μ_{ij} over $4d$)
- ▶ Map all monomials to $4d$ basis, $\mu_{ij} = f(k_i \cdot q_j, k_i \cdot w_j)$, then construct a linear system according to ordering of variables
- ▶ Solve the linear system for the independent monomials

Extracting $c_{i,\vec{\alpha}}$

$$\mathcal{A}^{(2),h} = \int [dk_1][dk_2] \sum_{i \in T} \frac{\Delta_i^h(k_i \cdot q_j, k_i \cdot w_j, \mu_{ij})}{(\text{propagators})_i} = \int [dk_1][dk_2] \sum_{i \in T} \frac{\sum_{\vec{\alpha}} c_{i,\vec{\alpha}}^h \cdot (\mathbf{m}_i(k_j))^{\vec{\alpha}}}{(\text{propagators})_i}$$

- ▶ Fit coefficients $c_{i,\vec{\alpha}}^h$ on multiple cuts

$$\mathcal{N}|_{8\text{xcut}} = \Delta(\text{diagram})|_{8\text{xcut}} \quad \mathcal{N}|_{7\text{xcut}} = \frac{\Delta(\text{diagram})}{(k_1 + k_2)^2} \Big|_{7\text{xcut}} + \Delta(\text{diagram})|_{7\text{xcut}}$$

- ▶ Numerator inputs:

⇒ product of trees (6d spinor helicity formalism [Cheung,O'Connell,2009] + 6d trees)

⇒ Feynman diagram input

- ▶ Momentum Twistor Variables [Hodges,2009]

⇒ rational function of $3n - 11$ variables (+1 overall dimension)

- ▶ Numerical evaluations over finite fields \mathcal{Z}_p [von Manteuffel,Schabinger,2014;Peraro,2016]

⇒ fast+exact, reconstruct \mathcal{Q} from several \mathcal{Z}_p

- ▶ Analytic reconstruction from finite-field evaluations [Peraro,arXiv:1608.01902]

From Integrand to Integrated Form

Integrands contain spurious terms

$$\int [dk_1][dk_2] \frac{k_i \cdot w_j}{(\text{propagators})} = 0$$

Express integrals containing μ_{ij} as higher dimensional integrals

for example:

[Bern,de Freitas,Dixon,2002]

$$\int [d^d k_1][d^d k_2] \frac{\mu_{11}}{D_1 \cdots D_n} = \epsilon \sum_{i \in \{2\} \cup \{12\}} \int [d^{(d+2)} k_1][d^{(d+2)} k_2] \frac{1}{D_1 \cdots D_i^2 \cdots D_n}$$

Evaluating integrals

► Proceed with IBP reduction + substitute Master Integrals

many new ideas for efficient solutions of IBP systems [Kosower, Kajda, Gluza, Schabinger, von Manteuffel, Ita Larsen, Zhang, Böhm, Georgoudis, Schnemann, Abreu, Page, Febres-Cordero, Zeng]
talk by Zhang

► Direct numerical evaluations with sector decomposition method
(Fiesta [Smirnov,etal], SecDec [Borowka,etal])

Status of 2-loop 5-point calculation

'All plus' 5-gluon case

- Planar integrand, analytic D -dim integrand [Badger,Frellesvig,Zhang,arXiv:1310.1051]
- Full-colour integrand [Badger,Mogull,O'Connell,Ochirov,arXiv:1507.08797]
- Local integrands (also for 6-gluon) [Badger,Mogull,Peraro,arXiv:1712.03946]
- Planar compact analytic [Gehrmann,Henn,LoPresti,arXiv:1511.05409][Dunbar,Perkins,arXiv:1603.07514]

2 \rightarrow 3 master integrals:

- planar, massless [Gehrmann,Henn,Lo Presti,arXiv:1511.05409]
- planar, massless+1 external mass [Papadopoulos,Tommasini,Wever,arXiv:1511.09404]
- (some) non-planar [Chicherin,Henn,Mitev,arXiv:1712.09610]

Recent progress on amplitudes (planar sector):

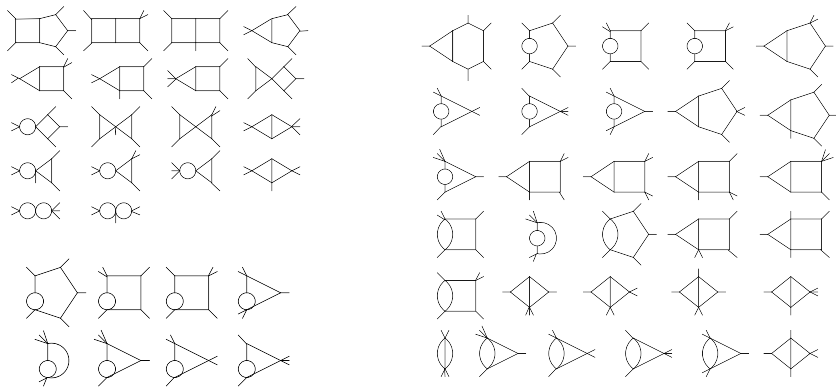
- *First look at two-loop five-gluon amplitudes in QCD*
[Badger,Brønnum-Hansen,HBH,Peraro,arXiv:1712.02229]
- *Planar two-loop five-gluon amplitudes from numerical unitarity* talk by S. Abreu
[Abreu,Febres-Cordero,Ita,Page,Zeng,arXiv:1712.03946]
- *Efficient integrand reduction for particles with spin* [Boels,Jin,Luo,arXiv:1802.06761]
- *Two-loop five-point massless QCD amplitudes within the IBP approach* talk by A. Mitov
[Chawdry,Lim,Mitov,arXiv:1805.09182]

Leading colour 2-loop 5-gluon amplitudes

[Badger, Brønnum-Hansen, HBH, Peraro, Phys.Rev.Lett. 120 (2018) no.9, 092001]

$$\mathcal{A}^{(L)}(1, 2, 3, 4, 5) = n^L g_s^3 \sum_{\sigma \in S_5/Z_5} \text{Tr}(T^{a\sigma(1)} \dots T^{a\sigma(5)}) A^{(L)}(\sigma(1), \sigma(2), \sigma(3), \sigma(4), \sigma(5))$$

$$n = m_\epsilon N_c \alpha_s / (4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$



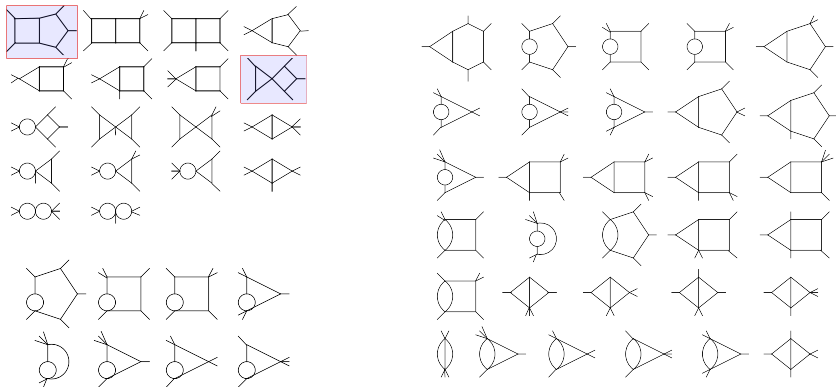
57 distinct topologies, 425 Δ (all permutations)

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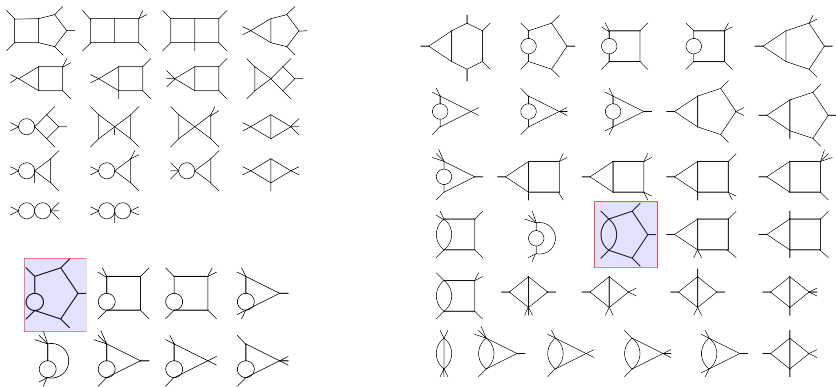
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Leading Colour 2-loop 5-gluon: Euclidean Region

$$A^{(2)} = A^{(2),[0]} + (d_s - 2)A^{(2),[1]} + (d_s - 2)^2 A^{(2),[2]}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{-+---}^{(2),[0]}$	12.5	27.7526	-23.7728	-168.1162	-175.2103
$P_{-+---}^{(2),[0]}$	12.5	27.7526	-23.7728	-168.1163	—
$\widehat{A}_{-+--+}^{(2),[0]}$	12.5	27.7526	2.5028	-35.8084	69.6695
$P_{-+--+}^{(2),[0]}$	12.5	27.7526	2.5028	-35.8086	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{+++++}^{(2),[1]}$	0	0	-2.5	-6.4324	-5.3107
$P_{+++++}^{(2),[1]}$	0	0	-2.5	-6.4324	—
$\widehat{A}_{-+----}^{(2),[1]}$	0	0	-2.5	-12.7492	-22.0981
$P_{-+----}^{(2),[1]}$	0	0	-2.5	-12.7492	—
$\widehat{A}_{-+---+}^{(2),[1]}$	0	-0.625	-1.8175	-0.4869	3.1270
$P_{-+---+}^{(2),[1]}$	0	-0.625	-1.8175	-0.4869	—
$\widehat{A}_{-+--+}^{(2),[1]}$	0	-0.625	-2.7759	-5.0018	0.1807
$P_{-+--+}^{(2),[1]}$	0	-0.625	-2.7759	-5.0018	—

	$\widehat{A}_{+++++}^{(2),[2]}$	$\widehat{A}_{-++++}^{(2),[2]}$	$\widehat{A}_{-+----}^{(2),[2]}$	$\widehat{A}_{-+---+}^{(2),[2]}$
ϵ^0	3.6255	-0.0664	0.2056	0.0269

$$s_{12} = -1, s_{23} = -\frac{37}{78}, s_{34} = -\frac{2023381}{3194997},$$

$$s_{45} = -\frac{83}{102}, s_{15} = -\frac{193672}{606645}$$

$$x_1 = -1, x_2 = \frac{79}{90}, x_3 = \frac{16}{61}, x_4 = \frac{37}{78}, x_5 = \frac{83}{102}$$

⇒ reduce $\mathcal{O}(10^4)$ integrals to $\mathcal{O}(10^2)$ MIs using FIRE & Reduze (mostly analytic, some high-rank integrals numerical)

⇒ Analytic MIs from [Gehrmann,Henn,Lo Presti] or alternatively [Papadopoulos,Tommasini,Wever] 6d/8d MIs to 4d MIs using LiteRed [Lee]

⇒ IR poles [Catani,Becher,Neubert,Gardi,Magnea] with d_s dependence, extracted from FDH results [Gnendiger,Signer,Stockinger]

⇒ verified by Abreu,Febris-Cordero,Ita,Page,Zeng [arxiv:1712.05721]

⇒ by-passing original numerical evaluation with Fiesta/SecDec

Leading Colour 2-loop 5-gluon: Physical Region

MIs from Gehrmann, Henn, Lo Presti

$$A^{(2)} = A^{(2),[0]} + (d_s - 2)A^{(2),[1]} + (d_s - 2)^2 A^{(2),[2]}$$

$$s_{12} = \frac{113}{7}, \quad s_{23} = -\frac{152679950}{96934257}, \quad s_{34} = \frac{1023105842}{138882415}, \quad s_{45} = \frac{10392723}{3968069}, \quad s_{15} = -\frac{8362}{32585}$$

$$x_1 = \frac{113}{7}, \quad x_2 = -\frac{2}{9} - \frac{i}{19}, \quad x_3 = -\frac{1}{7} - \frac{i}{5}, \quad x_4 = \frac{1351150}{13847751}, \quad x_5 = -\frac{91971}{566867}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{-+--+}^{(2),[0]}$	12.5	$-9.17716 + 47.12389 i$	$-107.40046 - 25.96698 i$	$17.24014 - 221.41370 i$	$388.44694 - 167.45494 i$
$\rho_{-+--+}^{(2),[0]}$	12.5	$-9.17716 + 47.12389 i$	$-107.40046 - 25.96698 i$	$17.24013 - 221.41373 i$	—
$\widehat{A}_{-+--+}^{(2),[0]}$	12.5	$-9.17716 + 47.12389 i$	$-111.02853 - 12.85282 i$	$-39.80016 - 216.36601 i$	$342.75366 - 309.25531 i$
$\rho_{-+--+}^{(2),[0]}$	12.5	$-9.17716 + 47.12389 i$	$-111.02853 - 12.85282 i$	$-39.80018 - 216.36604 i$	—

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{++++}^{(2),[1]}$	0	0	-2.5	$0.60532 - 12.48936 i$	$35.03354 + 9.27449 i$
$\rho_{++++}^{(2),[1]}$	0	0	-2.5	$0.60532 - 12.48936 i$	—
$\widehat{A}_{-+--+}^{(2),[1]}$	0	0	-2.5	$-7.59409 - 2.99885 i$	$-0.44360 - 20.85875 i$
$\rho_{-+--+}^{(2),[1]}$	0	0	-2.5	$-7.59408 - 2.99885 i$	—
$\widehat{A}_{-+--+}^{(2),[1]}$	0	-0.625	$-0.65676 - 0.42849 i$	$-1.02853 + 0.30760 i$	$-0.55509 - 6.22641 i$
$\rho_{-+--+}^{(2),[1]}$	0	-0.625	$-0.65676 - 0.42849 i$	$-1.02853 + 0.30760 i$	—
$\widehat{A}_{-+--+}^{(2),[1]}$	0	-0.625	$-0.45984 - 0.97559 i$	$1.44962 + 0.53917 i$	$-0.62978 + 2.07080 i$
$\rho_{-+--+}^{(2),[1]}$	0	-0.625	$-0.45984 - 0.97559 i$	$1.44962 + 0.53917 i$	—

	$\widehat{A}_{++++}^{(2),[2]}$	$\widehat{A}_{-+--+}^{(2),[2]}$	$\widehat{A}_{-+--+}^{(2),[2]}$	$\widehat{A}_{-+--+}^{(2),[2]}$
ϵ^0	$0.60217 - 0.01985 i$	$-0.10910 - 0.01807 i$	$-0.06306 - 0.01305 i$	$-0.03481 - 0.00699 i$

Leading colour 2-loop $qg\bar{g}g\bar{q}$: Euclidean Region

$$A^{(L)}(1_q, 2_g, 3_g, 4_g, 5_{\bar{q}}) = n^L g_s^3 \sum_{\sigma \in S_3} (T^{a_{\sigma(2)}} T^{a_{\sigma(3)}} T^{a_{\sigma(4)}})_{i_1}^{\bar{i}_5} A^{(L)}(1_q, \sigma(2)_g, \sigma(3)_g, \sigma(4)_g, 5_{\bar{q}})$$

$$n = m_\epsilon N_c \alpha_s / (4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$

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	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{+++++}^{(2)}$	0	0	-4	-13.5322768031	6.048656403
$P_{+++++}^{(2)}$	0	0	-4	-13.5322768028	—
$\widehat{A}_{+++--}^{(2)}$	8	7.9682909085	-52.3927085027	-140.1563714534	47.5687220127
$P_{+++--}^{(2)}$	8	7.9682909085	-52.3927085034	-140.1563714829	—
$\widehat{A}_{++-+-}^{(2)}$	8	7.9682909085	-32.2213536407	-47.9234973502	145.9720111187
$P_{++-+-}^{(2)}$	8	7.9682909085	-32.2213536403	-47.9234973889	—
$\widehat{A}_{+-+--}^{(2)}$	8	7.9682909084	-40.8851109385	-87.0299398048	101.2329971544
$P_{+-+--}^{(2)}$	8	7.9682909085	-40.8851109386	-87.0299398374	—

Fully numerical, IR poles checked in HV scheme, **preliminary!**

Leading colour 2-loop $q\bar{q}gQ\bar{Q}$: Euclidean Region

$$\mathcal{A}^{(L)}(1_q, 2_{\bar{q}}, 3_g, 4_Q, 5_{\bar{Q}}) = n^L g_s^3 \left[(T^{a_3})_{i_4}^{\bar{i}_2} \delta_{i_1}^{\bar{i}_5} A^{(L)}(1_q, 2_{\bar{q}}, 3_g, 4_Q, 5_{\bar{Q}}) + (1 \leftrightarrow 4, 2 \leftrightarrow 5) \right]$$

$$n = m_\epsilon N_c \alpha_s / (4\pi), \quad m_\epsilon = i(4\pi)^\epsilon e^{-\epsilon\gamma_E}$$

$$s_{12} = -1, \quad s_{23} = -\frac{37}{78}, \quad s_{34} = -\frac{2023381}{3194997}, \quad s_{45} = -\frac{83}{102}, \quad s_{15} = -\frac{193672}{606645}$$

$$x_1 = -1, \quad x_2 = \frac{79}{90}, \quad x_3 = \frac{16}{61}, \quad x_4 = \frac{37}{78}, \quad x_5 = \frac{83}{102}$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^0
$\widehat{A}_{+---+}^{(2)}$	4.5	2.2831548748	-32.0984879203	-41.3935016796	149.3305056672
$P_{+---+}^{(2)}$	4.5	2.2831548747	-32.0984879200	-41.3935017002	—
$\widehat{A}_{+----}^{(2)}$	4.5	2.2831548747	-4.6165799244	-6.3236903564	-32.0327897453
$P_{+----}^{(2)}$	4.5	2.2831548747	-4.6165799294	-6.3236903481	—
$\widehat{A}_{+--+}^{(2)}$	4.5	2.2831548747	-38.294786128	-43.5232972386	-56.7196838792
$P_{+--+}^{(2)}$	4.5	2.2831548747	-38.294786128	-43.5232972694	—
$\widehat{A}_{+----}^{(2)}$	4.5	2.2831548747	-26.7131604905	-69.7580529817	22.2365337061
$P_{+----}^{(2)}$	4.5	2.2831548747	-26.7131604895	-69.758052969	—

Fully numerical, IR poles checked in HV scheme, **preliminary!**

Can We Improve Integrand Basis?

So far ...

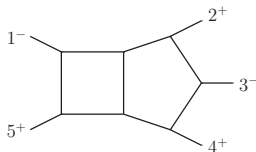
- ▶ Set up integrand reduction framework for massless 2-loop 5-point amplitudes
- ▶ Results for $ggggg$, $qggg\bar{q}$ and $q\bar{q}Q\bar{Q}g$ partonic channels

Issues ...

- ▶ analytic form of the integrands are very large
- ▶ large cancellation between topologies
- ▶ $\mathcal{O}(\epsilon)$ amplitudes are not manifest ($d_s = 2, +++++, -++++$)
- ▶ high-rank integrals
 - difficult to solve IBP systems analytically
 - very large analytic expressions

Improving Integrand Basis (1)

Pentagon-box topology



Old Basis

54 non-zero integrand coeffs

28 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

worst case:

$(k_2 \cdot p_2)^4 \mu_{11}$ (rank 4, 1 dot)

New Basis

49 non-zero integrand coeffs

4 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

$k_1 \cdot n_1$ $k_2 \cdot n_2$ (rank 2)

$k_1 \cdot n_1$ μ_{11} (rank 1, 1 dot)

$$\text{ISP} = \{k_1 \cdot p_1, k_2 \cdot p_2, k_2 \cdot p_3, \mu_{11}, \mu_{12}, \mu_{22}\}$$

$$I \left(\text{diagram} \right) [(k_2 \cdot p_2)^4 \mu_{11}] = \frac{\#}{\epsilon} + \#$$

but

$$I \left(\text{diagram} \right) [\langle 1|k_2|5 \rangle^4 \mu_{11}] = \mathcal{O}(\epsilon)$$

$$\langle 1|k_2|5 \rangle = A + B k_2 \cdot p_2 + C k_2 \cdot p_3 + \text{props}$$

New ISP basis

$$\{k_1 \cdot n_1, k_1 \cdot n_1^*, k_2 \cdot n_2, k_2 \cdot n_2^*, \mu_{11}, \mu_{12}, \mu_{22}\}$$

$$k_2 \cdot n_2 = \langle 1|k_2|5 \rangle$$

$$k_1 \cdot n_1 = \text{tr}_+(2(k_1 - p_2)(k_1 - p_{23})4)$$

$$\text{tr}_+(ijkl) = [i|j|k|l|i]$$

[Arkani-Hamed, Bourjaily, Cachazo, Trnka, 2012]

[Bourjaily, Hermann, Trnka, 2017] [Badger, Peraro, Mogull, 2016]

Improving Integrand Basis (2)

Old Basis

16 non-zero integrand coeffs

16 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

worst case:

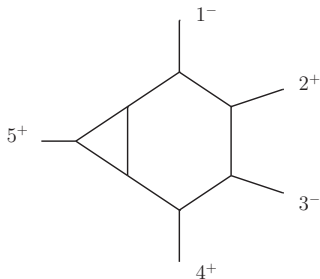
$(k_2 \cdot p_1)^3 \mu_{11}$ (rank 3, 1 dot)

New Basis

10 non-zero integrand coeffs

0 non-zero coeffs at $\mathcal{O}(\epsilon^0)$

Hexagon-triangle topology



$$\text{ISP} = \{k_2 \cdot p_1, k_2 \cdot p_2, k_2 \cdot p_3, \mu_{11}, \mu_{12}, \mu_{22}\}$$

New ISP basis

$$\{k_2 \cdot n_1, k_2 \cdot n_1^*, k_2 \cdot p_1, \mu_{11}, \mu_{12}, \mu_{22}\}$$

$$k_2 \cdot n_1 = \langle 1|k_2|5]$$

Summary:

- ▶ D -dimensional integrand reduction method at two loops + generalized unitarity cuts
- ▶ Analytic reconstructions of integrand coefficients from finite-field evaluations
- ▶ Benchmark results for massless 2-loop 5-point amplitudes
 $\Rightarrow ggggg, qggg\bar{q}, q\bar{q}gQ\bar{Q}$

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- ▶ Improve integrand representation (local basis?)
- ▶ Non-planar?, Off-shell leg?

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THANK YOU!!!