Two-loop five-point massless QCD amplitudes within the IBP approach

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Based on: arXiv:1805.09182 with Herschel Chawdhry and Matthew Lim

Intro: why the five-point QCD amplitudes?

- ✓ Phenomenology:
  - ✓ NNLO phenomenology at the LHC has cleared 2 ➛ 2 processes.
  - ✓ However, the 2 ➛ 3 ones, like 3 jet production, are still an open problem.
  - ✓ The existing methods can in principle compute such processes at NNLO. The corresponding 2-loop amplitudes are the only missing ingredient
- ✓ Amplitudes:
  - ✓ They are the basic building blocks of the theory:

    "knowing all correlation functions means knowing the theory"

  - ✓ A number of amplitude-specific "formal" motivations like:
    - ✓ Structure of the results
    - ✓ Functions and symmetries
    - ✓ Testing/developing approaches for their calculation

Five-point amplitudes in QCD: past results

- ✓ A number of parallel developments in the last several years

Badger, Frellesvig, Zhang '13
Ita '15
Badger, Mogull, Ochirov, O'Connell '15
Gehrmann, Henn, Presti '15
Dunbar, Perkins '16
Dunbar, Godwin, Jehu, Perkins '17
Badger, Bronnum-Hansen, Hartanto, Peraro '17
Abreu, Cordero, Ita, Page, Zeng '17
Boehm, Georgoudis, Larsen, Schoenemann, Zhang '18
Kosower '18

- ✓ Mostly based on approaches trying to utilize/extend unitarity to two loops. This makes sense:
  - ✓ Unitarity has been extremely successful at one-loop for QCD/collider applications
  - ✓ For multiloop developments in N=4 SUSY and related theories.

Five-point amplitudes in QCD: our interest in this

- ✓ Our interest in this problem is twofold

  1. Phenomenological applications at NNLO for 2 ➛ 3 processes at the LHC

  2. Our fascination with the IBP identities

Chetyrkin, Tkachov '81

- ✓ They contain incredible amount of information about the problem.

- ✓ Solving them however becomes a difficult problem even at 2 ➛ 2 at 2 loops (with many scales)

- ✓ Improvements in the way we solve the IBP identities is highly desirable if we want to increase the scope of problems that can be tackled with the IBP's

- ✓ Laporta algorithm has been a major breakthrough

- ✓ Approaches leading to abstract solution of the system of IBP's are likely needed

- ✓ Recent work in this direction: David Kosower '18

- ✓ This present work is a step in this direction. It adds to our understanding about how to think about this problem.

Five-point amplitudes in QCD: notation

✓ A generic bare massless amplitude M is expressed through Feynman integrals

$$M = \sum_{i=1}^{N} f_i I_i$$

- I_i are Feynman integrals
- N is large (10^4 – 10^5 in 2->3 case)
- f_i are some simple rational coefficients

✓ In this work we will consider squared amplitudes $M=\langle A^{(2)}|A^{(0)}\rangle$ but this is not essential

✓ IBPs relate generic integrals to O(100) master integrals

$$I_i = \sum_{m=1}^{\hat{N}} c_{i,m} \hat{I}_m$$

✓ As a result the amplitude is expressed through a small number of masters with (very large) rational coefficients

$$M = \sum_{m=1}^{\hat{N}} \hat{c}_m \hat{I}_m \,, \text{ with } \quad \hat{c}_m = \sum_{i=1}^{N} c_{i,m} f_i$$

✓ Solving the problem means:

1. Solve the IBPs
2. Compute the master integrals

Solving the IBPs: our approach

- ✓ I think it is well appreciated that a straightforward application of the Laporta algorithm with existing programs for solving IBPs is a very hard problem

AIR: Anastasiou, Lazoupoulos
FIRE: Smirnov, Smirnov
Reduze (2): von Manteuffel, Studerus
KIRA: Maierhöfer, Usovitsch, Uwer

Possibly related developments in KIRA – see talk by Johann Usovitsch

- ✓ We propose a slightly modified strategy which allows to solve the IBP identities for one MI at a time.

- ✓ One may wonder why would this even be useful?

  - ✓ In simple problems one can derive all masters in one go
  - ✓ In complicated problems the coefficients become very large so the memory requirements explode.
  - ✓ By solving for one master at a time one limits the memory requirement as well as the number of expressions that need to be simultaneously evaluated.

  - ✓ A negative: one needs to rerun the IBP identities many times, as many as there are masters. In practice this is not a problem because

    - This is easy to parallelize
    - The run times are vastly different so the whole problem takes almost as much as the most complicated master.

Solving the IBPs: our strategy

1. Determine the set of masters (ask Reduze, LiteRed or do numerics or something else). Not a problem
2. Set all masters but one to zero. Run the full set of IBPs. Once the solution is complete one gets the coefficient of the one master that was non-zero
3. Repeat the above step for all masters
4. The full solution is simply the sum of the above

✓ The above works like projecting the problem onto one master at a time and solving for (almost) just this one projection

✓ Why the above should work?

- The IBP system of equations is linear and homogeneous
- Assuming each integral is a linear combination of masters then at each step the IBP's do not mix the various projections. Thus setting one to zero (i.e. removing it from the problem) does not interfere with the remaining ones.

✓ This strategy us unrelated to how the IBP equations are solved. In practice we will use Laporta.

Solving the IBPs: our strategy

✓ Here are the reasons our strategy simplifies the solving of the IBPs:

1. Limits memory needs (mentioned above) due to smaller number of coefficients that are actively computed at the same time.

2. Many sectors become zero-sectors from the outset. This is a major simplification in practice. This should be used with some care in non-standard problems like propagators with non-integer powers.

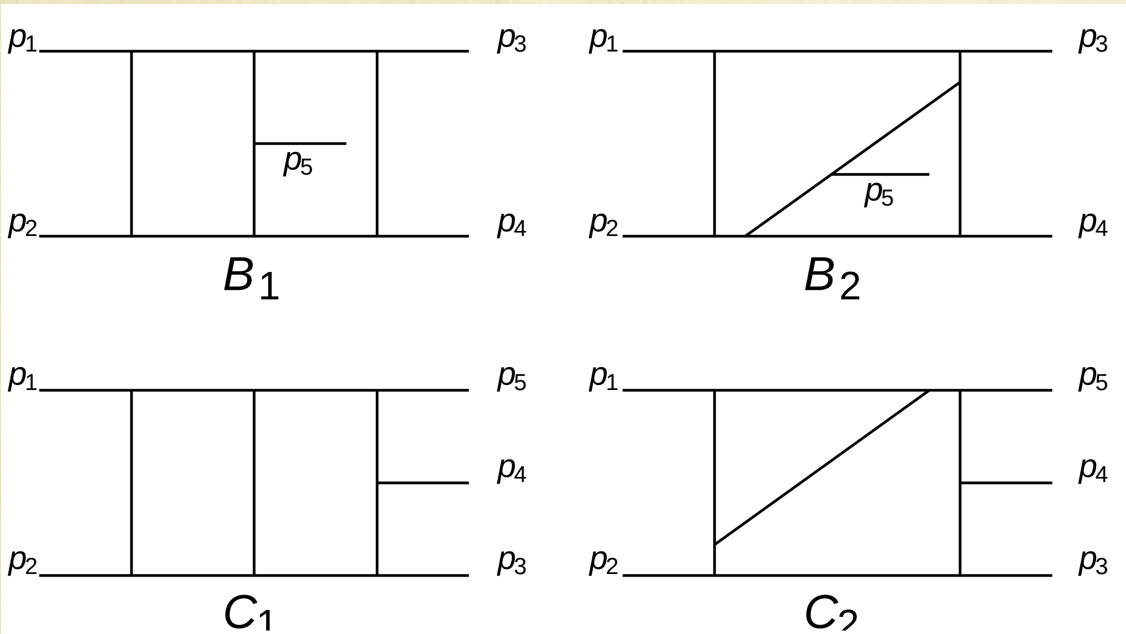
> Def: a zero-sector is a sector that has no masters or a sub-sector that has masters

3. The information about vanishing masters is incorporated into the solution early on in the solving (which makes it more efficient). This is because, using Laporta, master integrals and seeds for solving the equations are generated with the same algorithm.

4. The calculation of the various masters can be parallelized. This is restricted mainly by the amount of available memory. In practice we observe strong hierarchy among the masters. Basically, the time and RAM it takes to compute the slowest master can accommodate all others.

Results for 2 ➛ 3

- ✓ We consider the process qq ➛ q'q'g (no special reason – easy to generate)
- ✓ The amplitudes involve up to 8 propagators (11 needed for the reduction)
- ✓ There are four topologies with 8 propagators and several with less than that



Non-Planar

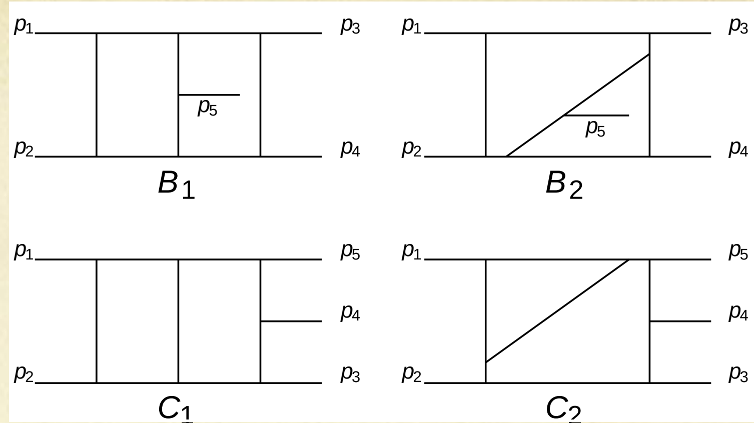
Planar

Hard

Easier

$$
\begin{aligned}
B = & \{k_1^2, k_2^2, (k_1+p_1)^2, (k_1+p_1+p_2)^2, \\
& (k_2-p_3)^2, (k_2-k_1-p_3)^2, \\
& (k_2-k_1-p_1-p_2+p_4)^2, (k_2+p_4)^2, \\
& (k_2+p_1+p_2)^2, (k_2+p_1)^2, (k_1+p_3)^2 \qquad (7) \\
C = & \{k_1^2, k_2^2, (k_1+p_1+p_2)^2, (k_1-k_2)^2, \\
& (k_2+p_1)^2, (k_2+p_1+p_2)^2, (k_2-p_3)^2, \\
& (k_1+p_1+p_2-p_3)^2, (k_1+p_1+p_2-p_3-p_4)^2, \\
& (k_2-p_3-p_4)^2, (k_1+p_1)^2 \quad . \qquad (8)
\end{aligned}
$$

Results for 2➛3

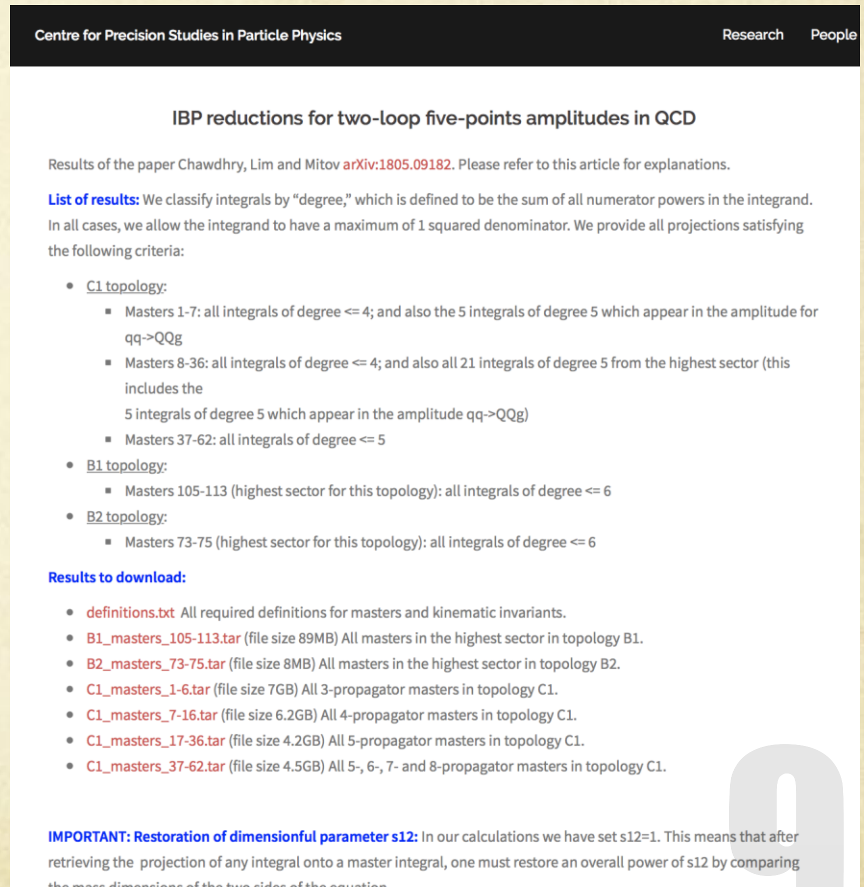


- ✓ We determined all masters. We find:
  - 113 masters in B1,
  - 75 in B2,
  - 62 in C1,
  - 28 in C2.

- ✓ We solved the IBPs for topologies C1 and C2 for all required numerator powers:
  - -5 for C1
  - -4 for C2

- ✓ The coefficients of the highest weight masters in:
  - B1 (9 masters)
  - B2 (3 masters)
  - Both for numerator powers of up to -6

- ✓ The results are available for download from here (22 GB in total):



Centre for Precision Studies in Particle Physics Research People

IBP reductions for two-loop five-points amplitudes in QCD

Results of the paper Chawdhry, Lim and Mitov arXiv:1805.09182. Please refer to this article for explanations.

List of results: We classify integrals by "degree," which is defined to be the sum of all numerator powers in the integrand. In all cases, we allow the integrand to have a maximum of 1 squared denominator. We provide all projections satisfying the following criteria:

- C1 topology:
  - Masters 1-7: all integrals of degree <= 4; and also the 5 integrals of degree 5 which appear in the amplitude for qq->QQg
  - Masters 8-36: all integrals of degree <= 4; and also all 21 integrals of degree 5 from the highest sector (this includes the 5 integrals of degree 5 which appear in the amplitude qq->QQg)
  - Masters 37-62: all integrals of degree <= 5
- B1 topology:
  - Masters 105-113 (highest sector for this topology): all integrals of degree <= 6
- B2 topology:
  - Masters 73-75 (highest sector for this topology): all integrals of degree <= 6

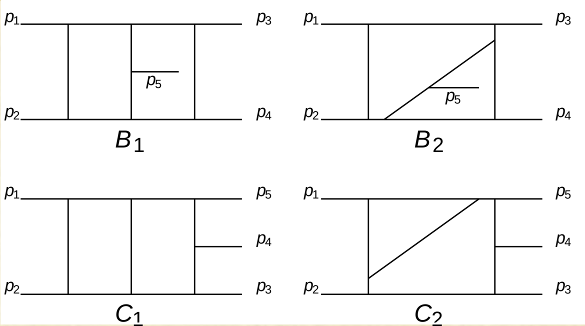
Results to download:

- definitions.txt All required definitions for masters and kinematic invariants.
- B1_masters_105-113.tar (file size 89MB) All masters in the highest sector in topology B1.
- B2_masters_73-75.tar (file size 8MB) All masters in the highest sector in topology B2.
- C1_masters_1-6.tar (file size 7GB) All 3-propagator masters in topology C1.
- C1_masters_7-16.tar (file size 6.2GB) All 4-propagator masters in topology C1.
- C1_masters_17-36.tar (file size 4.2GB) All 5-propagator masters in topology C1.
- C1_masters_37-62.tar (file size 4.5GB) All 5-, 6-, 7- and 8-propagator masters in topology C1.

IMPORTANT: Restoration of dimensionful parameter s12: In our calculations we have set s12=1. This means that after retrieving the projection of any integral onto a master integral, one must restore an overall power of s12 by comparing the mass dimensions of the two sides of the equation.

http://www.precision.hep.phy.cam.ac.uk/results/amplitudes/

Results for 2➛3: checks



✓ The first check was the recalculation of the 2 ➛ 2 amplitudes versus Reduze.

✓ The results of these papers can be used to relate the integrals in C1 with numerators of power -5 to numerators with lower power. We have checked this is consistent with our direct evaluation of the coefficients in C1

Gluza, Kajda, Kosower '10
Kosower '18

✓ Non-trivial checks in topology B2: in this paper the results for B2 were presented for powers up to -4. We checked that the coefficients of the highest weight masters (i.e. the top sector) agree.

Boehm, Georgoudis, Larsen, Schoenemann, Zhang '18

✓ To the best of my understanding, this reference claims that the planar topologies have been computed up to powers of -5. However no results are presented, or details given, so we cannot compare.

Boels, Jin, Luo '18

Some details about the calculation

- ✓ The calculation is implemented in our own private code
- ✓ Code written in C++. Runs are very stable.
- ✓ The bottleneck is the manipulation of rational expressions.
- ✓ To that end we use the program *Fermat* and find it indispensable (with many thanks to Robert Lewis for support)

R. H. Lewis, http://home.bway.net/lewis/

- ✓ The run times are vastly different
  - ✓ Easiest for the complicated masters: takes minutes
  - ✓ Hardest for the easiest masters (ones with 3 propagators): took several weeks
  - ✓ Even among the masters of weight 3 there is a run-times difference of factor of 10.
- ✓ Compressed solutions for C1 are about 20 GB. This includes integrals with squared denominators.
- ✓ Coefficients are not simplified in any way (fully expanded form)
- ✓ We have not attempted anything fancy in the definition of the IBP equations (defined in terms of the external and/or loop momenta). One can improve here.

Putting it all together: from IBPs to amplitudes

- ✓ In principle one can compute the planar amplitude now
  - ✓ The masters for C1 and C2 are known analytically

    Papadopoulos, Tommasini, Wever '15
  - ✓ They are available as a Mathematica code in terms of GPLs
  - ✓ The evaluation of the GPLs for complex argument can be done with GINAC
- ✓ We have tested all this and computed it in few points
- ✓ This is not yet a solution to the problem of the amplitude:
  - ✓ One has to derive the finite remainder (easy once the non-planar is also available)
  - ✓ Computing in Minkowski using the analytic continuation of GINAC is not optimal for practical evaluation of the amplitude.
  - ✓ There are many crossings. This significantly increases the number of integrals to be evaluated.
- ✓ The above mean that obtaining useable amplitude for phenomenological applications is another step after all is put together. We have certain ideas and are developing them at present.

Conclusions and outlook

- The calculation of the 2 ➛ 3 massless amplitudes in QCD is seriously underway and many new approaches are starting to make real progress

- The planar amplitudes are now available.

- Non-planar are still non-existent. In particular, the non-planar masters are not yet known.

- We have proposed a new strategy for the solving of the IBP identities which scales better for complicated problems (multiscale ones but we also suspect it will be useful in multiloop ones)

- We have been able to analytically evaluate the IBPs needed for any planar 2 ➛ 3 amplitude. Results publicly available for download.

- We are starting now the running for the non-planar topologies which is much more complicated

- Hope to have results to report at the next Loopfest!

- I am confident that 2 ➛ 3 phenomenology at NNLO will be available in time for the HL-LHC (and maybe even for the next LHC run).