Electroweak Corrections for Future e^+e^- Colliders: Low-Angle Bhabha Scattering and Charge Asymmetry Near the Z Resonance

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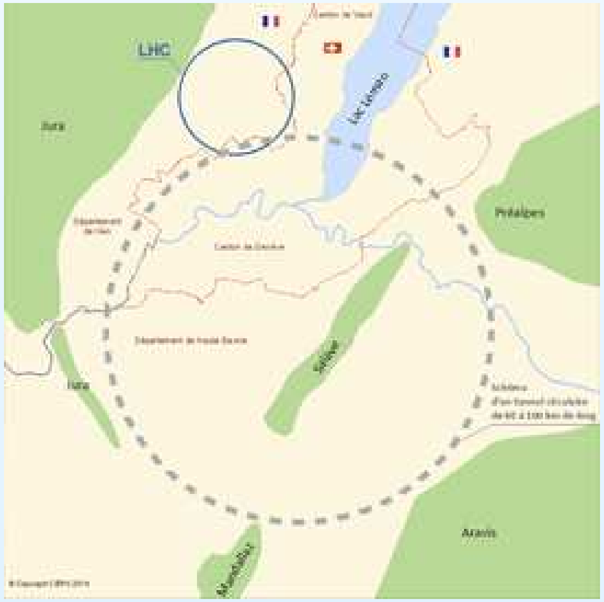
EW Precision Needs for FCC-ee

- This talk will look at EW precision needs for the proposed high-luminosity FCC-ee with circumference 100 km and energies from 90 GeV to 400 GeV.
- Focus on QED corrections for EW precision measurements at energies near M_Z.
    - I. low-angle Bhabha luminosity: the path toward 0.01%. (S. Jadach, W. Płaczek, M. Skrzypek, B.F.L. Ward and S. Yost, FCC-ee WG study to appear.)
    - II. IFI effects in a proposed measurement of $\alpha(M_Z)$. (See S. Jadach and S. Yost, arXiv:1801.08611 for much more detail.)



I. Low Angle Bhabha: LEP Legacy

	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $\mathcal{O}(\alpha^2)$	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization	0.04%	0.04%	0.10%	0.10%
(d) Light pairs	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.12%

- LEP1: energies near Z peak and angular range $1^\circ - 3^\circ$ ($18 - 52$ mrads),
- LEP2: energies up to 176 GeV and angular range 3°–6°.
- Technical precision included in (a).
- The target precision for FCC-ee will be 0.01%.

QED Update: 2018

Type of Correction / Error	1999	Update 2018 (LEP1)
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.015%
(c) Vacuum polariz.	0.040%	0.013%
(d) Light pairs	0.030%	0.010%
(e) Z and s-channel γ exchange	0.015%	0.015%
(f) Up-down interference	0.0014%	0.0014%
(g) Technical Precision	–	(0.027%)
Total	0.061%	0.038%

- Energies near Z peak, angular range $18 - 52$ mrad, $\sqrt{\langle|t|\rangle} \approx 1.75$ GeV.
- Large logarithm $L_e = \ln(|t|/m_e^2)$.
- Technical precision is not included in the total, because it is part of (a). This needs to be better separated in the future.
- Main improvements: Vacuum polarization (F. Jegerlehner) and light pairs (G. Montagna *et al.*).

The Path to 0.01% at FCC-ee

Type of correction / Error	Update 2018 (FCC-ee)	FCC-ee forecast
(a) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014%	0.6×10^{-4}
(d) Light pairs	0.010%	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090%	0.1×10^{-4}
(f) Up-down interference	0.009%	0.1×10^{-4}
(g) Technical Precision	(0.027%)	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

- The 2018 column shows the present state of the art for the proposed FCC-ee calorimeter parameters: energy near Z peak, angular range $64 - 86$ mrad, $\sqrt{\langle |t| \rangle} \approx 3.25$ GeV.
- Relative to LEP1, vacuum polarization is significantly less important, while Z and s-channel exchange are significantly more important.
- Photonic effects which were rendered irrelevant at LEP by vacuum polarization will come to the forefront at FCC-ee.

Photonic Corrections

Type of correction / Error	Update 2018	FCC-ee forecast
(a) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}

- Photonic corrections are large, soft/collinear resummation mandatory: consider BHLUMI.
- BHLUMI includes $\mathcal{O}(\alpha^1)$ and $\mathcal{O}(L_e^2\alpha^2)$ with YFS soft photon resummation.
- BHLUMI neglects photon interference between e^+, e^- lines – suppressed by $|t|/s = 1.3 \times 10^{-3}$ for $\sqrt{|t|} = 3.25$ GeV at the Z peak.
- $\mathcal{O}(L_e\alpha^2)$ and $\mathcal{O}(\alpha^3 L_e^3)$ known but not implemented yet ($\mathcal{O}(L_e\alpha^2)$ in unpublished version only).
- $\mathcal{O}(L_e^0\alpha^2) \sim 10^{-5}$ also known, not included in FCC error budget. (We ran tests with exact low-angle $\mathcal{O}(\alpha^2)$ residuals in the 1990's, but they are not in BHLUMI yet.)
- There have been many efforts at calculating exact $\mathcal{O}(\alpha^2)$ corrections beyond those relevant at low angles, but all of those are of order 10^{-5}. They should not affect the error budget for low-angle Bhabha, but this should be checked outside BHLUMI.
- Estimate $\mathcal{O}(L_e^4\alpha^4) \sim \gamma \times 0.015\% = 0.6 \times 10^{-5}$, $\mathcal{O}(L_e^2\alpha^3) \sim \gamma^2\alpha/\pi \sim 10^{-5}$. ($\gamma = \frac{\alpha}{\pi} L_e$)

Z and s-channel γ Exchange

Type of correction / Error	Update 2018	FCC-ee forecast
(e) Z and s-channel γ exchange	0.090%	0.1×10^{-4}

- Compared to dominant t channel exchange, all other contributions are suppressed by $|t|/s \sim 1.3 \times 10^{-3}$, instead of 0.4×10^{-3} at LEP.
- Resonant s-channel Z exchange ($\gamma_t \otimes Z_s$)included in BHLUMI at 1st order. Estimated uncertainty is 0.090%.
- Non-resonant s-channel γ exchange ($\gamma_t \otimes \gamma_s$) $\sim 0.1\%$ is included in BHLUMI. Estimated uncertainty is 0.01%.
- Not in BHLUMI: $|Z_s|^2 \sim 0.01\%, \gamma_t \otimes Z_t \sim 3 \times 10^{-5}, |\gamma_s|^2 \sim 10^{-6}, |Z_t|^2 \sim 10^{-6}$.
- Upgrading BHLUMI with EEX matrix element in BHWIDE will reduce these to $\sim 10^{-4}$.
- Upgrading BHLUMI with CEEX matrix element as in KKMC would reduce these to $\sim 10^{-5}$.

Vacuum Polarization

Type of correction / Error	Update 2018	FCC-ee forecast
(c) Vacuum polarization	0.014%	0.6×10^{-4}

- Fractional error due to hadronic vacuum polarization in $\alpha_{\rm eff}(t)$ is $\sim 1.3 \times 10^{-4}$ for FCC.
- This is based on obtaining the hadronic shift $\Delta\alpha^{(5)}_{\rm eff}(-s_0)$ at $s_0 = 2$ GeV from $\sigma_{\rm had}(s < 2.5{\rm GeV})$ using dispersion relations, and perturbative QCD to calculate $\Delta\alpha^{(5)}_{\rm eff}(t)$.
- Taking $\Delta\alpha^{(5)}_{\rm eff}(-2{\rm GeV}) = (64.09 \pm 0.63) \times 10^{-4}$ from F. Jegerlehner, arXiv:1711.06098 gives the above estimate.
- Improvement in $\sigma_{\rm had}$ should give factor of 2 improvement by FCC: estimate error at 0.65×10^{-4}.
- Alternatively: Measure $\alpha_{\rm eff}(M_Z^2)$ directly at FCC-ee and evolve down to $|t|$. Error is likely to be comparable, but could be useful for cross-checks.

Additional Light Fermion Pairs

Type of correction / Error	Update 2018	FCC-ee forecast
(d) Light pairs	0.010%	0.5×10^{-4}

- 2nd order correction: $e^+e^- \to e^+e^- f\overline{f}, f = e, \mu, \tau, u, d, s$, plus corresponding virtual loop on γ line.
- $f = e$ is biggest and well known, precision $\sim 0.5 \times 10^{-4}$
- Remaining f's contribute $\sim 0.8 \times 10^{-4}$ and counted as error for LEP. Can be included with precision $\ll 0.5 \times 10^{-4}$.
- $e^+e^- \to e^+e^- + 2(e^+e^-)$ and $e^+e^- \to 2(e^+e^-) + \gamma$ are calculated but negligible.
- Pair effects will probably have to be calculated outside BHLUMI.

Up-Down Interference

Type of correction / Error	1999	Update 2018
(f) Up-down interference	0.0014%	0.0014%

- This was calculated to be $\delta\sigma/\sigma \sim 0.07|t|/s = 0.91 \times 10^{-4}$.
- It will be included when BHLUMI M.E. is upgraded to BHWIDE or KKMC M.E.
- Conservative precision estimate once it's included is $2\gamma \sim 0.1 \times 10^{-4}$.

Technical Precision

Type of correction / Error	Update 2018	FCC-ee forecast
(g) Technical Precision	(0.027%)	0.1×10^{-4}

- Technical precision is the precision of the M.C. implementation, including control of numerical issues, bugs, etc.
- LEP estimate was based on comparison with semi-analytical calculations and other MC's: LUMLOG+OLDBIS and SABSPV.
- This was bundled with missing photonic corrections as 0.27%.
- Independent MC is needed. BabaYaga (C. Carloni Calame *et al*, based on photon shower algorithm, could provide a good independent test (needs to be validated near Z peak).
- The difficulty would be upgrading BabaYaga to the same precision as the upgraded BHLUMI for FCC-ee, which will require matching the shower to a hard NNLO process.

Low-Angle Bhabha Summary

- There are no hard obstacles to reaching 0.01% precision in Bhabha luminosity via upgrades to BHLUMI.
- It is essential to include $\mathcal{O}(L_e\alpha^2)$ corrections which are already in private versions of BHLUMI, but were not needed for LEP due to large VP uncertainty.
- Up-Down interference has been implemented in KKMC and BHWIDE since 1999 - the BHLUMI M.E. needs to be upgraded to this level as well.
- Z and s-channel γ exchange can also be incorporated with M.E. upgrade: not difficult.
- An auxilliary program will be needed for light pairs.
- A comparable MC would help to evaluate technical precision. Possible NNLO upgrade of BabaYaga for FCC-ee?

II. Direct Determination of $\alpha(M_Z^2)$

- Patrick Janot (arXiv:1512.05544) has proposed determining $\alpha(M_Z^2)$ from measurements of $A_{\rm FB}$ in $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s_\pm} = M_Z \pm 3.5$ GeV at FCC-ee.

- With the target FCC-ee luminosity, the sensitivity of $A_{\rm FB}$ to α provides a means to measure $\alpha(M_Z^2)$ to a fractional error of 2×10^{-5} with a year of data from each energy $M_Z \pm 3.5$ GeV.

- $A_{\rm FB}$ is a strong function of $\sqrt{s_\pm}$ near M_Z, so QED ISR plays a significant role.

- Interference of photons emitted from the initial and final fermions (IFI) affects the muon angular distribution. IFI contributes significantly to $A_{\rm FB}$ away from the Z peak, but is suppressed by Γ_Z/M_Z at the peak.

- The known photonic corrections in KKMC can calculate the effect both of ISR and IFI on $A_{\rm FB}(s_\pm)$, but is the accuracy sufficient?

- S. Jadach and S. Yost, arXiv:1801.08611 looks at the IFI contribution.

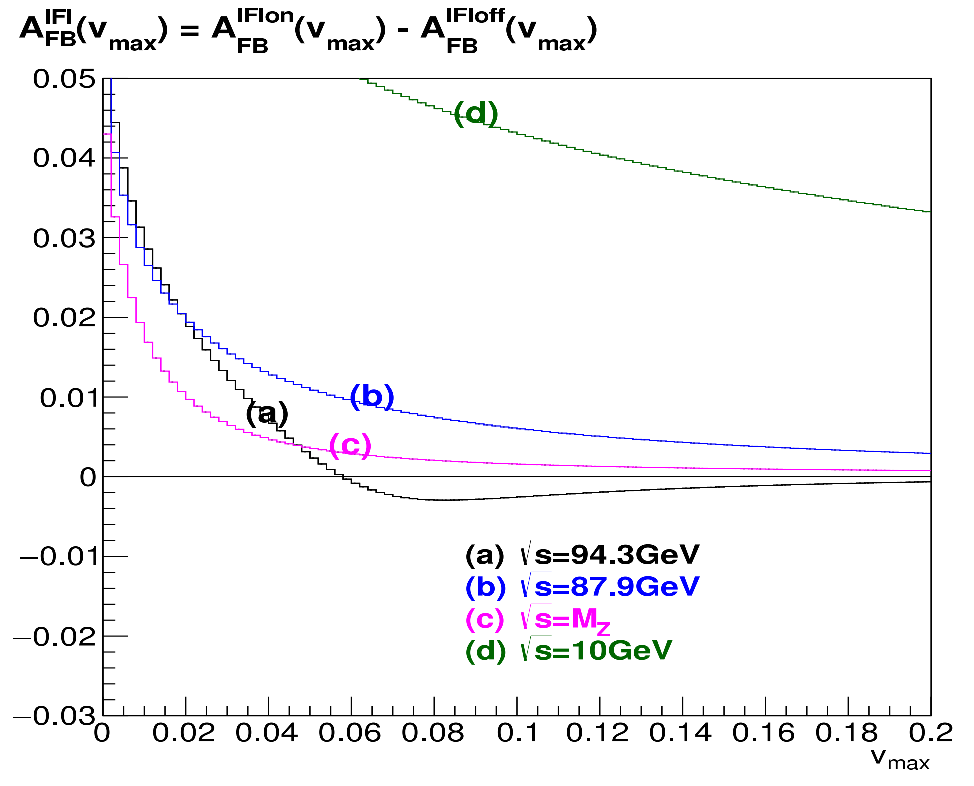
Precision of A_{FB} at $M_Z \pm 3.5$ GeV

- Janot's plan assumes an experimental precision $\delta A_{\mathrm{FB}} \approx 3 \times 10^{-5}$. The analysis will require unfolding QED IFI at the same precision.
- IFI is of order 1% at $\sqrt{s} = M_Z \pm 3.5$ GeV and grows with tighter cutoffs.
- KKMC includes soft photon resummation in the presence of a narrow resonance, including resummation of $\ln(\Gamma_Z/M_Z)$ factors. This was essential in the LEP2 analysis above the Z peak.
- The quoted LEP-era precision was estimated $\delta A_{\mathrm{FB}} \approx 0.1\%$ near the Z peak (and 0.3% far from it). It looks like we need a factor of 100 improvement for FCC-ee. Is this possible? Is this correct?
- The LEP estimate was believed to be overly conservative, but it was hard to quantify due to the absence of any other calculations of comparable precision.
- One of our goals was to produce a new semianalytical program to calculate IFI and provide a point of comparison.

Cutoff Dependence of IFI in A_{FB}

The graph shows the IFI contribution to A_{FB} in KKMC due to a cutoff $v_{\max}$ on the fraction of the total CMS energy radiated to photons in $e^+e^- \to Z/\gamma^* \to \mu^+\mu^-$.

- Note the increased importance of IFI for tight cutoffs on $v_{\max}$.



KKFoam Semi-analytical Calculation

KKFoam was developed as a semianalytic alternative to KKMC for calculating IFI. It begins with an amplitude-level CEEX resummation of the multi-photon emission amplitude in the semi-soft regime, $k_i^0 \ll \sqrt{s}/2$, but allowing photon energies $\geq \Gamma_Z$: for

$$e^-(p_1) + e^+(p_2) \to \mu^-(q_1) + \mu^+(q_2) + \gamma(k_1) + \cdots + \gamma(k_n),$$

$$\mathcal{M}^{\mu_1,\mu_2,\ldots,\mu_n}(p_i, q_j, k_l) = \sum_{V=\gamma,Z} \sum_{\mathcal{P}} e^{\alpha B_4^V(s_I,t,m_\gamma)} \prod_{i\in I} j_I^{\mu_i}(k_i) \prod_{r\in F} j_F^{\mu_r}(k_r)\, \mathcal{M}_0\big(s_I, t\big)$$

where the $\mathcal{P}$ sum is over the 2^n partitions of the photons into sets I and F of initial and final state emission and

$$s_I = P_I^2, \qquad P_I = p_1 + p_2 - \sum_{i\in I} k_j, \qquad \mathcal{M}_0 = \text{Born M.E.}$$

$$j_I^\mu(k) = eQ_e\left(\frac{p_1^\mu}{k\cdot p_1} - \frac{p_2^\mu}{k\cdot p_2}\right), \qquad j_F^\mu(k) = eQ_f\left(\frac{q_1^\mu}{k\cdot q_1} - \frac{q_2^\mu}{k\cdot q_2}\right).$$

B_4^V is the standard YFS virtual form factor when $V = \gamma$, but includes an added term which is nonzero only when $V = Z$:

$$\Delta B_4^Z(s, t, \overline{M}_Z^2) = -2Q_eQ_f\frac{\alpha}{\pi}\ln\left(\frac{t}{u}\right)\ln\left(\frac{\overline{M}_Z^2 - s}{\overline{M}_Z^2}\right), \qquad \overline{M}_Z^2 = M_Z^2 - iM_Z\Gamma_Z.$$

KKFoam Semi-analytical Calculation

When the amplitude on the previous slide is squared and summed over helicities, products of the j's produce eikonal factors for ISR, FSR, and IFI:

$$S_I(k) = -j_I(k) \cdot j_I(k), S_F(k) = -j_F(k) \cdot j_F(k), S_X(k) = -j_I(k) \cdot j_F(k).$$

A compact formula is obtained using a Fourier representation:

$$\sigma(s, v_{\max}) = \frac{3}{8}\sigma_0(s) \sum_{V,V'} \int_0^1 dv_I dv_F dr dr' \int \frac{d\cos\theta d\phi}{2} \theta(v_{\max} - v_I - r - r' - v_F)$$

$$\times \rho(\gamma_I, v_I)\rho(\gamma_X, r)\rho(\gamma_X, r')\rho(\gamma_F, v_F) e^{Y(p_i, q_i)}$$

$$\times \sum_{\epsilon,\tau} \mathrm{Re}\left\{ e^{\alpha \Delta B_4^V(s(1-v_I-r))} \mathcal{M}^V_{\epsilon,\tau}(v_I + r, c) \left[e^{\alpha \Delta B_4^{V'}(s(1-v_I-r'))} \mathcal{M}^{V'}_{\epsilon,\tau}(V_I + r', c) \right]^* \right\}$$

with $c = \cos\theta$ for the angle between the e^- and μ^- and

$$\rho(\gamma, v) = F(\gamma)\gamma v^{\gamma-1}, \quad F(\gamma) = \frac{e^{-\gamma C_E}}{\Gamma(1+\gamma)}, \quad \gamma_a = \int \frac{d^3k}{k^0} S_a(k) \delta\left(\frac{2k^0}{\sqrt{s}} - 1\right), a = I, F, X,$$

$$Y(p_i, q_i) = 2\alpha \mathrm{Re} B_4(s, t, m_\gamma) + \int_{k^0 < \sqrt{s}/2} \frac{d^3k}{k^0} \left[S_I(k) + 2S_X(k) + S_F(k) \right].$$

KKFoam Semi-analytical Calculation

The variable $v = v_I + v_F + r + r' = 1 - M^2_{\mu\mu}$ is extrapolated beyond the semi-soft regime using a multiplicative ansatz for the constraint,

$$1 - v = (1 - v_I)(1 - v_F)(1 - r)(1 - r')$$

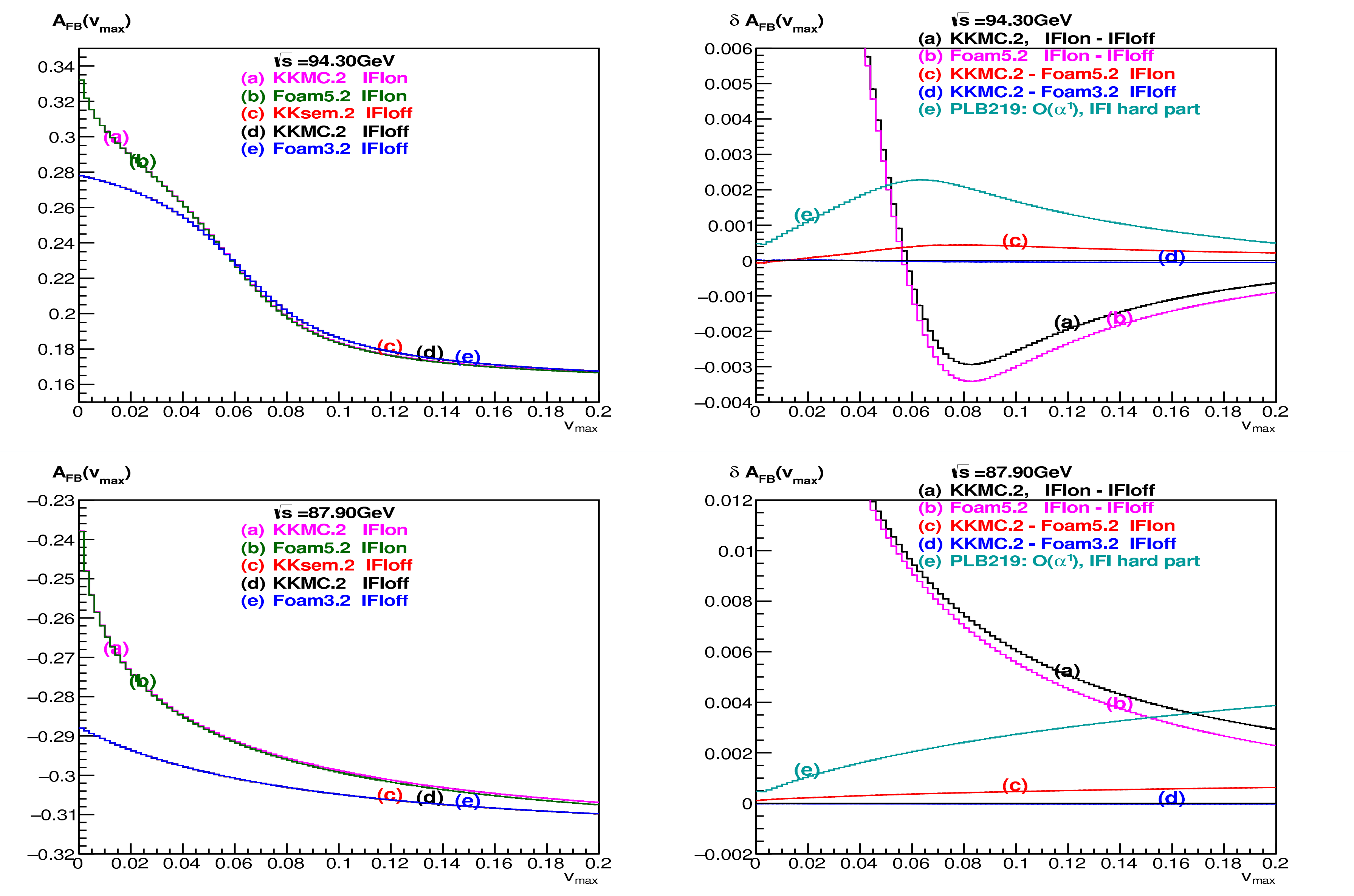
and for comparison to the $\mathcal{O}(\alpha^2)$ exponentiated formulas in KKMC, the radiator functions ρ are upgraded to $\mathcal{O}(\alpha^2)$. The complete $\mathcal{O}(\alpha^1)$ virtual IFI contribution (non-IR) is also added, and the 5-dimensional integral over v, v_I, v_F, r, and r' is done using the FOAM adaptive MC integrator.

A 3-dimensional version of KKFoam without r, r' was also programmed for comparison to KKMC with IFI off.

The difference between $A_{\rm FB}$ calculated with KKMC and KKFoam as a function of $v_{\rm max}$ was calculated for $\sqrt{s_+} = 94.3$ GeV and $\sqrt{s_-} = 87.9$ GeV.

The red curve (c) in the next slide shows that $\delta A_{\rm FB} < 5 \times 10^{-4}$, while curve (e) is included to show that the difference roughly tracks the hard $\mathcal{O}(\alpha^1)$ real contribution, which is omitted in KKFoam.

Comparisons of KKMC and KKFoam

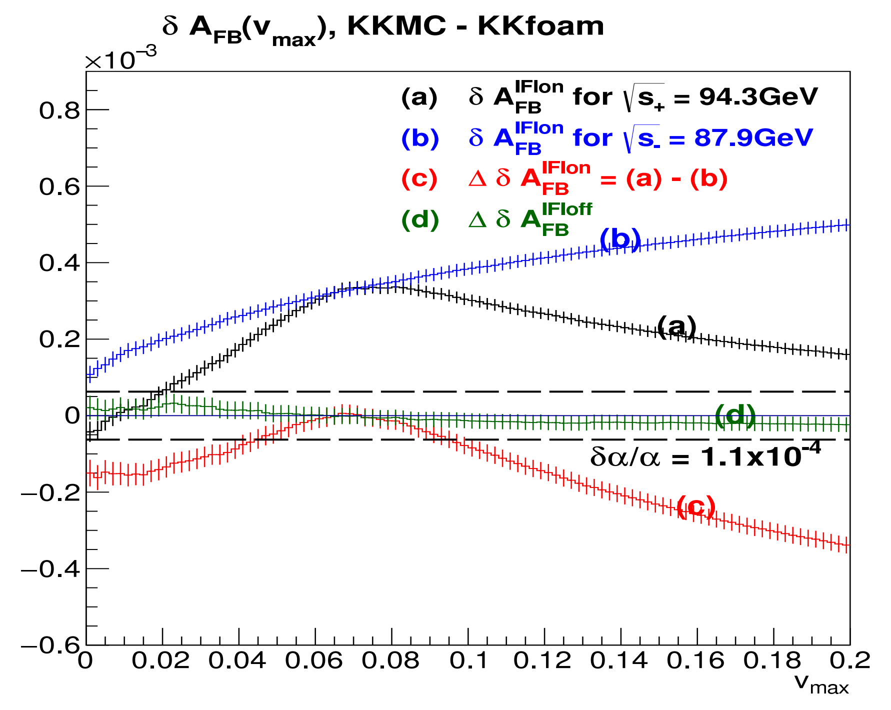


Error Propagation from $A_{\rm FB}$ to α

For the purposes of finding α_s via Janot's proposal, the error propagation based on the difference between KKMC and KKFoam can be well represented for small $\delta A_{\rm FB}$'s by the simple expression

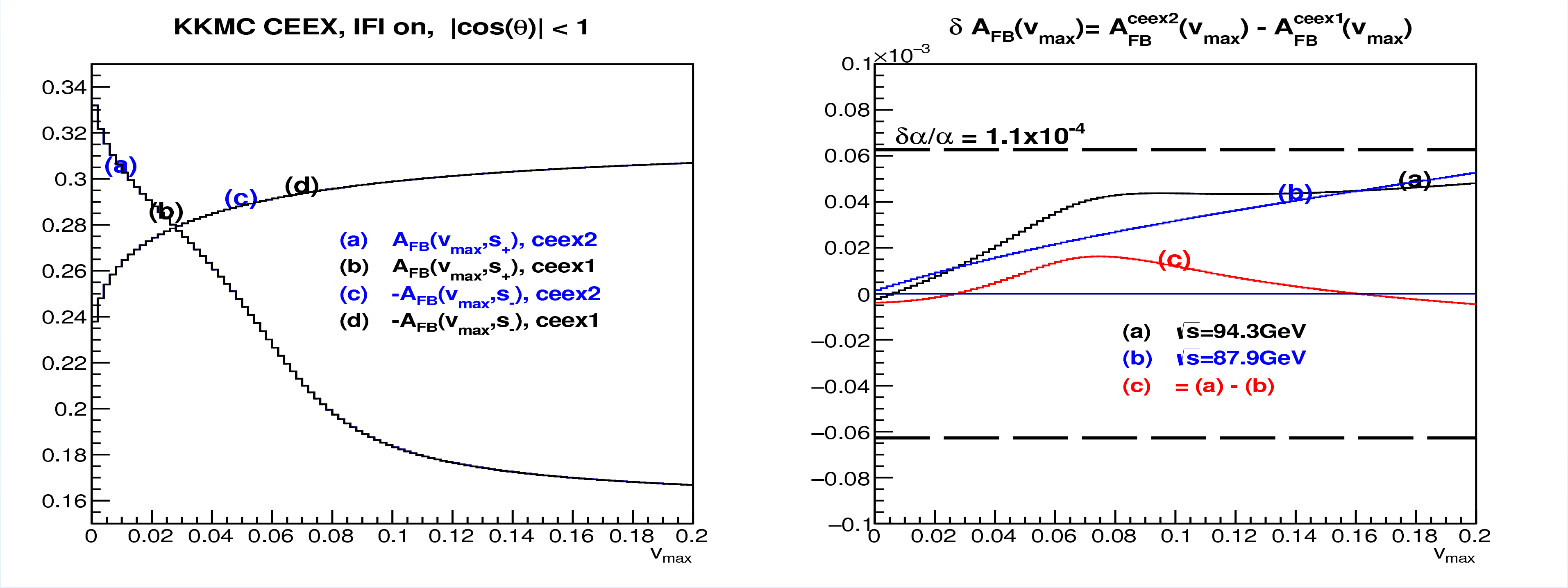
$$\frac{\delta\alpha}{\alpha} \approx \frac{\delta A_{\rm FB}(s_+) - \delta A_{\rm FB}(s_-)}{A_{\rm FB}(s_+) - A_{\rm FB}(s_-)} \equiv \frac{\Delta\delta A_{\rm FB}}{\Delta A_{\rm FB}} \qquad (\Delta A_{\rm FB} \sim 0.5)$$

We find that $\Delta\delta A_{\rm FB} \le 2\times 10^{-4}$ for $v_{\rm max} \le 0.1$. This is much bigger than 3×10^{-5}, which is the technical precision with IFI off. The dashed line shows the current precision for α.



KKMC Internal Precision Estimate

The main difference on the previous slide is likely due to approximations in KKFoam, so the error in KKMC is likely to be less. We could estimate it by comparing the size of $\mathcal{O}_{\exp}(\alpha^i)$ contributions for $i = 1, 2$. We see that the 2nd order contribution is $< 3 \times 10^{-5}$ for $v_{\max} < 0.06$, and that $\Delta\delta A_{\rm FB} < 2 \times 10^{-5}$. This is what is needed for the FCC-ee measurement, assuming it is the main source of error. (Other classes of EW error should be examined as well.)



A_{FB} Summary

- Our recent studies of A_{FB} in muon production suggest that the uncertainty in KKMC's prediction for the IFI component is less than 10^{-4}, which is much better than was claimed in the LEP era (10^{-3}).
- The precision near the Z resonance may, in fact, be at the level 3×10^{-5} required for the proposed FCC-ee measurement of $\alpha(M_Z^2)$.
- More work is needed for better confidence in this conclusion.
- Non-photonic corrections (*e.g.* pair emission and other EW corrections) also need to be estimated.
- An upgrade of KKFoam to include exact non-soft $\mathcal{O}(\alpha^1)$ real emission matched to the semi-soft analytic resummation would be useful.