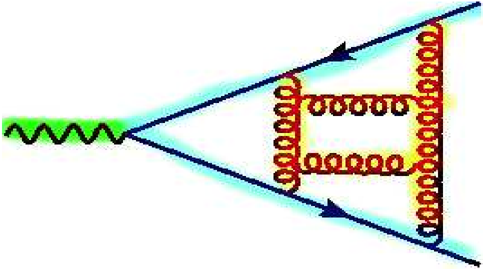
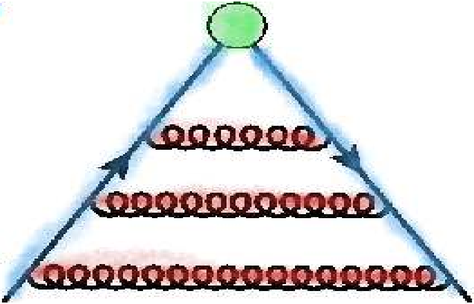
3- and 4-loop form factors

Matthias Steinhauser | Loopfest, Michigan State University, July 16-20, 2018

TTP KARLSRUHE





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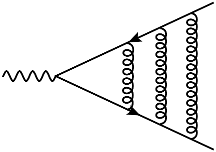
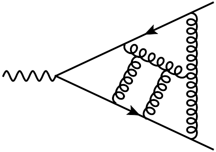
Photon-quark-anti-quark form factor





- Motivation
- Massless 4-loop form factor
- Massive 3-loop form factor
- Outlook

R. N. Lee, A. V. Smirnov, V. A. Smirnov, MS:
"Three-loop massive form factors: complete light-fermion and large-N_c corrections for vector, axial-vector, scalar and pseudo-scalar currents", 2018

R. N. Lee, A. V. Smirnov, V. A. Smirnov, MS:
"Three-loop massive form factors: complete light-fermion corrections for the vector current", 2018

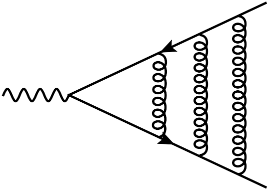
R. N. Lee, A. V. Smirnov, V. A. Smirnov, MS:
"The n_f^2 contributions to fermionic four-loop form factors", 2017

J. Henn, R. Lee, A. Smirnov, V. Smirnov, MS:
"Four-loop photon quark form factor and cusp anomalous dimension in the large-N_c limit of QCD", 2016

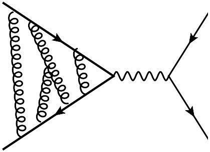
J. Henn, A. Smirnov, V. Smirnov, MS:
"A planar four-loop form factor and cusp anomalous dimension in QCD", 2016

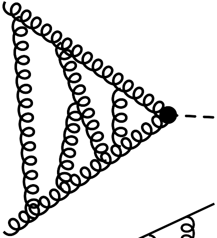
J. Henn, A. Smirnov, V. Smirnov, MS:
"Massive three-loop form factor in the planar limit", 2016

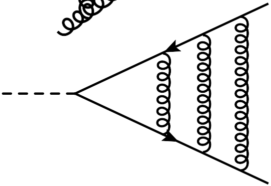
Form factors for physical processes



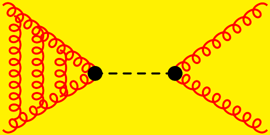
- $e^+e^- \to Q\bar{Q}$
- Forward-backward asymmetry
- static properties: $(g-2)_{\mathrm{quark}}$
- Γ_{cusp}
- Drell-Yan
- Higgs production
- Higgs decay: $H \to Q\bar{Q}$, $A \to Q\bar{Q}$



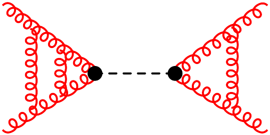


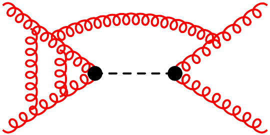


Example: Higgs production at the LHC

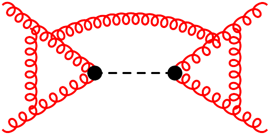


[Baikov,Chetyrkin,Smirnov,Smirnov, Steinhauser'09], [Gehrmann,Glover,Huber,Ikizlerli, Studerus'10]; [Lee,Smirnov'10]

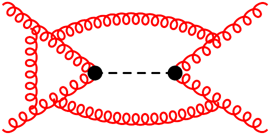




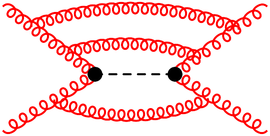
[Duhr,Gehrmann'13], [Li,Zhu'13], [Dulat,Mistlberger'14], [Duhr,Gehrmann,Jaquier'14]



[Anastasiou,Duhr,Dulat,Herzog, Mistlberger'13], [Kilgore'13]



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann, Herzog,Mistlberger'14], [Li,von Manteuffel,Schabinger,Zhu'14]

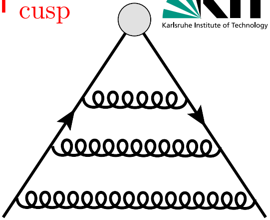


[Anastasiou,Duhr,Dulat,Mistlberger'13]

N^3LO: [Anastasiou,Duhr,Dulat,Herzog,Mistlberger'15]

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16; Mistlberger'18]

IR structure of massive form factor: $\Gamma_{\rm cusp}$



- F: UV-renormalized massive form factor

- $F = Z\,F^{\rm finite}$

$$Z = 1 + \frac{\alpha_s}{\pi}\left(-\frac{1}{2\epsilon}\Gamma^{(1)}_{\rm cusp}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\#}{\epsilon^2} - \frac{1}{4\epsilon}\Gamma^{(2)}_{\rm cusp}\right) + \left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{\#}{\epsilon^3} + \frac{\#}{\epsilon^2} - \frac{1}{6\epsilon}\Gamma^{(3)}_{\rm cusp}\right) + \ldots$$

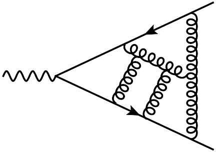
$$\Gamma^{(i)}_{\rm cusp} = \Gamma^{(i)}_{\rm cusp}(q^2/m^2)$$

➪ $\Gamma_{\rm cusp}$ from poles of F

- Note: $\Gamma_{\rm cusp}$ is universal

  ➪ same result for vector, axial-vector, scalar, pseudo-scalar current

IR structure of massless form factor



$$\log(F_q)|_{\text{pole part}} =$$

$$\frac{\alpha_s}{4\pi}\left\{\frac{1}{\epsilon^2}\left[-\frac{1}{2}C_F\gamma^0_{\text{cusp}}\right]+\frac{1}{\epsilon}\left[\gamma^0_q\right]\right\}$$

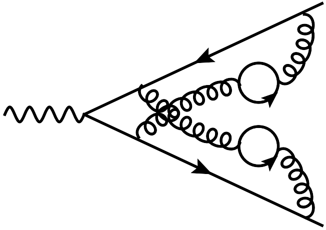
$$+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{1}{\epsilon^3}\left[\frac{3}{8}\beta_0 C_F\gamma^0_{\text{cusp}}\right]+\frac{1}{\epsilon^2}\left[-\frac{1}{2}\beta_0\gamma^0_q-\frac{1}{8}C_F\gamma^1_{\text{cusp}}\right]+\frac{1}{\epsilon}\left[\frac{\gamma^1_q}{2}\right]\right\}$$

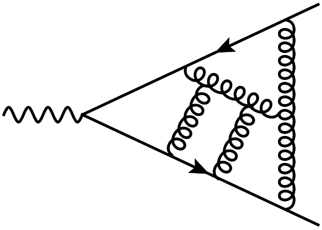
$$+\ldots$$

γ_{cusp}: light-like cusp anomalous dimension, from $1/\epsilon^2$ pole

γ_q: collinear anomalous dimension, from $1/\epsilon$ pole

III. Massless form factor





F_q

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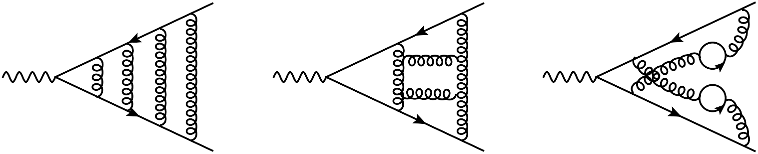
$$F_q(q^2) = -\frac{1}{4(1-\epsilon)q^2}\mathrm{Tr}\left(\not{q}_2\,\Gamma_q^\mu \not{q}_1\,\gamma_\mu\right)$$



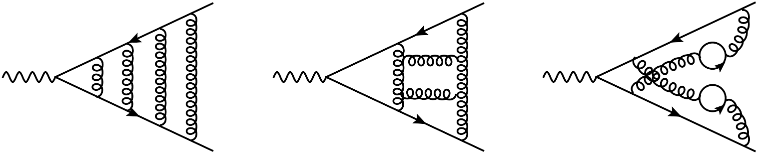
- All planar diagrams ➪ large-N_c

[Henn,Smirnov,Smirnov,Steinhauser'16; Henn,Lee,Smirnov,Smirnov,Steinhauser'16]

- All n_f^2 terms (planar and non-planar) [Lee,Smirnov,Smirnov,Steinhauser'17]

F_q

$$F_q(q^2) = -\frac{1}{4(1-\epsilon)q^2}\mathrm{Tr}\left(\not{q}_2\,\Gamma_q^{\mu}\not{q}_1\,\gamma_\mu\right)$$



- All planar diagrams ➪ large-N_c

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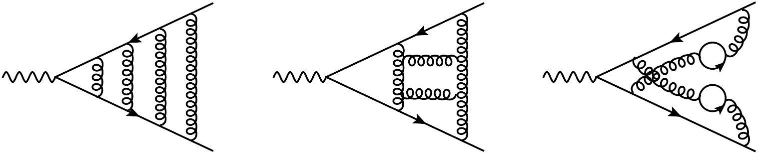
- All n_f^2 terms (planar and non-planar) [Lee,Smirnov,Smirnov,Steinhauser'17]

1. Reduction to master integrals
2. Compute master integrals

F_q

$$F_q(q^2) = -\frac{1}{4(1-\epsilon)q^2}\text{Tr}\left(\not{q}_2\,\Gamma_q^\mu \not{q}_1\,\gamma_\mu\right)$$



- All planar diagrams ➪ large-N_c

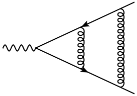
[Henn,Smirnov,Smirnov,Steinhauser'16; Henn,Lee,Smirnov,Smirnov,Steinhauser'16]

- All n_f^2 terms (planar and non-planar) [Lee,Smirnov,Smirnov,Steinhauser'17]

1. Reduction to master integrals ➪ bottleneck
2. Compute master integrals

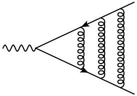
Status

- massless form factor



[Kramer,Lampe'87; Matsuura,van der Marck,van Neerven'88;

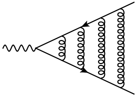
Harlander'00; Ravindran,Smith,van Neerven'05; Gehrmann,Huber,Maitre'05]



pole part: [Moch,Vermaseren,Vogt'05]

fermionic part: [Moch,Vermaseren,Vogt'05]

full: [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser'09]
[Gehrmann,Glover,Huber,Ikizlerli,Studerus'10]; [Lee,Smirnov'10]



all n_f terms, large-N_c: [Henn,Smirnov,Smirnov,Steinhauser'16]

n_f^3 terms Higgs-gluon and photon-quark FF: [von Manteuffel,Schabinger'16]

full large-N_c: [Henn,Lee,Smirnov,Smirnov,Steinhauser'16]

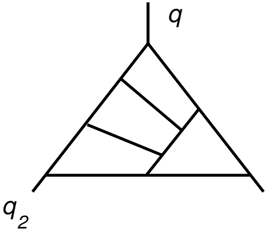
complete n_f^2 terms: [Lee,Smirnov,Smirnov,Steinhauser'17]

- $\gamma_{\rm cusp}$ $(1/\epsilon^2$ pole)

[Boels,Huber,Yang'17]: $\mathcal{N} = 4$ SYM, numerically

[Moch,Ruijl,Ueda,Vermaseren,Vogt'17'18] analytic and numerical results

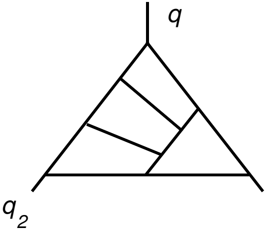
Computation of MIs



- reduction: `FIRE` [A. Smirnov], `LiteRed` [Lee]
- MIs: $q^2 \neq 0$ and $q_2^2 = (q_2 + q)^2 = 0$
- integrals simple if $q_2^2 = q^2$

- idea: introduce arbitray q_2^2 ➪ differential equations [Henn,Smirnov,Smirnov'14]
- boundary conditions: $q_2^2 = q^2$
- use canonical basis [Henn'13'14] [Lee'14] [Gituliar,Magerya'16; Meyer'16; Prausa'17]
  solution: iterated integrals ➪ harmonic polylogarithms (HPLs) [Remiddi,Vermaseren'99][Maitre'05]
- $q_2^2 \to 0$: $\left(q_2^2/q^2\right)^{a\,\epsilon}$ extract term with $a = 0$

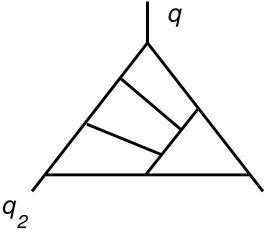
Computation of MIs: more details



➪ 76 MIs

$q_2^2 \neq 0$ ➪ 332 MIs $\qquad x = q_2^2/q^2$

Computation of MIs: more details



➪ 76 MIs

$q_2^2 \neq 0$ ➪ 332 MIs $\qquad x = q_2^2/q^2$

primary basis

canonical basis

$$f(x,\epsilon) \xrightarrow{f=T\cdot g \text{ [Lee'14]}} \boxed{\begin{aligned} g'(x,\epsilon) &= \epsilon A(x)\cdot g(x,\epsilon) \\ A(x) &= \frac{a}{x} + \frac{b}{x-1} \end{aligned}}$$

Computation of MIs: more details

primary basis

canonical basis

$$f(x,\epsilon) \xrightarrow{f=T\cdot g \text{ [Lee'14]}}$$

$$g'(x,\epsilon) = \epsilon A(x) \cdot g(x,\epsilon)$$

$$A(x) = \frac{a}{x} + \frac{b}{x-1}$$

$\downarrow$

solve in terms of HPLs

Computation of MIs: more details

primary basis

canonical basis

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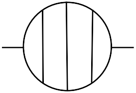
$$g'(x,\epsilon) = \epsilon\, A(x)\cdot g(x,\epsilon)$$
$$A(x) = \frac{a}{x} + \frac{b}{x-1}$$

↓

solve in terms of HPLs

boundary conditions for $x=1$:



4-loop

2-point functions

[Baikov,Chetyrkin,Kühn'05'08;...; Lee,Smirnov,Smirnov'11]

Computation of MIs: more details

primary basis

$f(x,\epsilon)$ $\xrightarrow{f=T\cdot g \text{ [Lee'14]}}$

canonical basis

$$g'(x,\epsilon) = \epsilon\, A(x)\cdot g(x,\epsilon)$$
$$A(x) = \frac{a}{x} + \frac{b}{x-1}$$

$\downarrow$

solve in terms of HPLs
boundary conditions for $x = 1$:

$\downarrow$

get $x = 0$ result ("naive"):

1. $g_{x\to 0} = x^{\epsilon a} h(\epsilon)$ a: 332×332 matrix

➪ $x^{\epsilon a}$ is 332×332 matrix; each element is linear combination of $x^{k\epsilon}$ terms with $k \leq 0$

Computation of MIs: more details

primary basis

canonical basis

$$f(x,\epsilon) \xrightarrow{f=T\cdot g \text{ [Lee'14]}} g(x,\epsilon)$$

$\downarrow$

solve in terms of HPLs
boundary conditions for $x = 1$:

$\downarrow$

get $x = 0$ result ("naive"):

1. $g_{x\to 0} = x^{\epsilon a} h(\epsilon)$
2. expand HPLs for $x \to 0$

match 1. and 2.

➪ $h(\epsilon)$

➪ get $x^{0\epsilon}$ terms $\widehat{=}$ "naive"

Computation of MIs: more details

primary basis | canonical basis

$f(x,\epsilon) \xrightarrow{f=T\cdot g \text{ [Lee'14]}} g(x,\epsilon)$

$\downarrow$

solve in terms of HPLs
boundary conditions for $x = 1$:

$\downarrow$

get $x = 0$ result ("naive"):
1. $g_{x\to 0} = x^{\epsilon a} h(\epsilon)$
2. expand HPLs for $x \to 0$

match 1. and 2.

➪ $h(\epsilon)$

➪ get $x^{0\epsilon}$ terms $\hat{=}$ "naive"

$f(0,\epsilon) \xleftarrow{f=T\cdot g}$

$f(0,\epsilon)$

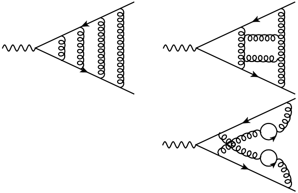
$\downarrow$

332 MIs

$\downarrow$

76 MIs

Results

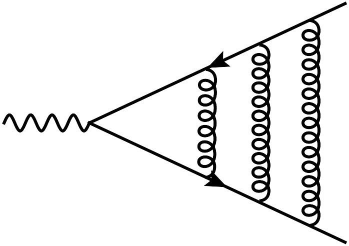


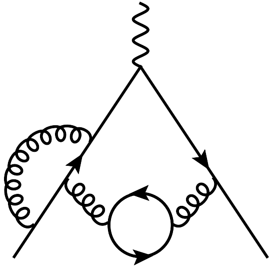
- analytic results (incl. ϵ^0) for:
  - full n_f^2 (vector and scalar current)
  - rest: large-N_c
- n_f^3 agrees with [von Manteuffel,Schabinger'16]
- $1/\epsilon^2$ pole: $\gamma_{\rm cusp}$ [Moch,Ruijl,Ueda,Vermaseren,Vogt'17]

$$\log(F_q)|^{(4)}_{\text{finite part}} =$$
$$N_c^4\Big(-14s_{8a}+10\pi^2\zeta_3^2-\frac{86647\zeta_3^2}{54}+766\zeta_5\zeta_3-\frac{251\pi^4\zeta_3}{6480}-\frac{57271\pi^2\zeta_3}{1296}+\frac{173732459\zeta_3}{23328}$$
$$+\frac{1517\pi^2\zeta_5}{216}-\frac{881867\zeta_5}{1080}-\frac{36605\zeta_7}{288}+\frac{674057\pi^8}{5443200}-\frac{135851\pi^6}{77760}+\frac{386729\pi^4}{31104}$$
$$-\frac{429317557\pi^2}{839808}-\frac{54900768805}{6718464}\Big)+\ldots N_c^3n_f+\ldots n_f^2C_AC_F+\ldots n_f^2C_F^2+\ldots n_f^3C_F$$

$$s_{8a}=\zeta_8+\zeta_{5,3}\approx 1.0417850291827918834$$

Massive form factor





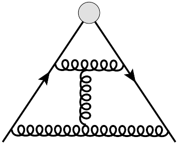
Currents and form factors

$$j_\mu^v = \bar{\psi}\gamma_\mu\psi$$ vector

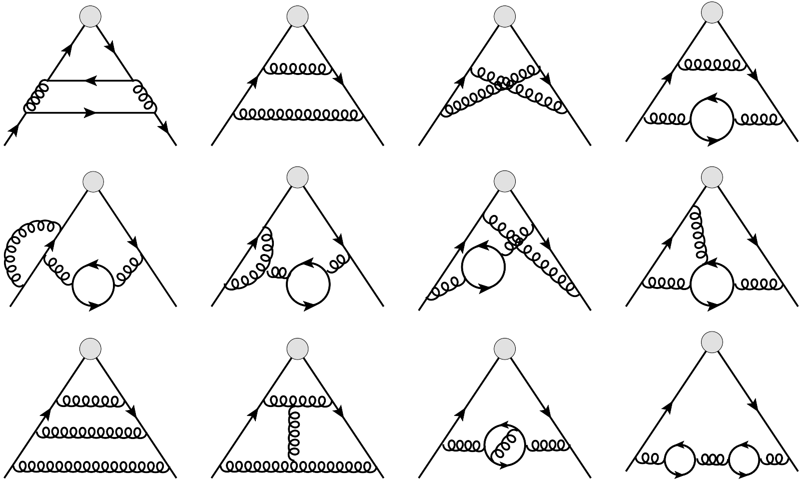
$$j_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\psi$$ axial-vector

$$j^s = m\bar{\psi}\psi$$ scalar

$$j^p = im\bar{\psi}\gamma_5\psi$$ pseudo-scalar



$$\begin{aligned}
\Gamma_\mu^v &= F_1^v(q^2)\gamma_\mu - \frac{i}{2m}F_2^v(q^2)\sigma_{\mu\nu}q^\nu \\
\Gamma_\mu^a &= F_1^a(q^2)\gamma_\mu\gamma_5 - \frac{1}{2m}F_2^a(q^2)q_\mu\gamma_5 \\
\Gamma^s &= mF^s(q^2) \\
\Gamma^p &= imF^p(q^2)\gamma_5\,,
\end{aligned}$$



Massive form factors at 2 and 3 loops

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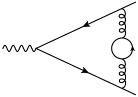
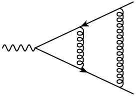
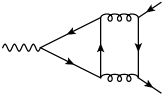


[Hoang,Teubner'97] fermionic corrections

[Mastrolia,Remiddi'03; Bonciani,Mastrolia,Remiddi'04]: QED

[Bernreuther,Bonciani,Gehrmann,Heinesch,Leineweber,Mastrolia,Remiddi'04-'06]





vector, axial-vector, scalar, pseudo-scalar current

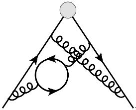
[Gluza,Mitov,Moch,Riemann'09] $+\mathcal{O}(\epsilon)$

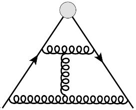
[Ahmed,Henn,Steinhauser'17; Ablinger,Behring,Blümlein,Falcioni,De Freitas,Marquard,Rana,Schneider'17] $+\mathcal{O}(\epsilon^2)$

3 loops:

large-N_c: [Henn,Smirnov,Smirnov,Steinhauer'16]

all-n_l: [Lee,Smirnov,Smirnov,Steinhauer'18]



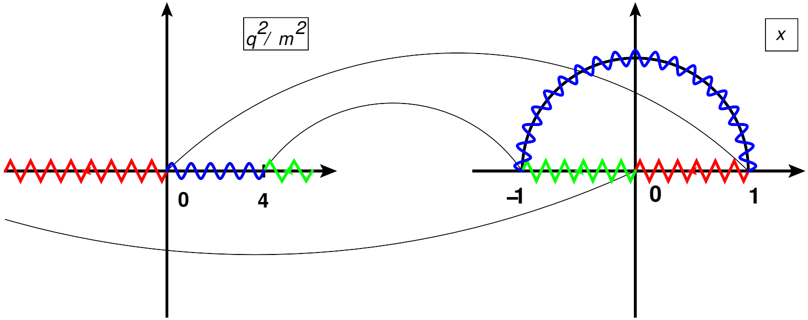


extension to axial-vector, scalar, pseudo-scalar current

[Lee,Smirnov,Smirnov,Steinhauer'18; Ablinger,Blümlein,Marquard,Rana,Schneider'18]

Analytic results

... in terms of Goncharov polylogarithms up to transcendelity weight 6.
[Henn,Smirnov,Smirnov,Steinhauser'16; Lee,Smirnov,Smirnov,Steinhauser'18]



Analytic results

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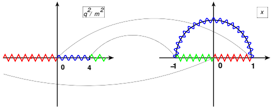

Analytic expansions in kinematical limits:

- $q^2 \to -\infty$, $x \to 0$
- $q^2 \to 0$, $x \to 1$
- $q^2 \to 4m^2$, $x \to -1$

Threshold: $q^2 \to 4m^2$, $x \to -1$

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- form factors ➪ cross sections and decay rates
- real radiation is suppressed by a relative order β^3

$\beta = \sqrt{1 - \frac{4m^2}{q^2}}$

$$\begin{aligned}
\sigma(e^+e^- \to Q\bar{Q}) &= \sigma_0 R^v + \ldots \\
\Gamma(H \to Q\bar{Q}) &= \frac{3G_F M_H M_Q^2}{4\sqrt{2}\pi} R^s + \ldots \\
\Gamma(A \to Q\bar{Q}) &= \frac{3G_F M_A M_Q^2}{4\sqrt{2}\pi} R^p + \ldots
\end{aligned}$$

$$R^v = \beta \left(|F_1^v + F_2^v|^2 + \frac{|(1-\beta^2)F_1^v + F_2^v|^2}{2(1-\beta^2)} \right)$$

$$R^a = \beta^3 |F_1^a|^2 \qquad R^s = \beta^3 |F^s|^2 \qquad R^p = \beta |F^p|^2$$

Threshold: $q^2 \to 4m^2$, $x \to -1$

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$$R^a = \beta^3|F_1^a|^2 \qquad R^s = \beta^3|F^s|^2 \qquad R^p = \beta|F^p|^2$$

$$R^v = \frac{3\beta}{2}\left[\left(1 - \frac{\beta^2}{3}\right) + \sum_{i\geq1}\left(\frac{\alpha_s}{4\pi}\right)^i \Delta^{(i),v}\right]$$

$$R^{a/s} = \beta^3\left[1 + \sum_{i\geq1}\left(\frac{\alpha_s}{4\pi}\right)^i \Delta^{(i),a/s}\right]$$

$$R^p = \beta\left[1 + \sum_{i\geq1}\left(\frac{\alpha_s}{4\pi}\right)^i \Delta^{(i),p}\right]$$

Threshold: $q^2 \to 4m^2$, $x \to -1$ (cont.)

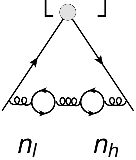
$$\Delta^{(1),s} = C_F \left[\frac{2\pi^2}{\beta} - 4 + 2\pi^2 \beta \right]$$

Threshold: $q^2 \to 4m^2$, $x \to -1$ (cont.)

$$\Delta^{(2),s} = \ldots$$

Threshold: $q^2 \to 4m^2$, $x \to -1$ (cont.)

$$
\begin{aligned}
\Delta^{(3),s} = \\
& N_c^3 \Bigg[\frac{\pi^4}{\beta^3} + \frac{1}{\beta^2}\Bigg(-\frac{44}{9}\pi^4 \log(2\beta) - \frac{44}{3}\pi^2 \log(2\beta) - \frac{88\pi^2\zeta(3)}{3} \\
& + \frac{176\pi^4}{27} + \frac{374\pi^2}{9} \Bigg) + \frac{1}{\beta}\Bigg(\frac{484}{9}\pi^2 \log^2(2\beta) - 8\pi^4 \log(2\beta) - 4\pi^4 l_2 \\
& - \frac{4484}{27}\pi^2 \log(2\beta) - \frac{104\pi^2\zeta(3)}{3} - \frac{\pi^6}{4} + \frac{1375\pi^4}{54} + \frac{11434\pi^2}{81} \Bigg) \Bigg] \\
& + C_F^2 T_F n_l \Big[\ldots \Big] + C_A C_F T_F n_l \Big[\ldots \Big] + C_F T_F^2 n_l^2 \Big[\ldots \Big] \\
& + C_F T_F^2 n_h n_l \left(\frac{640\pi^2}{27} - \frac{20096}{81} \right) \qquad \longleftarrow
\end{aligned}
$$



Threshold: $q^2 \to 4m^2$, $x \to -1$ (cont.)

$$\Delta^{(3),s} =$$

$$N_c^3 \left[\frac{\pi^4}{\beta^3} + \frac{1}{\beta^2} \left(-\frac{44}{9}\pi^4 \log(2\beta) - \frac{44}{3}\pi^2 \log(2\beta) - \frac{88\pi^2\zeta(3)}{3} \right.\right.$$

[Landau,Lifschitz: "Quantum mechnics"; Messiah: "Quantum mechnics 1", Fadin,Khoze'91]

$$\begin{aligned}
\Delta^\delta &= \frac{y}{1-e^{-y}} \left(1 + P^\delta \frac{y^2}{4\pi^2}\right) + \ldots \\
&= 1 + \frac{\alpha_s}{4\pi} C_F \frac{2\pi^2}{\beta} + \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{C_F^2}{\beta^2} \left(\frac{4\pi^4}{3} + P^\delta 4\pi^2\right) \\
&\quad + \left(\frac{\alpha_s}{4\pi}\right)^3 \frac{C_F^3}{\beta^3} P^\delta 8\pi^4 + \ldots .
\end{aligned}$$

$y = C_F \alpha_s \pi / \beta$ $\quad$ $P^\delta = 0$ for S-wave , $\quad$ $P^\delta = 1$ for P-wave

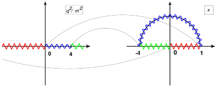
$P^v = P^p = 0$, $P^a = P^s = 1$

n_l $\quad$ n_h

Numerical example 1: F^s for $x \in [-1, 1]$

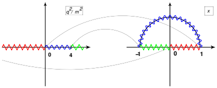
- GPLs ➪ `ginac` [Bauer,Frink,Kreckel'00; Vollinga,Weinzierl'04]
- subtract threshold and high-energy singularities

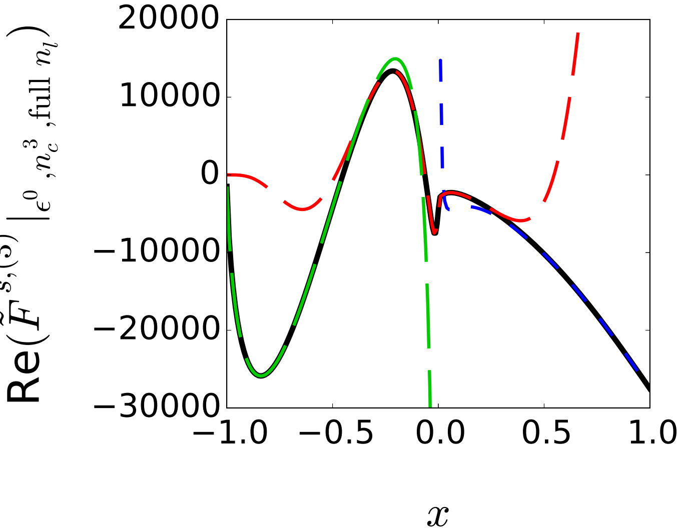
$$\tilde{F}_s(q^2) = (1+x)^4 \left[F_s(q^2) - F_s(q^2)\Big|_{q^2\to\infty} \right]$$



Numerical example 1: F^s for $x \in [-1, 1]$

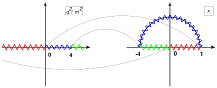
$$\tilde{F}_s(q^2) = (1+x)^4 \left[F_s(q^2) - F_s(q^2)\Big|_{q^2\to\infty} \right]$$

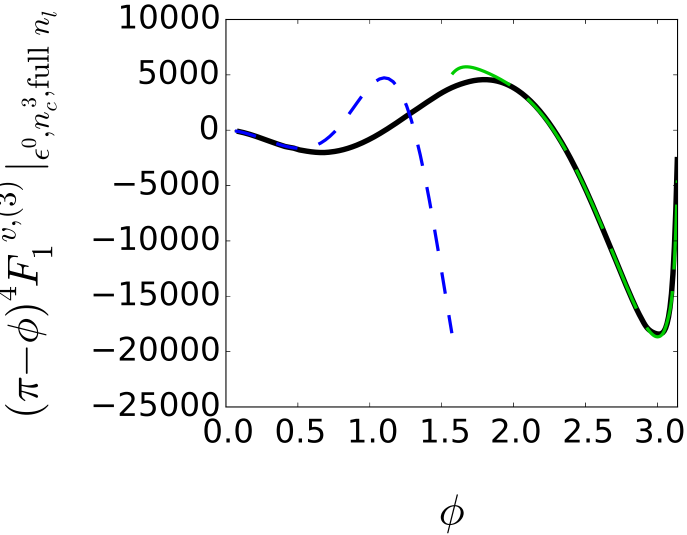




Numerical example 2: F_1^v for $\phi \in [0, \pi]$

$(\pi - \phi)^4 F_1^v$





Checks

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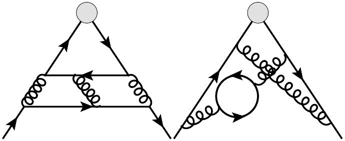
- $\Gamma_{\rm cusp}$ [Korchemsky,Radyushkin'87], [Grozin,Henn,Korchemsky,Marquard'14'15], [Grozin,Henn,Stahlhofen'17]
- $F_1^v = F_1^a$ and $F^s = F^p$ for $q^2 \to \infty$
- arbitray QCD gauge paramter ξ ➪ drops out for renormalized F
- numerical cross check of MIs with `FIESTA` [Smirnov'15]
- agrement for $F_2^v(0)$ [Grozin,Marquard,Piclum,Steinhauser'08] and $F_1^a(0)$ [Archambault,Czarnecki'04]
- Ward identity satisfied:

$$q^\mu \Gamma_\mu^a = 2i\Gamma^p \quad ➪ \quad F_1^a + \frac{q^2}{4m^2} F_2^a = F^p$$

$$\Gamma_\mu^a = F_1^a(q^2)\gamma_\mu\gamma_5 - \frac{1}{2m} F_2^a(q^2) q_\mu \gamma_5 \qquad \Gamma^p = imF^p(q^2)\gamma_5$$

Conclusions/Outlook

- massive 3-loop form factor
  - all n_l and N_c^3
  - vector, axial-vector, scalar, pseudo-scalar
  - next steps:
    massive closed quark loops,
    singlet contribution, . . .
    ➪ elliptic integrals



- massless 4-loop form factor
  - large-N_c limit for n_f^0 and n_f^1
  - planar and non-planar n_f^2 (also for Higgs-quark FF)
  - next steps:
    all n_f terms, singlet contribution, . . . , Higgs gluon FF
    ➪ effective reduction needed