High-Energy Limit of QCD beyond Sudakov Approximation

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Based on

T. Liu and A. A. Penin, Phys. Rev. Lett. 119, no. 26, 262001 (2017)

T. Liu, A. A. Penin and N. Zerf, Phys. Lett. B 771, 492 (2017)

A. A. Penin, Phys. Lett. B 745, 69 (2015)

Outline

Logarithms

Renormalization and factorization logs: $\alpha_s^n \ln^n \mu$

Reggie limit($s \gg t$): $(s/t)\alpha_s^{n+1} \ln^n(s/t)$

Sudakov logs: $\alpha_s^n \ln^{2n} Q$

originate from soft and collinear divergence

How about power corrections?

- phenomenological implications
- also interesting theoretical question

Beyond Sudakov approximation

- Light quark mass effects in Higgs production
  K.Melnikov and A.Penin, JHEP 1605(2016) 172
- Next-to-eikonal soft gluon radiation
  D.Bonocore, E.Laenen, L.Magnea, L.Vernazza, C.D.White JHEP 1612, 121(2016)
- Jets and jettiness
  R.Boughezal, X.Liu and F.Petriello JHEP 1703, 160(2017)
- Power corrections in SCET
  I.Moult, I.W.Stewart and G.Vita, JHEP 1707, 067(2017)
  M.Beneke, M.Garny, R.Szafron, J.Wang JHEP 1803(2018) 001
- many other studies ...

Form factor

$$\mathcal{F} = \bar{q}(p_2)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_q}F_2\right)q(p_1)$$

three-loop massless [Baikov,Chetyrkin,Smirnov^2,Steinhauser 2009]
[Gehrmann,Glover,Huber,Ikizlerli, Studerus 2010]

four-loop massless [Henn,Lee,Smirnov^2,Steinhauser 2016; Manteuffel,Schabinger 2016]
[Lee,Smirnov^2,Steinhauser 2017]

two-loop massive [Mastrolia,Remiddi 2003; Bernreuther et al 2005]

three-loop massive [Henn,Smirnov^2,Steinhauser 2016; Lee,Smirnov^2,Steinhauser 2018]

High energy limit

$$p_1^2 = p_2^2 = m_q^2 \ll Q^2, \quad \rho = m_q^2/Q^2$$

$$F_1 = \exp\left[-\frac{\alpha_s}{2\pi}\frac{C_F \ln\rho(1+\mathcal{O}(\rho^2))}{\varepsilon}\right]\sum_{n=0}^{\infty}\rho^n F_1^{(n)}$$

Leading power:

Sudakov logs [Sudakov 1956; Frenkel,Taylor 1976]
subleading logs also exponentiate [Muller 1979; Collins 1980; Sen 1981; ...]

$\mathcal{O}(\rho)$ corrections:

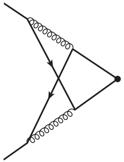
expansion by regions [Beneke, Smirnov 1998]
double logs come from soft quark exchange [Penin 2015; Liu,Penin,Zerf 2017]

Sudakov logarithms

- covarint gauge: soft glue exchange
- resummation: eikonal identity
  rainbow diagrams in physical gauge
- expansion by regions: $\tilde{p_1} p_2 \to p_1 p_2 (1 + \mathcal{O}(\rho^2))$
- cusp anomalous dimension: $\Gamma_{cusp} = -\frac{\alpha_s}{\pi} \ln \rho (1 + \mathcal{O}(\rho^2))$

- either $\mathcal{O}(\rho^2)$ or subleading logs

Two loop



- Sudakov parameterization: $l_i = u_i p_1 + v_i p_2 + l_{i\perp}$
- eikonal glue: $\frac{1}{(p_1+l_i)^2} \approx \frac{1}{Q^2 v_i}$
- soft quark: $\frac{m_q}{(l_i^2 - m_q^2)} \approx -i\pi m_q \delta(Q^2 u_i v_i + l_{i\perp}^2 - m_q^2)$
- $\eta_i = \ln v_i / \ln \rho$, $\xi_i = \ln u_i / \ln \rho$

Result:

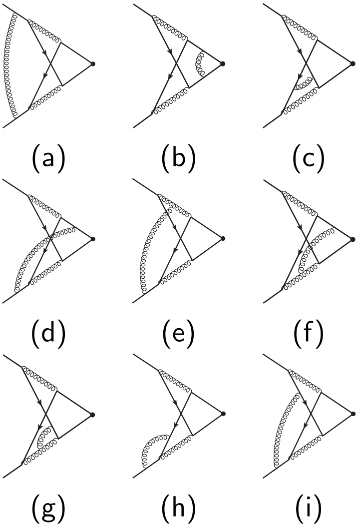
$$x = \frac{\alpha_s}{4\pi} \ln^2 \rho$$

$$K = \theta(1 - \eta_1 - \xi_1)\theta(1 - \eta_2 - \xi_2)\theta(\eta_2 - \eta_1)\theta(\xi_1 - \xi_2)$$

$$F_1^{(1,2l)} = 2\,(C_A - 2C_F)\,x^2 \times \int K(\eta_1, \eta_2, \xi_1, \xi_2) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

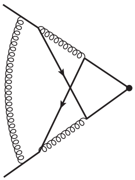
Three loop

Dressing the two-loop non-planar digram with a soft glue:



Others either have no proper region or vanshing color factor.

Dia(a)



- region: $v_3 \ll v_2$, $u_3 \ll u_1$
- eikonal propagator: $\frac{1}{(p_1+l_3)^2-m_q^2} \approx \frac{1}{Q^2(v_3+2\rho u_3)}$
- soft glue: $\frac{1}{l_3^2} \approx -i\pi\delta(Q^2 u_3 v_3 + l_{3\perp}^2)$

$$\propto \int_{\rho u_3}^{v_2} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3}^{u_1} \frac{\mathrm{d}u_3}{u_3}$$

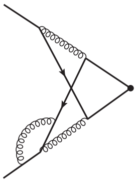
After subtractions one get infrared finite integrals:

$$\propto -\left(\int_{v_2}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3}^{u_1} \frac{\mathrm{d}u_3}{u_3} + \int_{\rho u_3}^{v_2} \frac{\mathrm{d}v_3}{v_3} \int_{u_1}^{1} \frac{\mathrm{d}u_3}{u_3} + \int_{v_2}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{u_1}^{1} \frac{\mathrm{d}u_3}{u_3}\right)$$

subtraction reproduces the factorized singular term.

Dia(h)

Sudakov parameterization: $l_3 = u_3 l_1 + v_3 p_2 + l_{3\perp}$



l_3 flow down at the vertex:

$$\propto \int_{\rho u_3/u_1}^{1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3/u_1}^{1} \frac{\mathrm{d}u_3}{u_3}$$

l_3 flow up at the vertex:

$$\propto - \int_{\rho u_3/u_1}^{v_1} \frac{\mathrm{d}v_3}{v_3} \int_{\rho v_3/u_1}^{1} \frac{\mathrm{d}u_3}{u_3}$$

The above integrals are divergent separately and the sum of them is finite.

Results

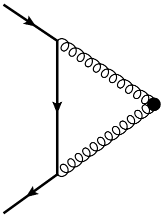
$$F_1^{(1,3l)} = \frac{C_F\,(C_A - 2C_F)}{2} \sum_\lambda c_\lambda d_\lambda\, x^3$$

$$d_\lambda = 4 \int w_\lambda(\eta,\xi) K(\eta_1,\eta_2,\xi_1,\xi_2) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

λ	w_λ	d_λ	c_λ
a	$-((\eta_2+2)\eta_2 + (\xi_1 - 2\eta_2 + 2)\xi_1 - 1)$	$-\frac{17}{45}$	$-C_F$
b	$2\xi_2\eta_1$	$\frac{1}{45}$	$-C_F$
c	$2(\xi_1 - \xi_2)(\eta_2 - \eta_1)$	$\frac{1}{15}$	$C_A - C_F$
d	$-\eta_1(\eta_1 - 2\xi_1 + 2)$	$-\frac{1}{10}$	$C_A - C_F$
e	$(\eta_2 - \eta_1)(2 - 2\xi_1 + \eta_1 + \eta_2)$	$\frac{8}{45}$	$-\frac{C_A}{2}$
f	$2\eta_1(\xi_1 - \xi_2)$	$\frac{1}{30}$	$-\frac{C_A}{2}$
g	$2\eta_2(\xi_1 - \xi_2)$	$\frac{1}{10}$	$-\frac{C_A}{2}$
h	$\eta_1(\eta_1 - 2\xi_1 + 2)$	$\frac{1}{10}$	$\frac{C_A}{2} - C_F$
i	$\eta_2(\eta_2 - 2\xi_1 + 2)$	$\frac{5}{18}$	$\frac{C_A}{2} - C_F$

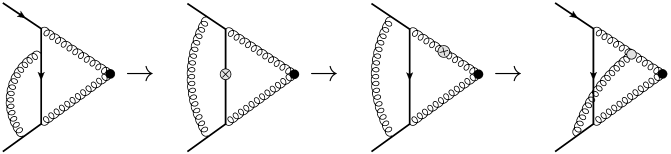
$C_A - 2C_F = 1/N_c$.
Our result agrees with the analysis of leading-color approximation. [Henn,Smirnov,Smirnov,Steinhauser 2016]

$h\bar{q}q$ amplitude



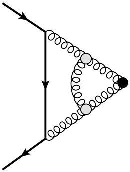
- local operator for the vertex
- $p_1^2 = p_2^2 = m_q^2$, $Q^2 = -(p_2 - p_1)^2$, $\rho \equiv m_q^2/Q^2$
- two-loop integrals are checked by the method of regions
- three-loop double-logarithmic terms are also evaluated

Resummation



- Ward identity: crossed circle on the soft quark line means $S(l) \to S(l) - S(l + l_g^+)$
- momentum shif: eikonal line becomes $\frac{1}{2p_1 l} - \frac{1}{2p_1(l+l_g^+)}$
- effective dipole coupling $2e_q p_1^\mu$
- eikonal factorization

Resummation



$e_q^2 \to C_F$ and glue self-coupling bring C_A

color weight $C_A - C_F$

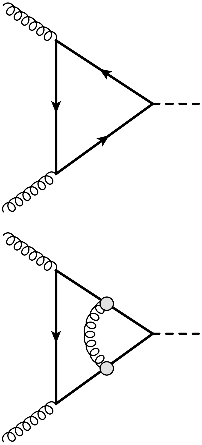
factorize and exponentiate [Frenkel Taylor 1984]

$$x = \frac{\alpha_s}{4\pi} \ln^2 \rho \quad z = (C_A - C_F)x$$

$$\mathcal{G} = Z_q^2 g(-z)\mathcal{G}^{(0)}, \quad Z_q^2 = \exp\left[-C_F\left(\frac{\alpha_s}{2\pi}\frac{\ln \rho}{\varepsilon} + x\right)\right]$$

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta e^{2z\eta\xi}$$

hgg amplitude



Two-loop analytical result available.

[Anastasiou ... 2007]

$$Z_g^2 = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\right]$$

$$\mathcal{M}^b_{gg\to H} = -Z_g^2 g(z)\left(\frac{3}{2}\ln^2\rho\,\rho\right)\mathcal{M}^{(0)}_{gg\to H}$$

Result

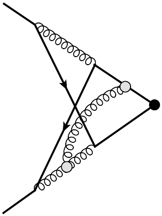
$$g(z) = {}_2F_2\,(1,1;3/2,2;z/2) = 2\sum_0^\infty \frac{n!}{(2n+2)!}(2z)^n$$

Asymptotic behavior at $z \to \infty$

$$g(-z) \sim \frac{\ln(2z)+\gamma_E}{z}, \quad g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{1/2}$$

Power suppressed or exponentially enhanced.

Vector form factor



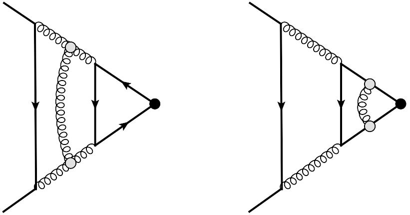
$+$ symmetric

$$F_1 = Z_q^2 \sum_n \rho^n F_1^{(n)}$$

$$F_1^{(1)} = \frac{C_F(C_A - 2C_F)}{6} x^2 f(-z)\,,$$

$$f(z) = 12 \int_0^1 \mathrm{d}\eta_1 \int_{\eta_1}^1 \mathrm{d}\eta_2 \int_0^{1-\eta_2} \mathrm{d}\xi_2 \int_{\xi_2}^{1-\eta_1} \mathrm{d}\xi_1 \, e^{2z\eta_1(\xi_1-\xi_2)} \times e^{2z\xi_2(\eta_2-\eta_1)}$$

Scalar form factor



$$F_S^{(1)} = -\frac{C_F T_F}{3} x^2 f_S(-z)\,,$$

$$f_S(z) = 24 \int_0^1 \mathrm{d}\eta_1 \int_0^{1-\eta_1} \mathrm{d}\xi_1 \int_{\eta_1}^{1-\xi_1} \mathrm{d}\eta_2 \int_{\xi_1}^{1-\eta_2} \mathrm{d}\xi_2 \, e^{2z\eta_2\xi_2} e^{-2z\eta_1\xi_1}$$

Form factors

$f_S(z) \equiv f(z)$.
Axial $f_A(z) = -f(z)$. Pseudoscalar $f_P(z) = f(z)$.

$f(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$

n	1	2	3	4	5	6	7
$2^n n^2 n! c_n$	$\frac{2}{5}$	$\frac{88}{105}$	$\frac{8}{7}$	$\frac{70144}{51975}$	$\frac{640}{429}$	$\frac{25344}{15925}$	$\frac{2727424}{1640925}$

Asymptotic behavior at $z \to \infty$

$$f(-z) \sim C_- \left(\frac{\ln z}{z}\right)^2, \quad f(z) \sim C_+ \ln z \left(\frac{e^z}{z^5}\right)^{1/2}$$

$C_- = 3.6\ldots,\ C_+ = 14.8\ldots$

Conclusion

- New universal source of double logs
- Factorization and resummation beyond Sudakov
- Two univsersal functions $g(z)$ and $f(z)$
- Still a lot of work to be done.

Thanks for your attention!