Testing reliability of the soft-gluon approximation in calculating radiative energy loss of high $p_\perp$ particles

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The soft-gluon approximation

- The soft-gluon approximation (*sg*) definition – radiated gluon carries away a small fraction of initial jet energy $x = \frac{\omega}{E} \ll 1$.
- **Widely-used assumption in calculating radiative energy loss of high $p_\perp$ particle traversing QGP**

ASW (PRD, 69:114003), BDMPS (NPB, 484:265), BDMPS-Z (JETP Lett., 65:615), GLV (NPB 594:371), HT (NPA 696:788);

M. Djordjevic, PRC , 80:064909 (2009), M. Djorjevic and U. Heinz, PRL, 101:022302 (2008)

Why do we reconsider the soft-gluon approximation validity?

- Significant medium induced radiative energy loss obtained by different models → inconsistent with *sg* approximation?
- Sg approximation also used in our Dynamical energy loss formalism.
- Our dynamical energy loss model reported robust agreement with extensive set of experimental R_{AA} data → implies model reliability

(M. Djordjevic and M. D. PLB 734:286 (2014), PRC 90:034910 (2014),

M. Djordjevic, M. D. and B. Blagojevic PLB 737:298 (2014); M. Djordjevic PRL 112:042302 (2014)

M. Djordjevic and M. D. PRC 92:024918 (2015))

  - It breaks-down for:
    - $5 < p_{\perp} < 10$ GeV
    - Primarily for gluon energy loss

Relaxing the soft-gluon approximation

1. **Beyond soft-gluon approximation (*bsg*) in DGLV: x finite**

DGLV formalism assumes:

Finite size (L) optically thin QGP medium

Static color-screened Yukawa potential:

(M. Gyulassy, P. Levai and I. Vitev, NPB 594:371 (2001))

Static scattering centers $V_n = 2\pi\delta(q_n^0)v(\vec{q}_n)e^{-i\vec{q}_n\cdot\vec{x}_n}T_{a_n}(R)\otimes T_{a_n}(n)$

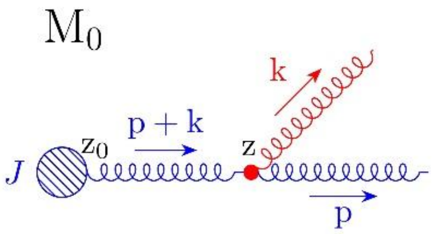
$$v(\vec{q}_n) = \frac{4\pi\alpha_s}{\vec{q}_n^2+\mu^2}$$

Gluons as transversely polarized partons with effective mass $m_g = \mu/\sqrt{2}$

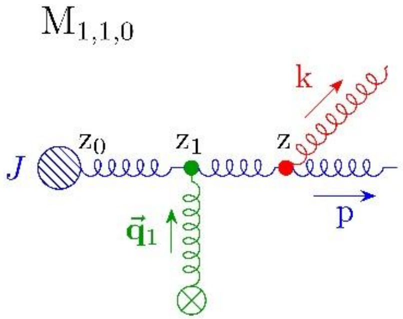
(M. Djordjevic and M. Gyulassy, PRC 68:034914 (2003))

Calculations beyond soft-gluon approximation

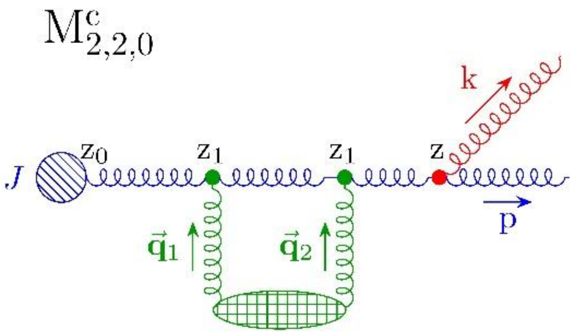
0th order



Interaction with one scatterer



Interaction with two scatterers in contact limit



Assumptions:

- Initial gluon propagates along the longitudinal axis
- The soft-rescattering (eikonal) approximation
- The 1st order in opacity approximation

(M. Gyulassy, P. Levai and I. Vitev, PLB 538:282 (2002))

Calculations beyond soft-gluon approximation

Symmetric under the exchange of radiated (k) and final gluon (p).

$$M_0 = J_a(p+k)e^{i(p+k)x_0}(-2ig_s)(1-x+x^2) \times \frac{\boldsymbol{\epsilon}\cdot\mathbf{k}}{\mathbf{k}^2+m_g^2(1-x+x^2)}(T^c)_{da}$$

No interaction with QGP medium

$$M_{1,1,0} = J_a(p+k)e^{i(p+k)x_0}(-i)(1-x+x^2)(T^cT^{a_1})_{da}T_{a_1}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times(-2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2+m_g^2(1-x+x^2)}e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1)^2+\frac{m_g^2(1-x+x^2)}{1-x})(z_1-z_0)}$$

$$M_{1,0,0} = J_a(p+k)e^{i(p+k)x_0}(-i)(1-x+x^2)(T^{a_1}T^c)_{da}T_{a_1}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times(2ig_s)\frac{\boldsymbol{\epsilon}\cdot\mathbf{k}}{\mathbf{k}^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{-\frac{i}{2\omega}\frac{x}{1-x}(\mathbf{k}^2-(\mathbf{k}-\mathbf{q}_1)^2)(z_1-z_0)}\right)$$

$$M_{1,0,1} = J_a(p+k)e^{i(p+k)x_0}(-i)(1-x+x^2)[T^c,T^{a_1}]_{da}T_{a_1}\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times(2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{\frac{i}{2\omega}(\mathbf{k}^2-(\mathbf{k}-\mathbf{q}_1)^2)(z_1-z_0)}\right)$$

One interaction with QGP medium

Recovers *sg* result for $x \ll 1$.

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Calculations beyond soft-gluon approximation

$$M^c_{2,2,0} = -J_a(p+k)e^{i(p+k)x_0}(T^cT^{a_2}T^{a_1})_{da}T_{a_2}T_{a_1}(1-x+x^2)(-i)\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}v(0,\mathbf{q}_1)v(0,\mathbf{q}_2)e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}$$
$$\times\frac{1}{2}(2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-x(\mathbf{q}_1+\mathbf{q}_2))}{(\mathbf{k}-x(\mathbf{q}_1+\mathbf{q}_2))^2+\chi}e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\frac{\chi}{1-x})(z_1-z_0)}$$

$$M^c_{2,0,3} = J_a(p+k)e^{i(p+k)x_0}[[T^c,T^{a_2}],T^{a_1}]_{da}T_{a_2}T_{a_1}(1-x+x^2)(-i)\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}v(0,\mathbf{q}_1)v(0,\mathbf{q}_2)e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}$$
$$\times\frac{1}{2}(2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)}{(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{\frac{i}{2\omega}(\mathbf{k}^2-(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2)(z_1-z_0)}\right)$$

$$M^c_{2,0,0} = J_a(p+k)e^{i(p+k)x_0}(T^{a_2}T^{a_1}T^c)_{da}T_{a_2}T_{a_1}(1-x+x^2)(-i)\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}v(0,\mathbf{q}_1)v(0,\mathbf{q}_2)e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}$$
$$\times\frac{1}{2}(2ig_s)\frac{\boldsymbol{\epsilon}\cdot\mathbf{k}}{\mathbf{k}^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{\frac{i}{2\omega}\frac{x}{1-x}((\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2-\mathbf{k}^2)(z_1-z_0)}\right)$$

$$M^c_{2,0,1} = J_a(p+k)e^{i(p+k)x_0}(T^{a_2}[T^c,T^{a_1}])_{da}T_{a_2}T_{a_1}(1-x+x^2)(-i)\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}v(0,\mathbf{q}_1)v(0,\mathbf{q}_2)e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}$$
$$\times(2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{\frac{i}{2\omega}(\mathbf{k}^2-\frac{(\mathbf{k}-\mathbf{q}_1)^2}{1-x}+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2)(z_1-z_0)}\right)$$

$$M^c_{2,0,2} = J_a(p+k)e^{i(p+k)x_0}(T^{a_1}[T^c,T^{a_2}])_{da}T_{a_2}T_{a_1}(1-x+x^2)(-i)\int\frac{d^2\mathbf{q}_1}{(2\pi)^2}(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}v(0,\mathbf{q}_1)v(0,\mathbf{q}_2)e^{-i(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{b}_1}$$
$$\times(2ig_s)\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_2)}{(\mathbf{k}-\mathbf{q}_2)^2+\chi}\left(e^{\frac{i}{2\omega}(\mathbf{k}^2+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2+\frac{\chi}{1-x})(z_1-z_0)}-e^{\frac{i}{2\omega}(\mathbf{k}^2-\frac{(\mathbf{k}-\mathbf{q}_2)^2}{1-x}+\frac{x}{1-x}(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2)(z_1-z_0)}\right)$$

Symmetric under the exchange of radiated (k) and final gluon (p).

Two interactions with QGP medium

Recovers *sg* result for $x \ll 1$.

Two negligible amplitudes are omitted.

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Calculations beyond soft-gluon approximation

$$\frac{xd^3N_g^{(0)}}{dxd\mathbf{k}^2} = \frac{\alpha_s}{\pi}\frac{C_2(G)\,\mathbf{k}^2}{(\mathbf{k}^2+m_g^2(1-x+x^2))^2} \times\frac{(1-x+x^2)^2}{1-x}.$$

Reduces to well-known Altarrelli-Parisi (G. Altarelli and G. Parisi, NPB 126:298 (1977)) **result in massless *sg* case.**

Introduction of effective gluon mass *bsg* radiative energy loss for the first time!

Single gluon radiation spectrum beyond soft-gluon approximation:

$$\frac{dN_g^{(1)}}{dx} = \frac{C_2(G)\alpha_s}{\pi}\frac{L}{\lambda}\frac{(1-x+x^2)^2}{x(1-x)}\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mu^2}{(\mathbf{q}_1^2+\mu^2)^2}\int d\mathbf{k}^2$$
$$\times\Big\{\frac{(\mathbf{k}-\mathbf{q}_1)^2+\chi}{(\frac{4x(1-x)E}{L})^2+((\mathbf{k}-\mathbf{q}_1)^2+\chi)^2}\Big(2\frac{(\mathbf{k}-\mathbf{q}_1)^2}{(\mathbf{k}-\mathbf{q}_1)^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q}_1)}{\mathbf{k}^2+\chi}-\frac{(\mathbf{k}-\mathbf{q}_1)\cdot(\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2+\chi}\Big)$$
$$+\frac{\mathbf{k}^2+\chi}{(\frac{4x(1-x)E}{L})^2+(\mathbf{k}^2+\chi)^2}\Big(\frac{\mathbf{k}^2}{\mathbf{k}^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2+\chi}\Big)+\Big(\frac{(\mathbf{k}-x\mathbf{q}_1)^2}{((\mathbf{k}-x\mathbf{q}_1)^2+\chi)^2}-\frac{\mathbf{k}^2}{(\mathbf{k}^2+\chi)^2}\Big)\Big\}$$

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Comparison of analytical expressions ($\frac{dN_g^{(1)}}{dx}$)

Beyond soft-gluon approximation:

$$f_{bsg}(\mathbf{k},\mathbf{q_1},x)=\frac{(1-x+x^2)^2}{x(1-x)}\left\{\left(2\frac{(\mathbf{k}-\mathbf{q_1})^2}{(\mathbf{k}-\mathbf{q_1})^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q_1})}{\mathbf{k}^2+\chi}-\frac{(\mathbf{k}-\mathbf{q_1})\cdot(\mathbf{k}-x\mathbf{q_1})}{(\mathbf{k}-x\mathbf{q_1})^2+\chi}\right)\frac{(\mathbf{k}-\mathbf{q_1})^2+\chi}{\left(\frac{4x(1-x)E}{L}\right)^2+\left((\mathbf{k}-\mathbf{q_1})^2+\chi\right)^2}+\right.$$

$$\left.+\frac{\mathbf{k}^2+\chi}{\left(\frac{4x(1-x)E}{L}\right)^2+\left(\mathbf{k}^2+\chi\right)^2}\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-x\mathbf{q_1})}{(\mathbf{k}-x\mathbf{q_1})^2+\chi}\right)+\left(\frac{(\mathbf{k}-x\mathbf{q_1})^2}{((\mathbf{k}-x\mathbf{q_1})^2+\chi)^2}-\frac{\mathbf{k}^2}{(\mathbf{k}^2+\chi)^2}\right)\right\}$$

$$\chi=m_g^2(1-x+x^2)$$

Only this term remains in *sg* and reduces to:

Soft-gluon approximation:

$$f_{sg}(\mathbf{k},\mathbf{q_1},x)=\frac{1}{x}\frac{(\mathbf{k}-\mathbf{q_1})^2+m_g^2}{\left(\frac{4xE}{L}\right)^2+((\mathbf{k}-\mathbf{q_1})^2+m_g^2)^2}2\left(\frac{(\mathbf{k}-\mathbf{q_1})^2}{(\mathbf{k}-\mathbf{q_1})^2+m_g^2}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q_1})}{\mathbf{k}^2+m_g^2}\right)$$

M. Djordjevic and M. Gyulassy, NPA 733:265(2004)

***Bsg* expression is quite different from *sg* analogon!**

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Comparison of numerical predictions between ***bsg*** and *sg*

1. Fractional radiative energy loss $\Delta E^{(1)}/E$ and number of radiated gluons $N_g^{(1)}$
2. Fractional differential radiative energy loss $\frac{1}{E}\frac{dE^{(1)}}{dx}$ and single gluon radiation spectrum $\frac{dN_g^{(1)}}{dx}$
3. Angular averaged nuclear modification factor R_{AA}

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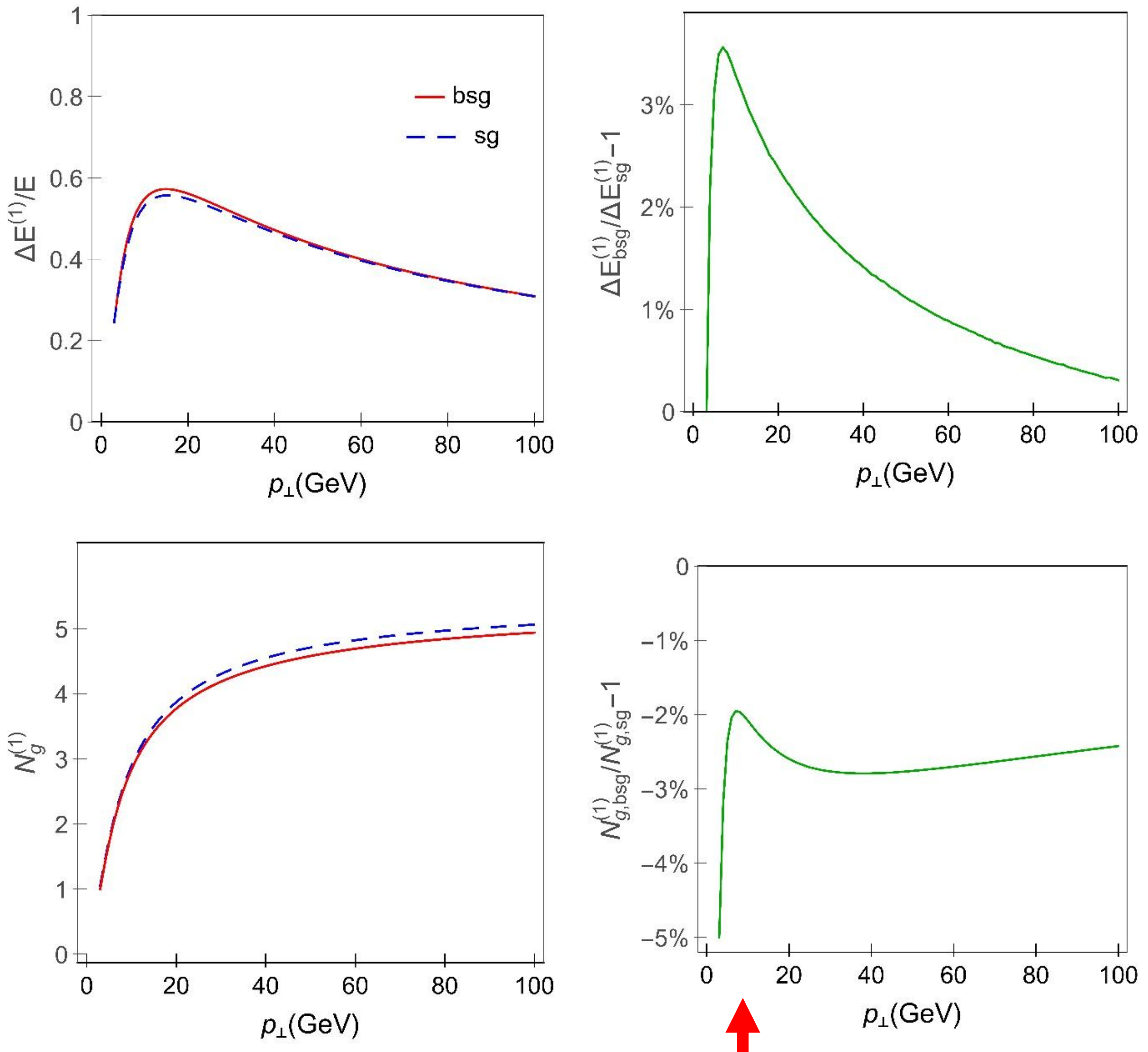
Effect of relaxing *sga* on integrated variables



≈ 10 GeV

Finite x slightly **increases** fractional radiative energy loss up to $\approx 3\%$ compared to *sg* .

Finite x slightly **decreases** number of radiated gluons $\approx -2\%$ compared to *sg* .

Effect on $\Delta E^{(1)}/E$ and $N_g^{(1)}$ is **very small** and of **opposite sign**!

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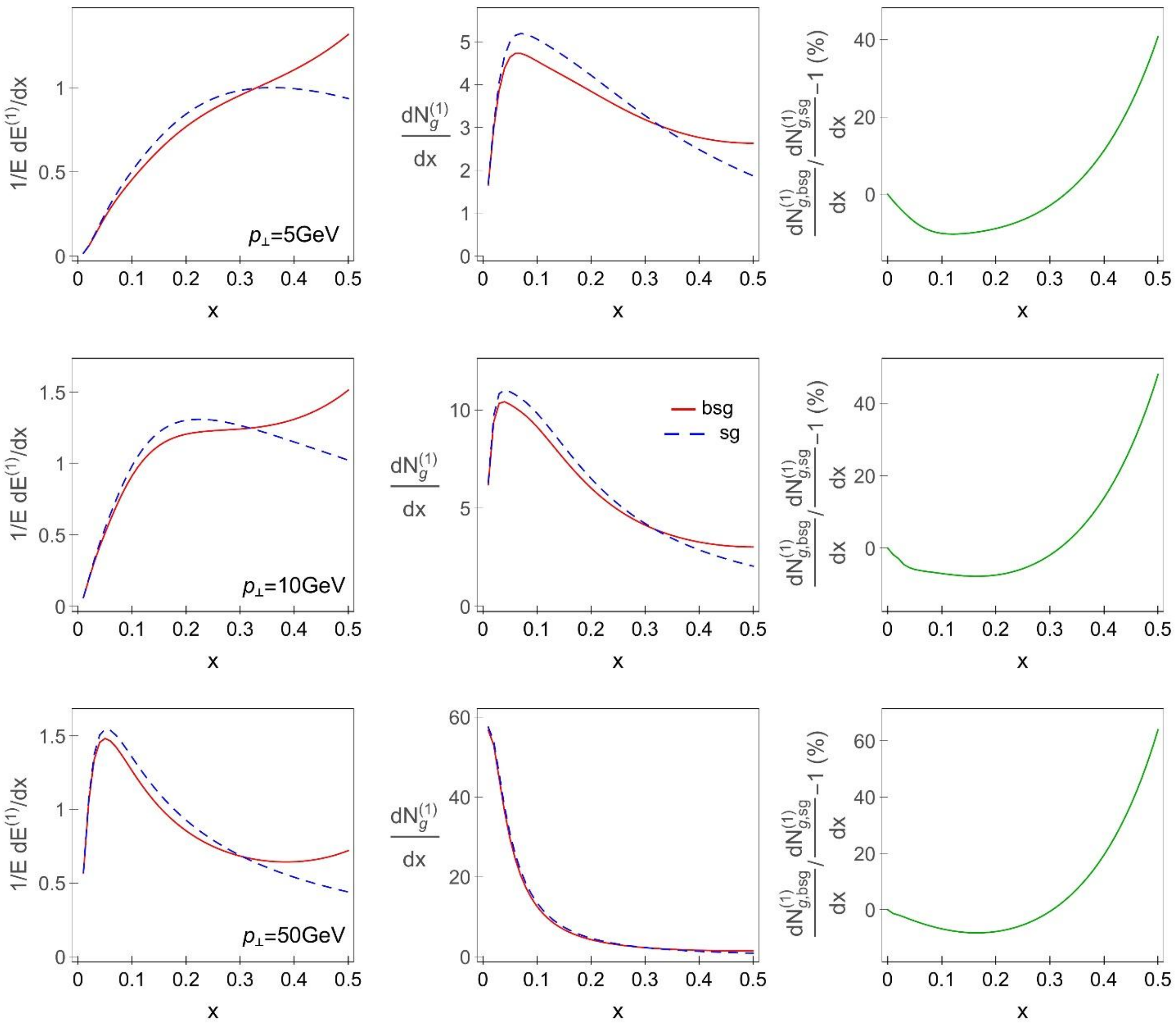
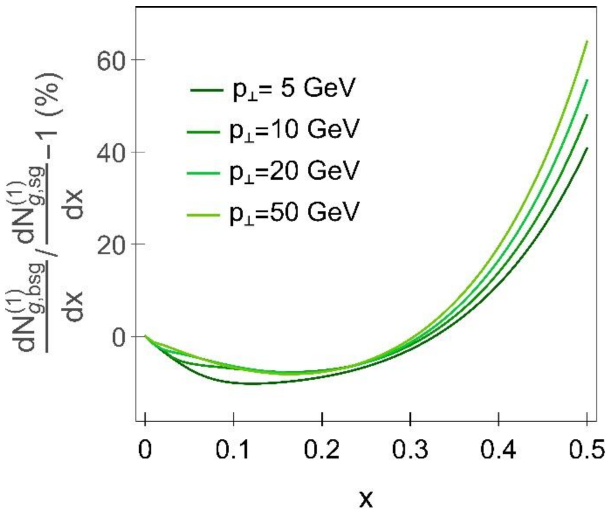
Effect of relaxing *sga* on differential variables



The effect on $dE^{(1)}/dx$ and $dN_g^{(1)}/dx$ is small for $x \leq 0.4$, while enhances to a notable value with increasing x above the "cross-over" point $x \approx 0.3$.



Nearly the same effect on $dN_g^{(1)}/dx$ for $0 < x \leq 0.4$ independently of $p_\perp$

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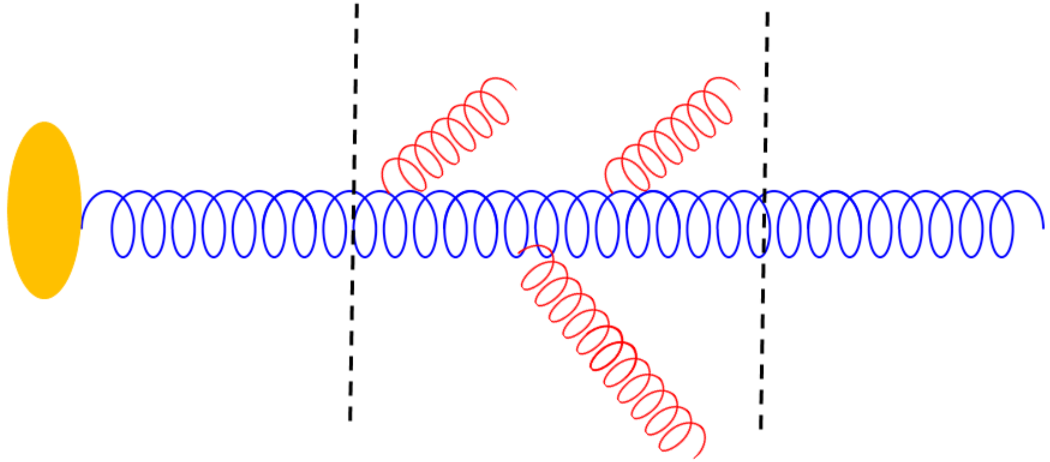
Computational formalism for bare gluon suppression



1. Initial gluon $p_\perp$ spectrum
2. Radiative energy loss

- **Gluon production**

(Z.B. Kang, I. Vitev and H. Xing, PLB 718:482 (2012); R. Sharma, I. Vitev and B.W. Zhang, PRC 80:054902 (2009))

- **Radiative energy loss in finite size static QGP medium *beyond soft gluon approximation***

(B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th/1804.07593 (2018))

- **Multi-gluon fluctuations**

(M. Gyulassy, P. Levai and I. Vitev, PLB 538:282 (2002))

- **Path-length fluctuations**

(S. Wicks, W. Horowitz, M. Djordjevic and M. Gyulassy, NPA 784:426 (2007); A. Dainese, EPJ C 33:495 (2004))

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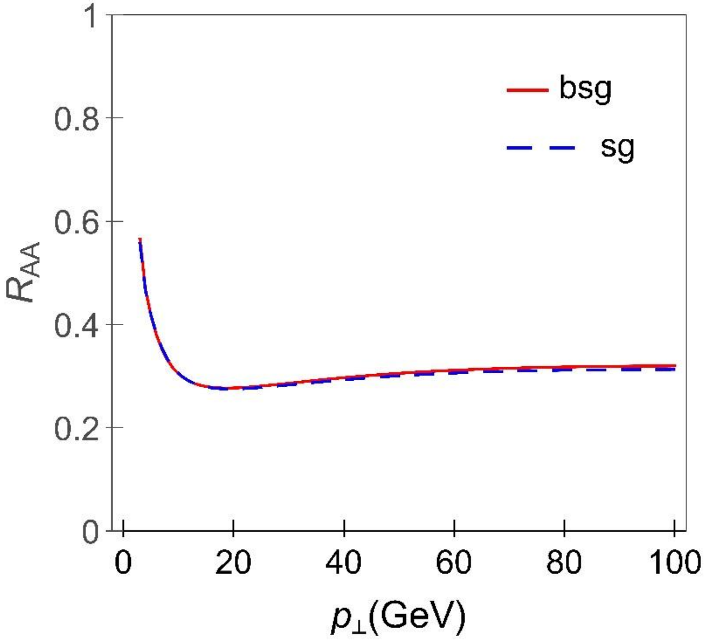
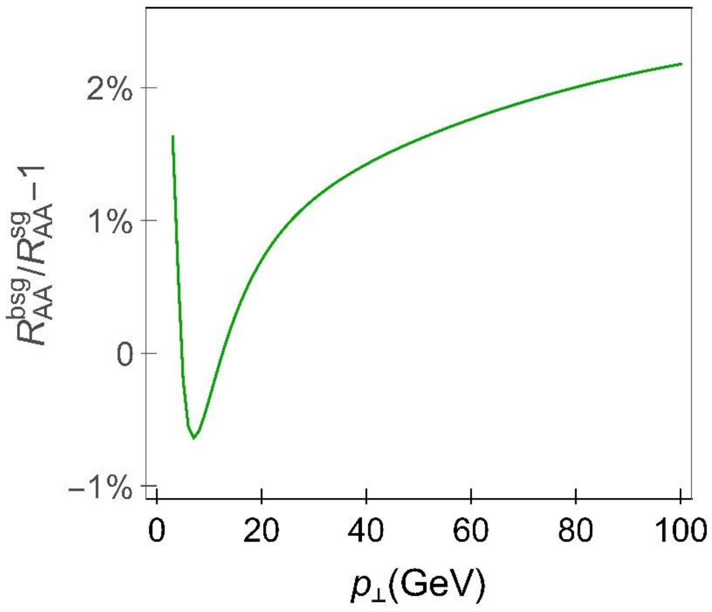
Effect of relaxing *sga* on R_{AA}

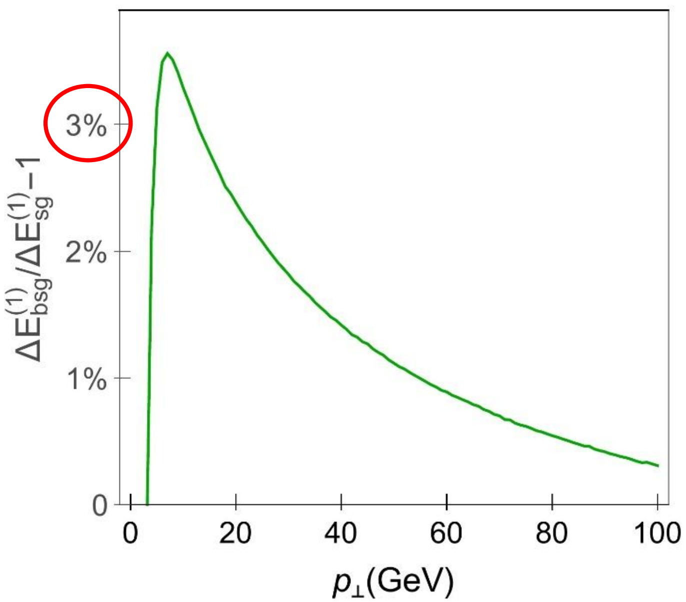


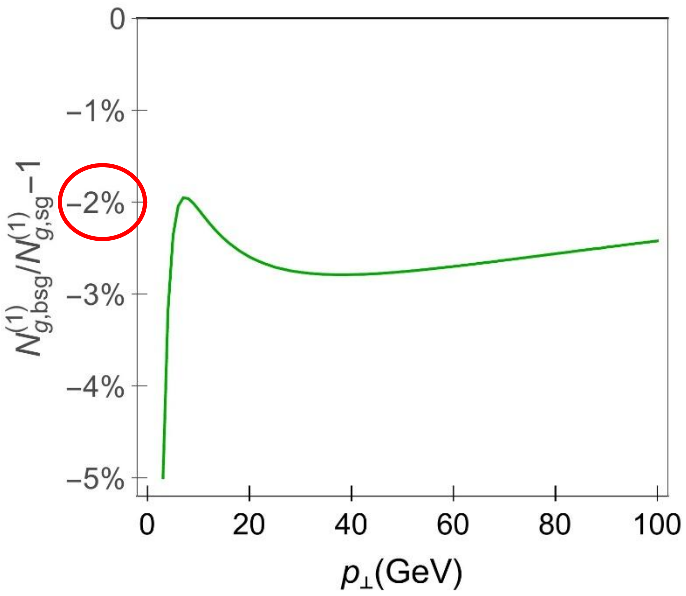


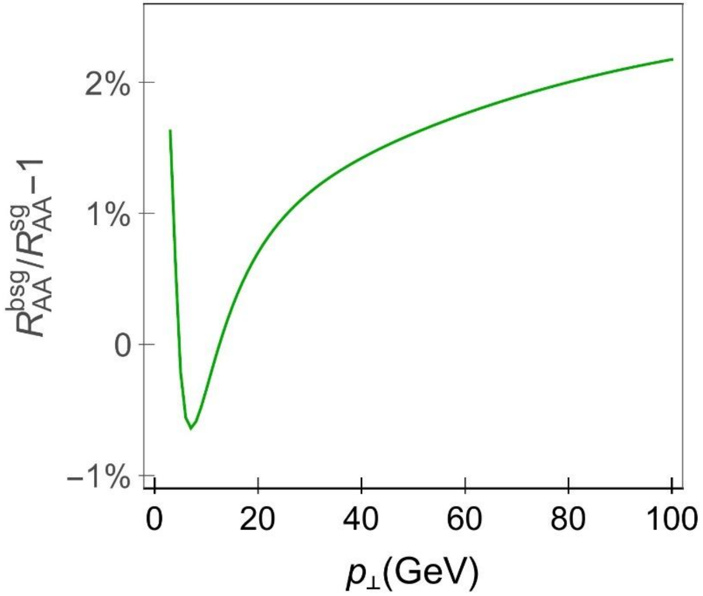
1. **Why is R_{AA} barely affected by this relaxation?**
2. **How the large differential variables discrepancies between *bsg* and *sg* cases at $x > 0.4$ do not influence R_{AA}?**

Even smaller effect on R_{AA} compared to all previous variables!

Explanation of negligible effect on R_{AA} (1)



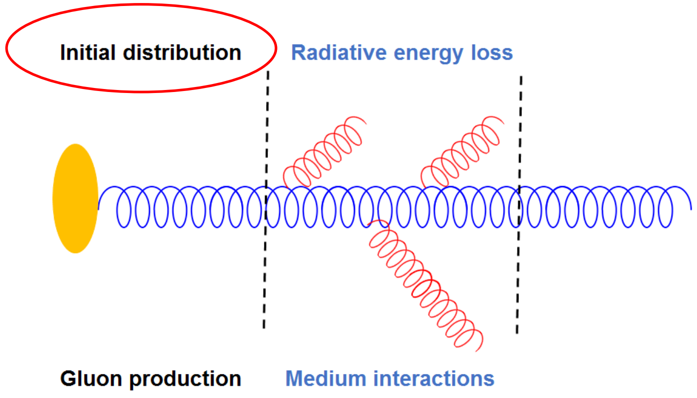


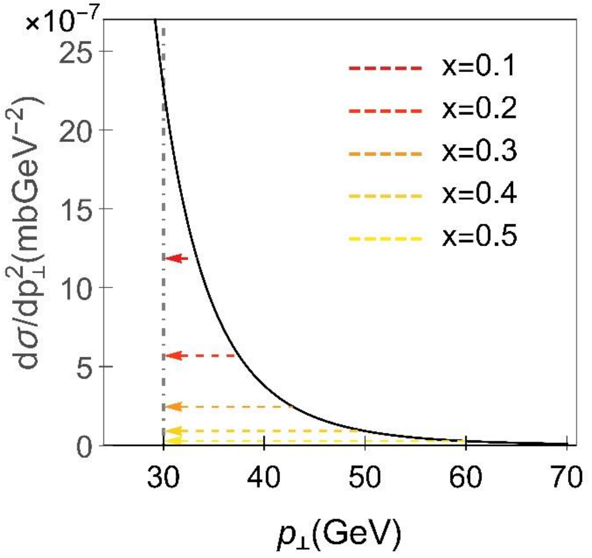


Both $\Delta E^{(1)}/E$ and $N_g^{(1)}$ non-trivially affect R_{AA}.

Interplay of the opposite effects on $\Delta E^{(1)}/E$ and $N_g^{(1)}$ is responsible for negligible effect on R_{AA}.

Explanation of negligible effect on R_{AA} (2)





Due to sharply decreasing initial gluon $p_\perp$ distribution, the $x \leq 0.4$ is the most relevant region for distinguishing *bsg* from *sg* R_{AA}.

In this region *bsg* and *sg* $\frac{dN_g^{(1)}}{dx}$ and $\frac{1}{E}\frac{dE^{(1)}}{dx}$ are within 10%.

Intuitively explains insignificant finite x effect on R_{AA}.

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Conclusions and outlook

Different theoretical models reported considerable radiative energy loss questioning the validity of the soft-gluon approximation.

We relaxed the approximation for high $p_\perp$ gluons, which are most affected by it, within DGLV formalism, and although analytical results differ to a great extent in *bsg* and *sg* cases, surprisingly the numerical predictions were nearly indistinguishable.

Consequently, this relaxation should have even smaller impact on high $p_\perp$ quarks.

This implies that soft gluon approximation is reliable within DGLV formalism

We expect that the soft-gluon approximation will remain well-founded within the dynamical energy loss formalism, which needs to be rigorously tested in the future.

Thank you for your attention!

BACK-UP

Generalization on dynamical medium

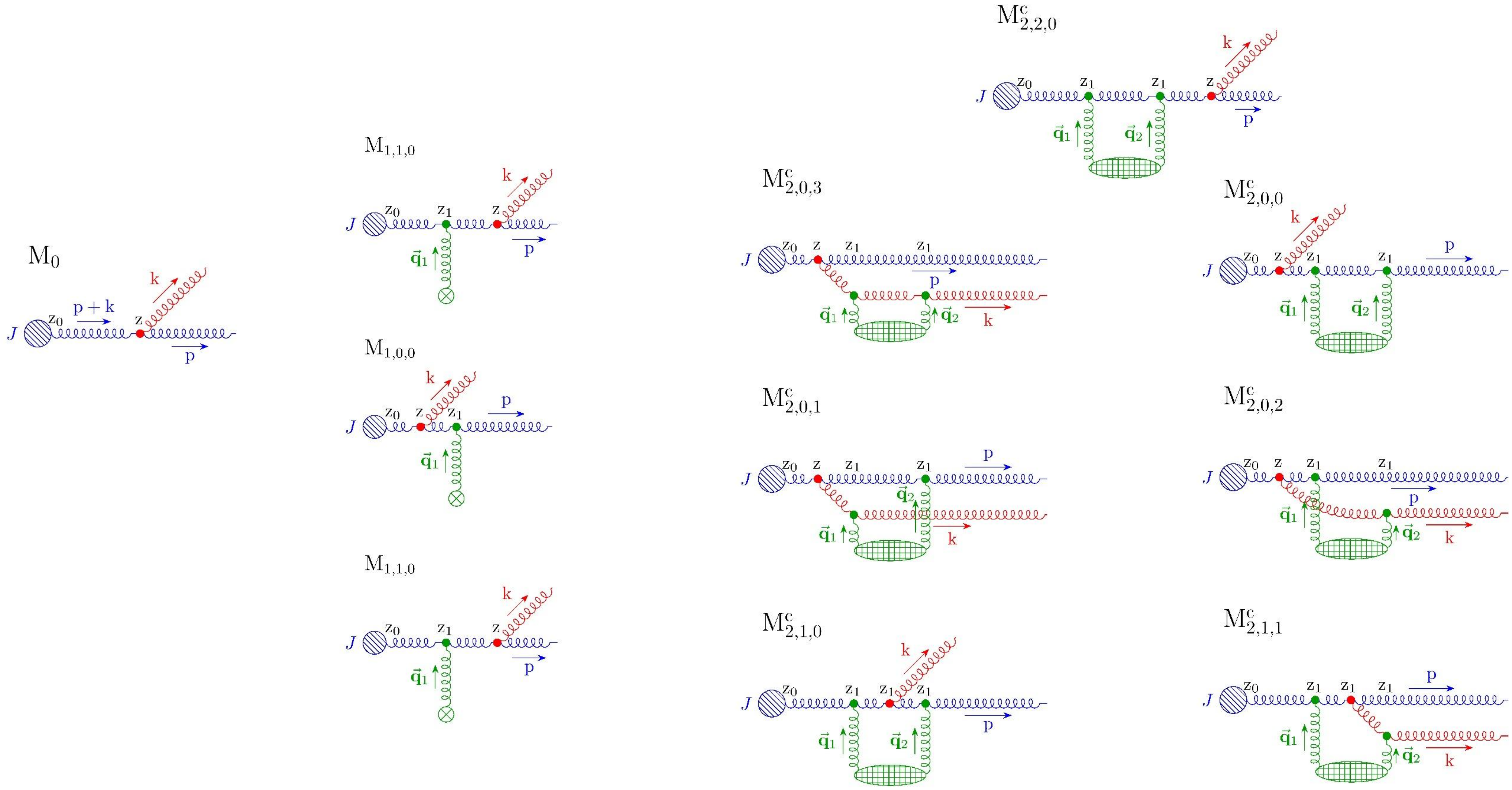
- **Implicitly suggested by robust agreement of our R_{AA} predictions with experimental data**
- **Only $f(\boldsymbol{k}, \boldsymbol{q}, x)$ depends on x**
- $f(\boldsymbol{k}, \boldsymbol{q}, x)$ in soft-gluon approximation is the same in static and in dynamical case

↓

We expect dynamical $f(\boldsymbol{k}, \boldsymbol{q}, x)$ to be modified in the similar manner to the static (DGLV) case.

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Calculations beyond soft-gluon approximation



B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Calculations beyond soft-gluon approximation

Longitudinal initial gluon direction:

No interactions with QGP medium (M_0)

$$p + k = [E^+, E^-, \mathbf{0}]$$

One interaction with QGP medium (M_1)

$$p + k - q_1 = [E^+ - q_{1z}, E^- + q_{1z}, \mathbf{0}]$$

Two interactions with QGP medium (M_2)

$$p + k - q_1 - q_2 = [E^+ - q_{1z} - q_{2z}, E^- + q_{1z} + q_{2z}, \mathbf{0}]$$

$$k = [xE^+, \frac{\mathbf{k}^2 + m_g^2}{xE^+}, \mathbf{k}] \qquad p = [(1-x)E^+, \frac{\mathbf{p}^2 + m_g^2}{(1-x)E^+}, \mathbf{p}]$$

Transverse momenta: $\boldsymbol{p} + \boldsymbol{k} = \mathbf{0}$

Transverse momenta: $\boldsymbol{p} + \boldsymbol{k} \neq \mathbf{0}$

Consistent with longitudinal propagation of initial particle!

Transverse momenta: $\boldsymbol{p} + \boldsymbol{k} = \mathbf{0}$ **in contact-limit**

Transverse gluon polarization: $n^\mu = [0,2,0]$

$$\epsilon(k) \cdot k = 0, \quad \epsilon(k) \cdot n = 0, \quad \epsilon(k)^2 = -1, \quad \epsilon(p+k) \cdot (p+k) = 0, \quad \epsilon(p+k) \cdot n = 0,$$

$$\epsilon(p) \cdot p = 0, \quad \epsilon(p) \cdot n = 0, \quad \epsilon(p)^2 = -1, \quad \epsilon(p+k)^2 = -1.$$

$$\epsilon_i(k) = [0, \frac{2\epsilon_i \cdot \mathbf{k}}{xE^+}, \epsilon_i], \quad \epsilon_i(p+k) = [0, 0, \epsilon_i], \quad \epsilon_i(p) = [0, \frac{2\epsilon_i \cdot \mathbf{p}}{(1-x)E^+}, \epsilon_i],$$

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593

Calculations beyond soft-gluon approximation

$$d^3N_g^{(1)}d^3N_J = \left(\frac{1}{d_T}\,\mathrm{Tr}\,\langle|M_1|^2\rangle + \frac{2}{d_T} Re\,\mathrm{Tr}\,\langle M_2 M_0^*\rangle\right)\frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2p^0}\frac{d^3\vec{\mathbf{k}}}{(2\pi)^3 2\omega}$$

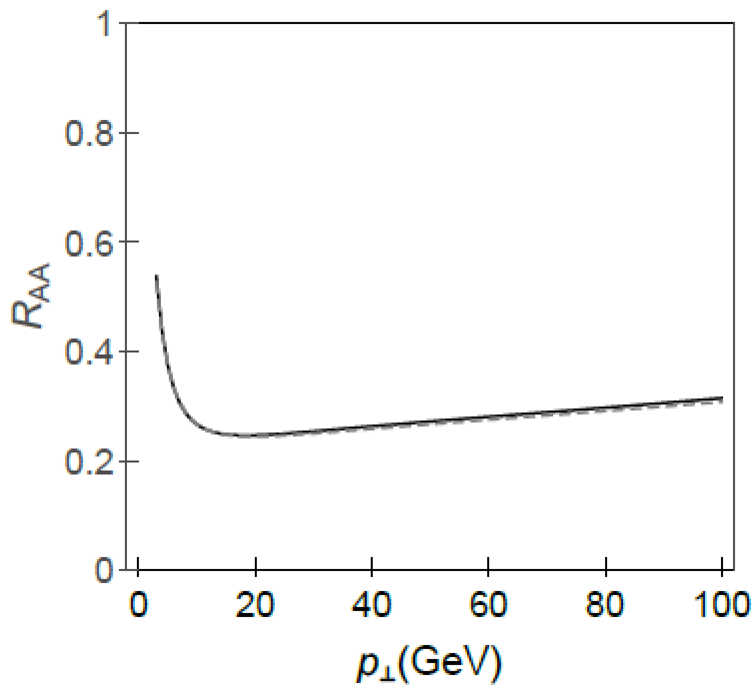
New!

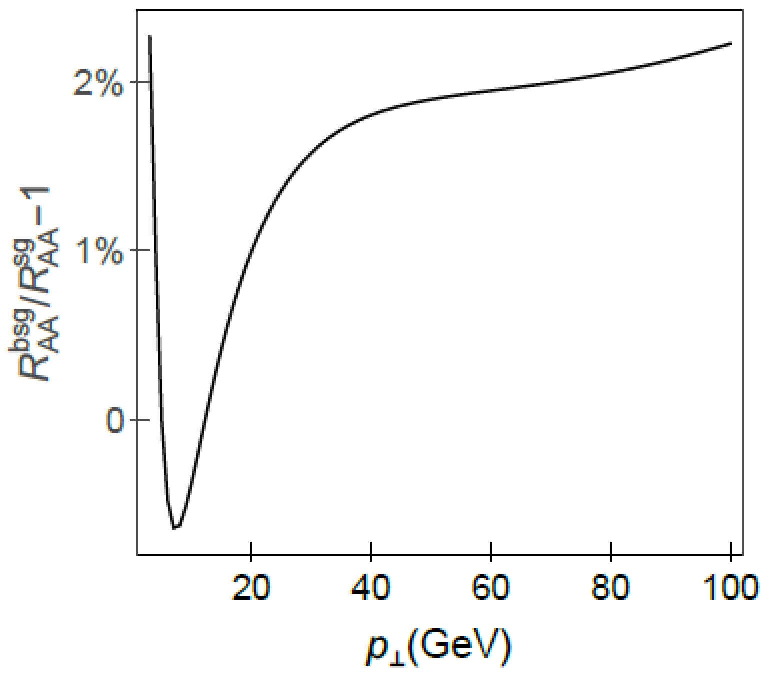
$$\frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2p^0}\frac{d^3\vec{\mathbf{k}}}{(2\pi)^3 2\omega} = \frac{d^3\vec{\mathbf{p}}_J}{(2\pi)^3 2E_J}\frac{dx d^2\mathbf{k}}{(2\pi)^3 2x(1-x)}$$

$$d^3N_J = d_G|J(p+k)|^2\frac{d^3\vec{\mathbf{p}}_J}{(2\pi)^3 2E_J}$$

$$\frac{x d^3N_g^{(0)}}{dx d\mathbf{k}^2} = \frac{\alpha_s}{\pi}\frac{C_2(G)\,\mathbf{k}^2}{(\mathbf{k}^2 + m_g^2(1-x+x^2))^2} \times \frac{(1-x+x^2)^2}{1-x}.$$

$$\frac{dN_g^{(1)}}{dx} = \frac{C_2(G)\alpha_s}{\pi}\frac{L}{\lambda}\frac{(1-x+x^2)^2}{x(1-x)}\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mu^2}{(\mathbf{q}_1^2+\mu^2)^2}\int d\mathbf{k}^2 \times\Bigg\{\frac{(\mathbf{k}-\mathbf{q}_1)^2+\chi}{(\frac{4x(1-x)E}{L})^2+((\mathbf{k}-\mathbf{q}_1)^2+\chi)^2}\left(2\frac{(\mathbf{k}-\mathbf{q}_1)^2}{(\mathbf{k}-\mathbf{q}_1)^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q}_1)}{\mathbf{k}^2+\chi}-\frac{(\mathbf{k}-\mathbf{q}_1)\cdot(\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2+\chi}\right) + \frac{\mathbf{k}^2+\chi}{(\frac{4x(1-x)E}{L})^2+(\mathbf{k}^2+\chi)^2}\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+\chi}-\frac{\mathbf{k}\cdot(\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2+\chi}\right)+\left(\frac{(\mathbf{k}-x\mathbf{q}_1)^2}{((\mathbf{k}-x\mathbf{q}_1)^2+\chi)^2}-\frac{\mathbf{k}^2}{(\mathbf{k}^2+\chi)^2}\right)\Bigg\}$$





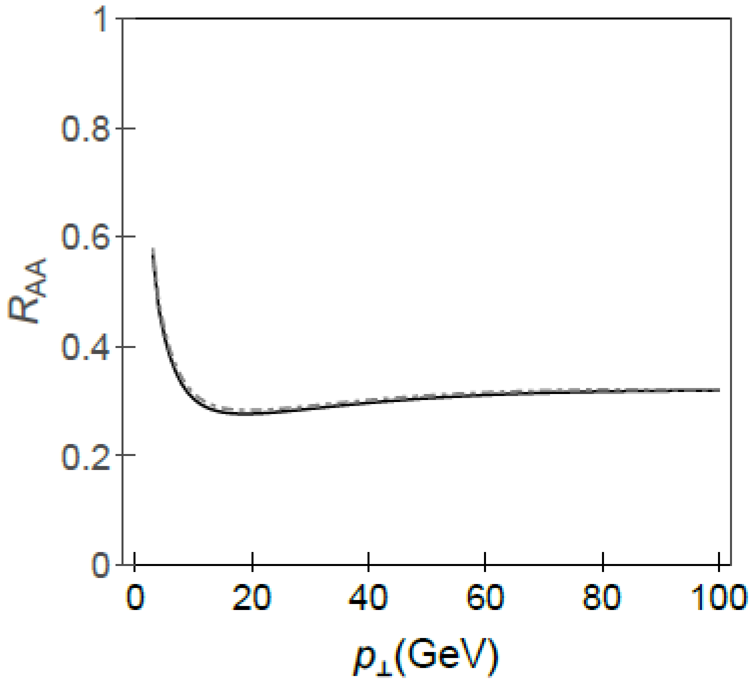
Uniform longitudinal distance distribution

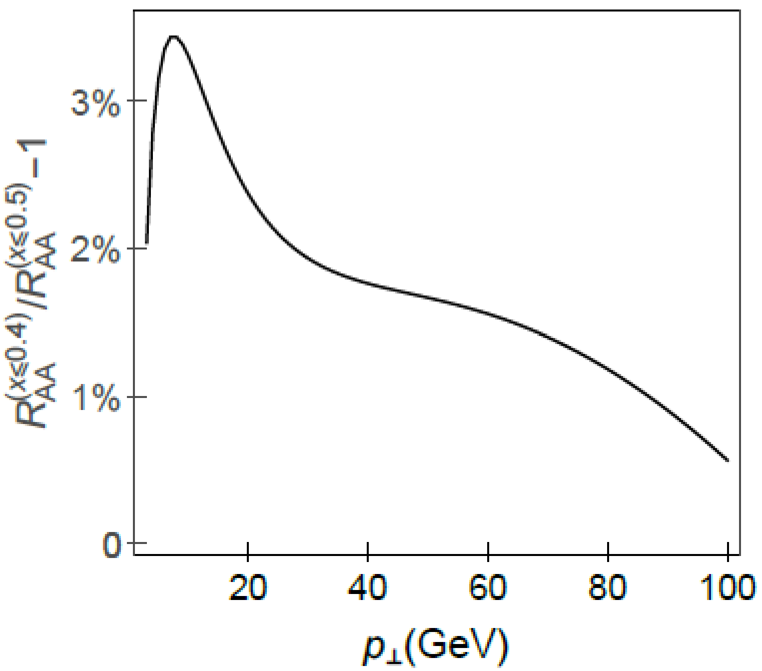
$$m_g = m_\infty = \sqrt{\Pi_T(p_0/|\vec{\mathrm{p}}| = 1)} = \mu_E/\sqrt{2}$$

Effective gluon mass

(M. Djordjevic and M. Gyulassy, PRC 68:034914 (2003))

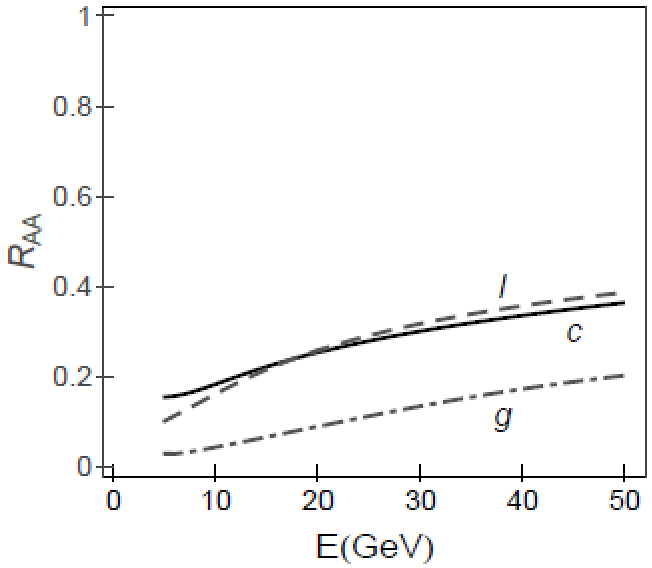
B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593





Non-relevance of $x > 0.4$ region for the importance of relaxing the soft-gluon approximation

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593



LHC 2.76 TeV

M. Djordjevic, PRL,112:042302 (2014).

B. Blagojevic, M. Djordjevic and M. Djordjevic, arXiv:nucl-th\1804.07593