



The fate of axial $U(1)$ in $2+1$ flavor QCD towards the chiral limit

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The $U_A(1)$ puzzle

- ▶ Nature of chiral phase transition for QCD with two light quark flavors is not yet completely resolved.
- ▶ Usually symmetries determine the order parameter across a phase transition.
- ▶ However, for 2 light quark flavors anomalous $U_A(1)$ may affect the nature of phase transition.
- ▶ How severely $U_A(1)$ is broken can be answered non-perturbatively.
- ▶ We want to investigate whether or not the anomalous $U_A(1)$ symmetry in the flavor sector is effectively restored along with the chiral symmetry.

Observables sensitive to $U_A(1)$ breaking

- ▶ $U_A(1)$ is not an exact symmetry \Rightarrow no unique order parameter!
- ▶ Instead, look at n-point correlation functions which become degenerate upon $U_A(1)$ rotation.
Start with 2-point correlation functions.

$$\begin{array}{ccccc}
 \chi_{5,\text{con}} & \pi : \bar{\mathbf{q}} \gamma_5 \frac{\tau}{2} \mathbf{q} & \xleftrightarrow{SU(2)_L \times SU(2)_R} & \sigma : \bar{\mathbf{q}} \mathbf{q} & \chi_{\text{con}} + \chi_{\text{disc}} \\
 & \updownarrow U(1)_A & & \updownarrow U(1)_A & \\
 \chi_{\text{con}} & \delta : \bar{\mathbf{q}} \frac{\tau}{2} \mathbf{q} & \xleftrightarrow{SU(2)_L \times SU(2)_R} & \eta : \bar{\mathbf{q}} \gamma_5 \mathbf{q} & \chi_{5,\text{con}} - \chi_{5,\text{disc}}
 \end{array}$$

$U_A(1)$ breaking and QCD eigenvalue density

- ▶ Observable of interest is [Shuryak, 1994]

$$\chi_\pi - \chi_\delta = \int d^4x [\langle i\pi^+(x)i\pi^-(0) \rangle - \langle i\delta^+(x)i\delta^-(0) \rangle]$$

- ▶ Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator

[Cohen, 1995, Hatsuda & Lee, 1995]

$$\chi_\pi - \chi_\delta \xrightarrow{V \rightarrow \infty} \int_0^\infty d\lambda \frac{4m_f^2 \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- ▶ Possible scenarios:

- ▶ $\lim_{m_f \rightarrow 0} \rho(0, m_f) \rightarrow 0 \Rightarrow U_A(1)$ trivially restored.
- ▶ $\lim_{\lambda \rightarrow 0} \rho(\lambda, m_f) = \delta(\lambda) m_f^\alpha$ with $1 < \alpha < 2 \Rightarrow U_A(1)$ broken.
- ▶ $\lim_{m_f \rightarrow 0} \rho(\lambda, m_f) \sim \lambda^3 \Rightarrow U_A(1)$ restored.

More on the eigenvalue spectrum and $U_A(1)$ breaking

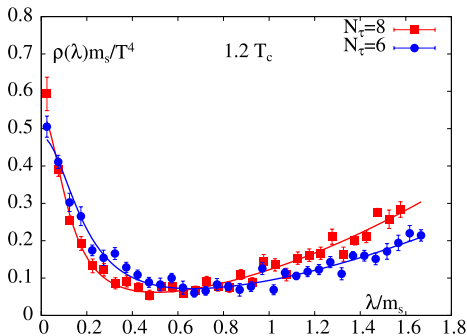
- ▶ Looking at higher n-point correlation functions is also important!
- ▶ $\rho(\lambda, m_f)$ has been investigated analytically using chiral Ward identities of n-point functions of scalar and pseudo-scalar currents, assuming $\rho(\lambda, m_f)$ to be analytic in m_f^2
[Aoki, Fukaya & Taniguchi, 2012].
- ▶ It was shown explicitly that $U_A(1)$ breaking is absent in upto six point correlation functions in the same scalar and pseudo-scalar sectors if $\rho(\lambda, m_f \rightarrow 0) \sim \lambda^3$.
- ▶ They found out that in the chiral limit $\rho(\lambda) \sim \lambda^n$ with $n > 2$.

Summary of the results till now

- ▶ Previous studies at almost physical quark masses:
Infrared part has both, non-analytic and analytic in λ

[Sayantan Sharma et al.,2015].

- ▶ $\rho(\lambda, m_f) \sim \lambda^2$ at $T = 1.2 T_c \Rightarrow U_A(1)$ broken.



[Viktor Dick et al.,2015]

Setup

- ▶ Gauge ensembles were generated within the Highly Improved Staggered Quark discretization scheme (HISQ) with 2+1 quark flavor.
- ▶ We used the overlap Dirac operator to measure the low-lying eigenvalue spectrum.
- ▶ 98 eigenvalues per configuration.

m_s/m_l	$N_s^3 \times N_\tau$	β	T/T_c	#conf
27	$32^3 \times 8$	6.390	0.97	78
27	$32^3 \times 8$	6.445	1.03	80
27	$32^3 \times 8$	6.500	1.09	109
40	$32^3 \times 8$	6.390	0.99	213
40	$32^3 \times 8$	6.423	1.03	103
40	$32^3 \times 8$	6.445	1.05	94

Overlap operator

- ▶ The overlap operator is given as,

$$D_{ov} = M [1 + \gamma_5 \text{sgn}(\gamma_5 D_W(-M))]$$

$$\text{sgn}(\gamma_5 D_W(-M)) = \gamma_5 D_W(-M) / \sqrt{D_W^\dagger(-M) D_W(-M)}$$

where D_W is the Wilson-Dirac operator with a negative mass parameter $M \in [0, 2)$.

- ▶ Zero-modes were not measured. Only eigenvalues with positive or negative chirality have been measured.
- ▶ Valence overlap quark mass has been tuned to the HISQ sea quark masses by matching the renormalized quantity

$$\Delta = \frac{m_s \langle \bar{\Psi} \Psi \rangle_l - m_l \langle \bar{\Psi} \Psi \rangle_s}{T^4}$$

Topological charge

- ▶ From left/right-handed fermion zero modes:

$$Q = n_+ - n_-.$$

- ▶ Gluonic definition of topological charge:

$$Q = \int d^4x q(x),$$

- ▶ with the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{tr} \{ F_{\mu\nu}(x) F_{\rho\sigma}(x) \}.$$

The Gradient Flow

- ▶ All gluonic definitions valid only on smooth configurations.
- ▶ Introduce extra coordinate t (flow-time) and define a t -dependent gauge field $B_\mu(x, t)$ [Lüscher, 2014]

$$\frac{d}{dt} B_\mu(x, t) = D_\nu G_{\nu\mu}(x, t),$$

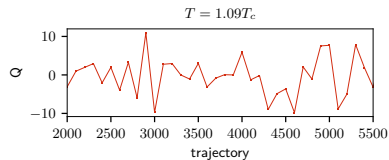
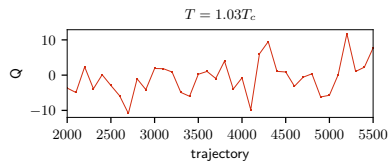
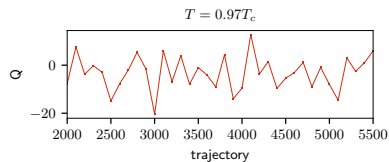
$$D_\mu = \partial_\mu + [B_\mu(x, t), \cdot],$$

$$G_{\mu\nu}(x, t) = \partial_\mu B_\nu(x, t) - \partial_\nu B_\mu(x, t) + [B_\mu(x, t), B_\nu(x, t)]$$

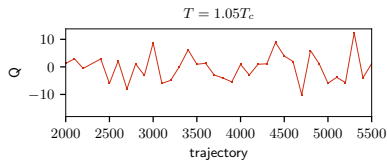
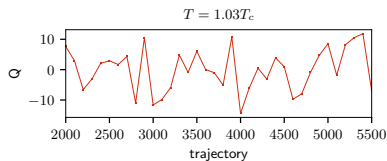
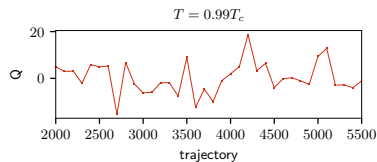
- ▶ with the initial condition: $B_\mu(x, t)|_{t=0} = A_\mu(x)$
- ▶ The flow smoothens the fields over a region of radius $\sqrt{8t}$

Is topological tunneling sufficient?

$$m_I = m_S/27:$$

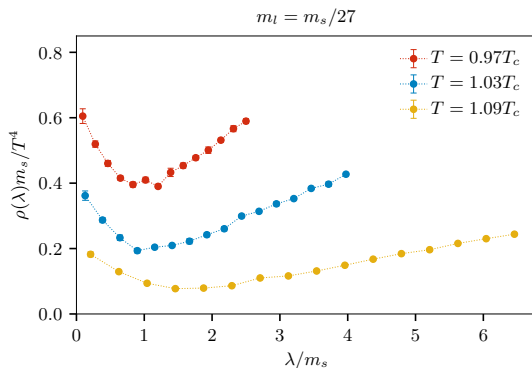


$$m_I = m_S/40:$$



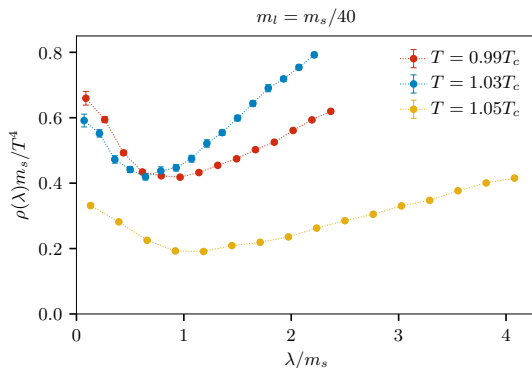
Dirac eigenvalue spectrum

- ▶ Analytic part goes as λ^γ with $\gamma \geq 1$.
- ▶ Non-analytic part reduces with temperature.
- ▶ Going towards smaller quark masses these features survive.
- ▶ Results are preliminary!



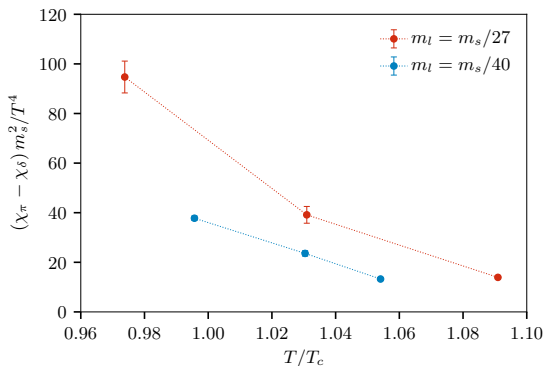
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Renormalized $U_A(1)$ breaking parameter as a function of quark mass

- ▶ $\chi_\pi - \chi_\delta$ reduces when m_f reduces from $m_s/27$ to $m_s/40$ for $T/T_c < 1.1$.
- ▶ Results are preliminary!



Conclusion

- ▶ $\rho(\lambda, m_f) \sim \lambda$ even when m_f reduces from $m_s/27$ to $m_s/40$ for $T/T_c < 1.1$.
- ▶ Non-analytic part also survives when we reduce the mass.
- ▶ Both of them contribute to the breaking of $U_A(1)$ above T_c after proper retuning of the valence quark masses and looking at renormalized quantities.
- ▶ We are looking at even smaller quark masses to check whether our conclusions survive in the chiral limit.