Anisotropic flow in the few collisions regime

Nina Kersting

in collaboration with Nicolas Borghini and Steffen Feld

Universität Bielefeld

Hot Quarks 2018, 10 September

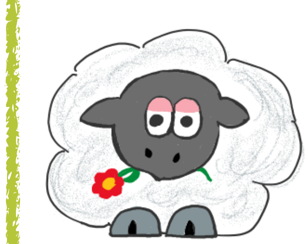
CRC–TR 211

Content

- Motivation
- Our set up
- Theoretical background
    - Anisotropic flow coefficient
    - Kinetic, relativistic Boltzmann equation
    - Single particle distribution function
- Results & discussion
    - Scaling behaviour
    - Bottomonia

denotes, that we can change something, e.g. modify our assumptions

throws light on terms and definitions



denotes a few papers on the respective topic

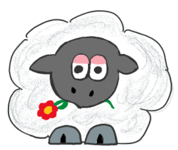
Motivation

Do something different.

Would even one collision in average per particle lead to sizeable anisotropic flow coefficients v_n ?

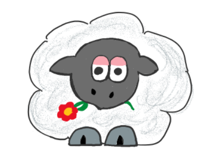
Few-hit-dynamics
Kinetic theory $Kn >> 1$
instead of hydrodynamics $Kn << 1$

Old stuff with new interest



Heiselberg & Levy, PRC 59 (1999) 2716

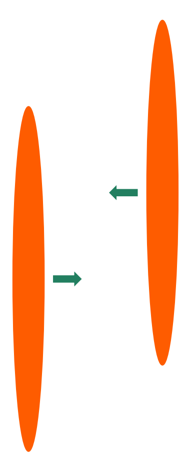
Borghini & Gombeaud, EPJC 71 (2011) 1612



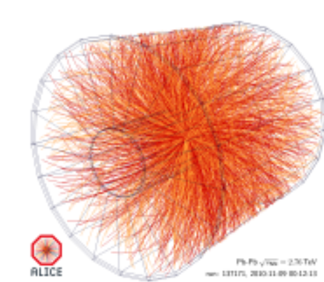
Kurkela, Wiedemann & Wu, arXiv:1805.04081

Romatschke, EPJC 78 (2018) 636

initial



final(ly)

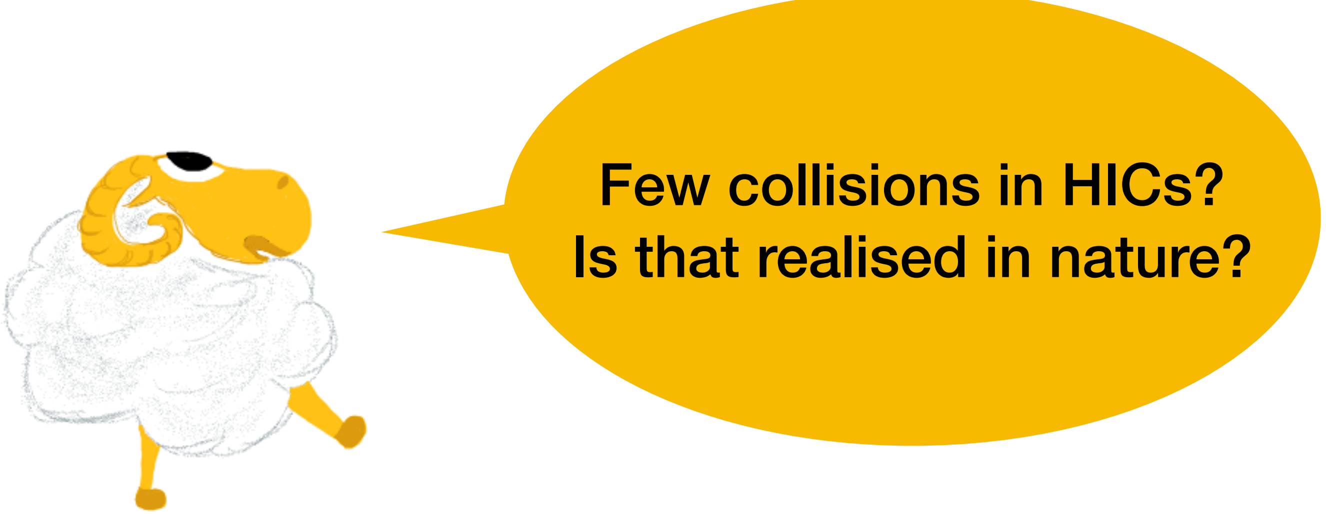


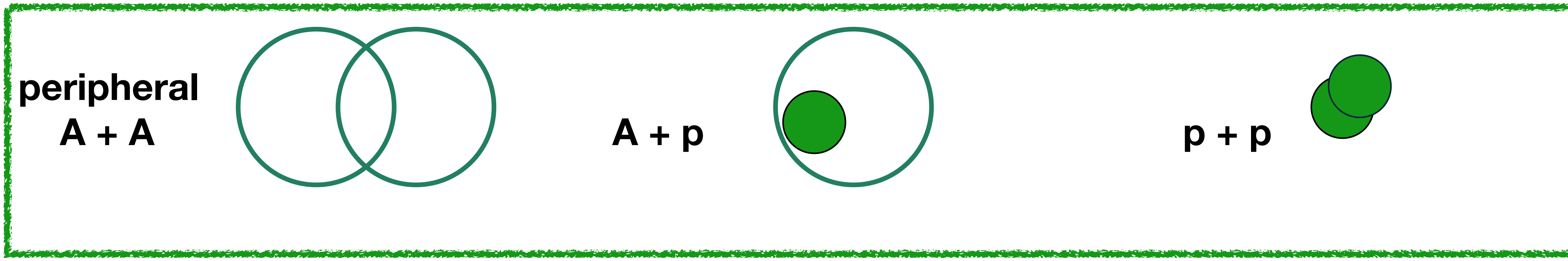
One of these observables are the harmonic flow coefficients

Introduce theoretical observables which are measurable

Comparison of experimental data and theoretical descriptions

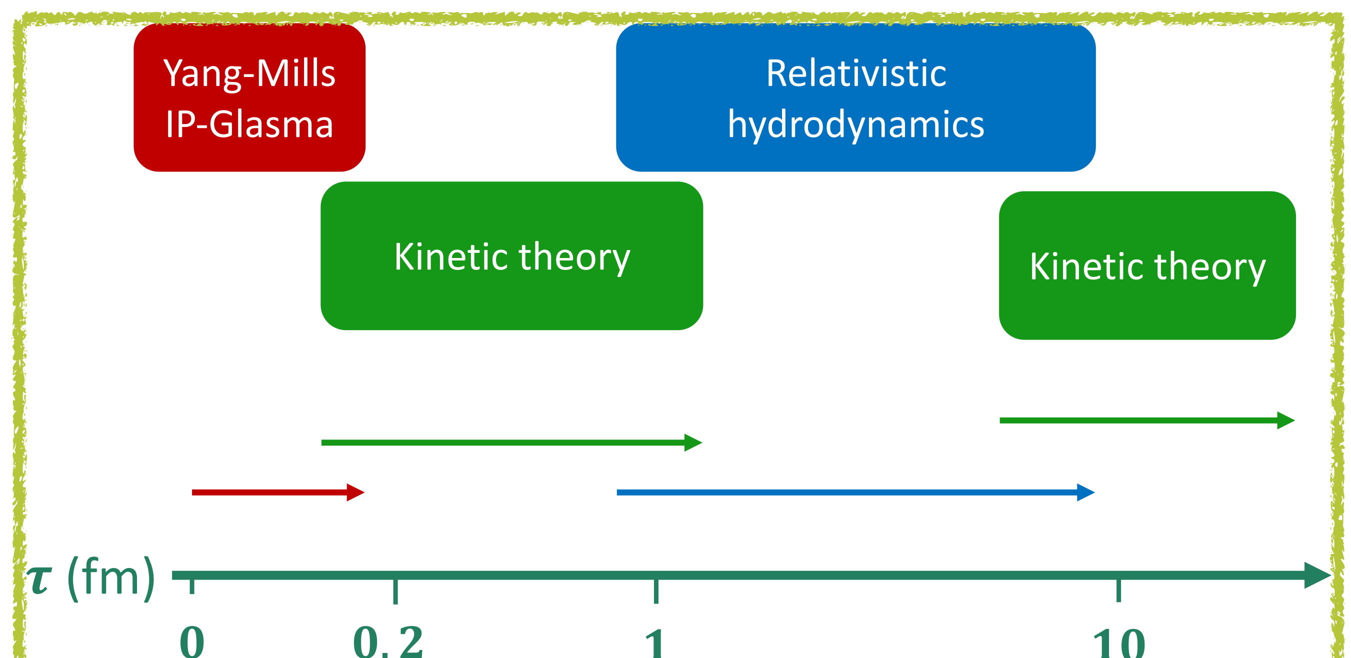
Understanding of the HIC and A+p- and p+p-collisions

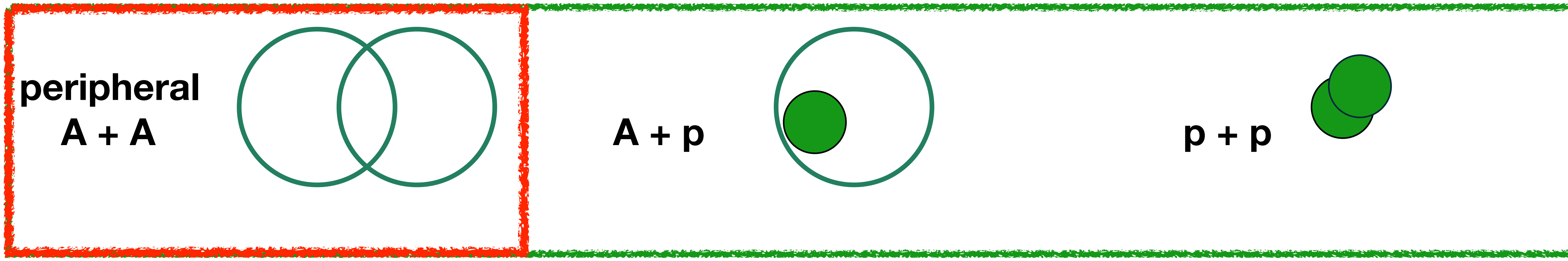




Small and dilute systems.
Like p+p, p+Pb and peripheral HIC

Early stage of all HIC's
(before equilibration)
&
Late stage of all HIC's
(around kinetic freeze-out)
in the standard picture





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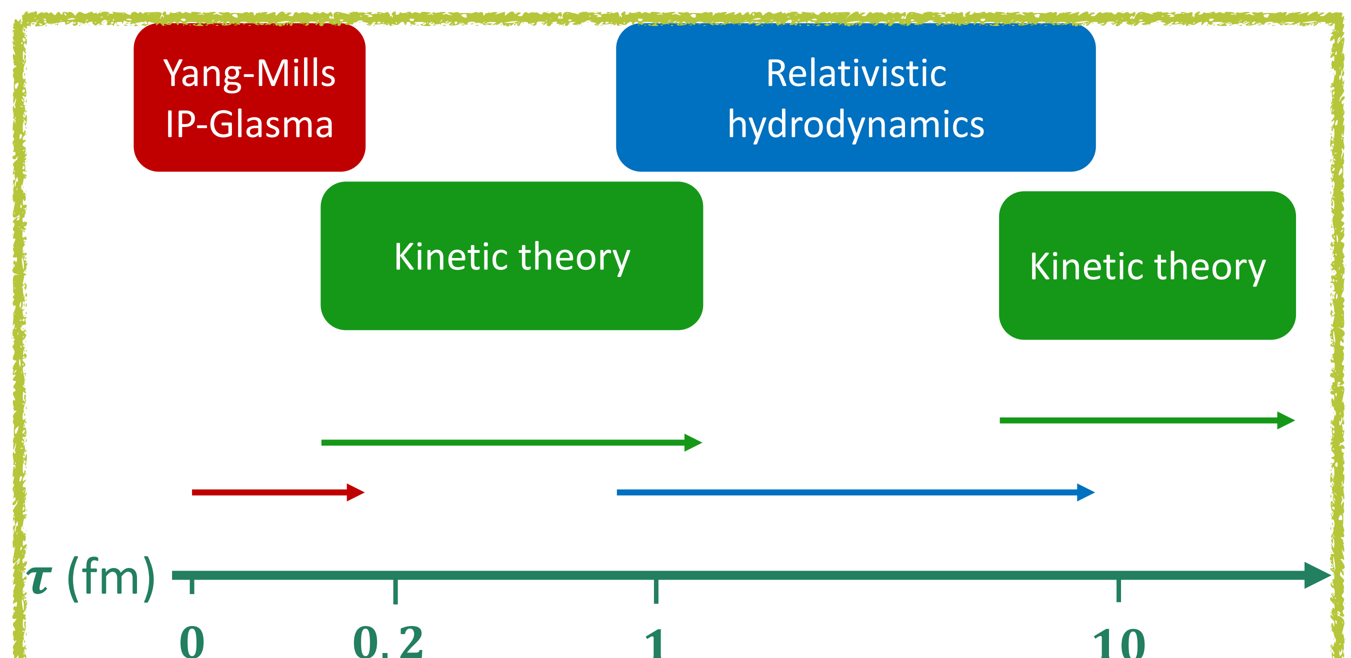
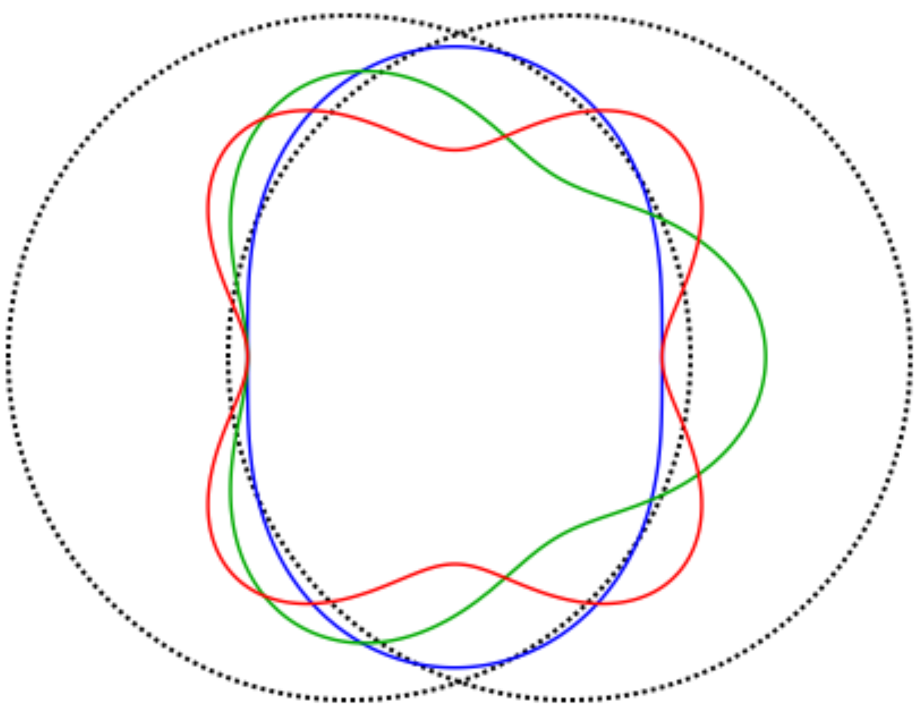


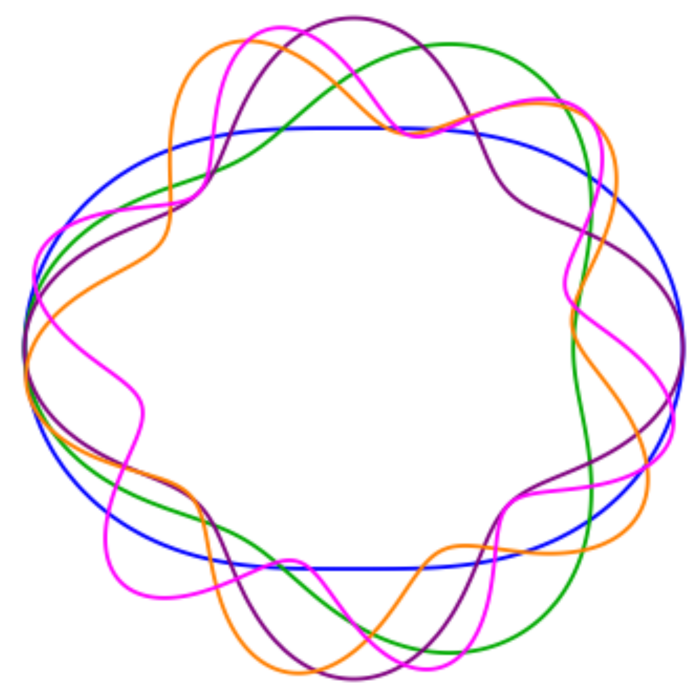
Link between eccentricities ϵ_n and anisotropic flow coefficients $v_n\left(p_T\right)$

Position space
- **Initial** $t = 0$
- **Eccentricities**

~~**Collective behaviour (Hydrodynamics)**~~
...
Few-hit-dynamics (kinetic theory) ✓
~~**Free-streaming**~~ $v_n = v_n(t) = v_n(t_0 = 0)$

Momentum space
- **Final** $t \to \infty$
- **Anisotropic flow**



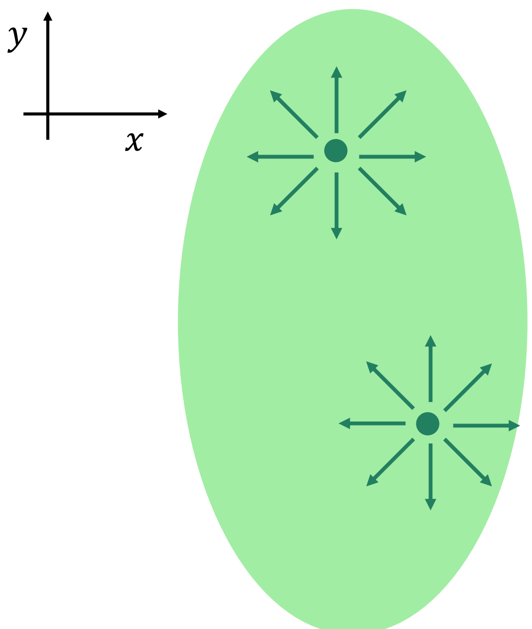


ϵ_2 ϵ_3 ϵ_4 ...

$v_2\left(p_T\right)$ $v_3\left(p_T\right)$ $v_4\left(p_T\right)$ $v_5\left(p_T\right)$ $v_6\left(p_T\right)$ ...

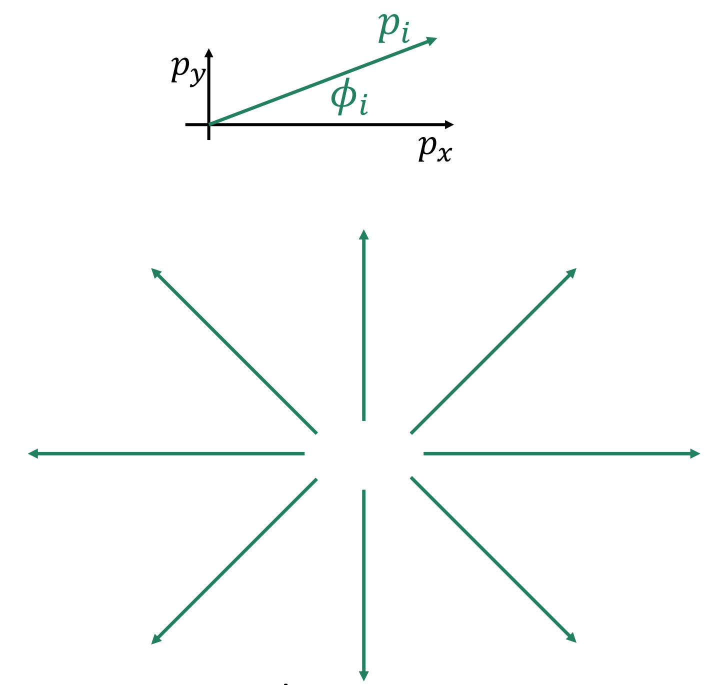
Set-up

Spatial asymmetry



initial

Anisotropic flow



later times

Evolving system far from equilibrium
Kinetic theory
Very few rescatterings in average per particle
(No continuous medium)

Expanding mixture of several species of relativistic (massive) particles

Start our description already with different particle-types i, k

Restriction to the transverse plane (two-dimensional)

Anisotropic flow for particles of type i

- **Measurements at $t=\infty$ of flow harmonics:**

$$v_{n,i}\left(p_i\right)=\frac{\int_0^{2\pi}\frac{d^2N_i}{d^2\mathbf{p}_i}\cos\left(n\left(\phi_i-\Psi_n\right)\right)\ d\phi_i}{\int_0^{2\pi}\frac{d^2N_i}{d^2\mathbf{p}_i}\ d\phi_i}$$

- **With the transverse momentum distribution of particles:**

$$\frac{d^2N_i}{d^2\mathbf{p}_i}\left(t,\mathbf{p}_i\right)=\int_{-\infty}^{\infty}f_i\left(t,\mathbf{x},\mathbf{p}_i\right)d\mathbf{x}$$

Time rate of chance of anisotropic flow coefficients

- **Time rate of change**

$$\frac{d}{dt} v_{n,i}\left(t, p_i\right) = \frac{\int_0^{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i}{\int_0^{2\pi} \frac{d^2 N_i}{d^2 \mathbf{p}_i}\ d\phi_i}$$

- **Measured** $(t = \infty)$ **anisotropic flow**

$$v_{n,i}\left(p_i\right) = \int_0^{\infty} \frac{d}{dt} v_{n,i}\left(t, p_i\right) dt = \int_0^{\infty} \frac{\int_0^{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i}{\int_0^{2\pi} \frac{d^2 N_i}{d^2 \mathbf{p}_i}\ d\phi_i}\, dt$$

- **with**

$$v_{n,i}\left(0, p_i\right) = 0$$

We need an on-shell single particle phase space distribution for particles of type i

What do we know about $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$?

- **Equation of motion is the classical relativistic Boltzmann equation (without external force):**

$$p_\mu \partial^\mu f_i(t, \mathbf{x}, \mathbf{p}_i) = C_{collision}\left[f_i(t, \mathbf{x}, \mathbf{p}_i)\right]$$

- **Distribution function** $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$ **obeys this equation**

What do we know about $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$?

- **Without collisions**

$$p_\mu \partial^\mu f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = 0$$

- **we find the free-streaming solution** $f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)$

- **The free-streaming distribution function depends on time in following way**

$$f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(0, \mathbf{x} - t \ \mathbf{v}_i, \mathbf{p}_i)$$

- **with the velocity** $\mathbf{v}_i$

$$\frac{d}{dt} v_{n,i}\left(t, p_i\right) = 0 \Rightarrow v_n = v_n(t) = v_n(t_0 = 0)$$

We have to include collisions.

What do we know about $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$?

- **We want to include few collisions, such that $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$ may develop anisotropies in momentum space.**
- **Expansion:**

$$f_i(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) + f_i^{(1)}(t, \mathbf{x}, \mathbf{p}_i) + \mathcal{O}(2)$$

same initial distribution function at $t = 0$

including higher orders is possible

→ **Collisions are due to $f_i^{(1)}(t, \mathbf{x}, \mathbf{p}_i)$**

- **With $f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) >> f_i^{(1)}(t, \mathbf{x}, \mathbf{p}_i)$**
- **and** $p_\mu \partial^\mu f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = 0$

$$\rightarrow \quad p_\mu \partial^\mu f_i^{(1)}(t, \mathbf{x}, \mathbf{p}_i) = C_{collision}\left[f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\right]$$

→ **We have to choose a collision term**

Choose a collision term $C_{collision}\left[f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i)\right]$

- **Elastic 2-to-2-collisions**

- **Particles of species i collide with particles of species k**

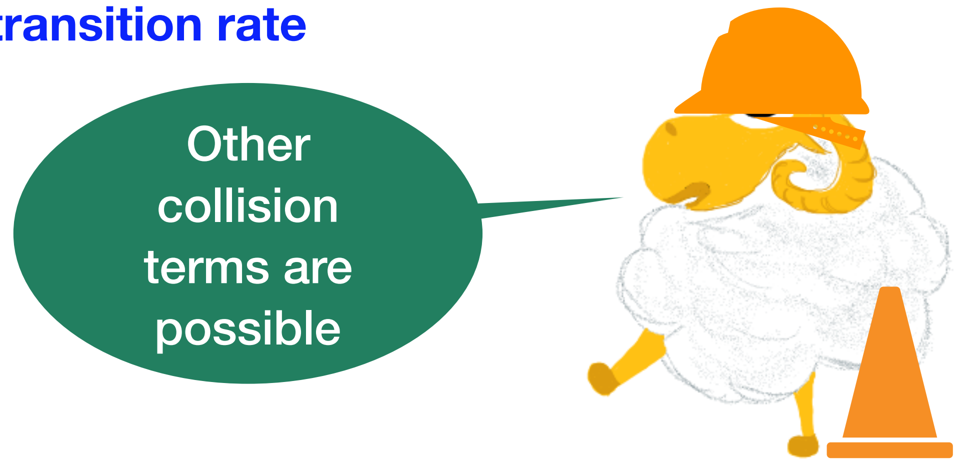
$$C_{collision}\left[f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i)\right] = \left(1-\frac{1}{2}\delta_{ik}\right)\int f_i^{(0)}(t,\mathbf{x},\tilde{\mathbf{p}}_i)\, f_k^{(0)}(t,\mathbf{x},\tilde{\mathbf{p}}_k)\; w\left(\tilde{\mathbf{p}}_i,\tilde{\mathbf{p}}_k \rightarrow \mathbf{p}_i,\mathbf{p}_k\right)$$

$$-f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i)\, f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\; w\left(\mathbf{p}_i,\mathbf{p}_k \rightarrow \tilde{\mathbf{p}}_i,\tilde{\mathbf{p}}_k\right)\;\; d^2\mathbf{p}_k\; d^2\tilde{\mathbf{p}}_k\; d^2\tilde{\mathbf{p}}_i$$

circumvent double counting
(if particle of type i is equal to k)

transition rate

- **Collision term is linear in transition rate**
- **Collision term is quadratic in distribution function**



Back to anisotropic flow coefficients

- **Remember:**

$$\frac{d}{dt} v_{n,i}\left(t, p_i\right) = \frac{\int_0^{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i}{\int_0^{2\pi} \frac{d^2 N_i}{d^2\mathbf{p}_i}\ d\phi_i}$$

- **free-streaming and correction due to collisions**
- **2-to-2 collision-term**

- **Consider the nominator.**
- **Our previous assumptions guide us to:**

$$\int_0^{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i = \int_0^{2\pi} \int_{-\infty}^{\infty} C_{collision}\left[f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\right] \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i$$

$$= \left(1 - \frac{1}{2}\delta_{ik}\right) \int f_i^{(0)}(t, \mathbf{x}, \tilde{\mathbf{p}}_i)\ f_k^{(0)}(t, \mathbf{x}, \tilde{\mathbf{p}}_k)\ w\left(\tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}_k \to \mathbf{p}_i, \mathbf{p}_k\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)$$

$$- f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\ f_k^{(0)}(t, \mathbf{x}, \mathbf{p}_k)\ w\left(\mathbf{p}_i, \mathbf{p}_k \to \tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}_k\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\ d^2\mathbf{p}_k\ d^2\tilde{\mathbf{p}}_k\ d^2\tilde{\mathbf{p}}_i\ d^2\mathbf{x}\ d\phi_i$$

- **Anisotropic flow coefficients are determined by the free-streaming solution.**
- **Transition rate → cross section**

We still need $f_i^{(0)}\left(0, \mathbf{x}, \mathbf{p}_i\right)$

Our free-streaming distribution function in polar coordinates (r, Θ)

$$f_i^{(0)}\left(0,\mathbf{x},\mathbf{p}_i\right) \propto F_i\left(p_i\right)\left[1+\epsilon_2\left(\frac{r}{R}\right)^2\exp\left(-\frac{r^2}{2R^2}\right)\cos\left(2\Theta\right)+\epsilon_3\left(\frac{r}{R}\right)^3\exp\left(-\frac{r^2}{2R^2}\right)\cos\left(3\left(\Theta-\Psi_3\right)\right)+\ldots\right]$$

factorised in position- and momentum space

isotropic momentum distribution function

contains only linear terms in eccentricities

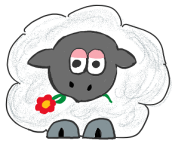
- **We assume nearly the same distribution function for particles of type** k
- **We only exchanged** $i \to k$ **(**$F_i\left(p_i\right) \to F_k\left(p_k\right)$ **and** $N_i \to N_k$**)**

It could also be interesting to assume different R for different species due to production processes

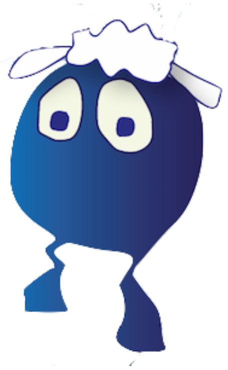
All we need to calculate our flow harmonics

Results - Scaling behaviour

Borghini & Feld & NK, arXiv:1804.05729



Interesting: Our ’“forecast”- hexagonal flow scales with different powers in σ_d



Our finding:

$$v_2 \propto \sigma_d \; \epsilon_2$$

$$v_3 \propto \sigma_d \; \epsilon_3$$

$$v_4 \propto \sigma_d \; \epsilon_2^2 + \sigma_d \; \epsilon_4$$

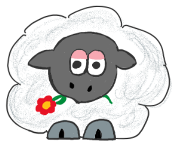
$$v_5 \propto \sigma_d \; \epsilon_2 \; \epsilon_3 + \sigma_d \; \epsilon_5$$

$$v_6 \propto \sigma_d \; \epsilon_2 \; \epsilon_4 + \sigma_d \; \epsilon_3^2 + \sigma_d \; \epsilon_6 + \sigma_d^2 \; \epsilon_2^3$$

- **Mean number of collision** $N_{scat.} \propto \sigma_d \propto Kn^{-1}$

Results - Scaling behaviour and comparison to hydrodynamics

Borghini & Feld & NK, arXiv:1804.05729



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$$v_2 \propto \sigma_d \; \epsilon_2$$

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$$v_4 \propto \sigma_d \; \epsilon_2^2 + \sigma_d \; \epsilon_4$$

$$v_5 \propto \sigma_d \; \epsilon_2 \; \epsilon_3 + \sigma_d \; \epsilon_5$$

$$v_6 \propto \sigma_d \; \epsilon_2 \; \epsilon_4 + \sigma_d \; \epsilon_3^2 + \sigma_d \; \epsilon_6 + \underline{\sigma_d^2 \; \epsilon_2^3}$$

Hydrodynamics:

$$v_2 \propto \kappa_{2,2} \; \epsilon_2$$

$$v_3 \propto \kappa_{3,3} \; \epsilon_3$$

$$v_4 \propto \kappa_{4,22} \; \epsilon_2^2 + \kappa_{4,4} \; \epsilon_4$$

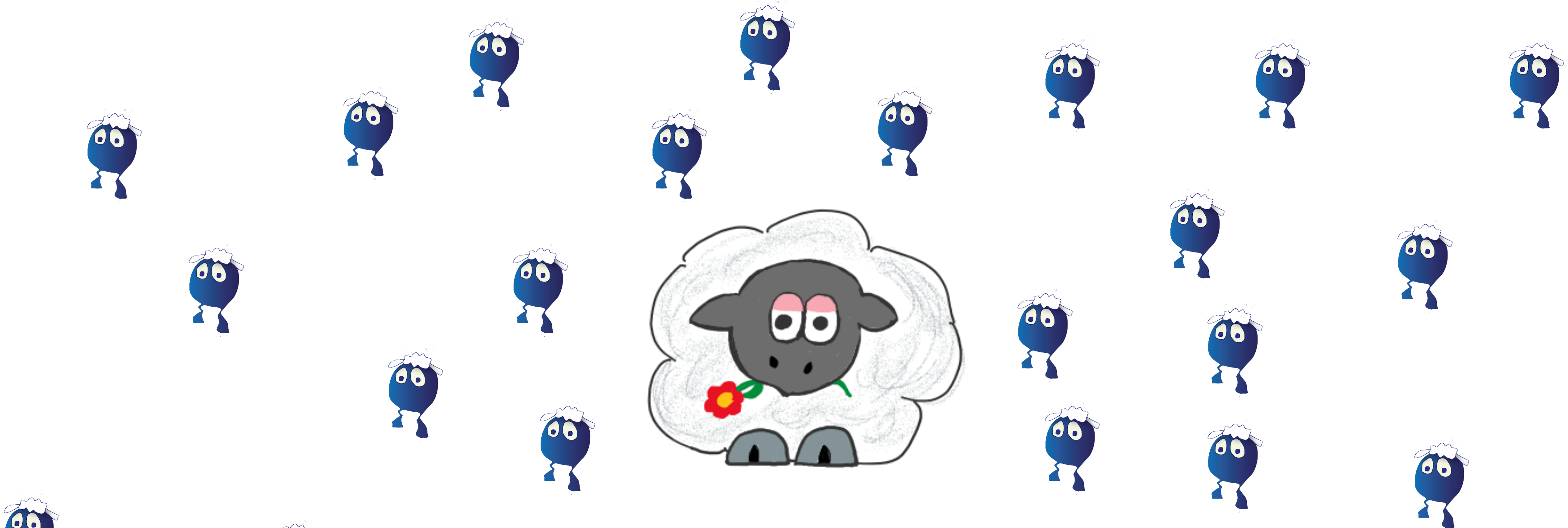
$$v_5 \propto \kappa_{5,23} \; \epsilon_2 \; \epsilon_3 + \kappa_{5,5} \; \epsilon_5$$

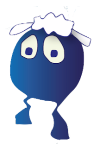
$$v_6 \propto \kappa_{6,24} \; \epsilon_2 \; \epsilon_4 + \kappa_{6,33} \; \epsilon_3^2 + \kappa_{6,6} \; \epsilon_6 + \kappa_{6,222} \; \epsilon_2^2$$

Thanks to Jasper's talk yesterday

When we consider e.g.
- 3-to-3 collisions
- additional terms in $f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)$

- Mean number of collision $N_{scat.} \propto \sigma_d \propto Kn^{-1}$
- Linear response coefficients $\kappa_{n,n}$
- Nonlinear response coefficients $\kappa_{n,lm...}$
  - They depend on transport properties like shear viscosity





Bottomonia (i)

and massless medium particles (k)

Collision term $C_{collision}\left[f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\right]$ for Bottomonia

- **Bottomonia** (i) **collide with massless medium particles** (k)
- **Inelastic collisions** ⟶ **restriction to loss term**

$$\int f_i^{(0)}(t, \mathbf{x}, \tilde{\mathbf{p}}_i)\, f_k^{(0)}(t, \mathbf{x}, \tilde{\mathbf{p}}_k)\, w\left(\tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}_k \to \mathbf{p}_i, \mathbf{p}_k\right) - f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\, f_k^{(0)}(t, \mathbf{x}, \mathbf{p}_k)\, w\left(\mathbf{p}_i, \mathbf{p}_k \to \tilde{\mathbf{p}}_i, \tilde{\mathbf{p}}_k\right)\; d^2\mathbf{p}_k\, d^2\tilde{\mathbf{p}}_k\, d^2\tilde{\mathbf{p}}_i$$

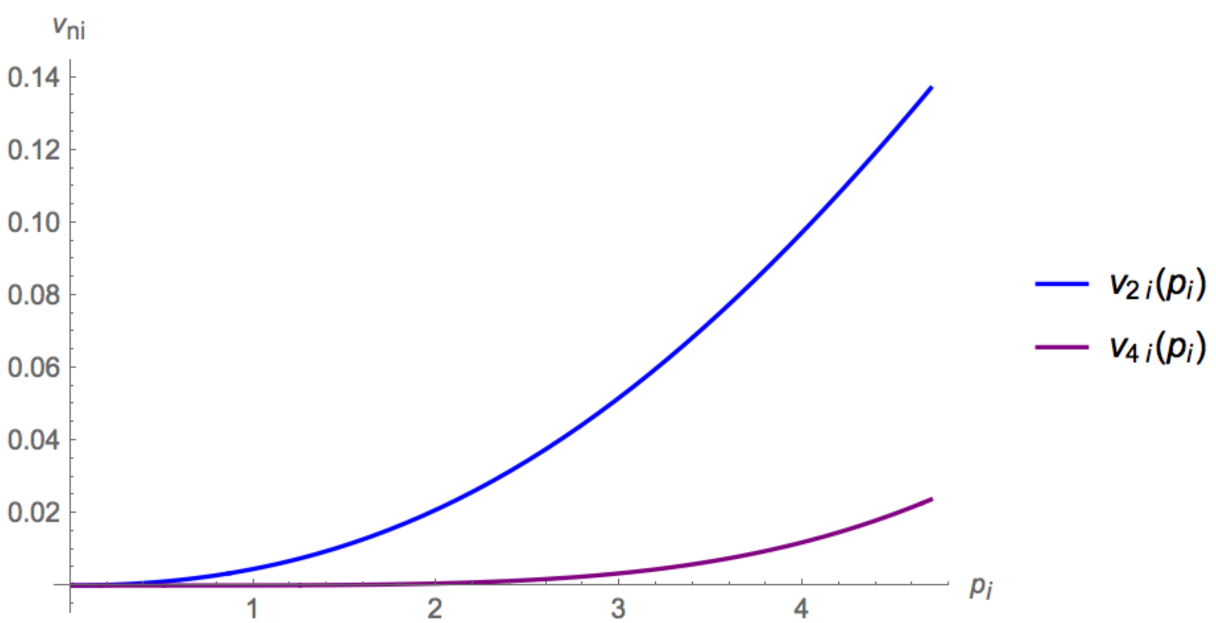
- **Transition rate** ⟶ **cross section**

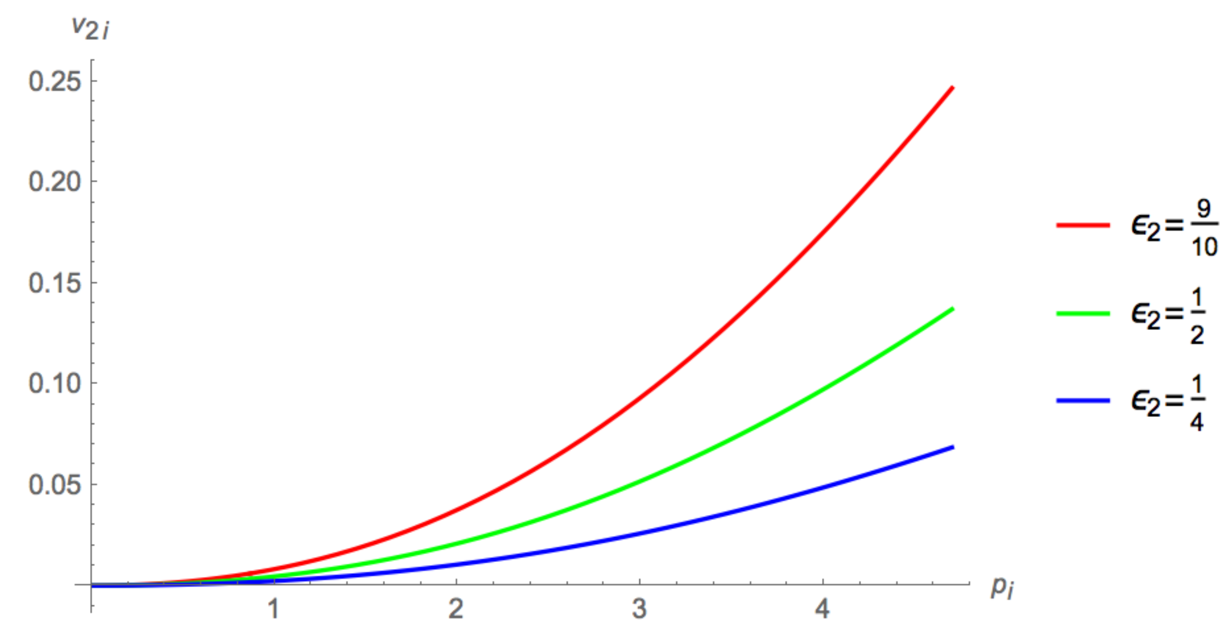
$$= -\,2\,\pi\,\sigma_d \int f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i)\, f_k^{(0)}(t, \mathbf{x}, \mathbf{p}_k)\;\; v_{ik}\; d^2\mathbf{p}_k$$

Comment: **Bottomonia are destroyed after a collision and cannot be created.**

Results - Bottomonia flow

Doubtful: We assume the maximal value of collisions (one collision per Bottomonia). That means that there is no Bottomonia left and no possibility to look at the $v_n{}' s$





$v_{2i}\left(p_i\right)$ **and** $v_{4i}\left(p_i\right)$ **are shown for** $0.01\ \frac{GeV}{c} \le p_i \le 4.7\ \frac{GeV}{c}$

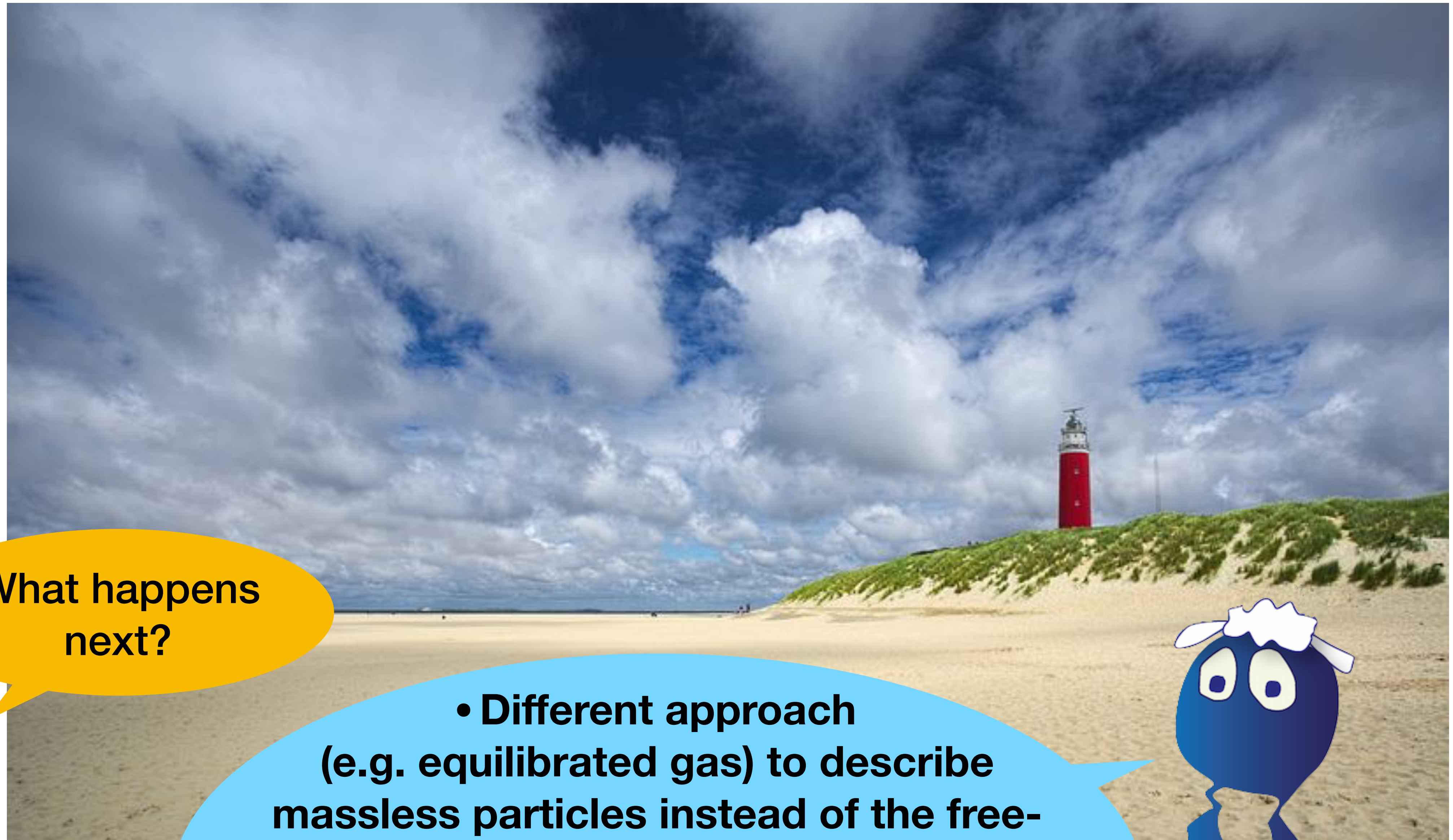
We choose $\epsilon_2 = \frac{1}{2}$

$v_{2i}\left(p_i\right)$ **is shown for** $0.01\ \frac{GeV}{c} \le p_i \le 4.7\ \frac{GeV}{c}$

We choose $\epsilon_2 = \frac{1}{4}, \frac{1}{2}, \frac{9}{10}$

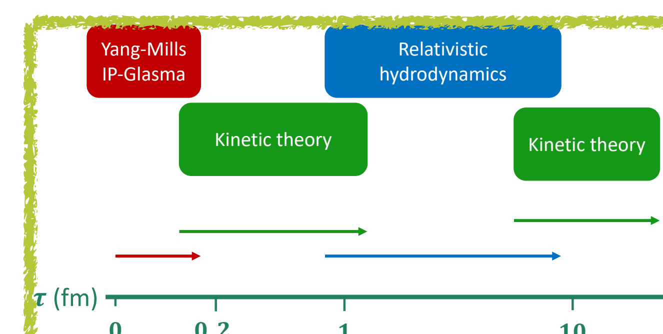
Even for at most 1 collision per Bottomonia we ”observe“ anisotropic flow

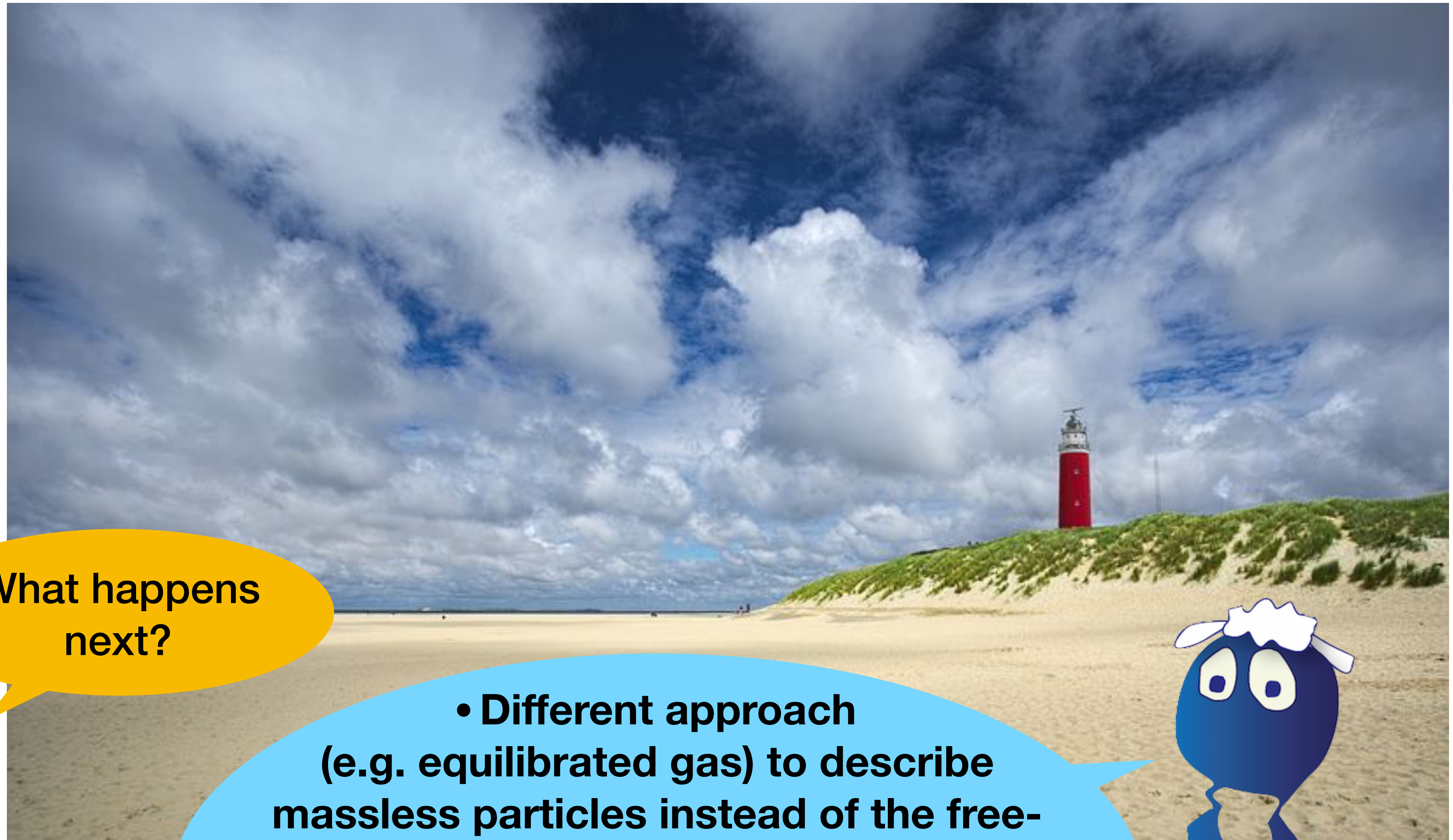
Comment: **Bottomonia are destroyed after a collision. That means that we have to assume that Bottomonia can collide at most once.**



What happens next?

- Different approach (e.g. equilibrated gas) to describe massless particles instead of the free-streaming ansatz.
- Comparison with numerical studies/ help to calibrate few collision regime
- Extension to 3-dimensional case





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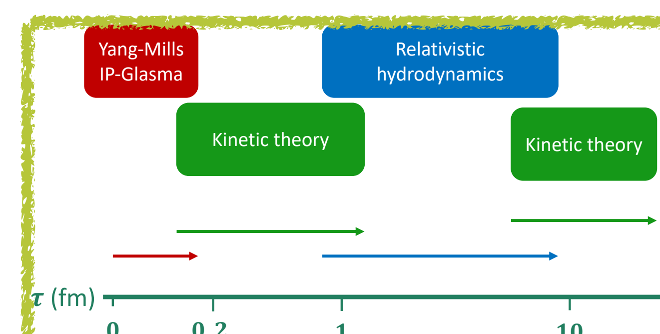


Thanks for your attention

Backup

Knudsen number Kn

- **Definition of dimensionless Knudsen number** $Kn = \frac{l_{mfp}}{L}$

- **with** L **typical length scale over which macroscopic system properties vary**

- **Small** $Kn \leftrightarrow$ **dense system/collisions occur relatively often**

- **Large** $Kn \leftrightarrow$ **free streaming limit**

- **Inverse Knudsen number** $Kn^{-1} \propto N_{scat.}$

- **is proportional to the number of rescatterings per particle** $N_{scat.} \approx \frac{\sigma_d N_k}{R} \leq 1$

Anisotropic flow coefficients

Measurement at $t = \infty$

$$v_{n,i}\left(p_i\right) = \frac{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i} \cos\left(n\left(\phi_i - \Psi_n\right)\right)\, d\phi_i}{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\, d\phi_i}$$

$v_{n,i}\left(p_i\right) := v_{n,i}\left(t = \infty, p_i\right)$

$$v_{n,i}\left(t, p_i\right) = \frac{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\left(t, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\, d\phi_i}{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\left(t, \mathbf{p}_i\right)\, d\phi_i}$$

$\frac{d^2N_i}{d^2\mathbf{p}_i}\left(t, \mathbf{p}_i\right) = \int_{-\infty}^{\infty} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) d\mathbf{x}$

$$v_{n,i}\left(t, p_i\right) = \frac{\int_0^{2\pi}\int_{-\infty}^{\infty} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\, d^2\mathbf{x}\, d\phi_i}{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\, d\phi_i}$$

$\frac{d}{dt}$

$$\frac{d}{dt}v_{n,i}\left(t, p_i\right) = \frac{\int_0^{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\, d^2\mathbf{x}\, d\phi_i}{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\, d\phi_i}$$

$v_{n,i}\left(p_i\right) := v_{n,i}\left(t = \infty, p_i\right)$

$v_{n,i}\left(0, p_i\right) = 0$

$$v_{n,i}\left(p_i\right) = \int_0^{\infty} \frac{d}{dt} v_{n,i}\left(t, p_i\right) dt = \int_0^{\infty} \frac{\int_0^{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} f_i\left(t, \mathbf{x}, \mathbf{p}_i\right) \cos\left(n\left(\phi_i - \Psi_n\right)\right)\, d^2\mathbf{x}\, d\phi_i}{\int_0^{2\pi} \frac{d^2N_i}{d^2\mathbf{p}_i}\, d\phi_i}\, dt$$

Left side of Boltzmann equation

$$p_\mu \partial^\mu f_i(t, \mathbf{x}, \mathbf{p}_i) = C_{collision}$$

$$\frac{d}{dt} f\left(t, \mathbf{x}, \mathbf{p}\right) = \left[\frac{\partial}{\partial t} + \frac{d\mathbf{x}}{dt}\partial_x + \frac{d\mathbf{p}}{dt}\partial_p\right] f\left(t, \mathbf{x}, \mathbf{p}\right) \overset{\text{no external force}}{=} \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_x\right] f\left(t, \mathbf{x}, \mathbf{p}\right)$$

for $f\left(t, \mathbf{x}, \mathbf{p}\right) = f^{(0)}\left(t, \mathbf{x}, \mathbf{p}\right) + f^{(1)}\left(t, \mathbf{x}, \mathbf{p}\right)$ **follows**

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_x\right] f^{(0)}\left(t, \mathbf{x}, \mathbf{p}\right) + \left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_x\right] f^{(1)}\left(t, \mathbf{x}, \mathbf{p}\right)$$

free streaming

vanishes,
when integrating
over space for
our choice of the
distribution function

Detailed evaluation for nominator of $v_{n,i}(t, p_i)$ for Bottomonia

$$
\begin{aligned}
&\int_0^{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} f_i(t,\mathbf{x},\mathbf{p}_i)\cos\left(n\left(\phi_i-\Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i \\
\overset{\frac{d}{dt}f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i)=0}{=}\ &\int_0^{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} f_i^{(1)}(t,\mathbf{x},\mathbf{p}_i)\cos\left(n\left(\phi_i-\Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i \\
\overset{\text{Boltzmann equation}}{=}\ &\int_0^{2\pi}\int_{-\infty}^{\infty} C_{collision}\left[f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i)\right]\cos\left(n\left(\phi_i-\Psi_n\right)\right)\ d^2\mathbf{x}\ d\phi_i \\
\overset{\text{only loss term}}{=}\ &-\int f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\cos\left(n\left(\phi_i-\Psi_n\right)\right) v_{ik}\ \frac{d\sigma_{ik}}{d\Theta}\ d\Theta\ d\phi_k\ p_k\ dp_k\ d^2\mathbf{x}\ d\phi_i \\
\overset{\text{cross section}}{=}\ &-2\pi\sigma_d\int f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\cos\left(n\left(\phi_i-\Psi_n\right)\right) v_{ik}\ d\phi_k\ d^2\mathbf{x}\ d\phi_i\ p_k\ dp_k
\end{aligned}
$$

differential cross section

relative velocity

How to compute the initial eccentricities ϵ_n ?

$$\epsilon_n = -\frac{\int_0^\infty \int_0^{2\pi} G(r,\Theta) r^{n+1} \cos\left(n\left(\Theta - \Psi_n\right)\right) d\Theta dr}{\int_0^\infty \int_0^{2\pi} G(r,\Theta) r^{n+1} d\Theta dr}$$

- **with** $G(\mathbf{r}) = G(r,\Theta)$

- **the position dependent part of initial distribution function**

$$f_i^{(0)}\left(t=0,\mathbf{r},\mathbf{p}_i\right) = G(\mathbf{r})F\left(\mathbf{p}_i\right)$$

Comment: Actually one has to average over the entropy density to determine the eccentricities. But computing the eccentricities with the distribution function will not lead to a difference. It is only a matter of convention.

How to calculate the number of collisions?
How to insure few collisions?
Part I

elastic coll.

$$N_i(t) = N_i$$

- **Scattering rate at time** t

$$\Gamma(t) = \int f_i\left(t, \mathbf{x}, \mathbf{p}_i\right)\ f_k\left(t, \mathbf{x}, \mathbf{p}_k\right)\ \sigma_d(\Omega)\ v_{ik}\ d\Omega\ d\mathbf{x}\ d\mathbf{p}_i\ d\mathbf{p}_k$$

- **With assumptions as before**

$$N_{scat.} = \int \Gamma(t) dt = \underbrace{2\ \pi\ \sigma_d \int f_i^{(0)}\left(t, \mathbf{x}, \mathbf{p}_i\right)\ f_k^{(0)}\left(t, \mathbf{x}, \mathbf{p}_k\right)\ v_{ik}\ d\mathbf{x}\ dt\ d\phi_i\ d\phi_k}_{\text{calculated}}\ p_i\ dp_i\ p_k\ dp_k$$

calculated

$$N_{scat.} = \int \frac{N_i\ N_k\ \pi^{\frac{5}{2}}\ \sigma_d}{R} \left[4 + \frac{3}{\sqrt{2}} \epsilon_2^2\right]\ F_i\left(p_i\right)\ F_k\left(p_k\right)\ F_D^{(2)}\left(\frac{1}{2}, -1, \frac{1}{2}, 1, x_a, x_b\right)\ p_k\ dp_k\ p_i\ dp_i$$

- **With** $\int F_k\left(p_k\right)\ p_k\ dp_k = 1$

- **follows further** $$N_{scat.} = \int \frac{N_i\ N_k\ \pi^{\frac{5}{2}}\ \sigma_d}{R} \left[4 + \frac{3}{\sqrt{2}} \epsilon_2^2\right]\ F_i\left(p_i\right)\ F_D^{(2)}\left(\frac{1}{2}, -1, \frac{1}{2}, x_a, x_b\right)\ p_i\ dp_i$$

- **We know** $\frac{2}{\pi} \leq F_D^{(2)}\left(\frac{1}{2}, -1, \frac{1}{2}, x_a, x_b\right) \leq 1$ **(in dependence of** $x_a \propto p_i$ **and** $x_b \propto p_i$ **)**

- **and therefore**

$$\frac{N_i\ N_k\ \pi^{\frac{3}{2}}\ \sigma_d}{2\ R} \left[4 + \frac{3}{\sqrt{2}} \epsilon_2^2\right] \leq N_{scat.} \leq \frac{N_i\ N_k\ \pi^{\frac{5}{2}}\ \sigma_d}{R} \left[4 + \frac{3}{\sqrt{2}} \epsilon_2^2\right]$$

How to calculate the number of collisions? How to insure few collisions? Part II

- **Again**

$$\frac{2\ N_i\ N_k\ \pi^{\frac{3}{2}}\ \sigma_d}{R}\left[4+\frac{3}{\sqrt{2}}\epsilon_2^2\right] \leq N_{scat.} \leq \frac{N_i\ N_k\ \pi^{\frac{5}{2}}\ \sigma_d}{R}\left[4+\frac{3}{\sqrt{2}}\epsilon_2^2\right]$$

- **At most one collisions per Bottomonia**

$$\Rightarrow \frac{2\ N_k\ \pi^{\frac{3}{2}}\ \sigma_d}{R}\left[4+\frac{3}{\sqrt{2}}\epsilon_2^2\right] \leq 1$$

$$\Rightarrow \frac{N_k\ \sigma_d}{R} \leq \frac{1}{2\ \pi^{\frac{3}{2}}\left[4+\frac{3}{\sqrt{2}}\epsilon_2^2\right]}$$

- **Maximal value for** $\epsilon_2 = 0$

- **Therefore** $\Rightarrow \frac{N_k\sigma_d}{R} \leq \frac{1}{8\pi^{\frac{3}{2}}}$

- **Insert this value in the calculations of anisotropic flow coefficients to ensure at most 1 collision per Bottomonia**

Our free-streaming distribution function in polar coordinates (r, Θ)

$$f_i^{(0)}(0,\mathbf{x},\mathbf{p}_i) = \frac{N_i}{2\,\pi\,R^2} F(p_i) \exp\left(-\frac{r^2}{2R^2}\right)\left[1 - 4\,\epsilon_2\,\exp\left(-\frac{r^2}{2R^2}\right)\left(\frac{r}{R}\right)^2\cos(2\Theta)\right]$$

particle number for particles of species i

isotropic momentum space distribution function

typical system size

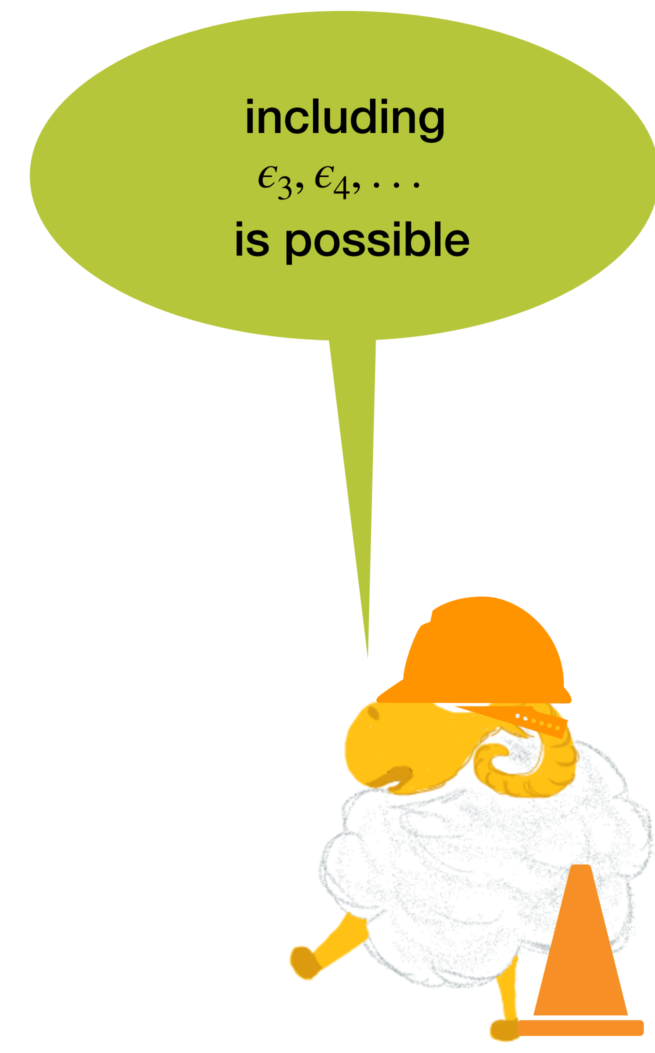
spacial eccentricity

cut-off

factorised in position- and momentum space

isotropic momentum distribution function

contains only linear terms in eccentricities



Further motivation

Increase the numbers of collisions to be more realistic (except Bottmonia)
and to compare results to different theoretical approaches