CRC-TR 211
Strong-interaction matter under extreme conditions

HOT QUARKS 2018

ITP
Institute for Theoretical Physics

GOETHE UNIVERSITÄT
FRANKFURT AM MAIN

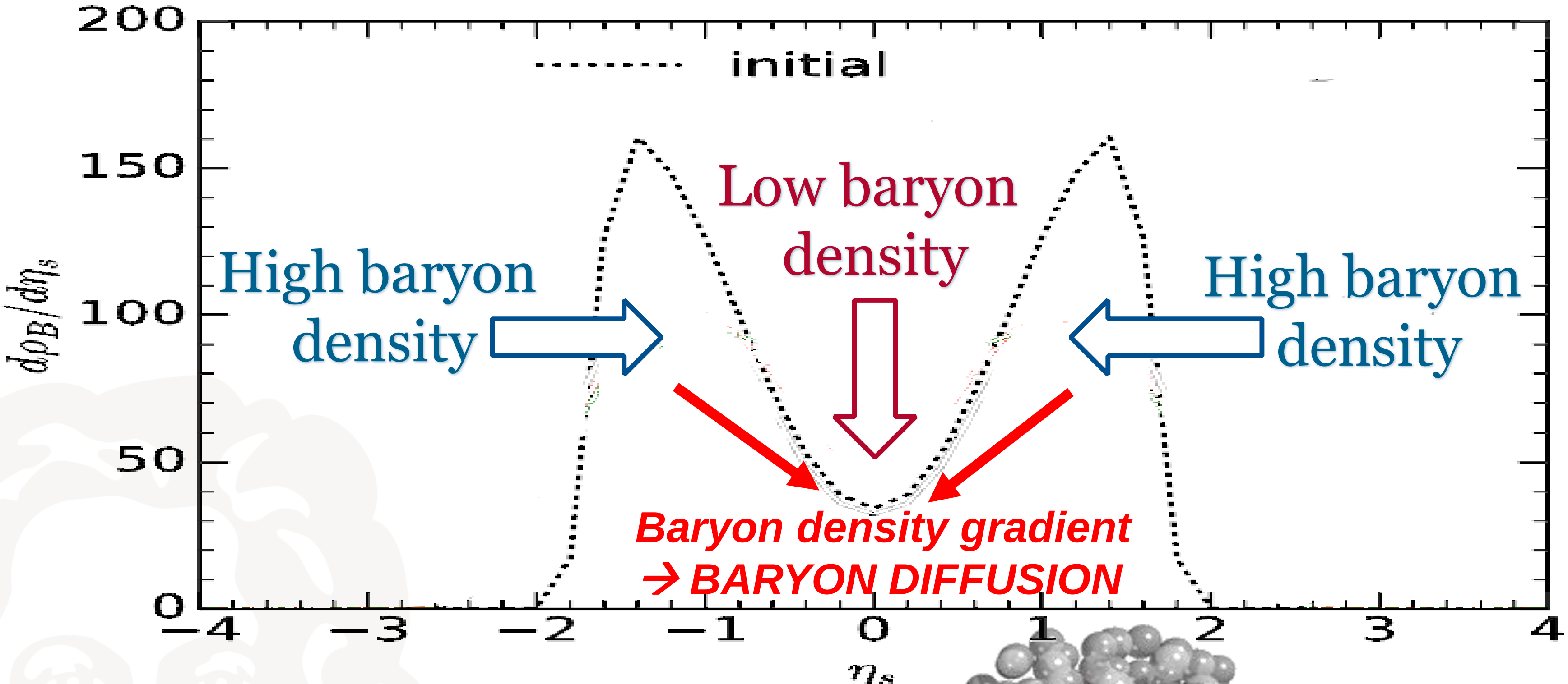
Diffusion of Conserved Charges in Relativistic Heavy Ion Collisions

Presented by **Jan Fotakis**

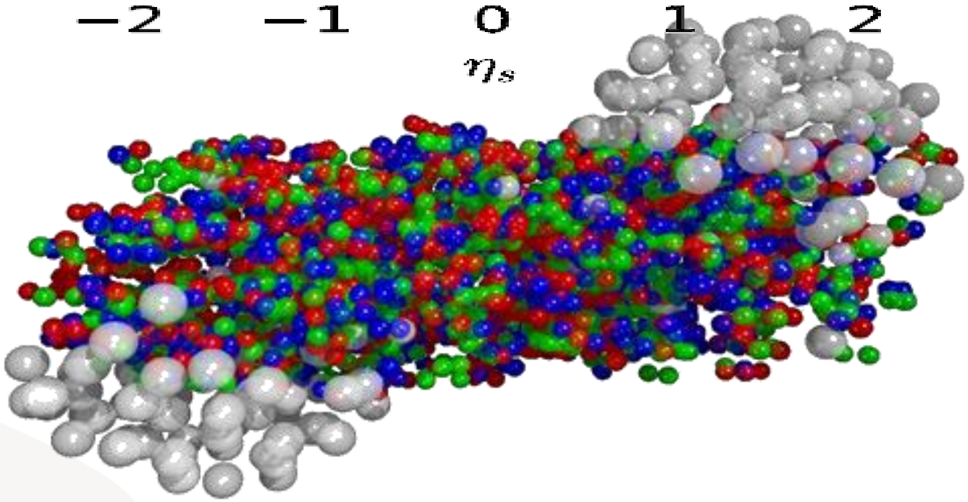
Collaborators
Moritz Greif, Gabriel Denicol and Carsten Greiner

Greif, Fotakis, Denicol, Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

Why could diffusion be important?

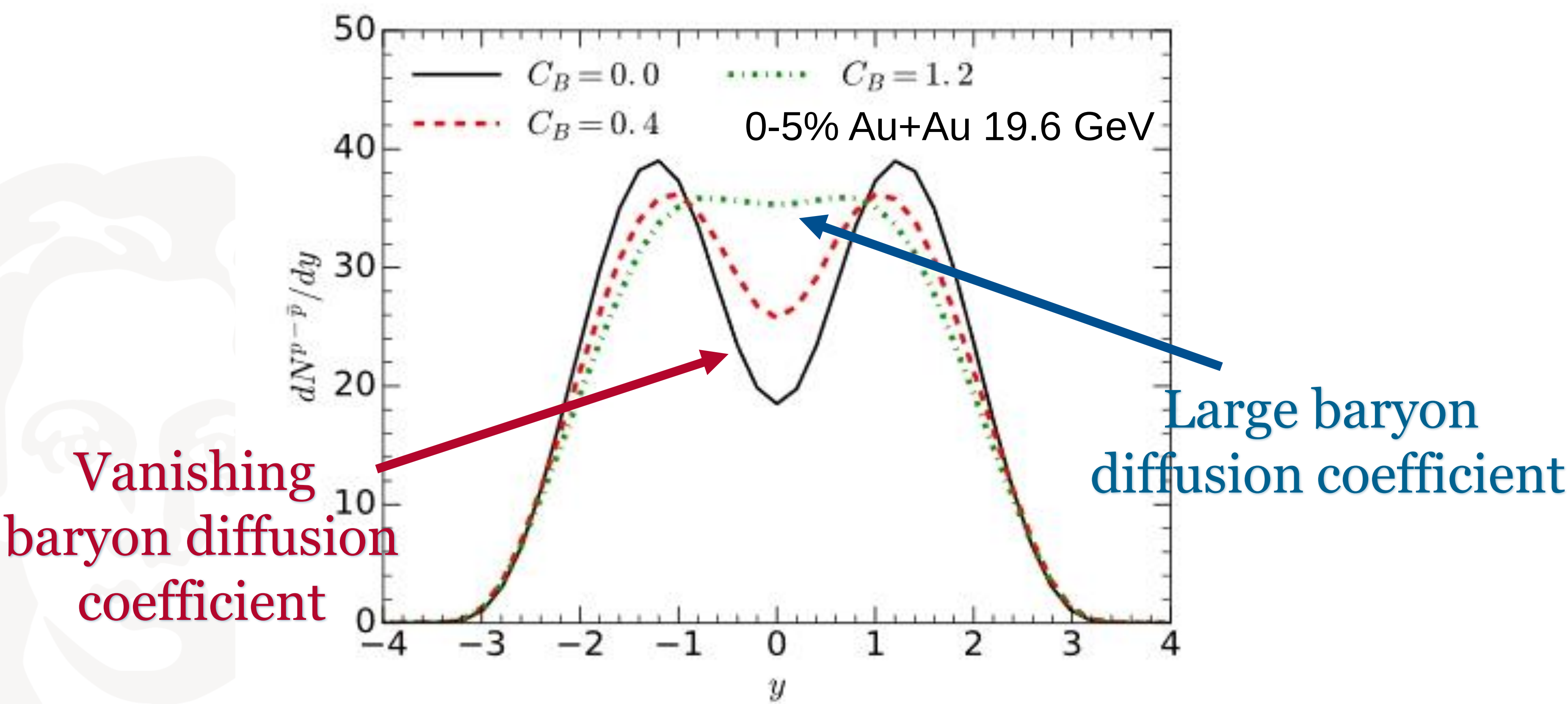


HIC



Why could diffusion be important?

- During low-energy HIC (e.g. RHIC BES): diffusion could have great impact on dynamic evolution



Chun Shen et al., Nucl. Phys. A 967 (2017) 796-799

Description of Diffusion

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

Description of Diffusion

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

Net charge 4-current:

$$N_q^\mu = n_q u^\mu + \kappa_q \nabla^\mu \left(\mu_q / T\right)$$

Description of Diffusion

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

Net charge 4-current:

$$N_q^\mu = \underbrace{n_q u^\mu}_{\text{Ideal flow}} + \kappa_q \nabla^\mu \left(\mu_q / T\right)$$

u^μ : flow velocity
n_q : net charge density

Description of Diffusion

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

j_q^μ : **Net charge diffusion current**

Ideal flow

Net charge 4-current:

$$N_q^\mu = n_q u^\mu + \kappa_q \nabla^\mu \left(\mu_q / T\right)$$

u^μ : flow velocity
n_q : net charge density

Description of Diffusion

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

j_q^μ : **Net charge diffusion current**

Ideal flow

Net charge 4-current:

$$N_q^\mu = n_q u^\mu + \kappa_q \nabla^\mu \left(\mu_q / T\right)$$

Gradient in thermal potential ~ Gradient in net charge density

u^μ : flow velocity

n_q : net charge density

Description of Diffusion

- Early dynamic evolution of HIC modeled in relativistic dissipative fluid dynamics
- Apply Navier-Stokes theory
- One conserved charge (q):

Net charge 4-current:

$$N_q^\mu = \underbrace{n_q u^\mu}_{\text{Ideal flow}} + \underbrace{\kappa_q \nabla^\mu (\mu_q / T)}_{j_q^\mu}$$

j_q^μ : **Net charge diffusion current**

κ_q: Net charge diffusion coefficient

$\nabla^\mu (\mu_q / T)$: Gradient in thermal potential ~ Gradient in net charge density

u^μ : flow velocity

n_q : net charge density

Description of Diffusion

- In multi-component system with multiple conserved charges: particles can have any combination of charges (e.g. proton: electric and baryon charge)

- Net-charge diffusion currents effect each other

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

Off-diagonal coefficients: gradients of given charge can effect diffusion currents of other charges

Are the off-diagonal coefficients important?

The Chapman-Enskog Expansion

- Assume dilute Boltzmann gas with N_s particle species and conserved baryon, strangeness, and electric charge close to local equilibrium → describe with kinetic theory

$$f_k^i = f_{0,k}^i + \epsilon f_{1,k}^i + \mathcal{O}(\epsilon^2)$$

book-keeping parameter
counts power of gradients

The Chapman-Enskog Expansion

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Assume dilute Boltzmann gas with N_s particle species and conserved baryon, strangeness, and electric charge close to local equilibrium → describe with kinetic theory

$$f_k^i = \boxed{f_{0,k}^i} + \epsilon \boxed{f_{1,k}^i} + \mathcal{O}(\epsilon^2)$$

small deviation from local equilibrium

local equilibrium term

book-keeping parameter counts power of gradients

The Chapman-Enskog Expansion

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Assume dilute Boltzmann gas with N_s particle species and conserved baryon, strangeness, and electric charge close to local equilibrium → describe with kinetic theory

small deviation from local equilibrium

$$f_k^i = \boxed{f_{0,k}^i} + \epsilon \boxed{f_{1,k}^i} + \mathcal{O}(\epsilon^2)$$

local equilibrium term

book-keeping parameter counts power of gradients

- Neglect non-linear contributions → Navier-Stokes limit

The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

$$k_i^\mu \partial_\mu f_k^i = -\sum_{j=1}^{N_s} C_{ij}[f_k^i]$$

The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

$$k_i^\mu \partial_\mu f_k^i = -\sum_{j=1}^{N_s} C_{ij}[f_k^i]$$

⇩ Chapman-Enskog expansion to first order

$$k_i^\mu \partial_\mu f_{0k}^i = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

$$k_i^\mu \partial_\mu f_k^i = -\sum_{j=1}^{N_s} C_{ij}[f_k^i]$$

$\Downarrow$ Chapman-Enskog expansion to first order

$$k_i^\mu \partial_\mu f_{0k}^i = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

With linearized collision term:

$$\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i] = \sum_{j=1}^{N_s} \gamma_{ij} \int \mathrm{d}K_j' \mathrm{d}P_i \mathrm{d}P_j' \boxed{W_{kk'\to pp'}^{ij}} f_{0k}^i f_{0k'}^j \left(\frac{f_{1k}^i}{f_{0k}^i} + \frac{f_{1k'}^j}{f_{0k'}^j} - \frac{f_{1p}^i}{f_{0p}^i} - \frac{f_{1p'}^i}{f_{0p'}^i} \right)$$

Transition rate: contains (isotropic) cross sections = information from microscopic interactions

The Chapman-Enskog Expansion

Diffusion currents in kinetic theory:

We want to calculate **THIS**

$$j_q^\mu = \sum_{i=1}^{N_s} q_i \int \mathrm{d}K \; k_i^{\langle\mu\rangle} f_{1k}^i \stackrel{!}{=} \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T} \right)$$

Navier-Stokes limit

In order to do so, we need to solve:

$$k_i^\mu \partial_\mu f_{0k}^i = - \sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

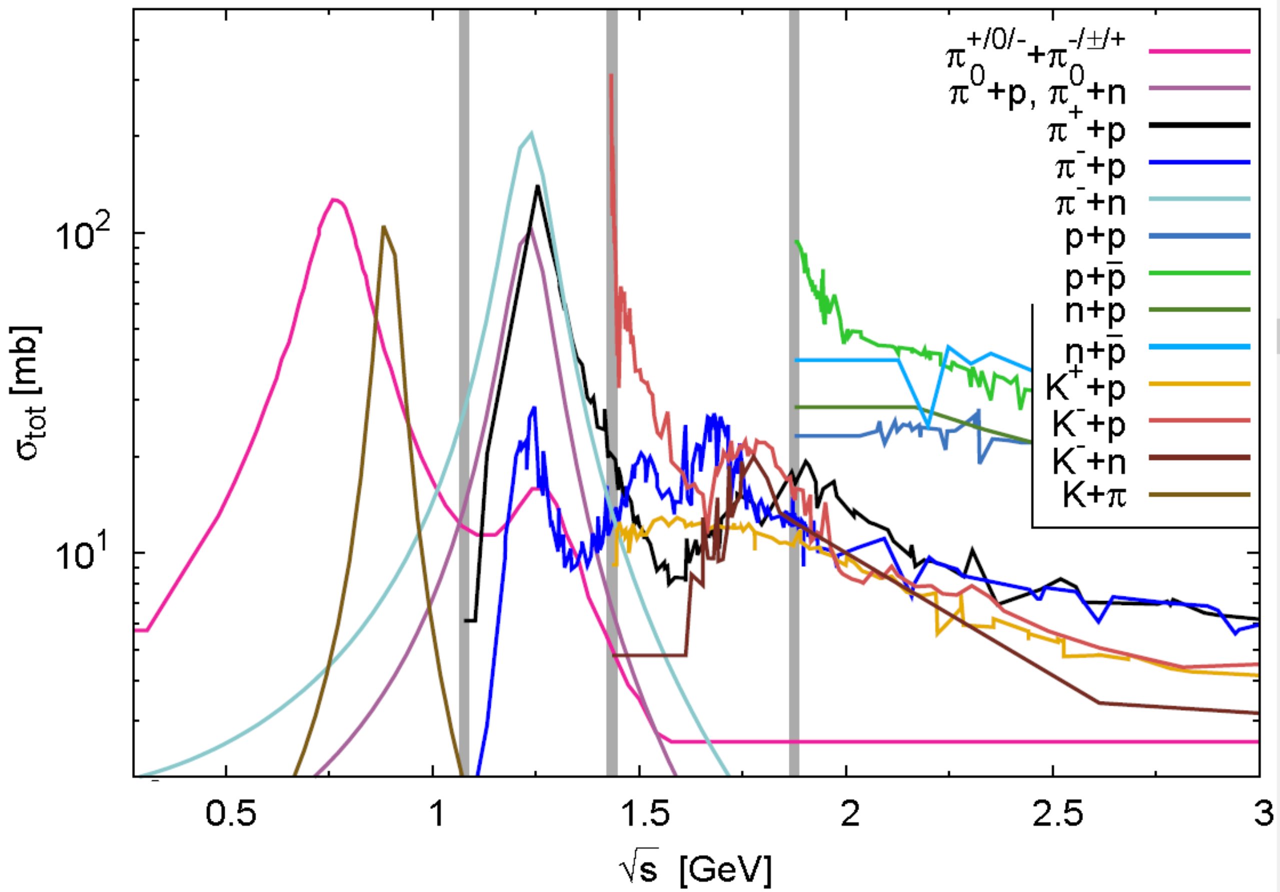
More details in: *Greif et al., Phys. Rev. Lett.* ***120****, 242301 (2018)*

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Hadronic resonance gas...

- Use 19 different massive species
- Elastic, isotropic cross sections

$$\pi^{0,\pm}, K^{\pm,0,\bar{0}}, p, \bar{p}, n, \bar{n}, \Sigma^{0,\pm}, \bar{\Sigma}^{0,\pm}, \Lambda, \bar{\Lambda}$$



- Use PDG data
- Other cross sections: GiBUU, UrQMD or constant

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Simplified (conformal) QGP model...

- Use 7 massless species $u, \bar{u}, d, \bar{d}, s, \bar{s}, g$
- Simplified approach: Fix shear viscosity to express isotropic cross section in terms of temperature

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma_{tot} \approx \frac{0.716}{T^2}$$

Bouras et al., Phys. Rev. Lett. ***103****, 032301 (2009)*

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Simplified (conformal) QGP model...

- Use 7 massless species $u, \bar{u}, d, \bar{d}, s, \bar{s}, g$
- Simplified approach: Fix shear viscosity to express isotropic cross section in terms of temperature

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma_{tot} \approx \frac{0.716}{T^2}$$

Bouras et al., Phys. Rev. Lett. ***103****, 032301 (2009)*

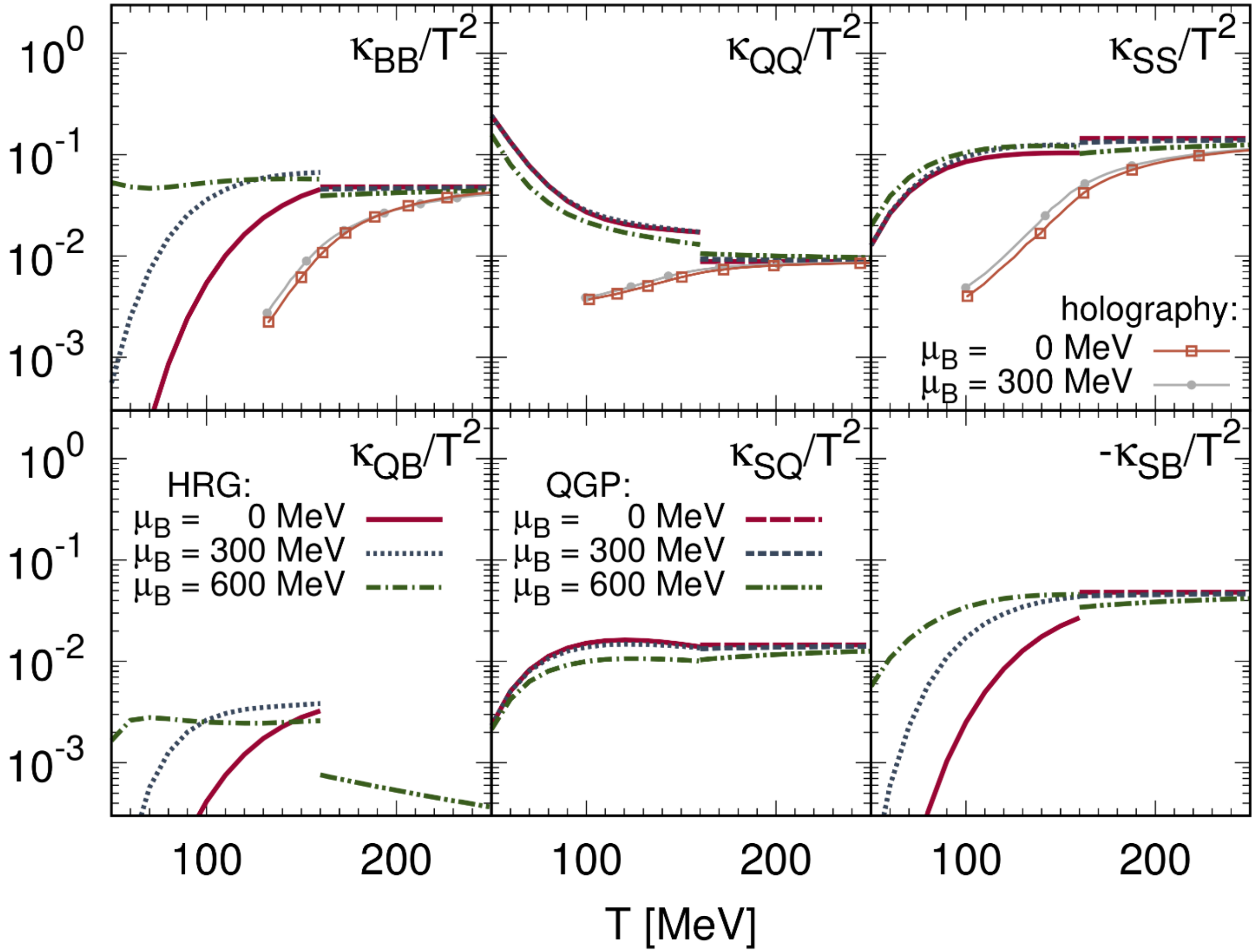
Two distinct systems:
T < 160 MeV: HRG
T >= 160 MeV: simple QGP model

→ phase transition area is **NOT** covered by our calculations

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix



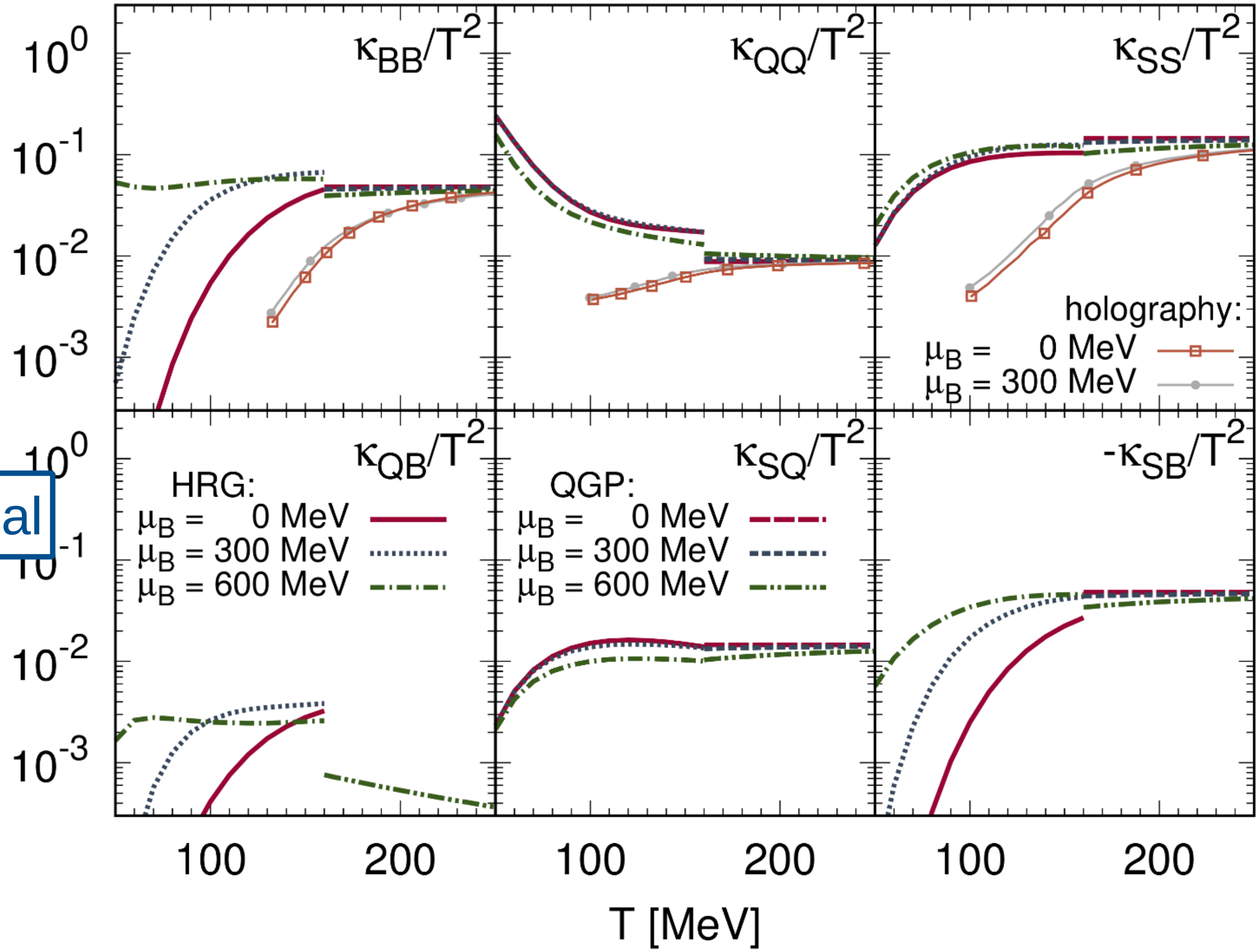
$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

diagonal

Off-diagonal



$n_S = 0$

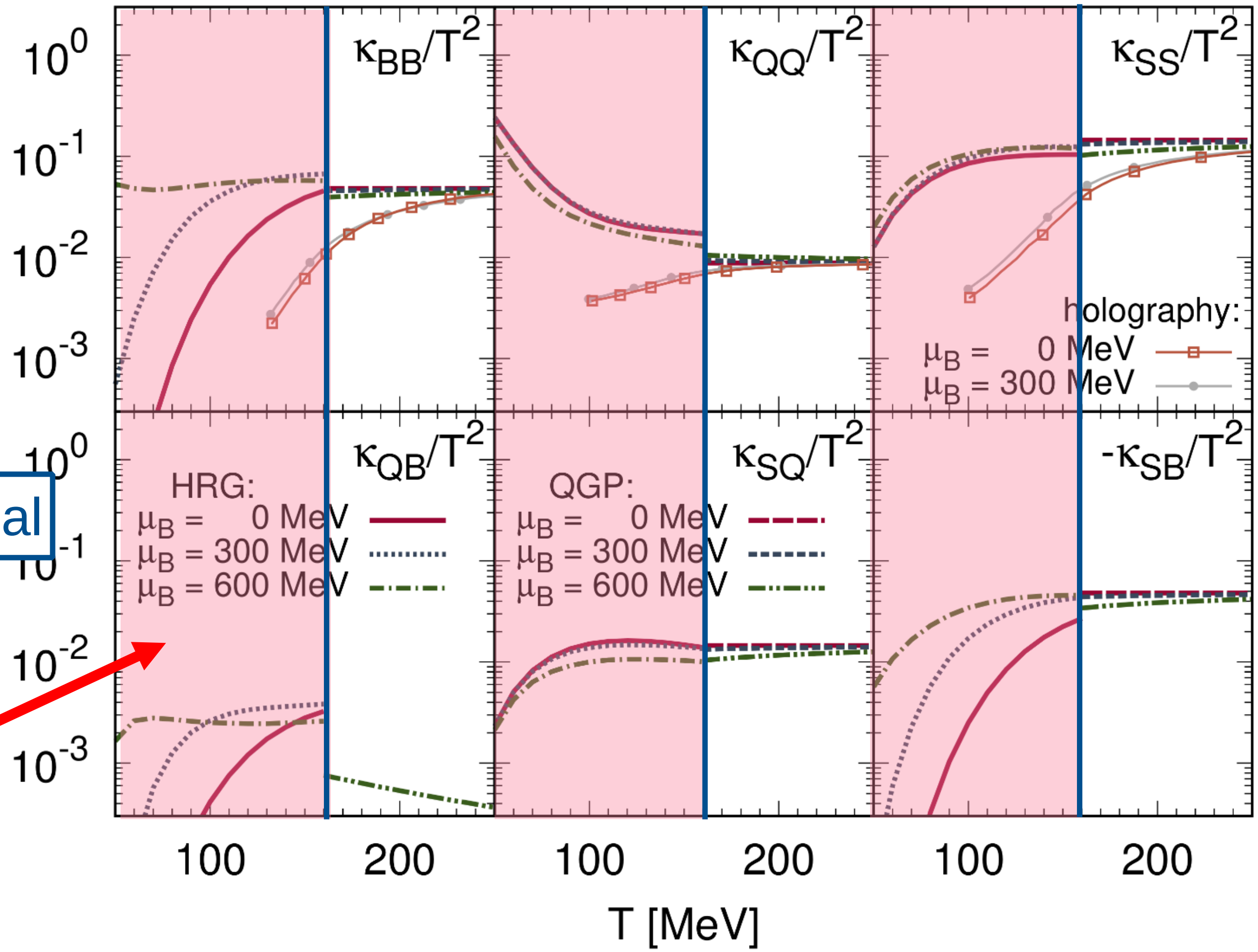
$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

diagonal

Off-diagonal

HRG



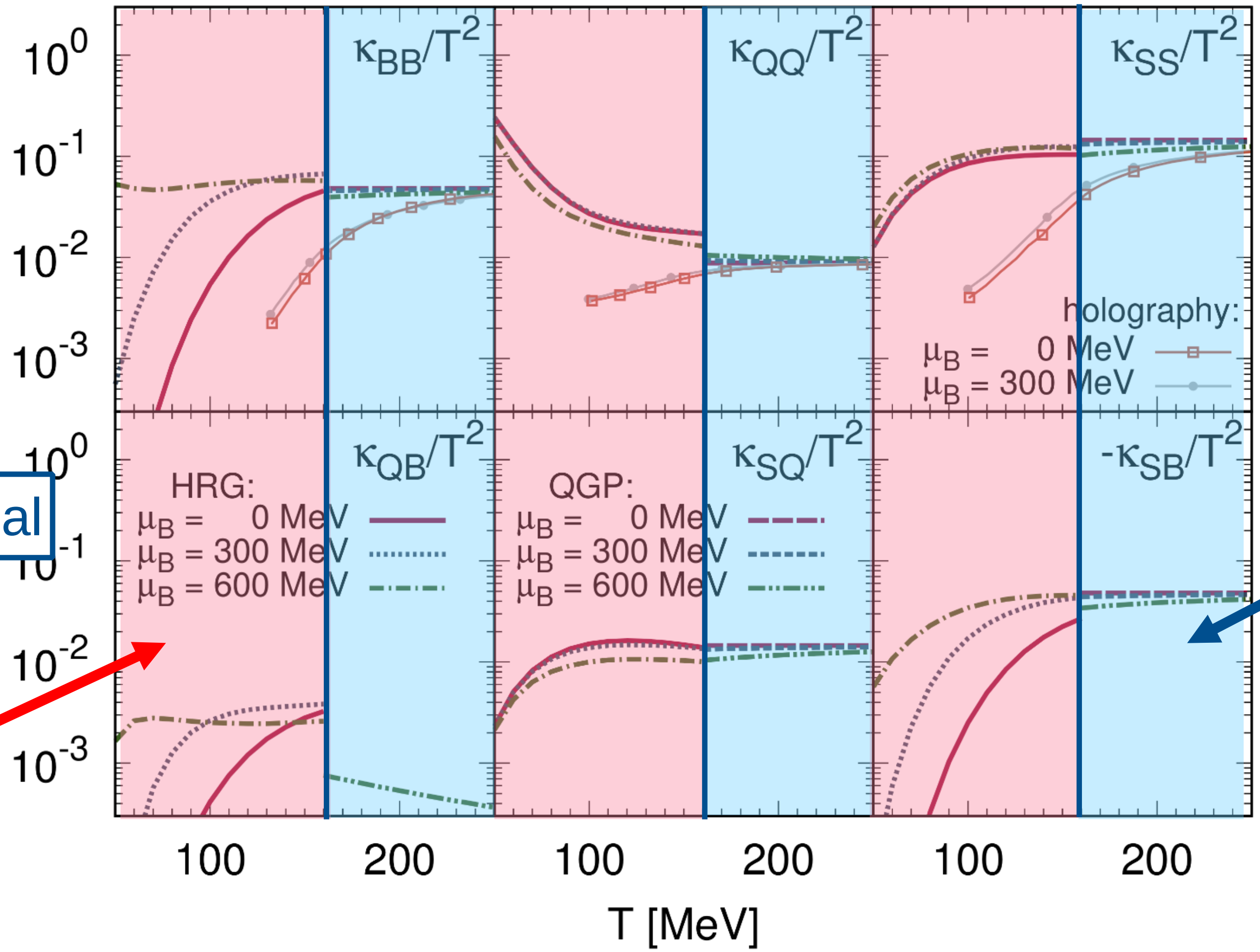
$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

diagonal

Off-diagonal



HRG

„simple“ QGP

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

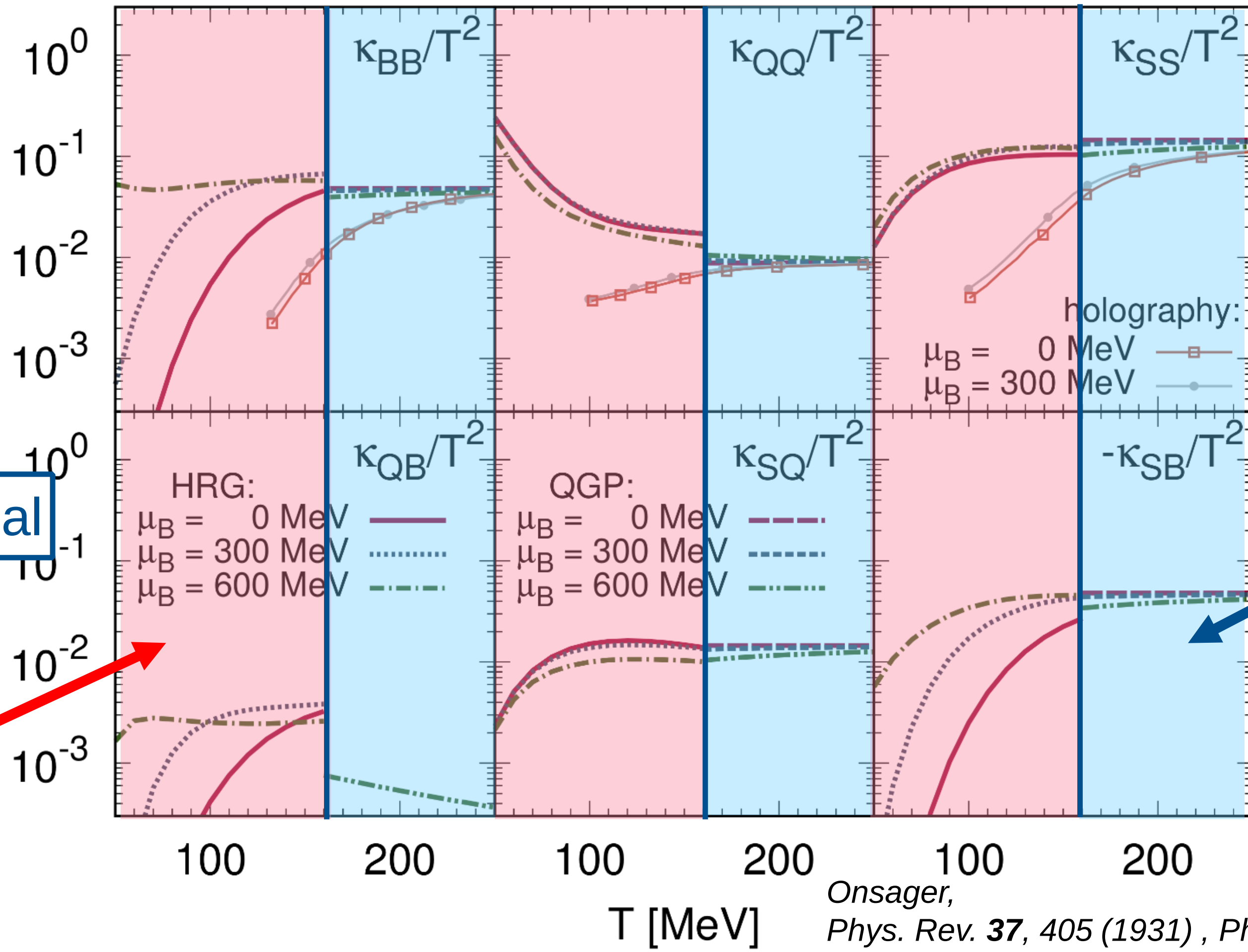
The diffusion matrix

diagonal

Off-diagonal

HRG

„simple“ QGP



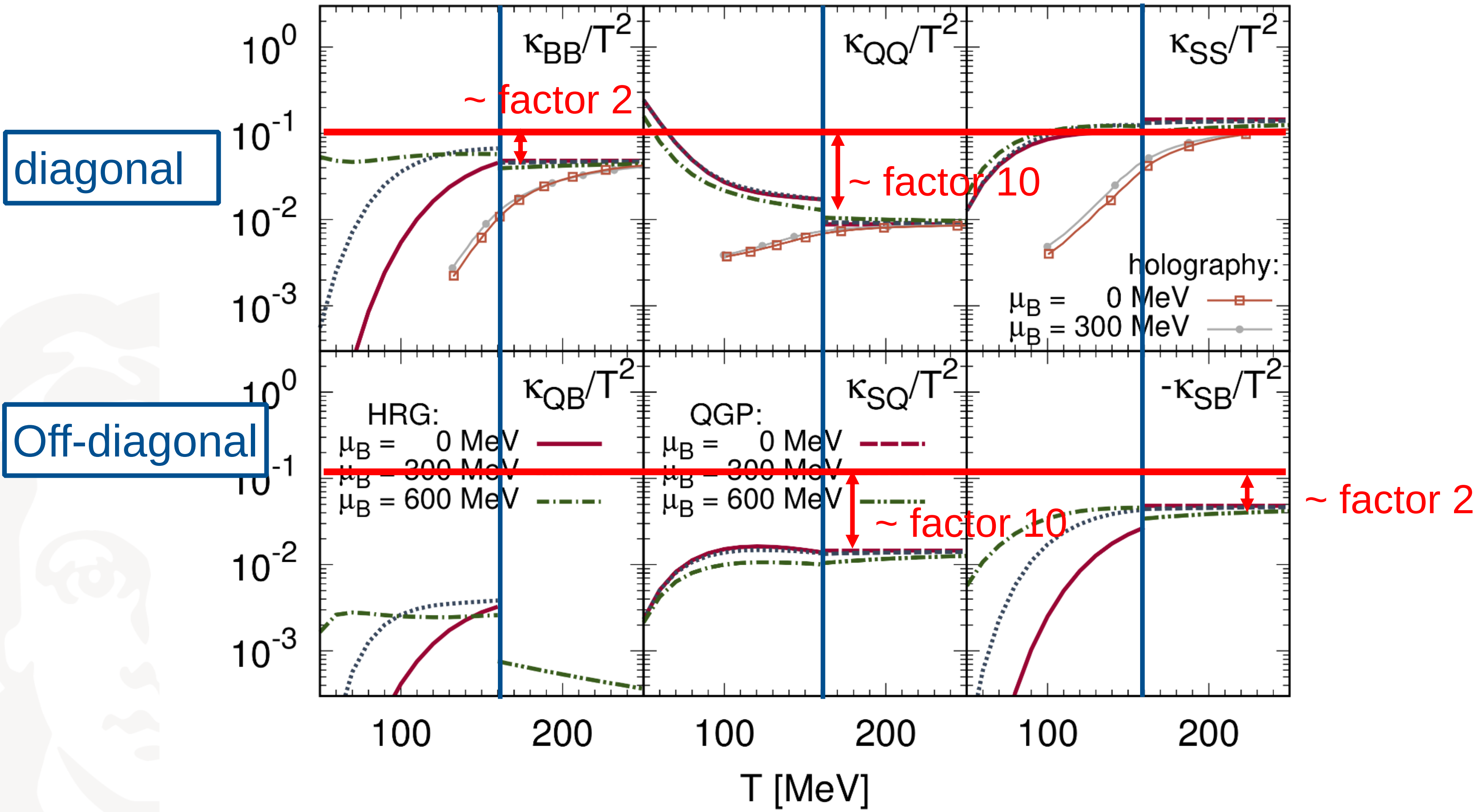
Onsager,
Phys. Rev. ***37****, 405 (1931) , Phys. Rev.* ***38****, 2265 (1931)*

Diffusion matrix is symmetric! → Onsager Theorem holds

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix



Off-diagonal contributions have similar magnitude as diagonal ones

→ effects of multi-carrying charges should not be neglected

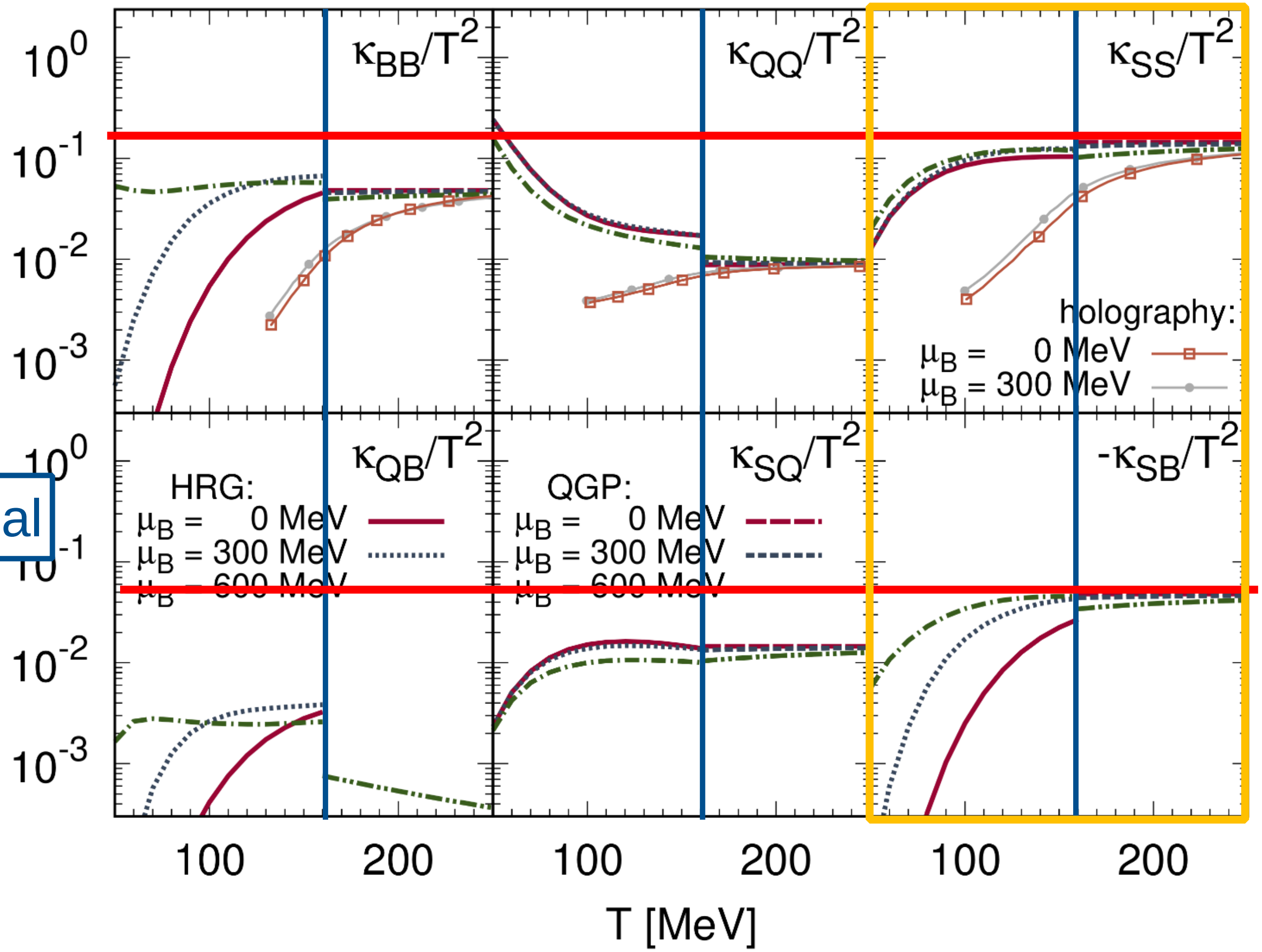
$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

diagonal

Off-diagonal



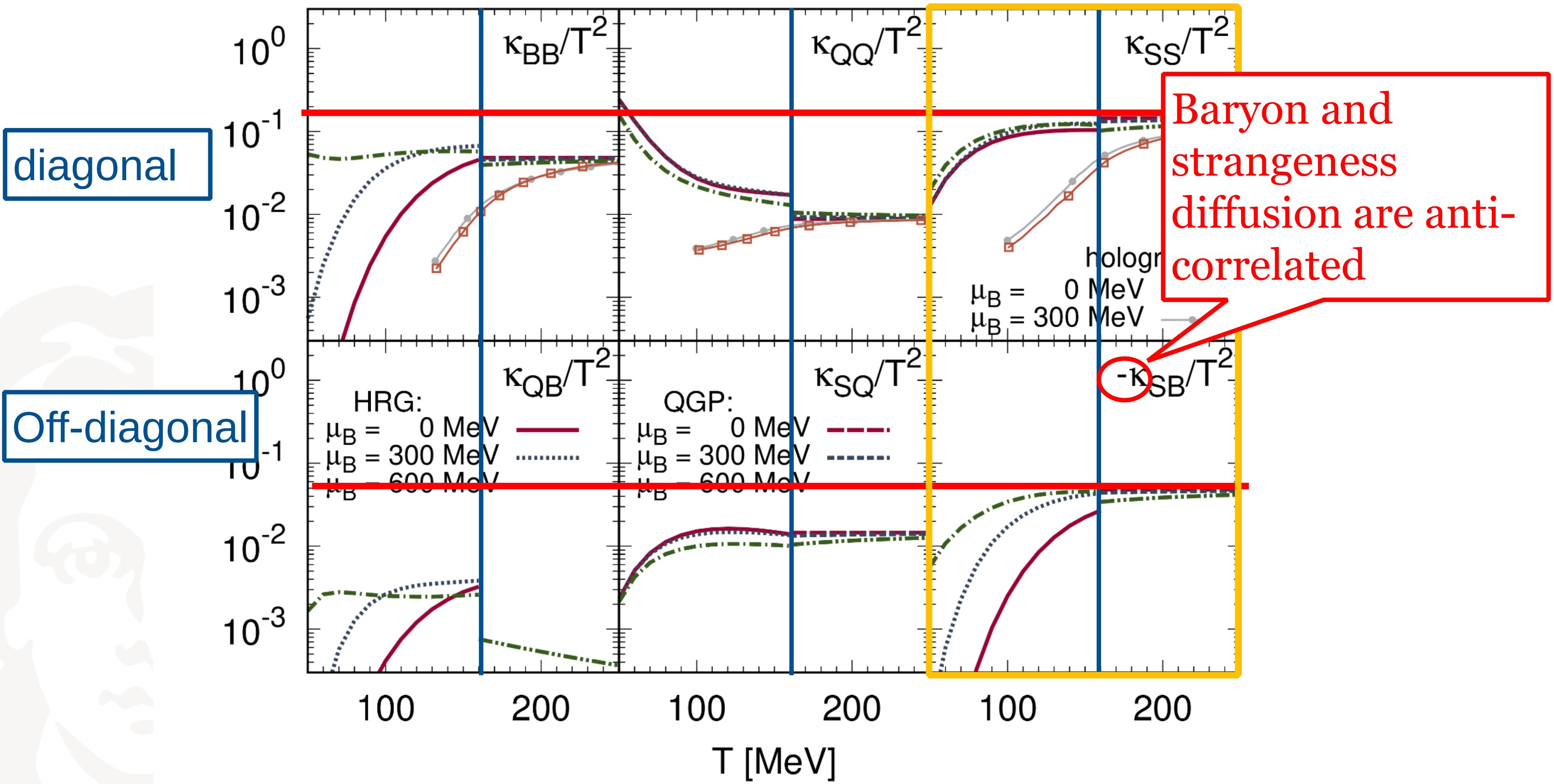
Coefficients in strangeness sector most dominant

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The diffusion matrix



Coefficients in strangeness sector most dominant

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

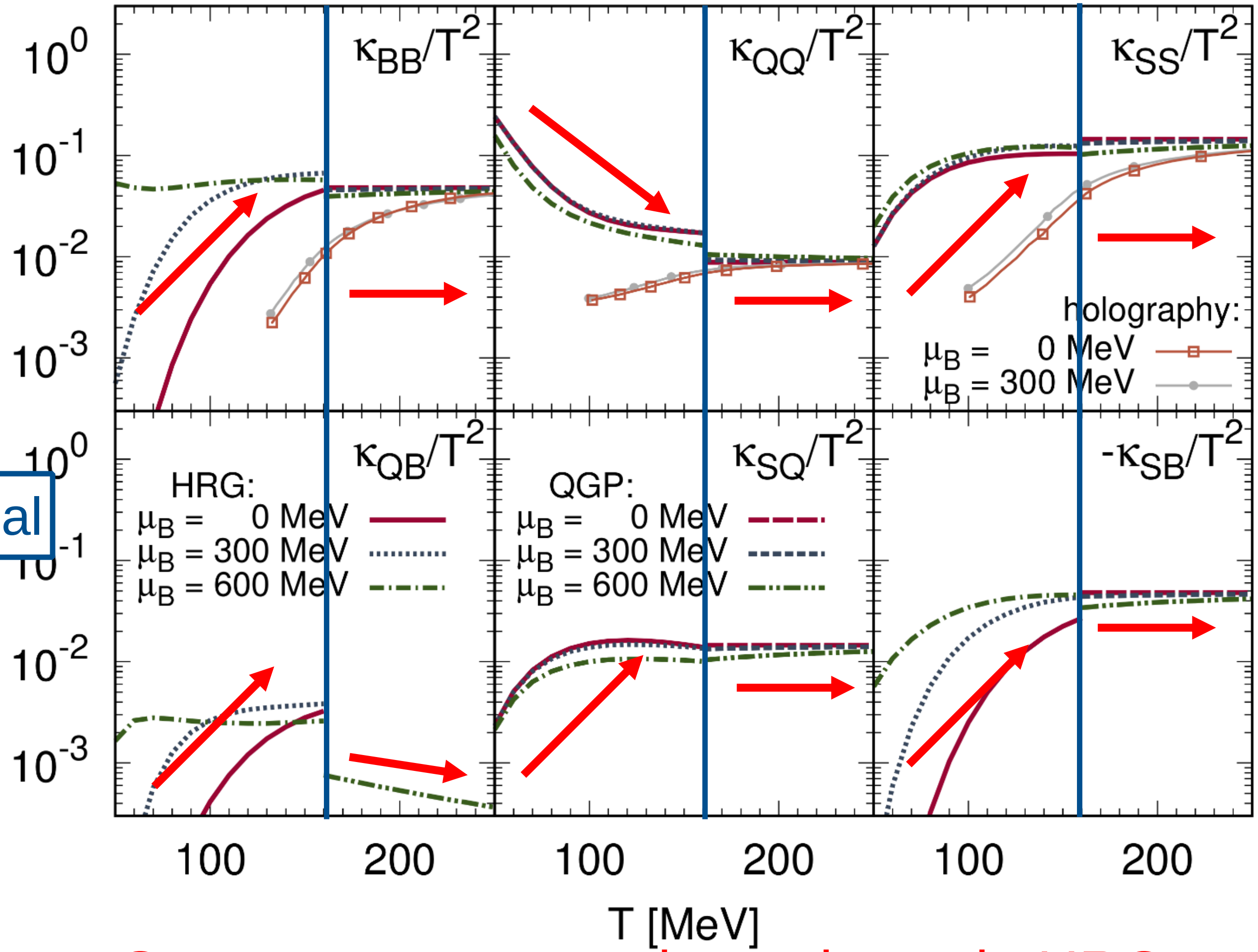
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The diffusion matrix

Holography:
Rougement et al.,
*Phys. Rev. D **96**, 014032 (2017)*

diagonal

Off-diagonal



Strong temperature dependence in HRG
Nearly constant in conformal QGP

$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

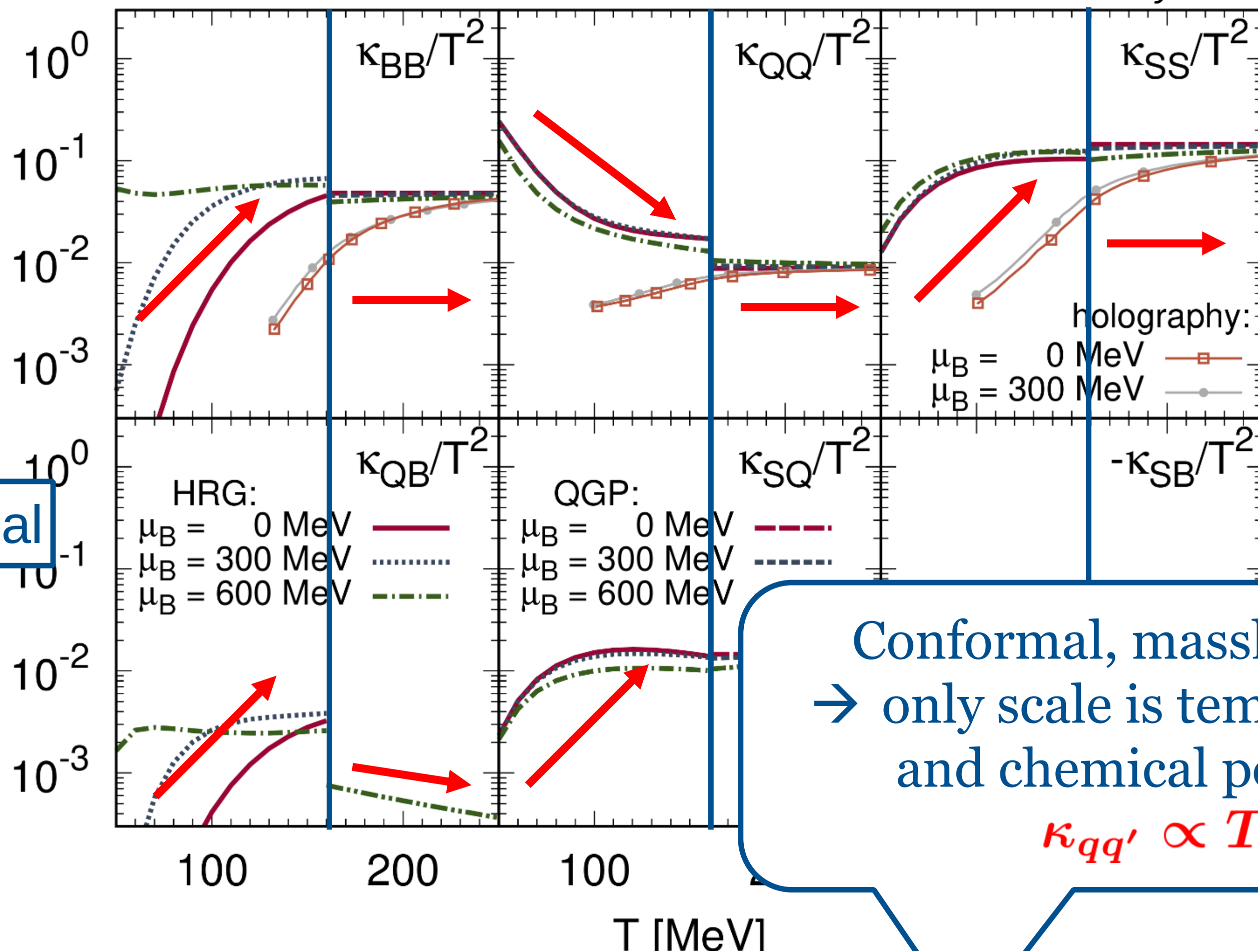
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The diffusion matrix

Holography:
Rougement et al.,
Phys. Rev. D ***96****, 014032 (2017)*

diagonal

Off-diagonal



Conformal, massless gas
→ only scale is temperature and chemical potentials
$\kappa_{qq'} \propto T^2$

Strong temperature dependence in HRG
Nearly constant in conformal QGP

$n_S = 0$

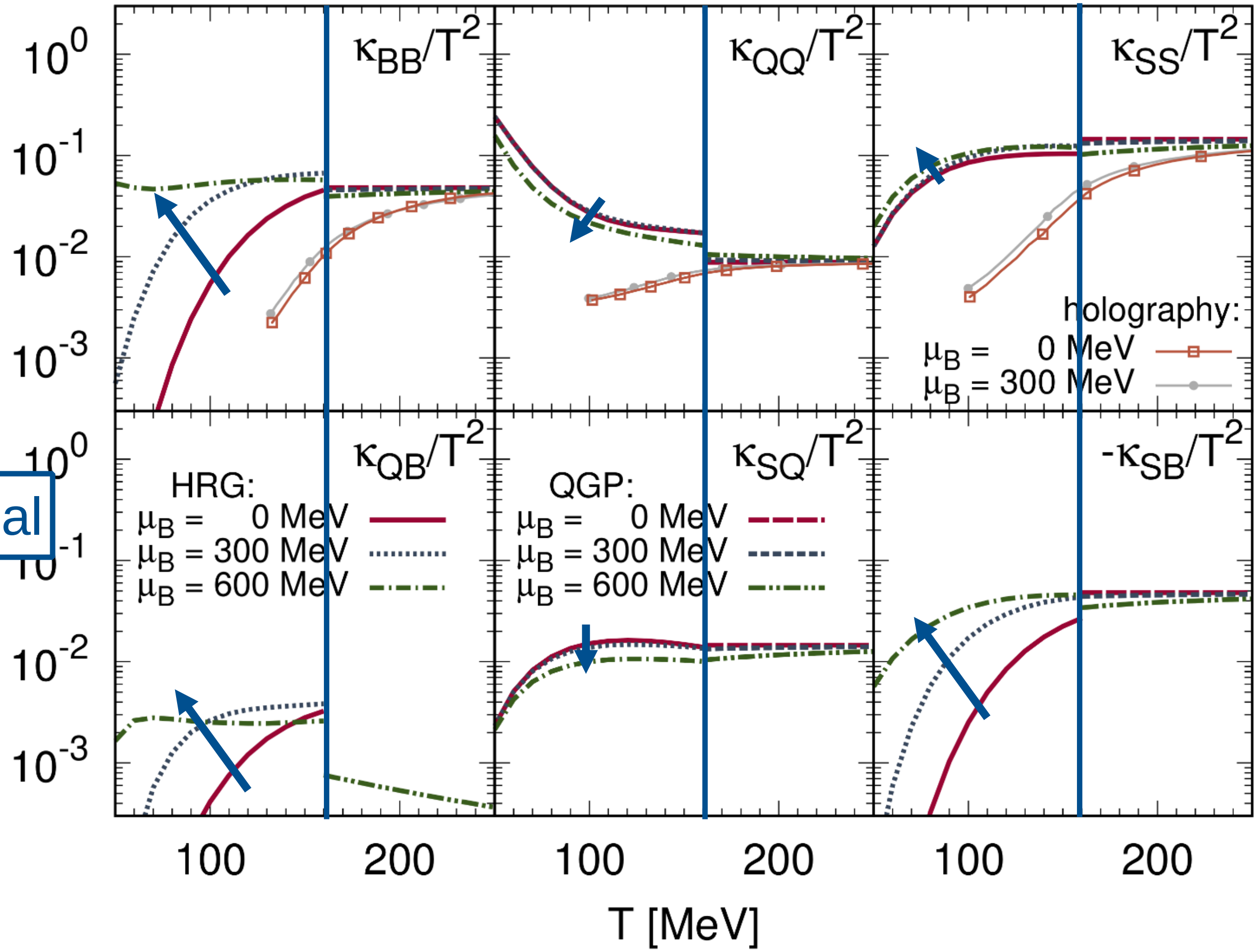
$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The diffusion matrix

diagonal

Off-diagonal



Dependence of coefficients in baryon sector on baryo-chemical potential in HRG

$n_S = 0$

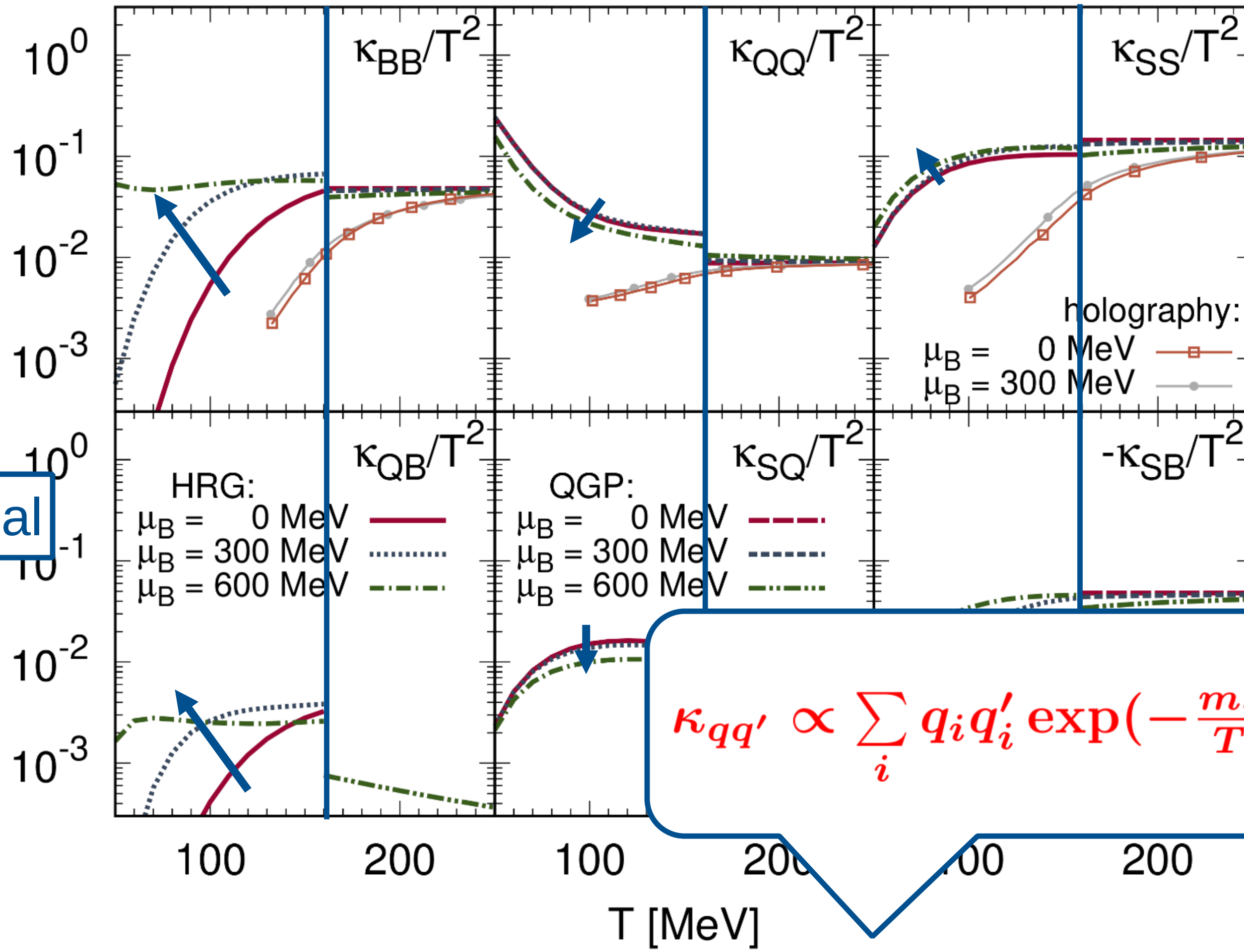
$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The diffusion matrix

diagonal

Off-diagonal



$$\kappa_{qq'} \propto \sum_i q_i q_i' \exp\left(-\frac{m_i}{T}\right)(\ldots)$$

Dependence of coefficients in baryon sector on baryo-chemical potential in HRG

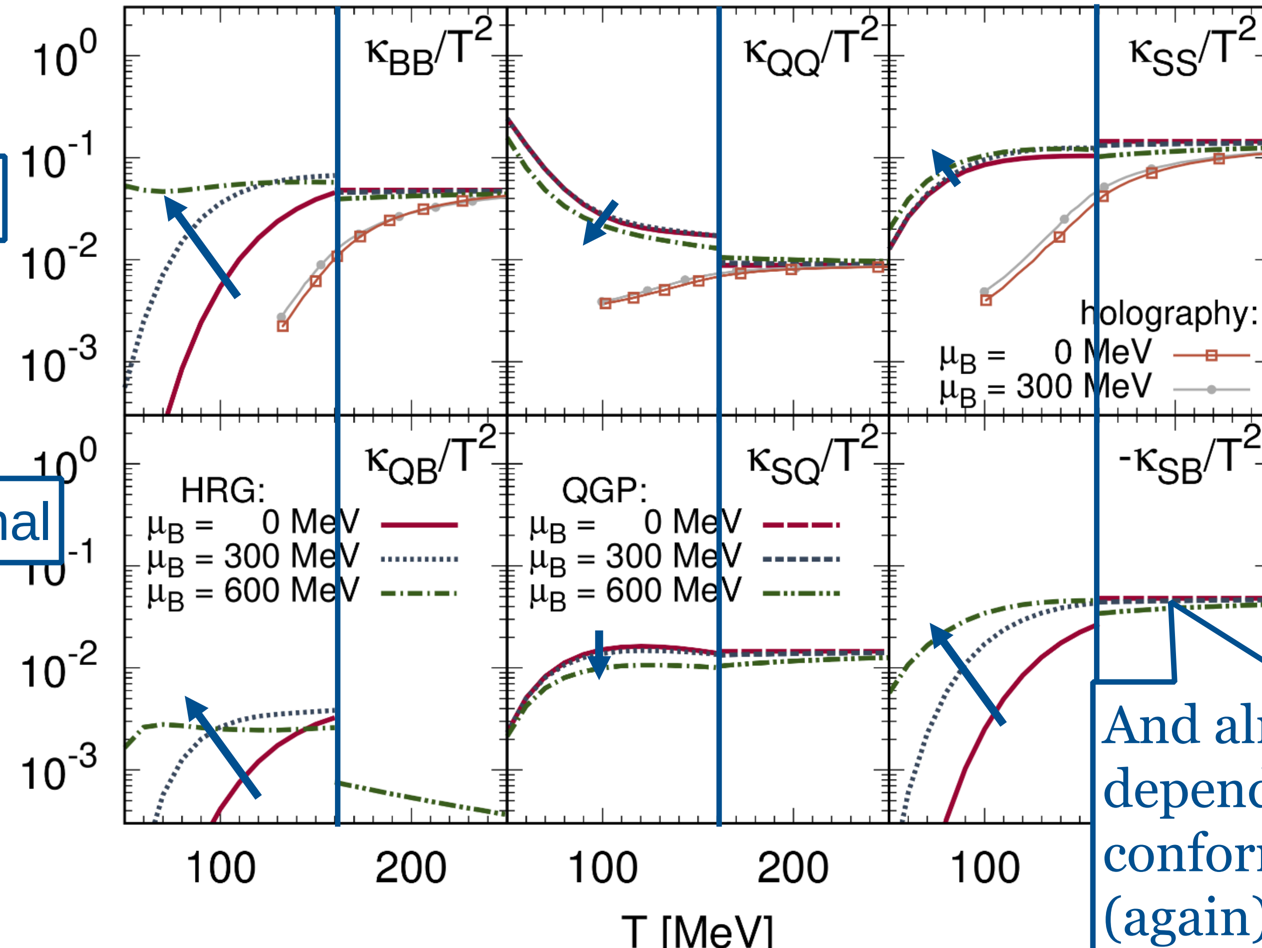
$n_S = 0$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

The diffusion matrix

diagonal

Off-diagonal



And almost no dependence in the conformal QGP (again)

Dependence of coefficients in baryon sector on baryo-chemical potential in HRG

Conclusion

- First calculation of complete diffusion matrix of baryon, electric and strangeness charges in Navier-Stokes limit with first order Chapman-Enskog expansion
- Classical hadron gas with realistic isotropic cross sections and simple conformal QGP model were used

Conclusion

- First calculation of complete diffusion matrix of baryon, electric and strangeness charges in Navier-Stokes limit with first order Chapman-Enskog expansion
- Classical hadron gas with realistic isotropic cross sections and simple conformal QGP model were used

- HRG: dependence of coefficients on temperature and baryo-chemical potential
- Strong coupling of all gradients to (almost) all currents → large off-diagonal coefficients
- Suggestion: Off-diagonal terms should not be neglected!
- Can be used in (hydro) models

Outlook

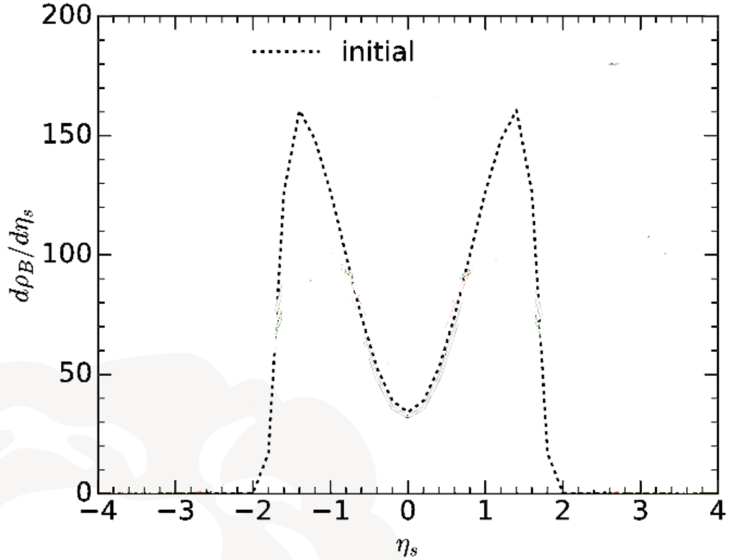
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Investigate effects of (off-diagonal) diffusion coefficients in viscous hydro simulations:
  - Measurable effects on rapidity distribution (on strangeness?)
  - Initial state correlations (flow harmonics?)
  - Investigate dependence of calculation in terms of new species with higher masses and more carried charge
- Parametrize coefficients
- Compare to other models: SMASH? BAMPS? lQCD?

BACKUP

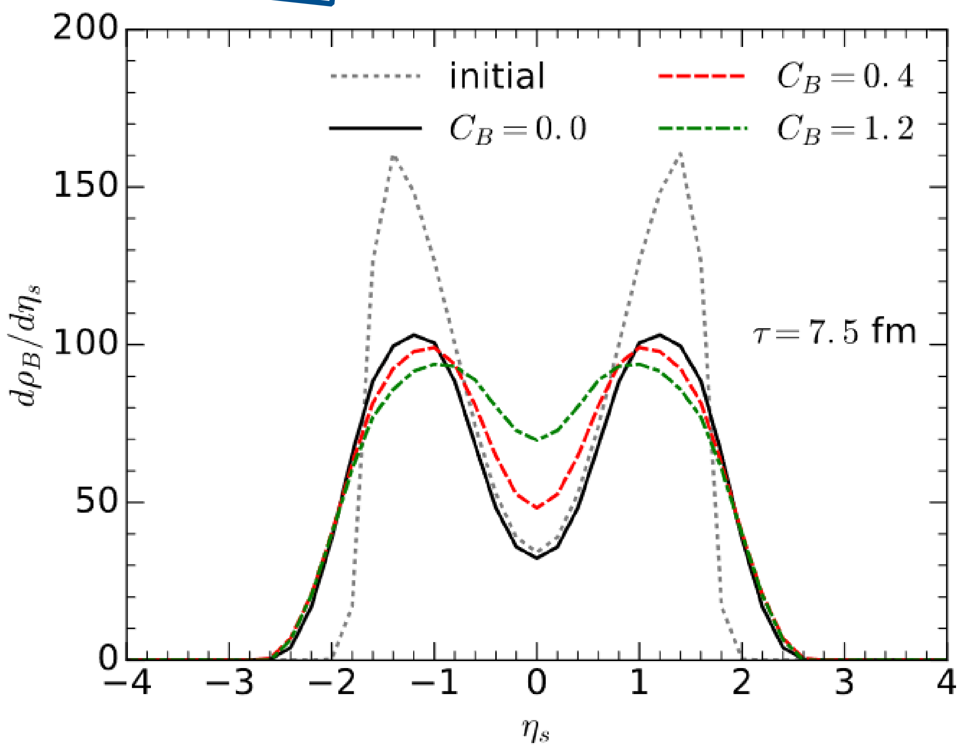
The Evolution in (3+1)-Viscous Hydro

Chun Shen et. al. Nucl. Phys. A 967 (2017) 796-799



0-5% Au-Au collision at 19.7 GeV

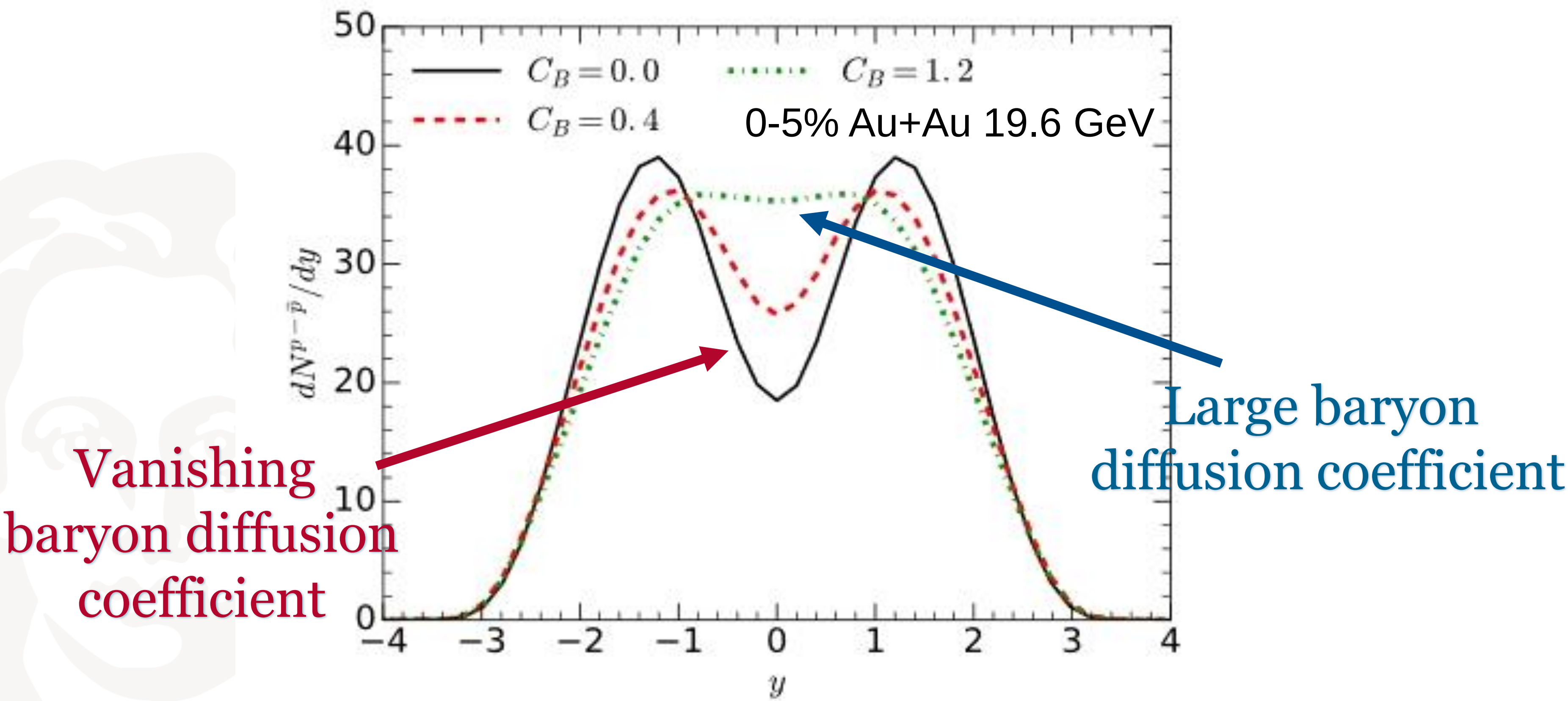
Hydrodynamical evolution after 7.5 fm



Hadronization on next slide

Why could diffusion be important?

- During low-energy HIC (e.g. RHIC BES): diffusion could have great impact on dynamic evolution



Chun Shen et al., Nucl. Phys. A 967 (2017) 796-799

The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

$$k_i^\mu \partial_\mu f_k^i = -\sum_{j=1}^{N_s} C_{ij}[f_k^i]$$

Chapman-Enskog expansion

$$\epsilon k_i^\mu \partial_\mu \left(f_{0k}^i + \epsilon f_{0k}^i\right) \approx \epsilon k_i^\mu \partial_\mu f_{0k}^i = -\epsilon \sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

With linearized collision term:

$$\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i] = \sum_{j=1}^{N_s} \gamma_{ij} \int \mathrm{d}K_j' \mathrm{d}P_i \mathrm{d}P_j' \boxed{W_{kk' \to pp'}^{ij}} f_{0k}^i f_{0k'}^j \left(\frac{f_{1k}^i}{f_{0k}^i} + \frac{f_{1k'}^j}{f_{0k'}^j} - \frac{f_{1p}^i}{f_{0p}^i} - \frac{f_{1p'}^i}{f_{0p'}^i} \right)$$

Transition rate: contains (isotropic) cross sections
= information of microscopic interactions

The Chapman-Enskog Expansion

Evaluating derivatives leads to source equation for deviation f_{1k}^{i}

$$k_i^\mu \partial_\mu f_{0k}^i = - \sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

$$\sum_{q \in \{B,S,Q\}} f_{0k}^i k_i^\mu \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left(\frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

Gradient in thermal potential

Sum over all conserved charges
→ coupling of diffusion currents

L.H.S. of eq. ~ force term due to gradients in particle density → Navier Stokes currents

The Chapman-Enskog Expansion

Diffusion currents in kinetic theory:

We want to calculate **THIS**

$$j_q^\mu = \sum_{i=1}^{N_s} q_i \int \mathrm{d}K \; k_i^{\langle\mu\rangle} f_{1k}^i \overset{!}{=} \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

Navier-Stokes limit

In order to do so, we need to solve:

$$\sum_{q\in\{B,S,Q\}} f_{0k}^i k_i^\mu \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i\right) \nabla_\mu \left(\frac{\mu_q}{T}\right) = -\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i]$$

The Chapman-Enskog Expansion

$$\sum_{q\in\{B,S,Q\}} f_{0k}^{i} k_i^{\mu} \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left(\frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij}[f_{1k}^{i}]$$

Since collision term is linear in f_{1k}^{i} the solutions have the general form:

scalar function in energy

$$f_{1k}^{i} = \sum_q a_q^i k_i^{\mu} \nabla_\mu \left(\frac{\mu_q}{T} \right)$$

Expand coefficients in power series in energy:

$$a_q^i = \sum_{m=0}^{\infty} a_{q,m}^i E_{ik}^m$$

The Chapman-Enskog Expansion

$$\sum_{q\in\{B,S,Q\}} f^i_{0k} k^\mu_i \left(\frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left(\frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij} [f^i_{1k}]$$

Truncate series at finite integer M and calculate n-th moment of source equation → set of linear equations for expansion Coefficients

Solutions of matrix equation → gives us f^i_{1k}

$$\sum_{m=0}^{M} \sum_{j=1}^{N_s} \left(A^i_{nm} \delta^{ij} + C^{ij}_{nm} \right) a^j_{q,m} = b^i_{q,n}$$

moments of collision term → complicated integrals with information about microscopic interactions

Source term for diffusion

The Chapman-Enskog Expansion

$$j_q^\mu = \sum_{i=1}^{N_s} q_i \int \mathrm{d}K \; k_i^{\langle\mu\rangle} f_{1k}^i \stackrel{!}{=} \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

By comparing both sides we find:

$$\kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_s} q_i \sum_{m=0}^{M} a_{q',m}^i \int \mathrm{d}K_i E_{ik}^m (m^2 - E_{ik}^2) f_{0k}^i$$

In our most detailed calculation: M = 1 and N_s = 19

The Relaxation Time Approximation

Calculated for $p\ n\ \bar{p}\ \bar{n}\ K\ \pi$ gas (11 hadron species)

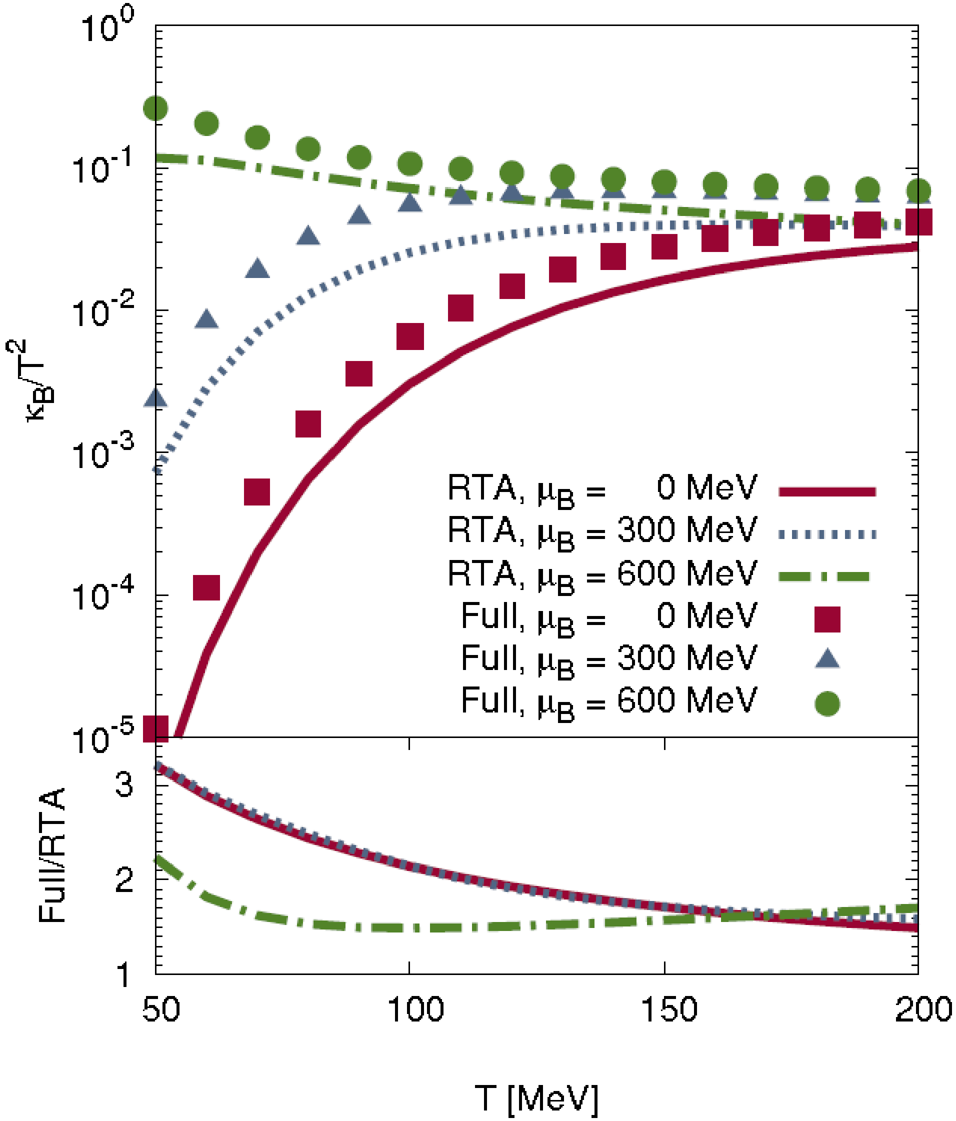
$$\sum_{j=1}^{N_s} C_{ij}[f_{1k}^i] = -\frac{E_{ik}}{\tau} f_{1k}^i$$

Relaxation time:

Total baryon density

$$\tau^{-1} = \frac{2}{3} n_{B,\mathrm{tot}}\, \sigma_0$$

Constant cross section

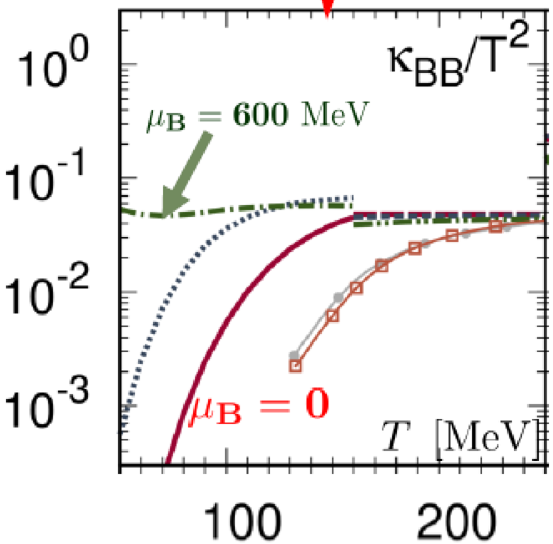


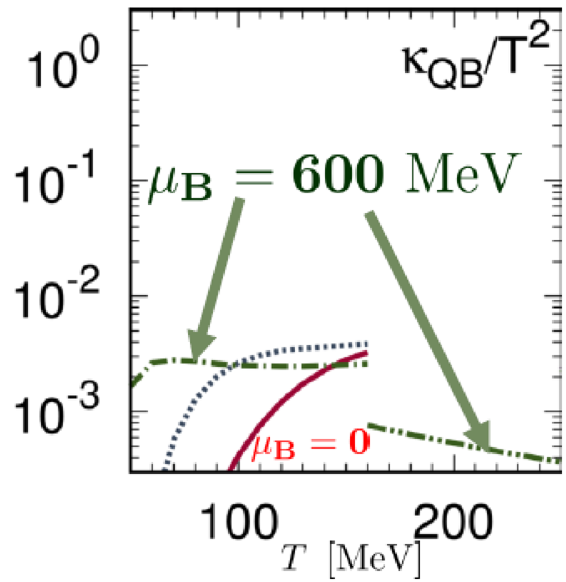
Baryon current

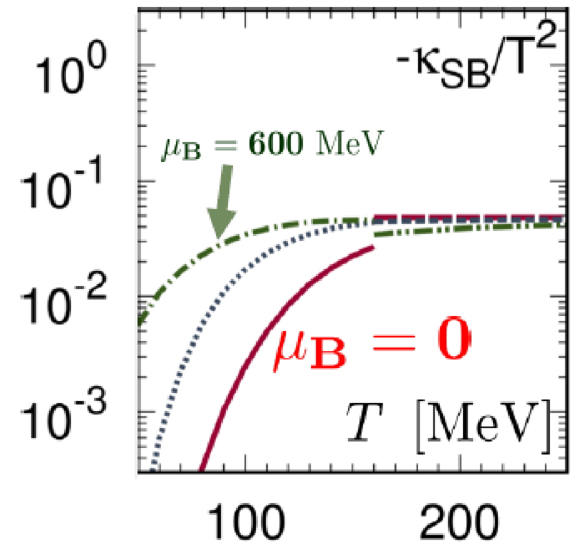
$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

$$j_B^\mu = \kappa_{BB} \nabla^\mu \alpha_B + \kappa_{BQ} \nabla^\mu \alpha_Q + \kappa_{BS} \nabla^\mu \alpha_S$$







- Largest contribution
- Nearly constant at $\mu_B = 600\ MeV$
- So far only used coefficient

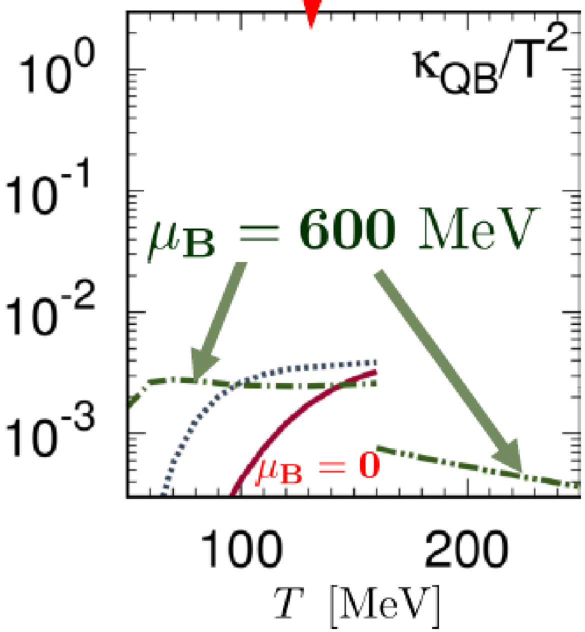
- Much smaller than others
- QGP-part vanishes at $\mu_B = 0$
- Strong μ_B dependence

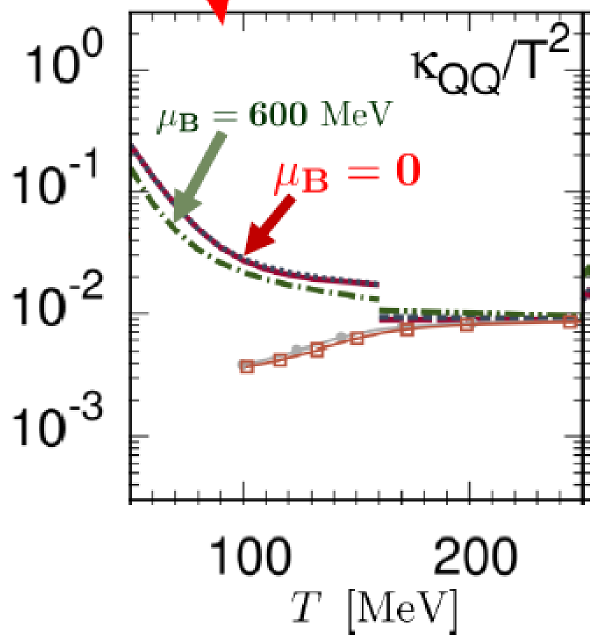
- Negative contribution!
- Similar strength as κ_{BB}
- Could drastically reduce baryon current

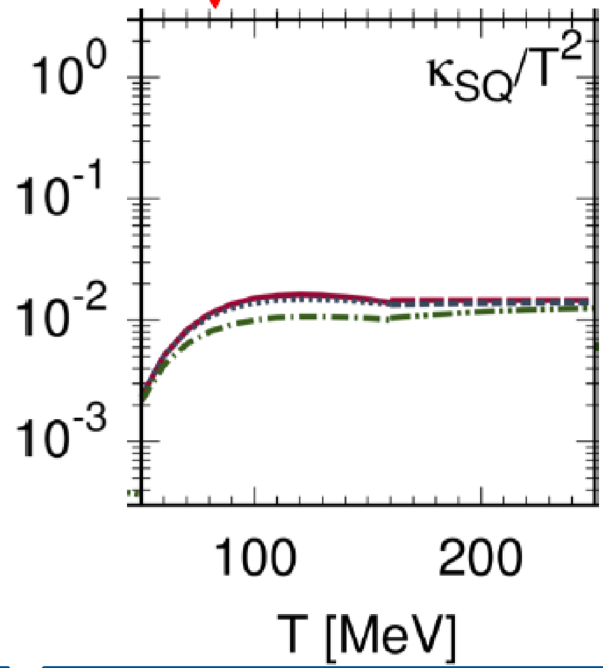
Electric current

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

$$j_Q^\mu = \kappa_{QB} \nabla^\mu \alpha_B + \kappa_{QQ} \nabla^\mu \alpha_Q + \kappa_{QS} \nabla^\mu \alpha_S$$







- Smaller than others
- QGP-part vanishes at $\mu_B = 0$
- Strong μ_B dependence

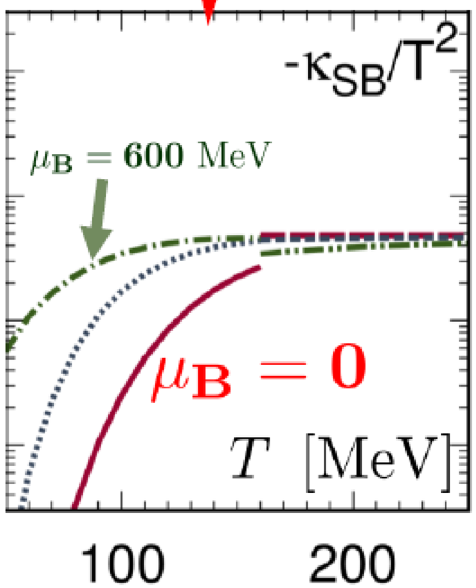
- $\mu_B = 0$ same as electric conductivity
- Only decreasing behavior in T

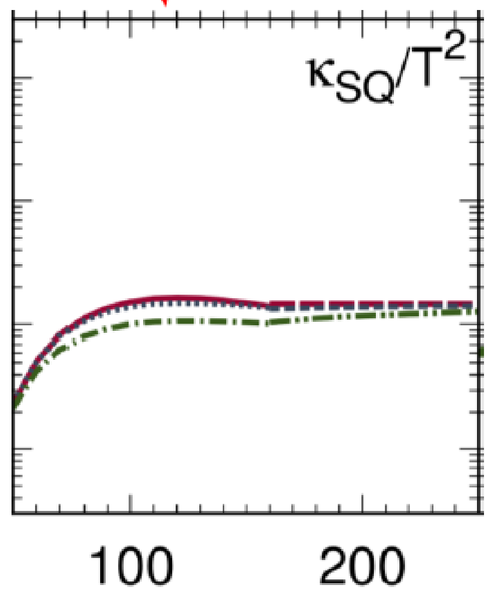
- QGP: strongest contribution

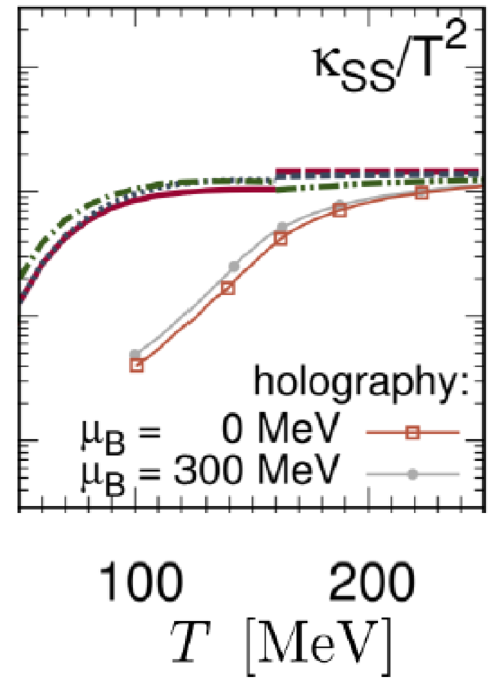
Strangeness current

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

$$j_S^\mu = \kappa_{SB} \nabla^\mu \alpha_B + \kappa_{SQ} \nabla^\mu \alpha_Q + \kappa_{SS} \nabla^\mu \alpha_S$$







- Negative contribution
- Could also drastically reduce strange currents

- 1 Magnitude smaller than κ_{SS}
- Charged Kaons contribute to electric currents (see κ_{QQ})

- By far most important contribution