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Locating the QCD Critical Point using Holographic Black Holes

Israel Portillo

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Collaborators:

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Romulo Rougemont (UFRN)

Phys.Rev.D**96**(2017)9,096026.

Exploring The QCD Phase Diagram

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Heavy Ion Collisions

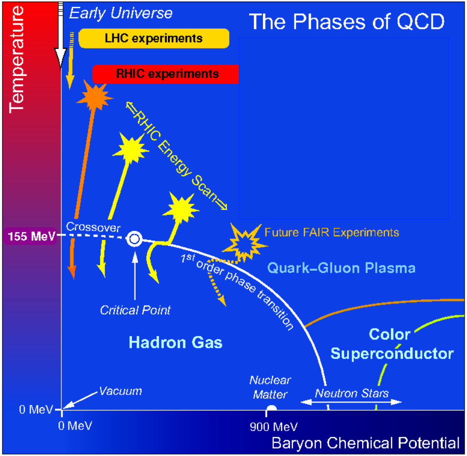
Explore the phase diagram by systematically changing $\sqrt{s}$ of colliding ion beam

Lattice QCD

Perform calculations at $\mu_B = 0$, and extrapolate via Taylor expansion to finite μ_B

Black Hole Engineering

Through the holographic correspondence, properties of the sQGP can be determined



Black Hole Engineering

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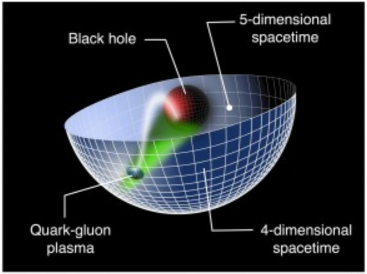
Holography (gauge/string duality at Strong Coupling)

Quantum Field Theory in 4-dimensions $\Longleftrightarrow$ **Classical Gravity in 5-dimensions**

- Vanishing string coupling in GR
  $\rightarrow$ Coupling >> 1 in QFT
- Black Hole solution
  $\rightarrow$ (T, μ_B) in QFT
- Holography
  $\rightarrow$ Near Perfect fluidity



J M Maldacena 1999 Int. J. Theor. Phys. (**38**) 1113

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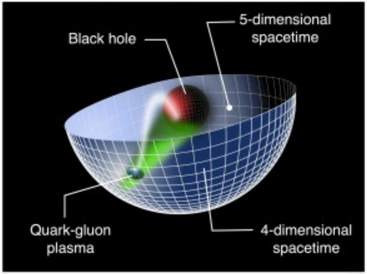

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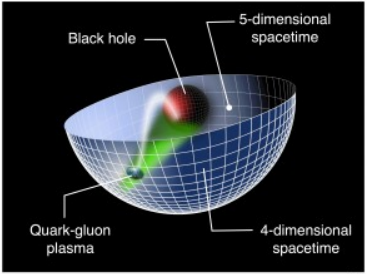

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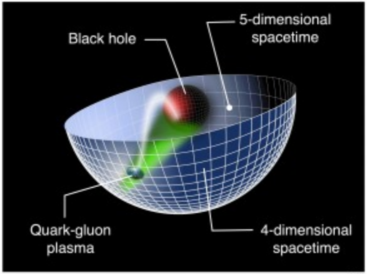

J M Maldacena 1999 Int. J. Theor. Phys. (**38**) 1113

Holographic Model

Non-conformal holographic gravity dual in 5 dimensions $\Longrightarrow$ **Black Hole Solution**

$$\mathcal{S} = \frac{1}{16\pi G_5}\int \mathrm{d}x^5\sqrt{-g}\,[\mathcal{R} - \frac{1}{2}(\partial_M\phi)^2 - \underbrace{V(\phi)}_{\text{nonconformal}} - \frac{1}{4}\underbrace{f(\phi)F_{MN}^2}_{\mu_B\neq 0}]$$

- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and $\mu_B \rightarrow$ Predictions

Holographic Model

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- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and $\mu_B \rightarrow$ Predictions

Holographic Constructions with Critical Point

- O DeWolfe, et al. Phys. Rev. D **83**, (2011)
- R. Critelli, I.P. et al. Phys .Rev. D **96**, (2017)
- J. Knauteab, et al. Phy. Let. B **778**, (2018)

Holographic Model

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PHYSICAL REVIEW D **83,** 086005 (2011)

A holographic critical point

Oliver DeWolfe,[1] Steven S. Gubser,[2] and Christopher Rosen[1]
[1]*Department of Physics, 390 UCB, University of Colorado, Boulder, Colorado 80309, USA*
[2]*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA*
(Received 28 December 2010; published 8 April 2011)

$$V(\phi) = \frac{-12\cosh\gamma\phi + b\phi^2}{L^2} \quad \text{with} \quad \gamma = 0.606 \quad \text{and} \quad b = 2.057,$$

$$f(\phi) = \frac{\operatorname{sech}[\frac{6}{5}(\phi - 2)]}{\operatorname{sech}\frac{12}{5}},$$

$$T_c = 143\ \text{MeV} \quad \mu_c = 783\ \text{MeV}.$$

- Simple $V(\phi)$ and $f(\phi)$
- Fitting outdated Lattice Calculations
- **Main results:** location of CP, critical line and calculated critical exponents

Black Hole Engineering

R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Free Parameters of the Holographic Model

$$G_5 = 0.46 \qquad \Lambda = 1058.83\ \mathrm{MeV}$$

$$V(\phi) = -12\cosh(0.63\phi) + 0.65\phi^2 - 0.05\phi^4 + 0.003\phi^6$$

$$f(\phi) = \frac{\mathrm{sech}(c_1\phi + c_2\phi^2)}{1+c_3} + \frac{c_3}{1+c_3}\mathrm{sech}(c_4\phi)$$

where $c_1 = -0.27 \quad c_2 = 0.4 \quad c_3 = 1.7 \quad c_4 = 100$

Holographic Model

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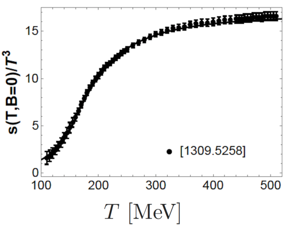
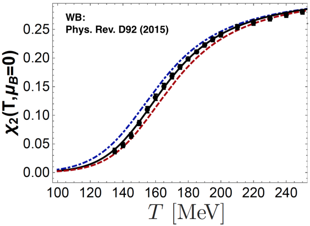
$$\mathcal{S} = \frac{1}{16\pi G_5}\int dx^5\sqrt{-g}\,[\mathcal{R} - \tfrac{1}{2}(\partial_M\phi)^2 - \underbrace{V(\phi)}_{\text{nonconformal}} - \tfrac{1}{4}\underbrace{f(\phi)F_{MN}^2}_{\mu_B\neq 0}]$$





- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and μ_B and higher χ_n $\rightarrow$ Predictions

R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Model Fields

For a static charged black hole backgrounds that are **spatially isotropic** and **translationally invariant**, which can be described by the following Ansatz for the EMD fields

$$ds^2 = e^{2A(r)}\left[-h(r)dt^2 + d\vec{x}^2\right] + \frac{e^{2B(r)}dr^2}{h(r)}$$

$$\phi = \phi(r)$$
$$\mathrm{A}_\mu dx^\mu = \Phi(r)dt$$

$$h = h(r)$$
$$A = A(r)$$
$$B = B(r)$$

The background function $B(r)$ has no dynamics and can be fixed to zero.

Equations of Motion

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Equations of Motion:

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r)\right]\phi'(r) - \frac{1}{h(r)}\left[\frac{\partial V(\phi)}{\partial \phi} - \frac{e^{-2A(r)}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial \phi}\right] = 0 \quad (1)$$

$$\Phi''(r) + \left[2A'(r) + \frac{d\left[\ln\left(f(\phi)\right)\right]}{d\phi}\phi'(r)\right]\Phi'(r) = 0 \quad (2)$$

$$A''(r) + \frac{\phi'(r)^2}{6} = 0 \quad (3)$$

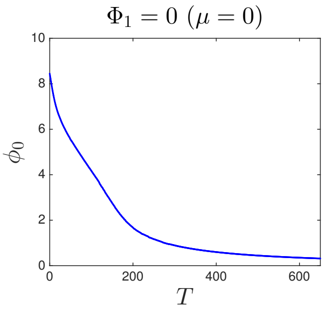
$$h''(r) + [4A'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0 \quad (4)$$

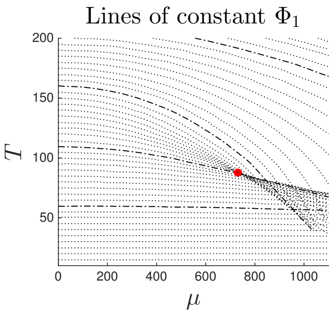
Constraint:

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r)V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0$$

Near-horizon asymptotic

- The initial value for the fields can be parametrized in terms of (ϕ_0, Φ_1)
- The mapping from (ϕ_0, Φ_1) to (T, μ) plane is highly non-linear.





From Black Holes to the QCD Phase Diagram

Far-Region asymptotic:

$$
\begin{aligned}
A(r) &= \alpha(r) + \mathcal{O}\left(e^{-2\nu\alpha(r)}\right) , \\
h(r) &= h_0^{\mathrm{far}} + \mathcal{O}\left(e^{-4\alpha(r)}\right) , \\
\phi(r) &= \phi_A e^{-\nu\alpha(r)} + \mathcal{O}\left(e^{-2\nu\alpha(r)}\right) , \\
\Phi(r) &= \Phi_0^{\mathrm{far}} + \Phi_2^{\mathrm{far}} e^{-2\alpha(r)} + \mathcal{O}\left(e^{-(2+\nu)\alpha(r)}\right) ,
\end{aligned}
$$

Thermodynamics:

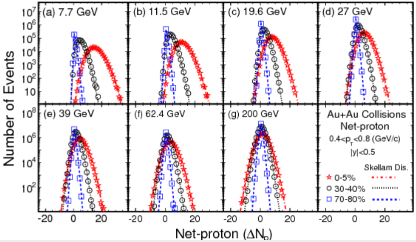
$$
T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{\mathrm{far}}}}\Lambda \qquad s = \frac{2\pi}{\kappa_5^2\,\phi_A^{3/\nu}}\Lambda^3
$$

$$
\mu_B = \frac{\Phi_0^{\mathrm{far}}}{\phi_A^{1/\nu}\sqrt{h_0^{\mathrm{far}}}}\Lambda \qquad \rho_B = -\frac{\Phi_2^{\mathrm{far}}}{\kappa_5^2\,\phi_A^{3/\nu}\sqrt{h_0^{\mathrm{far}}}}\Lambda^3
$$

Susceptibilities of Conserved Charges

- Baryonic Susceptibilities:
  $\chi_n^B(T,\mu_B) = \frac{\partial^n}{\partial(\mu_B/T)^n}\left(\frac{P}{T^4}\right)$
- The susceptibilities (χ_n) are related directly to the moments of the distribution.
- The volume-independent ratios are useful quantities to compare to experimental data.

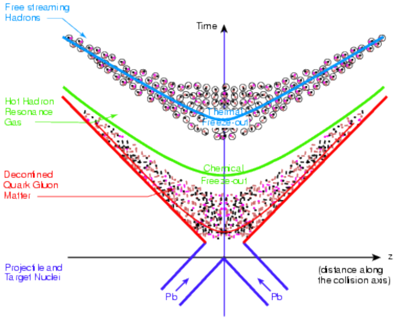
$$\begin{aligned} \text{mean}: \quad & M = \chi_1 \\ \text{variance}: \quad & \sigma^2 = \chi_2 \\ \text{skewness}: \quad & S = \chi_3/\chi_2^{3/2} \\ \text{kurtosis}: \quad & \kappa = \chi_4/\chi_2^2 \end{aligned}$$



$$\begin{aligned} M/\sigma^2 &= \chi_1/\chi_2 \\ S\sigma &= \chi_3/\chi_2 \\ \kappa\sigma^2 &= \chi_4/\chi_2 \\ S\sigma^3/M &= \chi_3/\chi_1 \end{aligned}$$

Evolution of heavy ion collisions

- **Chemical freeze-out:** all inelastic interactions cease. The chemical composition of the system is fixed.
- **Susceptibilities of conserved charges:** are fixed at the freeze-out.



- We study susceptibilities of conserved charges:
  - → They can be measured and calculated
  - → They can be used to extract freeze-out parameters
  - → They are sensitive to the critical point

Model Predictions at $\mu_B = 0$

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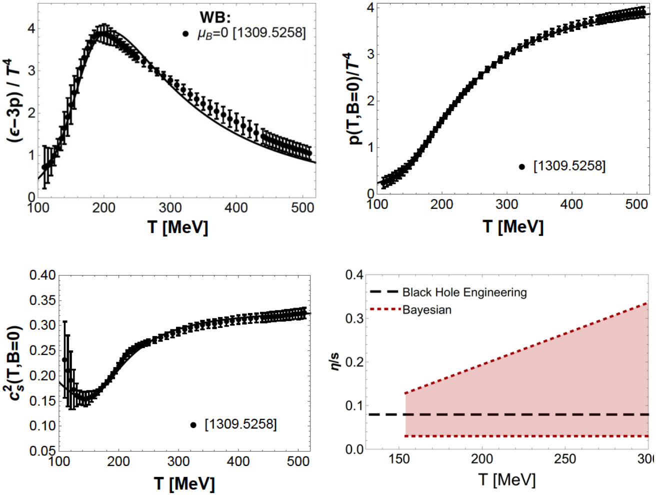
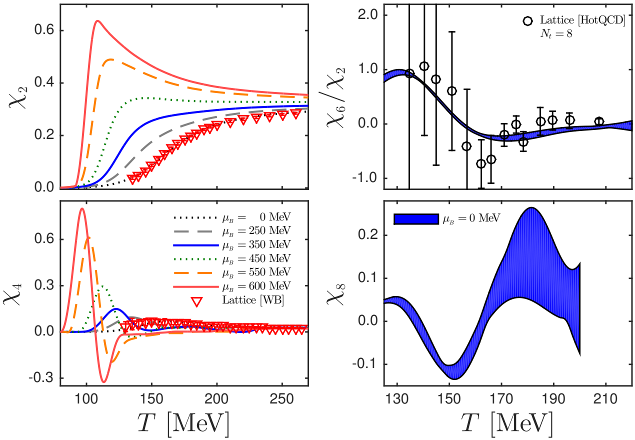


R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Black Hole Susceptibilities



BH curves: R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Lattice results: [WB] Phys.Rev.D**92**(2015).

[HotQCD] Phys.Rev.D**95**(2017).

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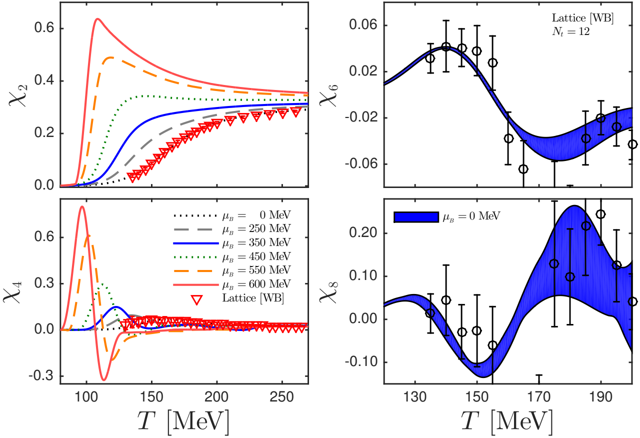
Baryon Susceptibilities on the lattice

- χ_2^B, χ_4^B, and χ_6^B on coarse lattice, $N_\tau = 4$. 2 flavors (2005).
  C.R. Allton, et al. Phys.Rev.D.71,054508
- Continuum extrapolation of χ_2^B. 2+1 flavors (2012).
  S. Borsanyi, et al., JHEP 08 (2012) 053 [1204.6710].
- χ_4^B on a finite lattice, $N_\tau = 8, 6$. 2+1 flavors (2014).
  Prasad Hegde, J.nuclphysa.2014.08.089
- Continuum limit for χ_6^B. 2+1 flavors (2016).
  J. Gunther, J.nuclphysa.2017.05.044
- Continuum limit for χ_6^B. 2+1 flavors (2017).
  A. Bazavov, et al. Phys. Rev. D 95, 054504
- Two points of χ_8^B on finite lattice, $N_\tau = 8$. 2+1 flavors (2018).
  Massimo D Elia, et al.
- χ_6^B, and χ_8^B on finite lattice, $N_\tau = 12$. 2+1+1 flavors (2018).
  S Borsanyi, I.P. et al. arXiv:1805.04445v1

Black Hole Susceptibilities



BH curves: R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Lattice results: [WB] Phys.Rev.D**92**(2015).

[WB] S Borsanyi, I.P. et al. arXiv:1805.04445v1.

Observables at finite μ_B

Taylor expansion of observables in terms of susceptibilities
$\chi_n = \chi_n^B(T, \mu_B = 0)$

- **Pressure**

$$\frac{p(T, \mu_B) - p(T, \mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

- **Baryonic density**

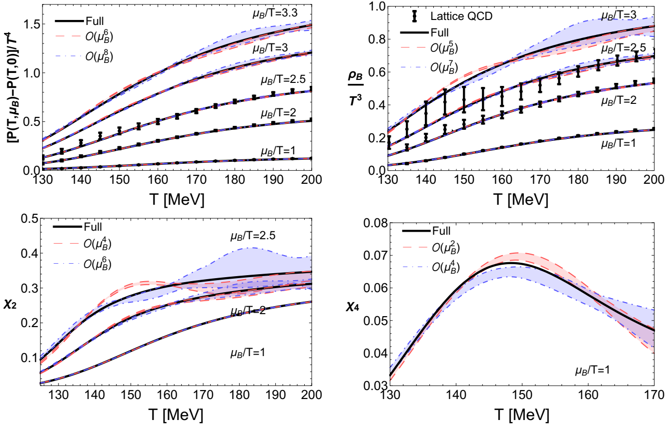
$$\frac{\rho_B(T, \mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

- **Susceptibilities χ_2 and χ_4**

$$\chi_2(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_4(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

Taylor Reconstruction up to $\mathcal{O}(\mu_B^8)$

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.



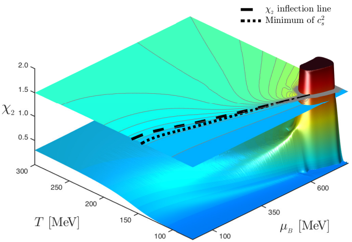
BH curves: R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

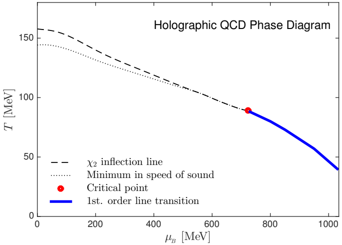
Lattice results: [HotQCD] Phys.Rev.D**95**(2017).

Black Hole Critical End Point

The black hole model contains a critical end point at

- $\mu_B = 724$ MeV
- $T = 89$ MeV





R. Critelli, I. P. et al., Phys.Rev.D**96**(2017).

Connection to Experiment

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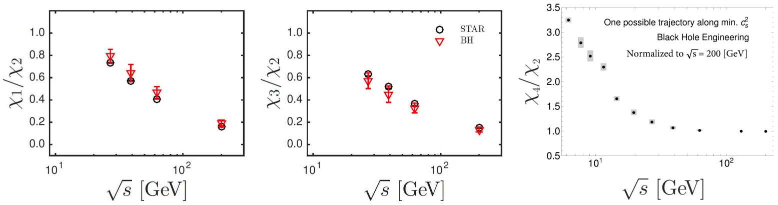
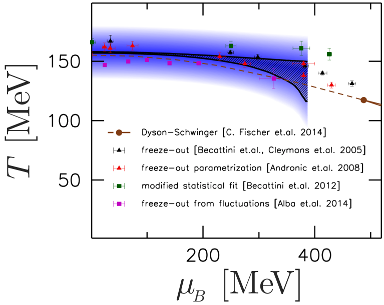
- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for χ_1/χ_2 and χ_3/χ_2
- χ_4/χ_2 predicted at the minimum of speed of sound



[BH] R.Critelli, I.P. et al., Phys.Rev.D**96**(2017). [STAR] Phys.Rev.Lett.**112**(2014).

Freeze-out Line

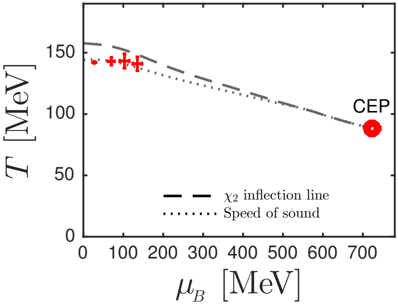
Lattice QCD



[WB:] R. Bellwied *et. al.*,

Phys.Lett.B**751**(2015).

BH Model

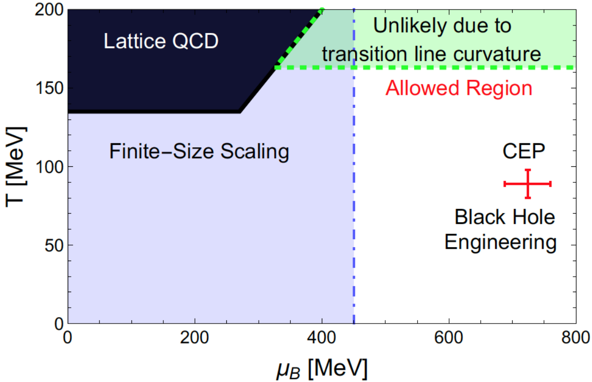


R. Critelli, I. P. et al.,

Phys.Rev.D**96**(2017).

Black Hole Model Critical Point

- $\mu_B = 724 \pm 36$ MeV
- $T = 89 \pm 11$ MeV

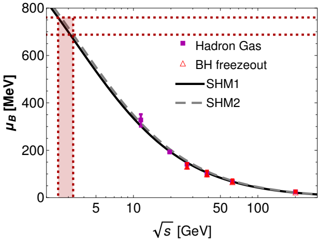


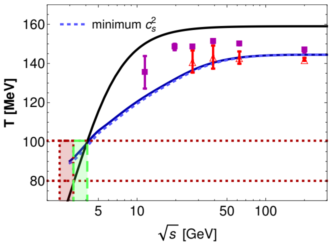
[Fig] R.Critelli, I.P. et al., Phys.Rev.D**96** (2017).
[Lattice] A.Bazavov, et al., Phys.Rev.D**95** 054504.
[FSS] E.S. Fraga, et al., Phys.Rev.C**84** 011903.
[Tran Line] R. Bellwied, et al., Phys.Lett.B**751** (2015) 053

Collision Energy Estimates

We estimate a collision energy needed to hit the CEP

- $\sqrt{s} = 2.5 - 4.1$ GeV





- The collision energy is reachable by the next generation of experiments.

[BH] R.Critelli, I.P. et al., Phys.Rev.D**96**(2017).
[HRG] Paolo Alba et al. Phys.Lett.B**738**(2014),
[SHM1] A. Andronic et al. Phys.Lett.B**673**(2009).
[SHM2] J. Cleymans et al. Phys.Rev.C**73**(2006).

Conclusions

The holographic Black Hole Model

- Reproduces the available lattice data at small μ_B.
- Predictions confirmed for χ_6 and χ_8
- Contains a critical end point which is located at $\mu_B = 724 \pm 36$ MeV and $T = 89 \pm 11$ MeV
- Allows us to compute transport coefficients, baryonic susceptibilities, extract freeze-out parameters, and out of equilibrium phenomena.
- Estimates that the collision energy needed to hit the CEP should be $\sqrt{s} = 2.5 - 4.1$ GeV

Outlook: Critical Behavior

Universality:

- Global Symmetries
- Spatial Dimensions

Critical Exponets:

α : $C_\rho \sim |T - T_c|^{-\alpha}$ along first-order axis.

β : $\Delta\rho \sim |T_c - T|^{\beta}$ along first-order line.

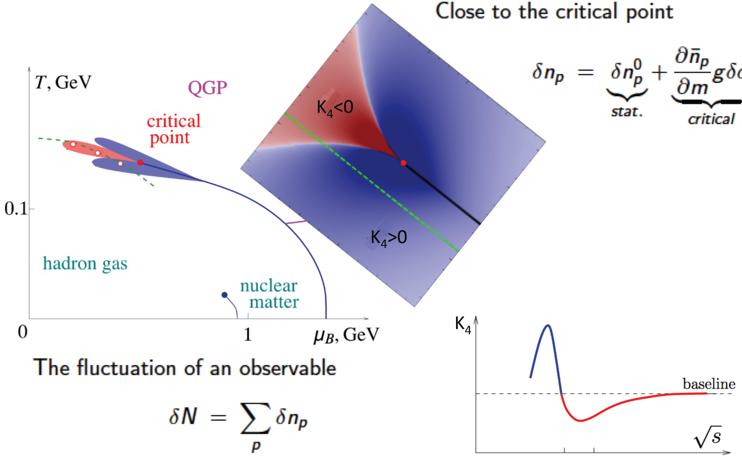
γ : $\chi_2 \sim |T - T_c|^{-\gamma}$ along first-order axis.

δ : $\rho - \rho_c \sim |\mu - \mu_c|^{1/\delta}$ for $T = T_c$

	3D Ising	Mean Field	Ref[1]	Our Model
α :	0.110	0.0	0.0	?
β :	0.325	0.5	0.482	?
δ :	1.240	1.0	0.942	?
γ :	4.820	3.0	3.034	?

[1] O DeWolfe, S S Gubser, and C Rosen, Phys.Rev.D **83**, (2011)

Outlook: Kurtosis at the Critical Point



Close to the critical point

$$\delta n_p = \underbrace{\delta n_p^0}_{stat.} + \underbrace{\frac{\partial \bar{n}_p}{\partial m} g \delta\sigma}_{critical}$$

The fluctuation of an observable

$$\delta N = \sum_p \delta n_p$$

M. A. Stephanov, Phys. Rev. Lett. **107** (2011) 052301

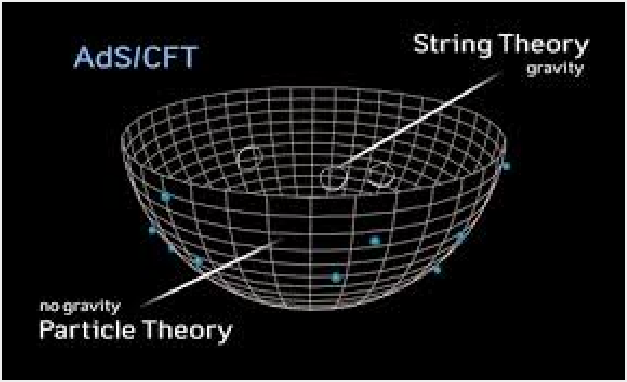
Hot Quarks 2018...!!!

Thanks to the Organizing Committee:

Javier Albacete, Universidad de Granada (Spain)
Jana Bielcikova, Nuclear Physics Institute CAS (Czech Republic)
Jorge Noronha, University of Sao Paulo (Brazil)
Alessandro Grelli, Utrecht University (The Netherlands)
Jiangyong Jia, Stony Brook University (USA)
Hannah Petersen, FIAS (Germany)
Lijuan Ruan, Brookhaven National Laboratory (USA)
Sevil Salur, Rutgers University (USA)
Bjoern Schenke, Brookhaven National Laboratory (USA)
Anthony Timmins, University of Houston (USA)

Backup Slides

Backup Slides — Locating the QCD Critical Point using Holographic Black Holes



Susceptibilities

The Baryonic Susceptibilities χ_n^B are defined as

$$\chi_n^B(T,\mu_B) = \frac{\partial^n}{\partial(\mu_B/T)^n}\left(\frac{P}{T^4}\right)$$

where:

$$P = \frac{T}{V}\ln \mathrm{Z} \qquad \mathrm{Z} = \mathrm{Tr}\left[-\frac{H-\sum_i \mu_i Q_i}{T}\right]$$

χ_1^B is proportional to the baryonic density ($\chi_1^B = \rho_B/T^3$)

χ_2^B measures the equilibrium response of the baryonic density to a change in the chemical potential

Moments of the Distribution

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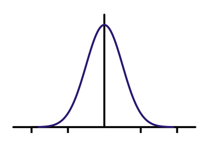
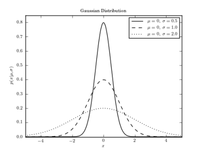
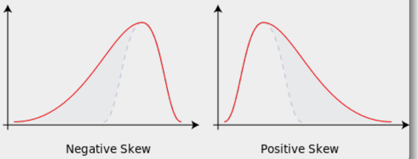
mean : $M = \int dx\, x\, f(x)$



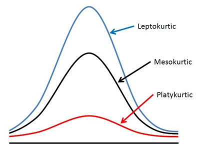
variance : $\sigma^2 = \int dx\, x^2 f(x)$



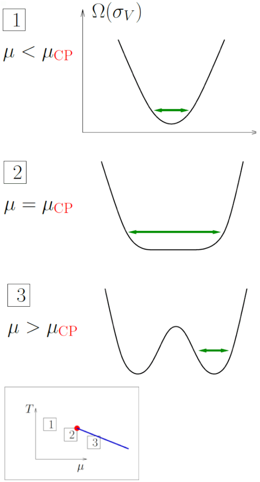
skewness : $S = \int dx\, x^3 f(x)$



kurtosis : $\kappa = \int dx\, x^4 f(x) - 3$



Fluctuations at the Critical End Point



The probability distribution for the order parameter

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma_2 + \frac{\lambda_3}{3}\sigma^3 + \cdots\right]$$

The correlation length ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$

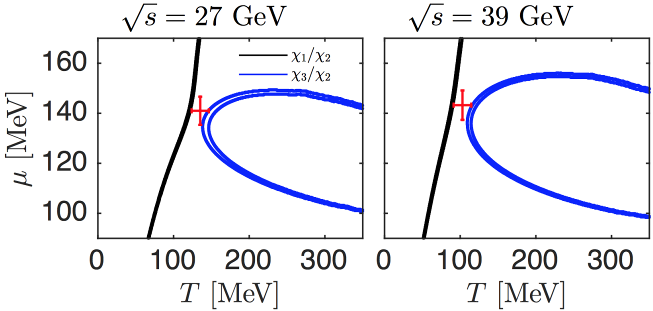
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

M. A. Stephanov, Phys.Rev.Lett.**102**(2009)032301.

Freeze out parameters from the Black Hole model

Trajectories in the $[T - \mu]$ plane that satisfy the experimental values



- Freeze out points $[T - \mu_B]$ are extracted from the line made by the closest points between χ_1/χ_2 and χ_3/χ_2

Holographic Model

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Holographic QCD phase diagram with critical point from Einstein–Maxwell-dilaton dynamics

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$\lambda_T = \lambda_\mu$	$\lambda_s = \lambda_n$
1148.07 MeV	$(513.01\ \text{MeV})^3$

$$V(\phi) = \begin{cases} -12\exp\left\{\frac{a_1}{2}\phi^2 + \frac{a_2}{4}\phi^4\right\} & : \phi < \phi_m \\ a_{10}\cosh\left[a_4(\phi - a_5)\right]^{a_3/a_4} \times \exp\left\{a_6\phi + \frac{a_7}{a_8}\tanh\left[a_8(\phi - a_9)\right]\right\} & : \phi \geq \phi_m \end{cases}$$

ϕ_m	a_1	a_2	a_3	a_4	a_5
1.7058	0.2840	−0.0089	0.7065	0.4951	0.1761

a_6	a_7	a_8	a_9	a_{10}
−0.0113	−0.4701	2.1420	4.3150	−10.0138

$$f(\phi) = c_0 + c_1\tanh\left[c_2(\phi - c_3)\right] + c_4\exp\left[-c_5\phi\right]$$

c_0	c_1	c_2	c_3	c_4	c_5
0.1892	−0.1659	1.5497	2.1820	0.6219	112.7136

$$T_{CEP} = (111.5 \pm 0.5)\ \text{MeV} \qquad \mu_{CEP} = (611.5 \pm 0.5)\ \text{MeV}.$$

- Good Fitting to updated Lattice Calculations
- Main results: location of CP and critical line