Insight into thermal modifications of quarkonia from a comparison of continuum-extrapolated lattice results to perturbative QCD

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[arXiv:1710.08858]

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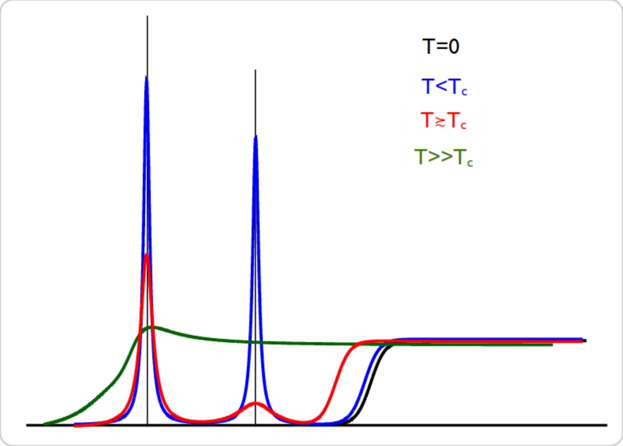
Strong-interaction matter under extreme conditions

- Information about in-medium properties of quarkonia encoded in spectral function

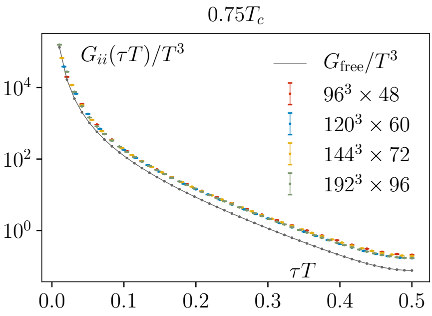


Correlators

- Lattice: Correlator
- Pseudoscalar $G_P(\tau) = M_B^2 \int_{\vec{x}} \langle (\bar{\psi} i\gamma_5 \psi)(\tau,\vec{x})(\bar{\psi} i\gamma_5 \psi)(0,\vec{0})\rangle_c$
- Vector: $G_{ii}(\tau) = \langle \psi^\dagger(\tau,\vec{x})\gamma_i\psi(\tau,\vec{x})\psi^\dagger(0,\vec{0})\gamma_i\psi(0,\vec{0})\rangle$
- Related to ρ: $G(\tau) = \int_0^\infty d\omega \rho(\omega) K(\omega,\tau)$

with $K(\omega,\tau) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$

$\Rightarrow$ ill-posed inversion problem



- Pseudoscalar: No transport peak $\Rightarrow$ easier
- Vector:
  - Information on heavy quark diffusion coefficients

$$D = \frac{\pi}{3\chi_q} \lim_{\omega \to \infty} \sum_{i=3}^{3} \frac{\rho_{ii}(\omega, T)}{\omega}$$

  - Information on quark number susceptibility

$$G^{00} = \chi_q T$$

Continuum Extrapolation of Correlators

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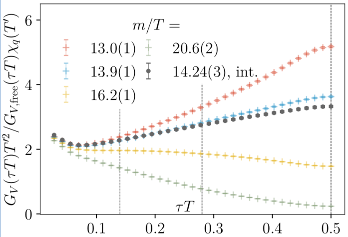
Lattice Correlators

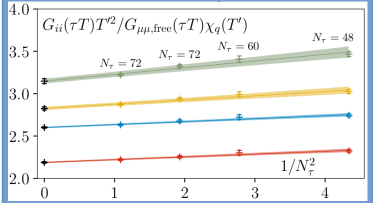
Renormalization

Mass Interpolation



Normalization with G_{free} and B-Spline Interpolation

Continuum Extrapolation



Renormalization

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Perturbatively determined renormalization constants in pseudoscalar channel

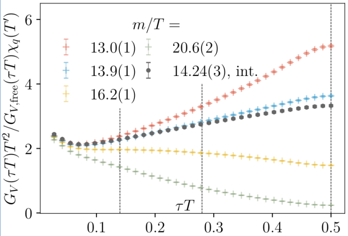
Different Options in vector channel

- Perturbative renormalization constants (1 and 2loop available)
- Non-perturbative renormalization constants [Lüscher et al., Phys.Lett. B372(1996)275-282]
  $Z_V^m(g^2,\kappa) = \frac{1-0.7663g^2+0.0488g^4}{1-0.6369g^2}(1+b_V(g^2)am_q)$ with
  $b_V(g^2) = \frac{1-0.6518g^2-0.1226g^4}{1-0.8467g^2}$
- Renormalization independent ratio with quark number susceptibility χ_q
  $\chi_q T$ given by G^{00}, Choice: χ_q at $T' = 2.25\,T_c$

Lattice Correlators

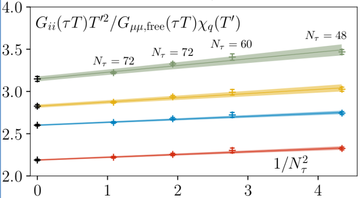
Renormalization

Mass Interpolation

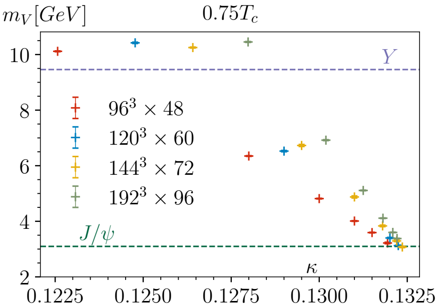


Normalization with G_{free} and B-Spline Interpolation

Continuum Extrapolation

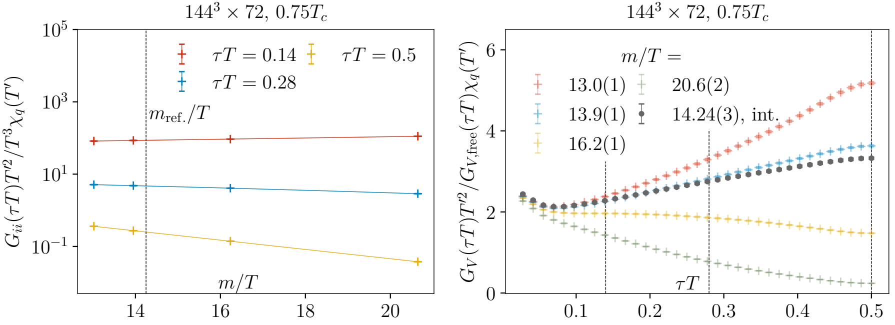


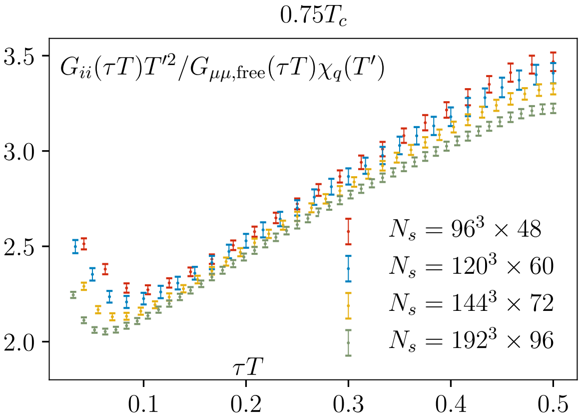
Mass Interpolation



- Problem: Meson mass not equal on different lattices
- Solution: Interpolation of correlators between different masses to a fixed m/T for $T = 0.75\,T_c$
- Ansatz: $\frac{G^{ii}(\tau T, m/T) T'^2}{T^3 \chi_q} = \exp(p(m/T)^2 + qm/T + r)$

Mass Interpolation





$0.75T_c$
$G_{ii}(\tau T)T'^2/G_{\mu\mu,\mathrm{free}}(\tau T)\chi_q(T')$
$N_s = 96^3 \times 48$
$N_s = 120^3 \times 60$
$N_s = 144^3 \times 72$
$N_s = 192^3 \times 96$
τT
3.5
3.0
2.5
2.0
0.1
0.2
0.3
0.4
0.5

Lattice Correlators

Renormalization

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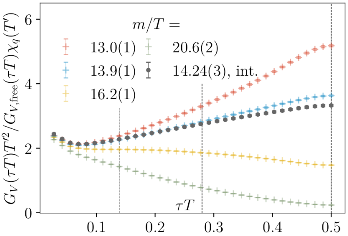
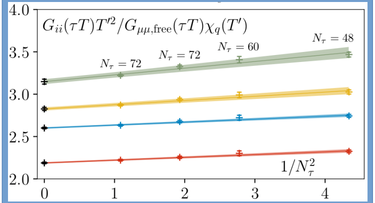


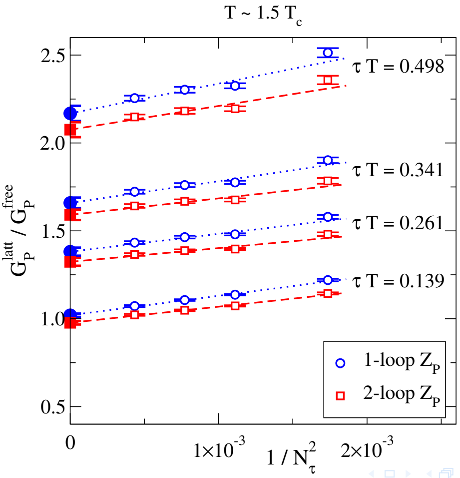
Normalization with G_{free} and B-Spline Interpolation

Continuum Extrapolation

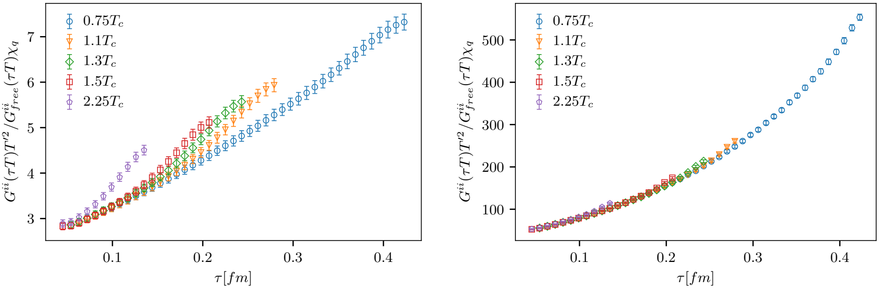


Continuum Extrapolation

- Ansatz: $\frac{G^{ii}(\tau T, a) T'^2}{G_{free} T^3 \chi_q} = G^{cont}(\tau T) + \frac{c}{N_\tau^2}$
  or $\frac{G(\tau T, a)}{G_{free}} = G^{cont}(\tau T) + \frac{c}{N_\tau^2}$



Continuum Correlators - Vector

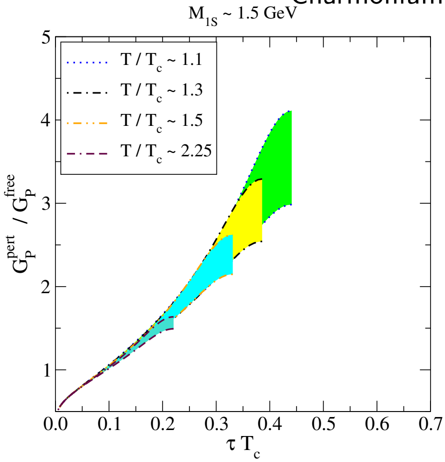


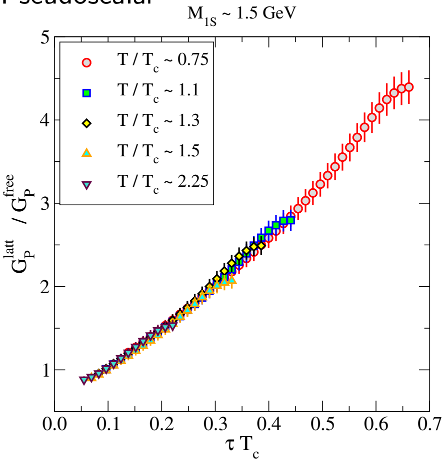
Perturbative Spectral Function

- Ultraviolet asymptotics available up to 5-loop level
  [Y. Burnier, M. Laine, Eur.Phys.J.C 72 (2012) 1902]
- Threshold region using a finite-temperature real-time static potential
  [M.Laine, JHEP 0705:028,2007]
- Matching between regions
  [Y. Burnier, H.-T. Ding, O. Kaczmarek, A.-L. Kruse, M. Laine, H. Ohno, H. Sandmeyer, JHEP 1711 (2017) 206]

$$G(\tau) = \int_0^\infty d\omega K(\omega, \tau)\rho(\omega)$$

Charmonium - Pseudoscalar

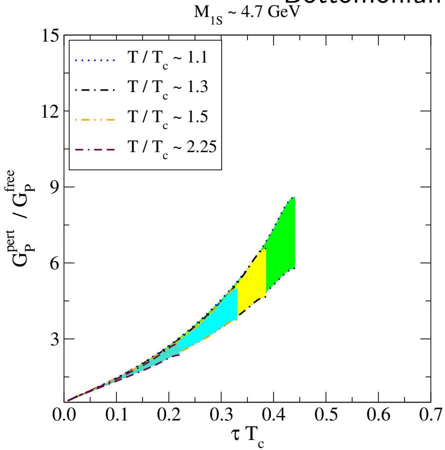


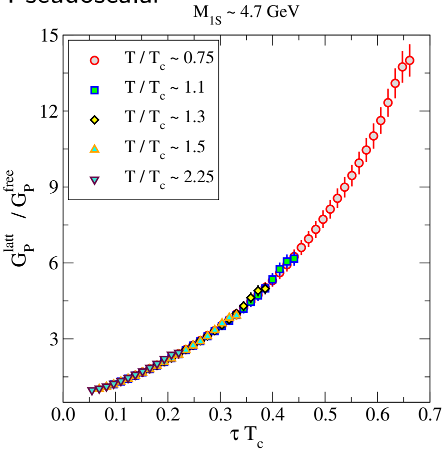


Comparison of Lattice and Perturbation Theory - A first look

$$G(\tau) = \int_0^\infty d\omega K(\omega,\tau)\rho(\omega)$$

Bottomonium - Pseudoscalar





Fits to the Correlator

Qualitatively similar results

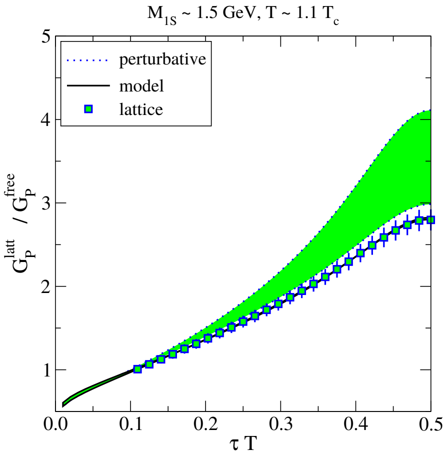
- On Lattice side: Uncertainties in renormalization constants
  $\Rightarrow$ Relative normalization might be off
  $\Rightarrow$ Introduce overall normalization factor A

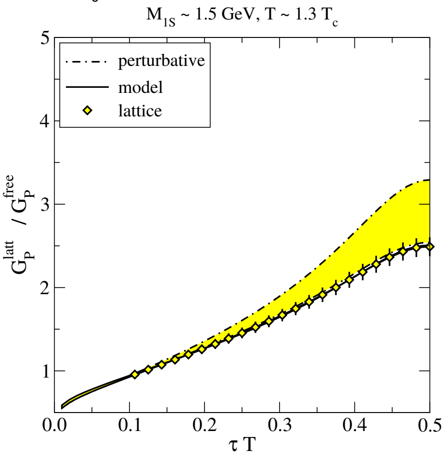
- On perturbative side: Relation between M and $m(\bar{\mu}_{ref})$ poorly known
  $\Rightarrow$ Threshold location might be off
  $\Rightarrow$ Introduce frequency shift B

- Ansatz: $\rho^{model}(\omega) = A\rho^{pert}(\omega - B)$

Fit Results

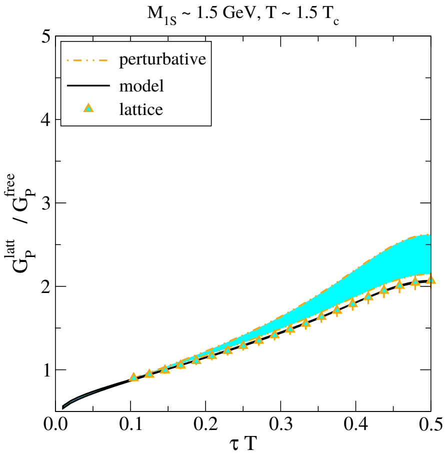
$$\rho^{model}(\omega) = A\rho^{pert}(\omega - B), \quad G(\tau) = \int_0^\infty d\omega K(\omega,\tau)\rho(\omega)$$

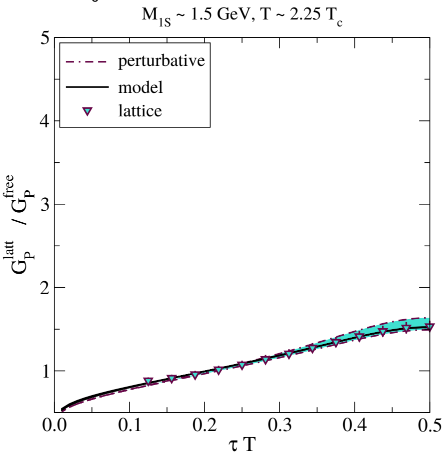




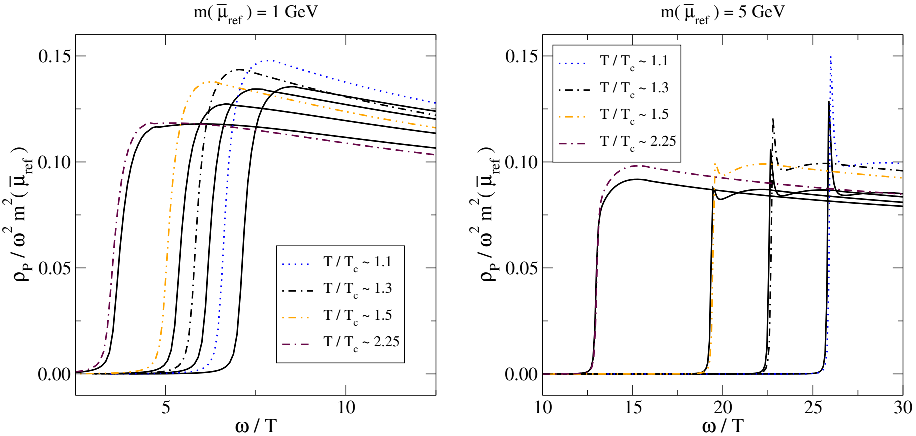
Fit Results

$$\rho^{model}(\omega) = A\rho^{pert}(\omega - B), \quad G(\tau) = \int_0^\infty d\omega K(\omega,\tau)\rho(\omega)$$





Fit Results



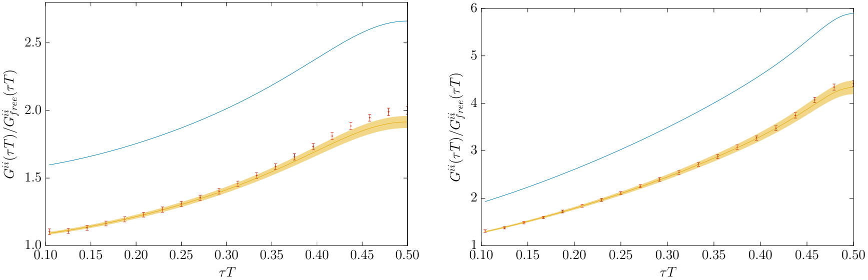
dashed: perturbative spectral function

solid: fitted model spectral function

Results

- Quite good agreement:
  A close to 1
  B small
- Now moving on to the vector channel: Shape of the transport peak not known!
  But: Integral can be calculated
  $\Rightarrow$ Constant contribution to correlator $\frac{G_{ii}^{const}}{T^3}$ is known.

$1.5\,T_c$



Fit to small τT compared to lattice data. Difference $\Rightarrow$ Transport peak!

Conclusion & Outlook

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- Good agreement in pseudoscalar channel
- Differences explained by systematic uncertainties
- Transport contribution visible in vector channel

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Future:

- Use suitable ansätze to fit the transport peak
- Crosscheck using Bayesian reconstruction methods

  [arXiv:1807.06315, arXiv:1712.03341]
- Full QCD study required in future