Thermodynamics and fluctuations-correlations of conserved charges in a hadron resonance gas model with attractive and repulsive interaction within S-matrix formalism

Subhasis Samanta

National Institute of Science Education and Research

Jatni - 752050, India

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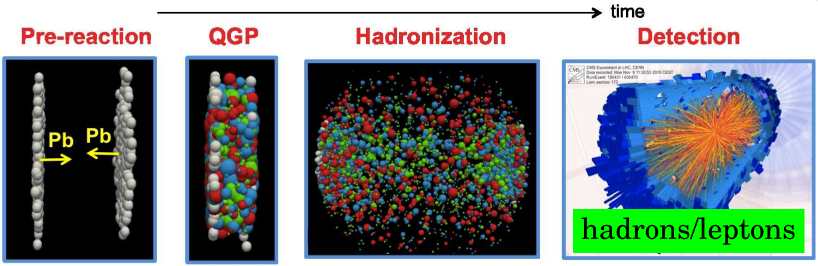
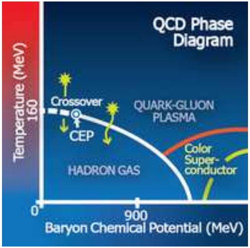
Overview

- Motivation
- IDHRG model
- Interaction within S-matrix formalism
- Results
- Summary

Study of matter under extreme conditions





The major goals

- The mapping of QCD phase diagram in terms of T and μ_B
- Locating the QCD critical point

Facility	$\sqrt{s_{NN}}$ (GeV)	μ_B (MeV)	Status
LHC	2760	0	Running
RHIC	7.7 - 200	420-20	Running
NA61/ SHINE	8	400	Running
FAIR	2.7-4.9	800-500	Future
NICA	4-11	600-300	Future

HRG models have been used to study hadronic phase

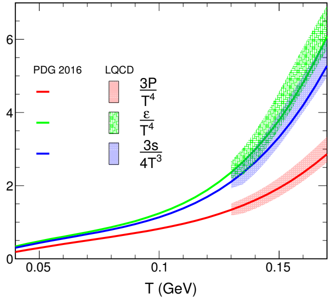
Ideal Hadron Resonance Gas model

- Statistical thermal model
- System consists of all the hadrons including resonances (non-interacting)
- Hadrons are in thermal and chemical equilibrium
- The grand canonical partition function of a hadron resonance gas: $\ln Z = \sum_i \ln Z_i$
- For i th hadron/resonance,

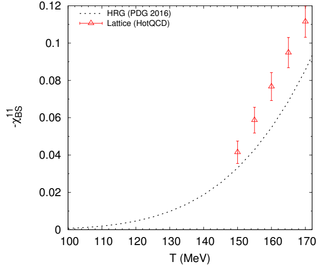
$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

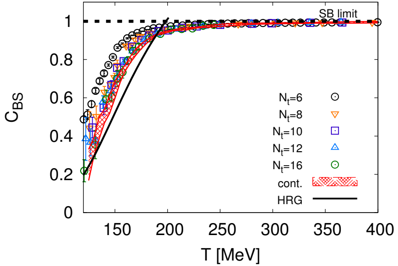
The upper and lower sign corresponds to baryons and mesons respectively.

$$E_i = \sqrt{p^2 + m_i^2}, \quad \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$



- IDHRG provides a satisfactory description of EOS in the hadronic phase of continuum LQCD data
- IDHRG fails to describe χ_S^2, χ_{BS}^{11}, $C_{BS} = -3\chi_{BS}^{11}/\chi_S^2$ etc.





LQCD data: Phys. Rev. D 90, 094503 (2014), JHEP01, 138 (2012)

Interaction is needed

van der Waals interaction in HRG model (VDWHRG model)

$$\left(P + \left(\frac{N}{V}\right)^2 a\right)(V - Nb) = NT,$$

$$P(T,n) = \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 \equiv \frac{nT}{1 - bn} - an^2$$

where $n \equiv N/V$ is the number density of particles.

$$P(T,\mu) = P_{id}(T,\mu^*) - an^2, \qquad \mu^* = \mu - bP(T,\mu) - abn^2 + 2an$$

$$n = \frac{n_{id}(T,\mu^*)}{1 + bn_{id}(T,\mu^*)}$$

Extra parameters

- $a = 0 \quad \Rightarrow$ EVHRG
- $a = b = 0 \quad \Rightarrow$ IDHRG

Classical Virial Expansion (Non-relativistic)

$$P = \frac{NT}{V}\left(1 + \frac{NB(T)}{V} + \frac{N^2C(T)}{V^2} + ..\right)$$

- The first term in the expansion corresponds to an ideal gas
- The second term is obtained by taking into account the interaction between pairs of particles and subsequent terms involve the interaction between groups of three,four, etc. particles
- $B, C, ...$ are called second, third, etc., virial coefficients

Second virial coefficient

$$B(T) = \frac{1}{2}\int(1 - e^{-U_{12}/T})dV$$

U_{12} is the two body interaction energy

Relativistic Virial Expansion

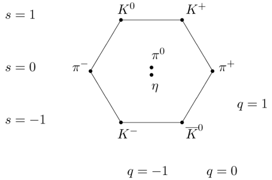
$$\ln Z = \ln Z_0 + \ln Z_{int} = \ln Z_0 + \sum_{i_1,i_2} z_1^{i_1} z_2^{i_2} b(i_1,i_2)$$

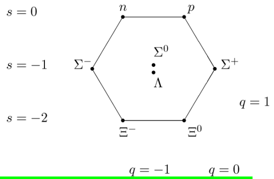
$$b(i_1,i_2) = \frac{V}{4\pi i}\int \frac{d^3p}{(2\pi)^3}\int d\varepsilon \exp\left(-\beta(p^2+\varepsilon^2)^{1/2}\right)\left[\left\{S^{-1}\frac{\partial S}{\partial \varepsilon} - \frac{\partial S^{-1}}{\partial \varepsilon}S\right\}\right]$$

- z_1 and z_2 are fugacities of two species ($z = e^{\beta\mu}$)
- The labels i_1 and i_2 refer to a channel of the S-matrix which has an initial state containing $i_1 + i_2$ particles
- We ignore contributions from bound states

Second virial coefficient

$b_2 = b(i_1,i_2)/V$ where $i_1 = i_2 = 1$





$\ln Z_0 \Rightarrow$ Non interacting stable hadrons

$\ln Z_{int} \Rightarrow$ Scattering between two hadrons

Interacting part of pressure

b_2 in terms of phase shift

$b_2 = \frac{1}{2\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{l,I}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}$

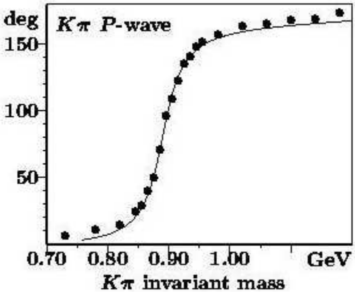
M is the invariant mass of the interacting pair at threshold

$$
\begin{aligned}
\mathrm{P}_{\mathrm{int}} &= \frac{1}{\beta}\frac{\partial \ln Z_{int}}{\partial V} = \frac{1}{\beta}z_1 z_2 b_2 \\
&= \frac{z_1 z_2}{2\pi^3\beta^2}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{I,l}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}
\end{aligned}
$$

- Interaction is attractive (repulsive) if derivative of the phase shift is positive (negative)

Ideal gas limit

- For a very narrow width resonance, δ_l^I changes rapidly through 180° around $\varepsilon = m_R$
- δ_l^I can be approximated by a step function: $\delta_l^I \sim \Theta(\varepsilon - m_R)$
- $\partial\delta_l^I/\partial\varepsilon \approx \pi\delta(\varepsilon - m_R)$



$$b_2^R = \frac{1}{2\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{l,I}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}$$
$$= \frac{g_{I,l}}{2\pi^2} m_R^2 T K_2(\beta m_R)$$

$$\mathrm{P}_{\mathrm{int}}^{\mathrm{R}} = T z_1 z_2 b_2^R = \mathrm{P}_{\mathrm{id}}^{\mathrm{R}}$$

- Narrow resonance behaves like a stable hadron of mass m_R
- This establishes the fundamental premise of the IDHRG

K-matrix formalism (Attractive part of the interaction)

Scattering amplitude: $S_{ab\to cd} = \langle cd|S|ab\rangle$

Scattering operator (matrix)

$$S = I + 2iT$$

S is unitary

$$SS^\dagger = S^\dagger S = 1$$

$$(T^{-1} + iI)^\dagger = T^{-1} + iI$$

$K^{-1} = T^{-1} + iI, \qquad K = K^\dagger$
(i.e., K martix is real and symmetric)

Phase shift in K-matrix formalism

$$\mathrm{Re}\, T = K(I + K^2)^{-1}, \quad \mathrm{Im}\, T = K^2(I + K^2)^{-1}$$

$$\Rightarrow \mathrm{Im}\, T / \mathrm{Re}\, T = K$$

$$K_{ab\to R\to ab} = \sum_R \frac{m_R \Gamma_{R\to ab}(\sqrt{s})}{m_R^2 - s}$$

Resonances appear as sum of poles in the K matrix

Partial wave decomposition

$$S_l = \exp(2i\delta_l) = 1 + 2iT_l$$
$$\Rightarrow T_l = \exp(i\delta)\sin(\delta_l)$$
$$\mathrm{Re}\, T_l = \sin(\delta_l)\cos(\delta_l), \quad \mathrm{Im}\, T_l = \sin^2(\delta_l)$$

$$K = \tan(\delta_l), \quad \delta_l = \tan^{-1}(K)$$

Transition amplitude: K-matrix

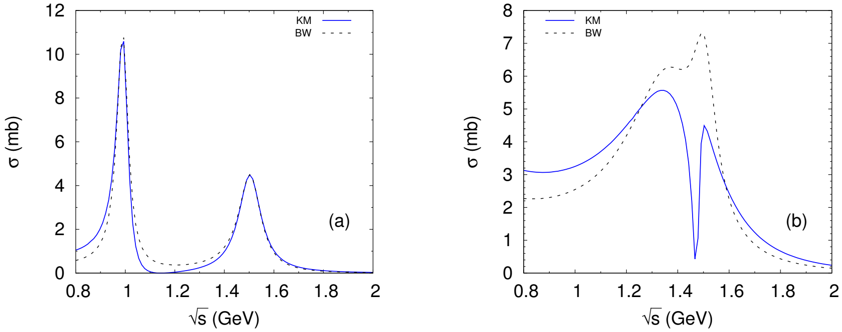
Example: $\pi\pi \to r_1(m_1) \to \pi\pi$
$\pi\pi \to r_2(m_2) \to \pi\pi$ (r_1, r_2 have same l, I)

$$K = \frac{m_1\Gamma_1(\sqrt{s})}{m_1^2 - s} + \frac{m_2\Gamma_2(\sqrt{s})}{m_2^2 - s}$$

$$T = \frac{m_1\Gamma_1(\sqrt{s})}{(m_1^2 - s) - im_1\Gamma_1(\sqrt{s}) - i\frac{m_1^2-s}{m_2^2-s}m_2\Gamma_2(\sqrt{s})} + \frac{m_2\Gamma_2(\sqrt{s})}{(m_2^2 - s) - im_2\Gamma_2(\sqrt{s}) - i\frac{m_2^2-s}{m_1^2-s}m_1\Gamma_1(\sqrt{s})}$$

Comparison between K-matrix and Breit-Wigner approach

$$T \approx \frac{m_1\Gamma_1(\sqrt{s})}{(m_1^2-s)-im_1\Gamma_1(\sqrt{s})} + \frac{m_2\Gamma_2(\sqrt{s})}{(m_2^2-s)-im_2\Gamma_2(\sqrt{s})} \text{ (Separated)}$$



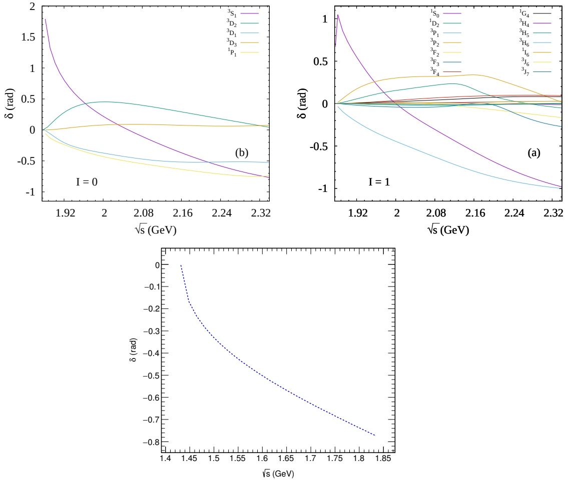
f_0 (980) (m_1 = 990 MeV Γ_1 = 55 MeV)

and f_0 (1500) (m_2 = 1505 MeV Γ_2 = 109 MeV)

f_0 (1370) (m_1 = 1370 MeV Γ_1 = 350 MeV)

and f_0 (1500) (m_2 = 1505 MeV Γ_2 = 109 MeV)

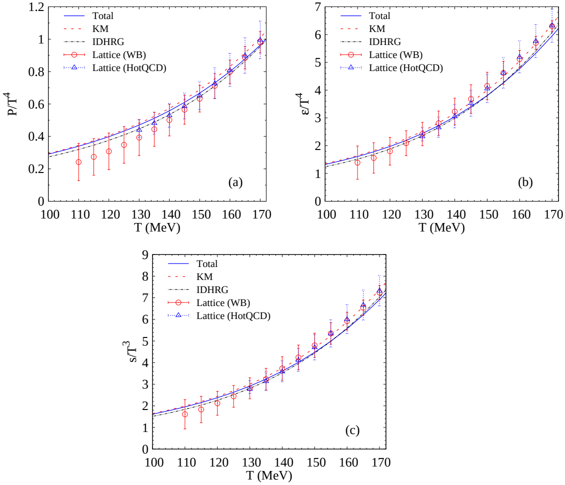
Input from experimental data of phase shift



Data: Scattering Analysis Interactive Database (SAID) partial wave analysis

- NN interaction: All available data ($|B| = 2$)
- πN interaction: S_{31} ($l_{2I,2J}$) ($\Delta(1620)$), $\Delta(1910)$, $N(1720)$ etc.
- KN interaction: $S_{11}(l_{I,2J})$ ($\Sigma(1660)$)
- $\pi\pi$ interaction: δ_0^2

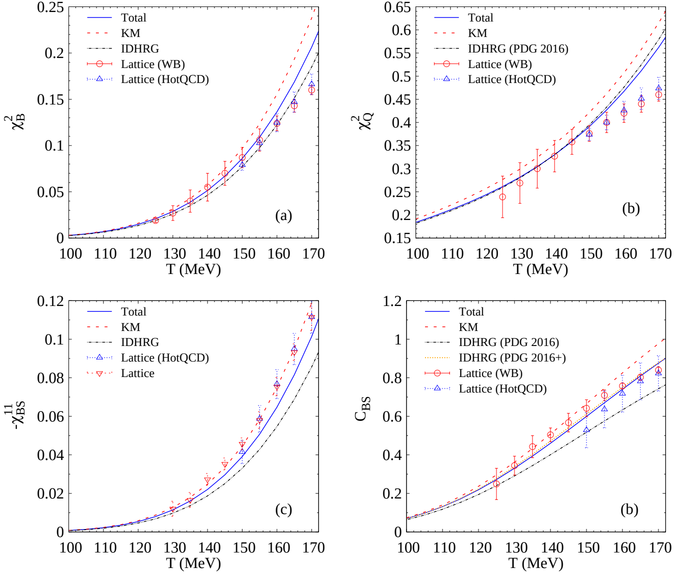
Result: EOS



A. Dash et al., arXiv:1806.02117 [hep-ph]

- KM: Attractive interaction (scattering between two hadrons)
- Total: Attractive + repulsive
- Both KM and Total contain non-interacting part as well
- IDHRG: PDG 2016
- Repulsive interactions suppress the bulk variables

Result: Fluctuations and corrections



Improvement

A. Dash et al., arXiv:1806.02117 [hep-ph]

Summary

- An extension of HRG model is constructed to include interactions using S-matrix formalism. Interaction in the S-matrix formalism is a genuine interaction.
- We have considered all the stable hadrons and resonances which have two-body decay channels
- Interacting part of the partition function depends on the derivative of the phase shift
- The attractive part of the interaction is calculated by parameterizing the two body phase shifts using K-matrix formalism
- The repulsive part is included by fitting to experimental phase shifts
- A good agreement between EOS calculated in S-matrix formalism and LQCD simulations is observed
- Effect of interaction is more visible in χ_Q^2, χ_B^2, C_{BS} etc.
- We find a good agreement for the C_{BS} (without adding extra resonances) and lattice QCD simulations

Collaborator

Prof. Bedangadas Mohanty
Mr. Ashutosh Dash

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Thank you