Axion-photon coupling: models vs. experiments

The strong CP puzzle & Axions
LPSC Grenoble - 15.05.18

Luca Di Luzio

[Based on: LDL, F. Mescia (Barcelona U.), E. Nardi (LNF, Frascati),1610.07593 + 1705.05370]

Outline

1. Strong CP & the axion solution
2. Current limits and search strategies
3. Axion-photon coupling
4. Beyond standard DFZS/KSVZ benchmarks

The strong CP problem

- CP violation in QCD

$$\mathcal{L}_{\text{QCD}} = \sum_q \overline{q} \left(i \not{D} - m_q e^{i\theta_q} \right) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a \qquad \left(\tilde{G}_{\mu\nu}^a = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a,\rho\sigma} \right)$$

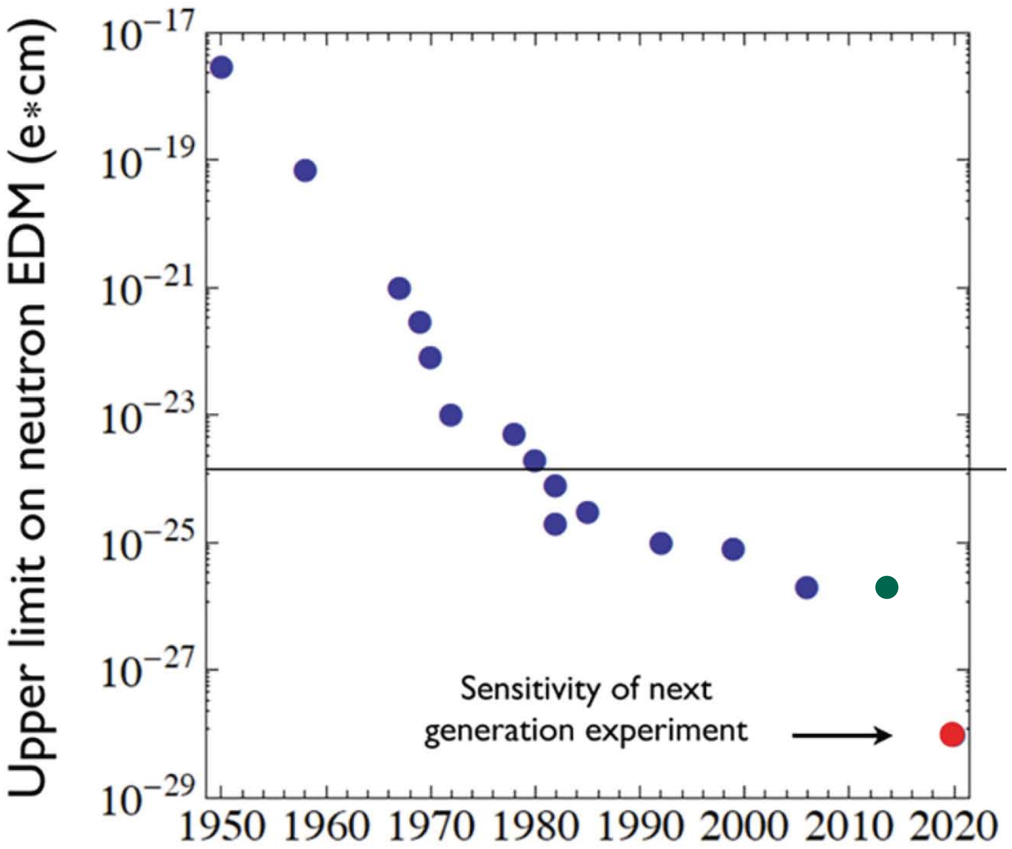
$\overline{\theta} = \theta - \sum_q \theta_q$ invariant under a chiral transformation $(q \to e^{i\gamma_5 \alpha} q)$

The strong CP problem

- CP violation in QCD

$$\mathcal{L}_{\rm QCD} = \sum_q \overline{q}\left(i\not{D} - m_q e^{i\theta_q}\right) q - \frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}^a_{\mu\nu}$$

- Non-zero neutron EDM



[B. Fillipone]

$$d_n \approx \frac{e\,|\overline{\theta}|\,m_\pi^2}{m_n^3} \approx 10^{-16}\,|\overline{\theta}|\,e\,\mathrm{cm}$$

[Baluni PRD 19 (1979), Crewther et al. PLB 88 (1979)]

$\rightarrow$ $|\overline{\theta}| \lesssim 10^{-10}$ why so small ?

"Small value" problems

- Strong CP: qualitatively different from other small value problems of the SM

  - $\overline{\theta} \propto J_{\rm CKM} \log \Lambda_{\rm UV}$ radiatively stable (unlike $m_H^2 \ll \Lambda_{\rm UV}^2$) [Ellis, Gaillard NPB 150 (1979)]

    [Khriplovich, Vainshtein NPB 414 (1994)]

Fig. 9. Generic topology of a class of divergent *CP* violating 14th-order diagrams in the Kobayashi-Maskawa model [21,22].

  - it evades explanations based on environmental selection (unlike $y_{e,u,d} \sim 10^{-6} \div 10^{-5}$)

    nuclear physics and BBN practically unaffected for $\overline{\theta} \lesssim 10^{-2}$ [Ubaldi, 0811.1599]

    [Kaloper, Terning, 1710.01740, Dine et al, 1801.03466]

→ theoretically motivated to look for an explanation of strong CP *independently* of other small value problems

Axion solution

- PQ mechanism [Peccei, Quinn PRL 38 (1977), PRD 16 (1997)]

- assume a global $U(1)_{PQ}$: i) QCD anomalous and ii) spontaneously broken

- axion: PGB of $U(1)_{PQ}$ breaking [Weinberg PRL 40 (1978), Wilczek PRL 40 (1978)]

$$a(x) \to a(x) + \delta\alpha \, f_a$$

$$\mathcal{L}_{\text{eff}} = \left(\overline{\theta} + \frac{a}{f_a}\right) \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a - \frac{1}{2} \partial^\mu a \partial_\mu a + \mathcal{L}(\partial_\mu a, \psi)$$

- theta term *dynamically* relaxed to zero on the axion ground state: $\langle a(x) \rangle = -\overline{\theta} f_a$

Axion solution

- PQ mechanism [Peccei, Quinn PRL 38 (1977), PRD 16 (1997)]

  - assume a global $U(1)_{PQ}$: i) QCD anomalous and ii) spontaneously broken

  - axion: PGB of $U(1)_{PQ}$ breaking [Weinberg PRL 40 (1978), Wilczek PRL 40 (1978)]

- UV completions: $f_a \gg v$ → invisible axion (phase of a SM singlet)

  - DFSZ axion: [Zhitnitsky SJNP 31 (1980), Dine, Fischler, Srednicki PLB 104 (1981)]

    SM quarks charged under PQ (requires 2HDM)

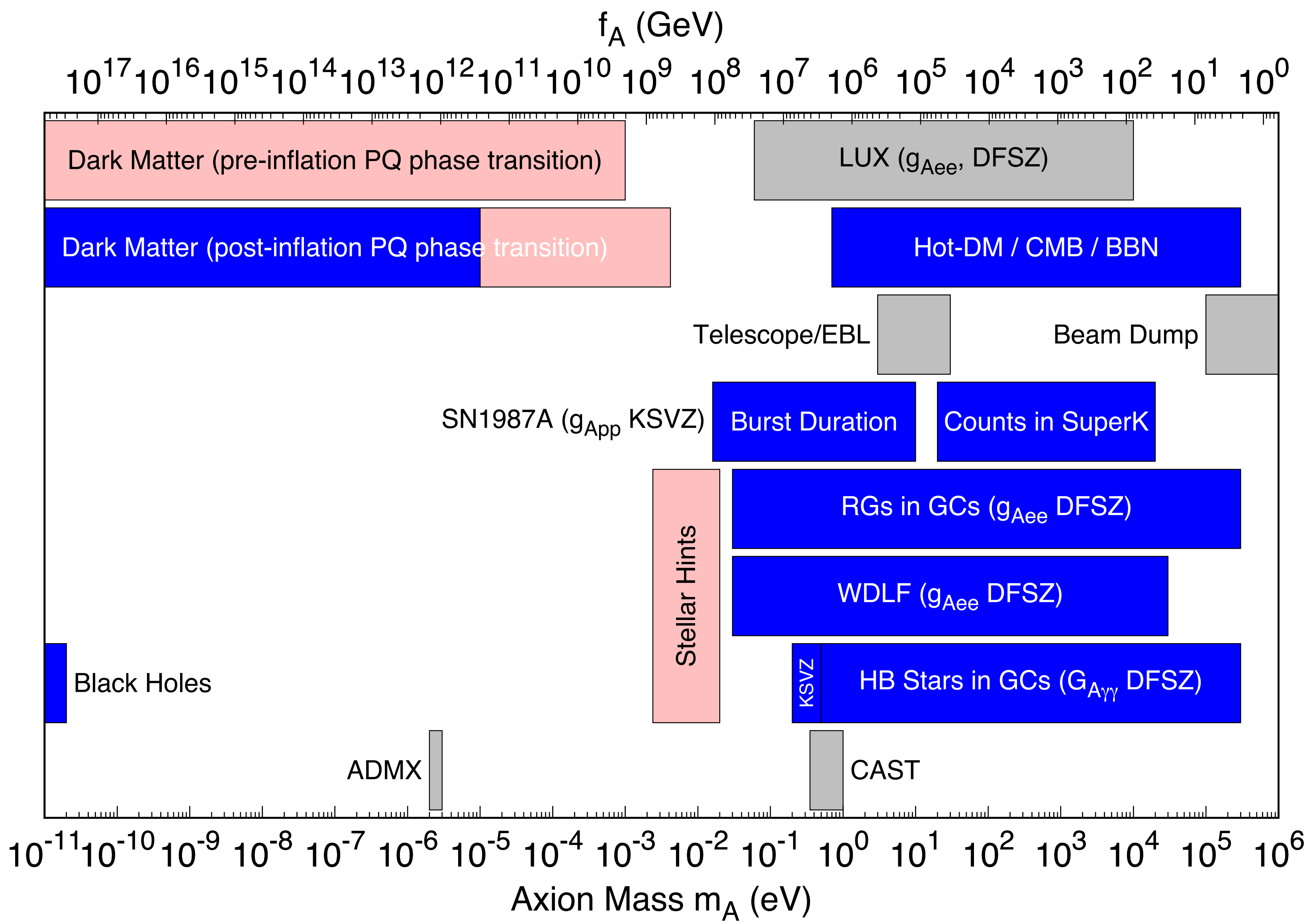
  - KSVZ axion: [Kim PRL 43 (1979), Shifman, Vainshtein, Zakharov NPB 166 (1980)]

    new vector-like quarks charged under PQ

Axion landscape

- axion mass $m_a \simeq m_\pi \frac{f_\pi}{f_a} \simeq 6\ \text{meV}\ \frac{10^9\ \text{GeV}}{f_a}$

- axion couplings $\sim 1/f_a$



[Ringwald, Rosenberg, Rybka, Particle Data Group (2017)]

Lab exclusions

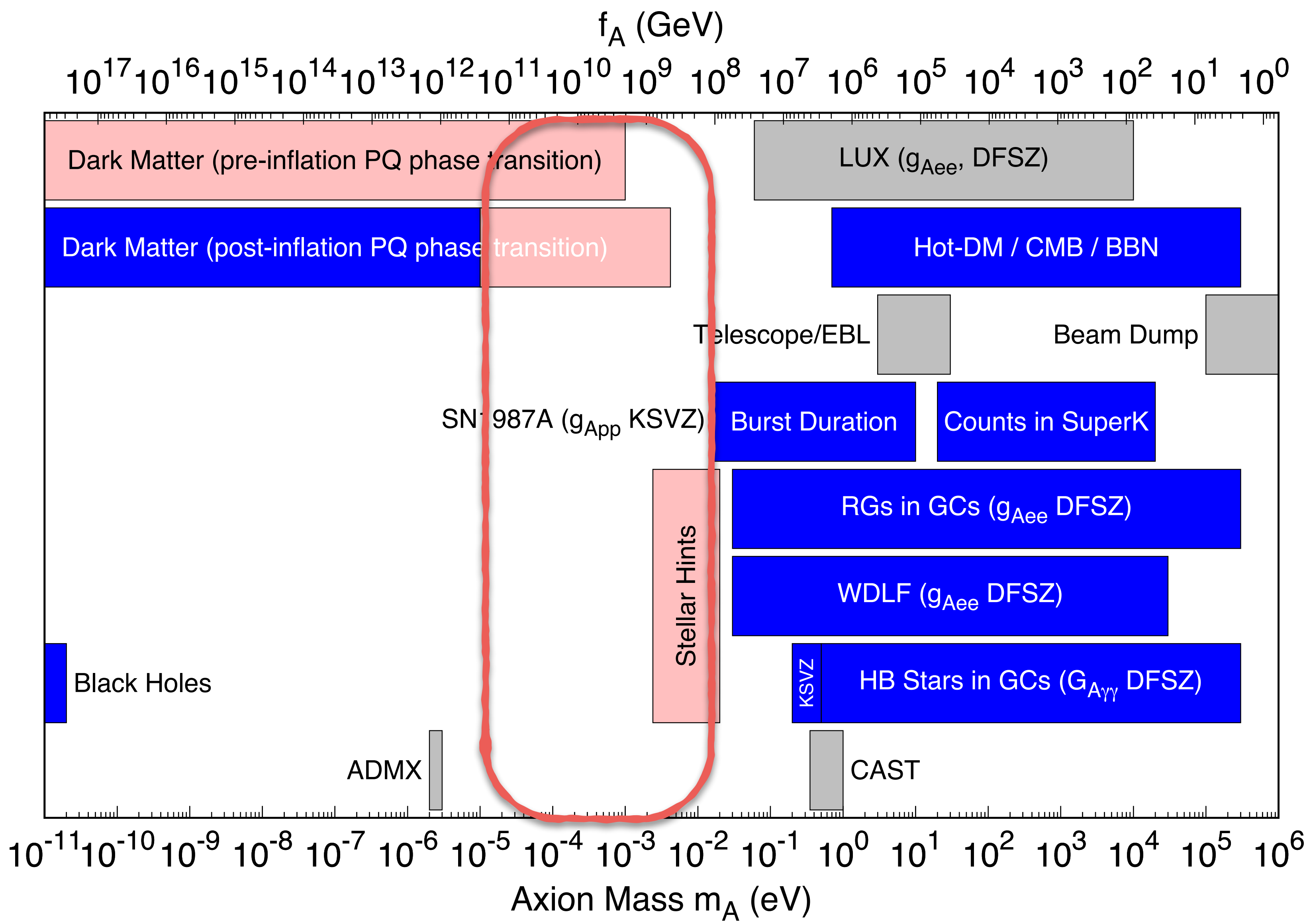
Astro/cosmo exclusions

DM explained / Astro Hints

Axion landscape

- axion mass $m_a \simeq m_\pi \frac{f_\pi}{f_a} \simeq 6\ \text{meV}\ \frac{10^9\ \text{GeV}}{f_a}$

- axion couplings $\sim 1/f_a$



[Ringwald, Rosenberg, Rybka, Particle Data Group (2017)]

Lab exclusions

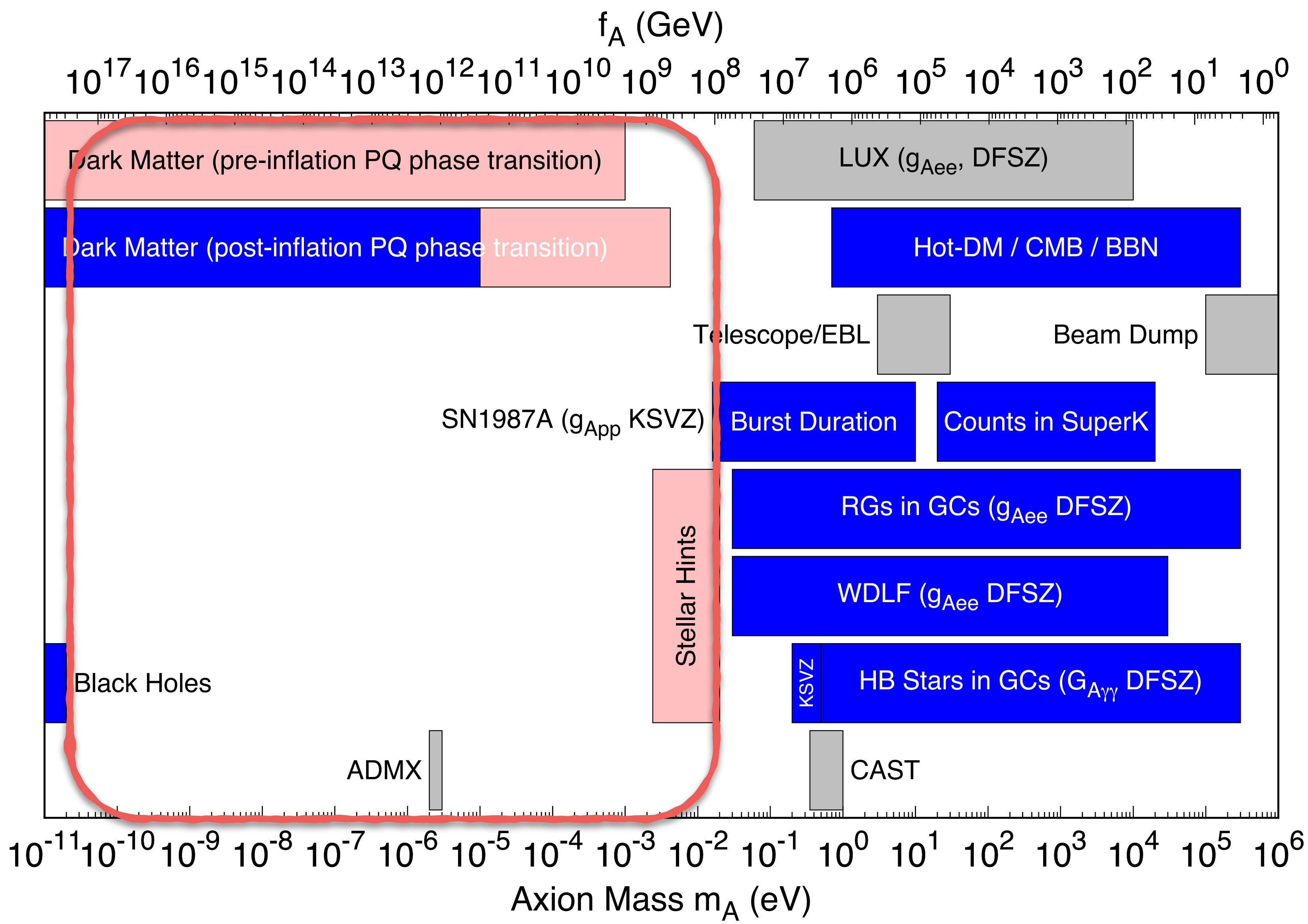
Astro/cosmo exclusions

DM explained / Astro Hints

Axion landscape

- axion mass $m_a \simeq m_\pi \frac{f_\pi}{f_a} \simeq 6 \text{ meV} \frac{10^9 \text{ GeV}}{f_a}$

- axion couplings $\sim 1/f_a$



[Ringwald, Rosenberg, Rybka, Particle Data Group (2017)]

Lab exclusions

Astro/cosmo exclusions

DM explained / Astro Hints

Current limits and search strategies

- Astrophysical bounds

[For a collection see e.g. Raffelt, hep-ph/0611350, Giannotti et al., 1708.02111]

- Star evolution $g_{a\gamma\gamma} \lesssim 6.6 \times 10^{-11}\,\mathrm{GeV}^{-1}$
- White dwarf cooling $g_{aee} \lesssim 1.3 \times 10^{-13}\,\mathrm{GeV}^{-1}$
- Supernova SN1987A $g_{aNN} \lesssim 3 \times 10^{-7}\ \mathrm{GeV}^{-1}$ (KSVZ) → $f_a \gtrsim 4 \times 10^{8}\ \mathrm{GeV}$

- The translation of the bound on f_a requires the specification of a UV completion

- e.g. axion-nucleon coupling can be sizeably suppressed in the presence of a generation dependent PQ symmetry (astrophobic axion) [LDL, Mescia, Nardi, Panci, Ziegler, 1712.04940]

[☞ see R. Ziegler's talk]

Current limits and search strategies

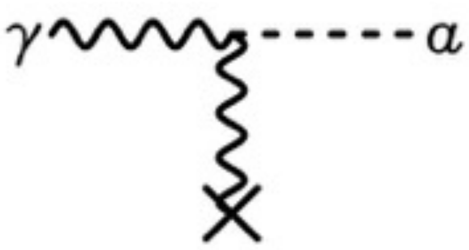
- Astrophysical bounds

[For a collection see e.g. Raffelt, hep-ph/0611350, Giannotti et al., 1708.02111]

- Most laboratory search techniques are sensitive to $g_{a\gamma\gamma}$

Primakoff effect: axion-photon transition in external static E or B field

$$\mathcal{L}_{a\gamma\gamma} = -\frac{1}{4} g_{a\gamma\gamma}\, a\, F \cdot \tilde{F} = g_{a\gamma\gamma}\, a\, \mathbf{E} \cdot \mathbf{B}$$



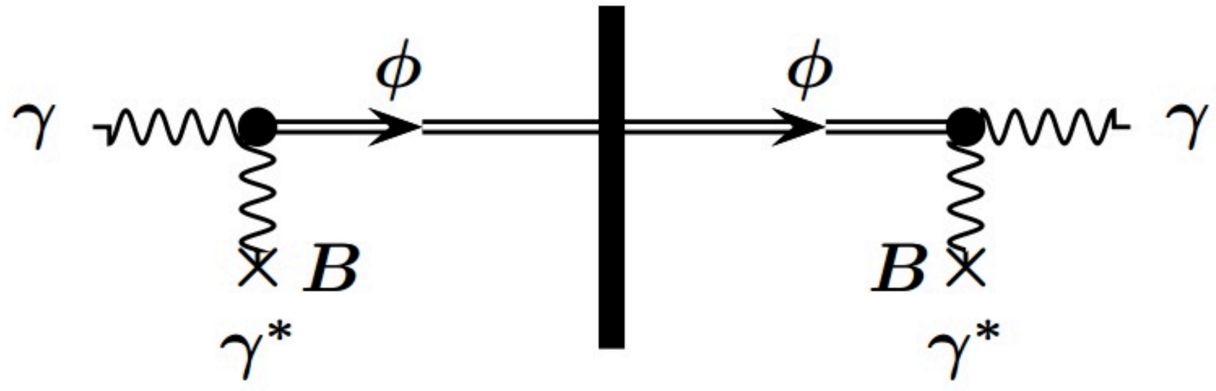
- Light Shining through Walls [See e.g. Redondo, Ringwald hep-ph/10113741]

- Haloscopes (axion Dark Matter) [Sikivie PRL 51 (1983)]

- Helioscopes (axions from the Sun)

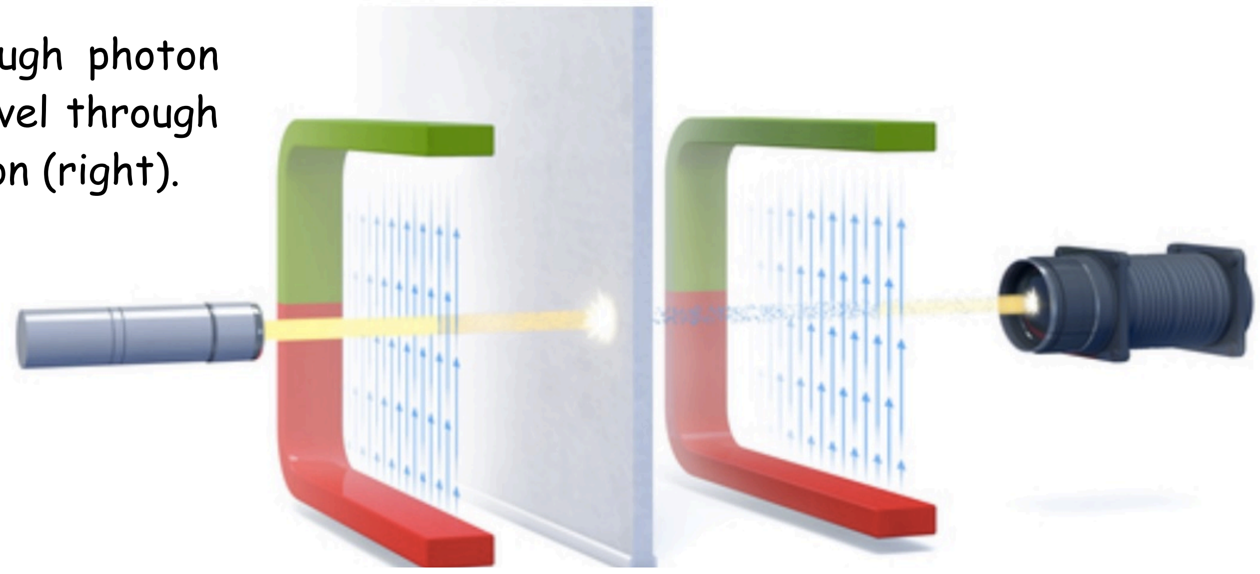
Light Shining through Walls (LSW)

- Any Light Particle Search (DESY): ALPS-I (2007-2010) and ALPS-II (2013-…)



Schematic view of axion (or ALP) production through photon conversion in a magnetic field (left), subsequent travel through a wall, and final detection through photon regeneration (right).

Artist view of a light shining through a wall experiment



- LSW experiments pay a $g^4_{a\gamma\gamma}$ suppression

Haloscopes

- Look for DM axions with a microwave resonant cavity [☞ see N. Crisosto's talk]

  - power of axions converting into photons in an EM cavity

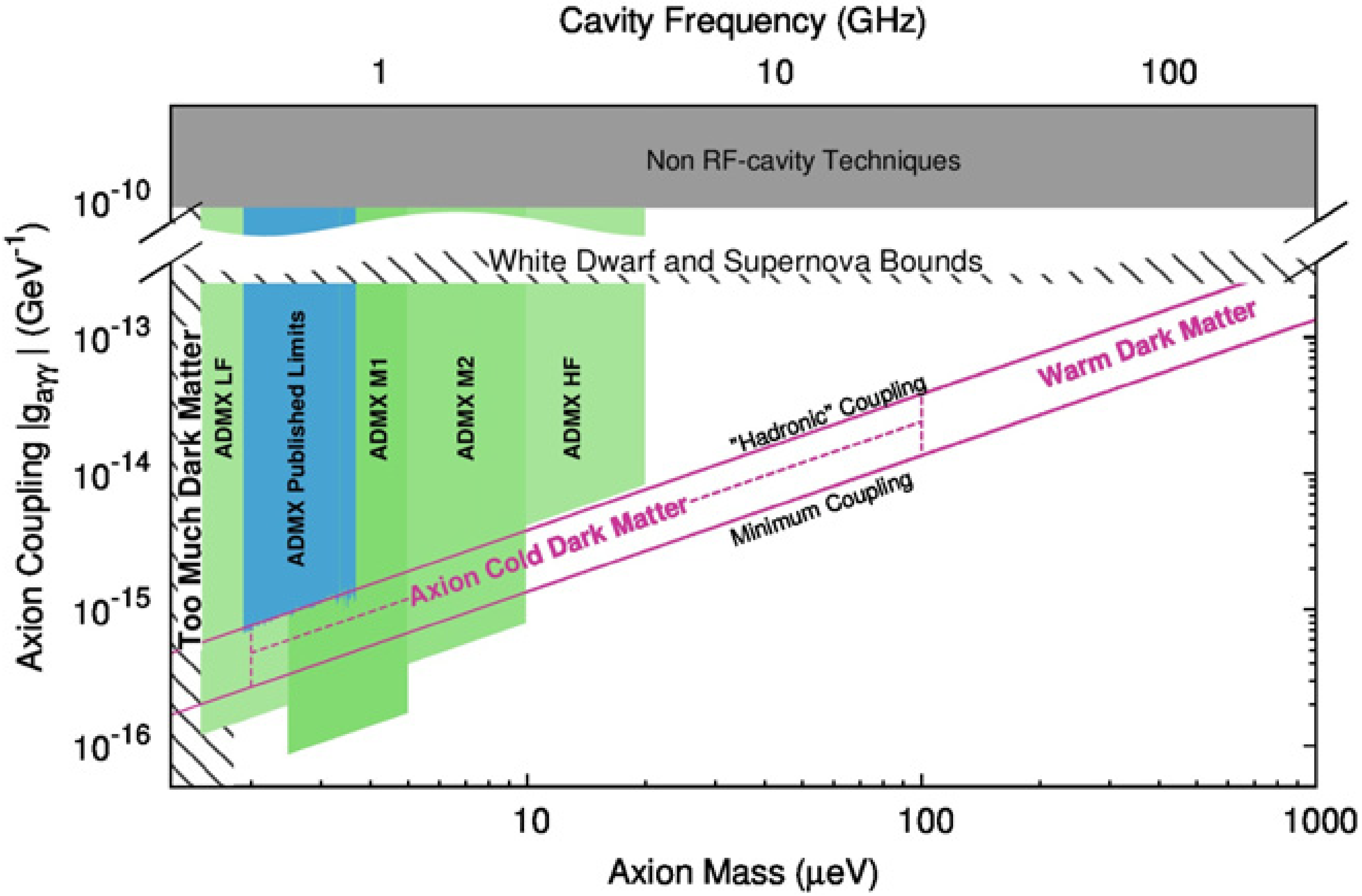
$$P_a = C g_{a\gamma\gamma}^2 V B_0^2 \frac{\rho_a}{m_a} Q_{\rm eff}$$

  - resonance condition: need to tune the frequency of the EM cavity on the axion mass

Haloscopes

- Look for DM axions with a microwave resonant cavity [☞ see N. Crisosto's talk]

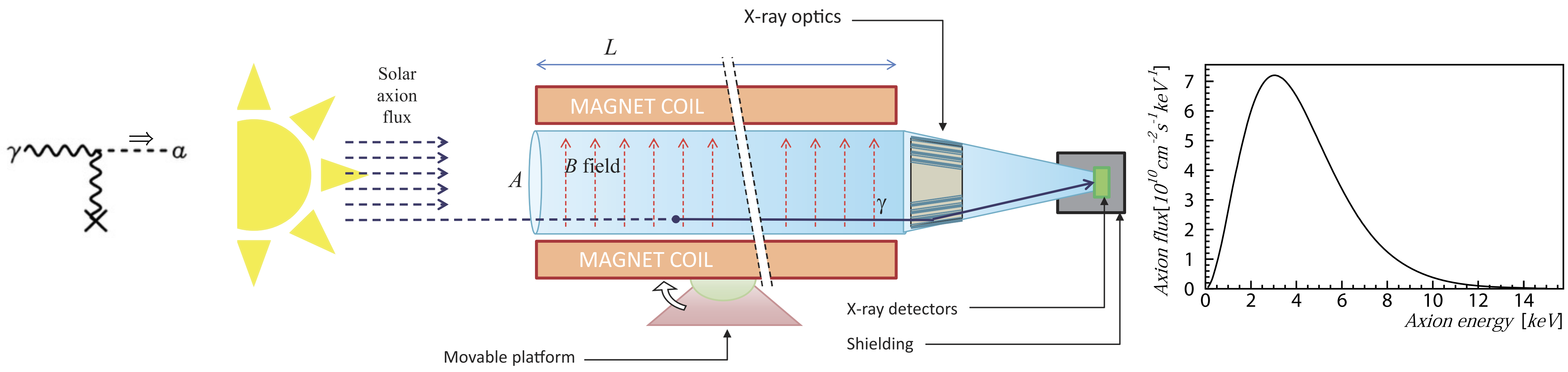
  - Axion Dark Matter eXperiment (ADMX) (U. of Washington)



[ADMX Collaboration, Phys. Dark Univ. 4 (2014)]

Helioscopes

- The Sun is a potential axion source



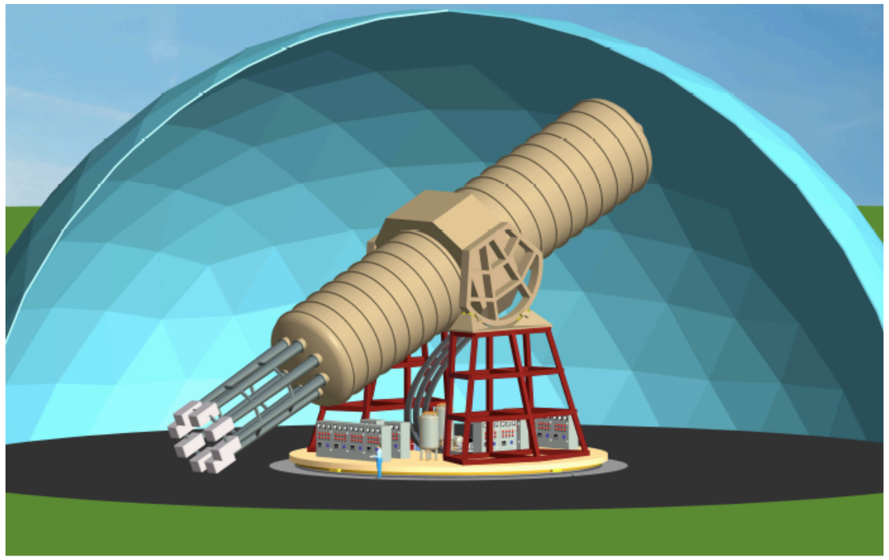
- macroscopic B-field can provide a coherent axion-photon (x-ray) conversion rate over a big volume

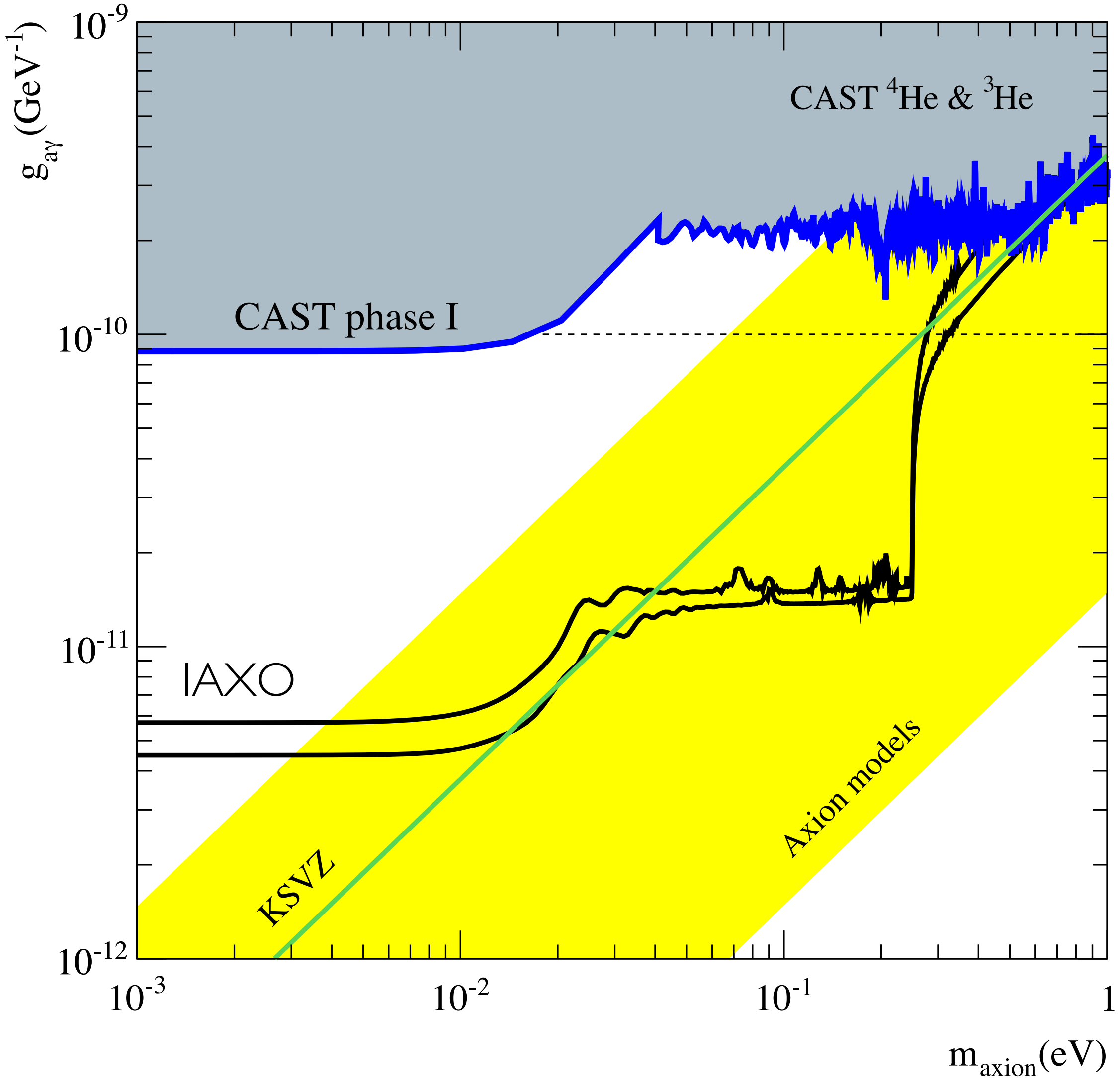
Helioscopes

- The Sun is a potential axion source

  - CERN Axion Solar Telescope (CAST)

  - International AXion Observatory (IAXO)





[IAXO "Letter of intent", CERN-SPSC-2013-022]

New exp. proposals [incomplete list!]

PHYSICAL REVIEW X 4, 021030 (2014)

Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr)

Dmitry Budker,[1,5] Peter W. Graham,[2] Micah Ledbetter,[3] Surjeet Rajendran,[2] and Alexander O. Sushkov[4]

PRL 113, 161801 (2014) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2014

Resonantly Detecting Axion-Mediated Forces with Nuclear Magnetic Resonance

Asimina Arvanitaki[1] and Andrew A. Geraci[2,*]

PRL 117, 141801 (2016) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2016

Broadband and Resonant Approaches to Axion Dark Matter Detection

Yonatan Kahn,[1,*] Benjamin R. Safdi,[2,†] and Jesse Thaler[2,‡]

PRL 118, 091801 (2017) PHYSICAL REVIEW LETTERS week ending 3 MARCH 2017

Dielectric Haloscopes: A New Way to Detect Axion Dark Matter

Allen Caldwell,[1] Gia Dvali,[1,2,3] Béla Majorovits,[1] Alexander Millar,[1] Georg Raffelt,[1] Javier Redondo,[1,4] Olaf Reimann,[1] Frank Simon,[1] and Frank Steffen[1]
(MADMAX Working Group)

Searching for galactic axions through magnetized media: The QUAX proposal

R. Barbieri[a,b], C. Braggio[c], G. Carugno[c], C.S. Gallo[c], A. Lombardi[d], A. Ortolan[d], R. Pengo[d], G. Ruoso[d,*], C.C. Speake[e]

PHYSICAL REVIEW D 91, 084011 (2015)

Discovering the QCD axion with black holes and gravitational waves

Asimina Arvanitaki[*]
Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

Masha Baryakhtar[†] and Xinlu Huang[‡]
Stanford Institute for Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 16 December 2014; published 7 April 2015)

PHYSICAL REVIEW D 91, 011701(R) (2015)

Search for dark matter axions with the Orpheus experiment

Gray Rybka,[*] Andrew Wagner,[†] Kunal Patel, Robert Percival, and Katleiah Ramos
University of Washington, Seattle, Washington 98195, USA

Aryeh Brill
Yale University, New Haven, Connecticut 06520, USA
(Received 16 November 2014; published 21 January 2015)

CULTASK, The Coldest Axion Experiment at CAPP/IBS/KAIST in Korea

Woohyun Chung*
Center for Axion and Precision Physics Research, Institute for Basic Science (IBS), Republic of Korea

New exp. proposals [incomplete list!]

PHYSICAL REVIEW X **4,** 021030 (2014)

Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr)

Dmitry Budker,[1,5] Peter W. Graham,[2] Micah Ledbetter,[3] Surjeet Rajendran,[2] and Alexander O. Sushkov[4]

PRL **113,** 161801 (2014) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2014

Resonantly Detecting Axion-Mediated Forces with Nuclear Magnetic Resonance

Asimina Arvanitaki[1] and Andrew A. Geraci[2,*]

g_{aNN}

Searching for galactic axions through magnetized media: The QUAX proposal

R. Barbieri[a,b], C. Braggio[c], G. Carugno[c], C.S. Gallo[c], A. Lombardi[d], A. Ortolan[d], R. Pengo[d], G. Ruoso[d,*], C.C. Speake[e]

g_{aee}

PHYSICAL REVIEW D **91,** 084011 (2015)

Discovering the QCD axion with black holes and gravitational waves

Asimina Arvanitaki[*]

Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

Masha Baryakhtar[†] and Xinlu Huang[‡]

Stanford Institute for Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305, USA

(Received 16 December 2014; published 7 April 2015)

PRL **117,** 141801 (2016) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2016

Broadband and Resonant Approaches to Axion Dark Matter Detection

Yonatan Kahn,[1,*] Benjamin R. Safdi,[2,†] and Jesse Thaler[2,‡]

PRL **118,** 091801 (2017) PHYSICAL REVIEW LETTERS week ending 3 MARCH 2017

Dielectric Haloscopes: A New Way to Detect Axion Dark Matter

Allen Caldwell,[1] Gia Dvali,[1,2,3] Béla Majorovits,[1] Alexander Millar,[1] Georg Raffelt,[1] Javier Redondo,[1,4] Olaf Reimann,[1] Frank Simon,[1] and Frank Steffen[1]
(MADMAX Working Group)

PHYSICAL REVIEW D **91,** 011701(R) (2015)

Search for dark matter axions with the Orpheus experiment

Gray Rybka,[*] Andrew Wagner,[†] Kunal Patel, Robert Percival, and Katleiah Ramos

University of Washington, Seattle, Washington 98195, USA

Aryeh Brill

Yale University, New Haven, Connecticut 06520, USA

(Received 16 November 2014; published 21 January 2015)

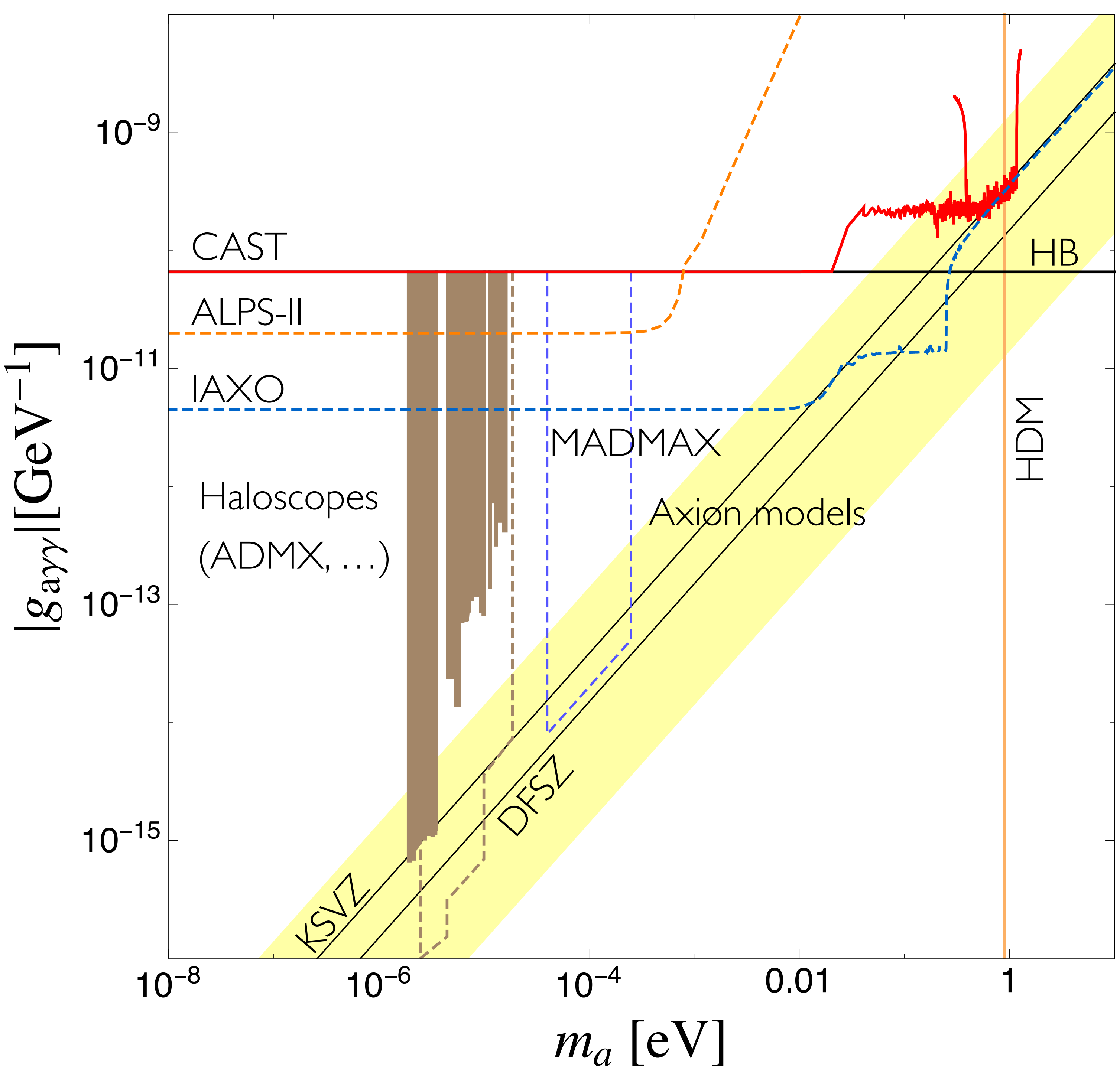
CULTASK, The Coldest Axion Experiment at CAPP/IBS/KAIST in Korea

Woohyun Chung*

Center for Axion and Precision Physics Research, Institute for Basic Science (IBS), Republic of Korea

$g_{a\gamma\gamma}$

Need to know where to search



$$g_{a\gamma\gamma} = \frac{m_a}{\mathrm{eV}} \frac{2.0}{10^{10}\ \mathrm{GeV}} \left(\frac{E}{N} - 1.92 \right)$$

E/N anomaly coefficients, depend on UV completion

$$|E/N - 1.92| \in [0.07, 7]$$

[Particle Data Group (since end of 90's). Chosen to include some representative KSVZ/DFSZ models e.g. from:
- Kaplan, NPB 260 (1985),
- Cheng, Geng, Ni, PRD 52 (1995),
- Kim, PRD 58 (1998)]

KSVZ axions

- Field content

Field	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
Q_L	1/2	$\mathcal{C}_Q$	$\mathcal{I}_Q$	$\mathcal{Y}_Q$	$\mathcal{X}_L$
Q_R	1/2	$\mathcal{C}_Q$	$\mathcal{I}_Q$	$\mathcal{Y}_Q$	$\mathcal{X}_R$
Φ	0	1	1	0	1

- PQ charges carried by a vector-like quark $Q = Q_L + Q_R$

- original KSVZ model assumes Q ~ (3,1,0), but in general only $\mathcal{C}_Q \neq I$ required

$$\partial^\mu J_\mu^{PQ} = \frac{N\alpha_s}{4\pi} G \cdot \tilde{G} + \frac{E\alpha}{4\pi} F \cdot \tilde{F}$$

$$N = \sum_Q (\mathcal{X}_L - \mathcal{X}_R)\, T(\mathcal{C}_Q)$$

$$E = \sum_Q (\mathcal{X}_L - \mathcal{X}_R)\, \mathcal{Q}_Q^2$$

} anomaly coeff.

and a SM singlet **Φ** containing the "invisible" axion $(f_a \gg v)$

$$\Phi(x) = \frac{1}{\sqrt{2}} [\rho(x) + f_a]\, e^{ia(x)/f_a}$$

KSVZ axions

- Field content

Field	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
Q_L	$1/2$	$\mathcal{C}_Q$	$\mathcal{I}_Q$	$\mathcal{Y}_Q$	$\mathcal{X}_L$
Q_R	$1/2$	$\mathcal{C}_Q$	$\mathcal{I}_Q$	$\mathcal{Y}_Q$	$\mathcal{X}_R$
Φ	0	1	1	0	1

- Lagrangian

$$\mathcal{L}_a = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm PQ} - V_{H\Phi} + \mathcal{L}_{Qq} \qquad |\mathcal{X}_L - \mathcal{X}_R| = 1$$

- $\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c.})$ → $m_Q = y_Q f_a / \sqrt{2}$
- $V_{H\Phi} = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \lambda_{H\Phi} |H|^2 |\Phi|^2$ → $m_\rho \sim f_a$
- $\mathcal{L}_{Qq}$ d ≤ 4 mixing with SM quarks (depends in Q-gauge quantum numbers)

Q stability

- Symmetry of the kinetic term

$$U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\Phi \xrightarrow{y_Q \neq 0} U(1)_{\rm PQ} \times U(1)_{\rm Q}$$

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q - (y_Q \overline{Q}_L Q_R \Phi + {\rm H.c.})$$

- $U(1)_Q$ is the Q-baryon number: <u>if exact, Q would be stable</u>

↓

cosmological issue if thermally produced
in the early universe !

Q stability

- Symmetry of the kinetic term

$$U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_{\Phi} \xrightarrow{y_Q \neq 0} U(1)_{\rm PQ} \times U(1)_{\rm Q}$$

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c.})$$

- $U(1)_Q$ is the Q-baryon number: <u>if exact, Q would be stable</u>

- if $\mathcal{L}_{Qq} \neq 0$ $U(1)_Q$ is further broken and Q-decay is possible [Ringwald, Saikawa, 1512.06436]

- decay also possible via d>4 operators (e.g. Planck-induced)

→ stability depends on Q representations

Selection criteria

- We require: [for $T_{reheating} > m_Q \sim f_a$ (post-inflat. PQ breaking)]

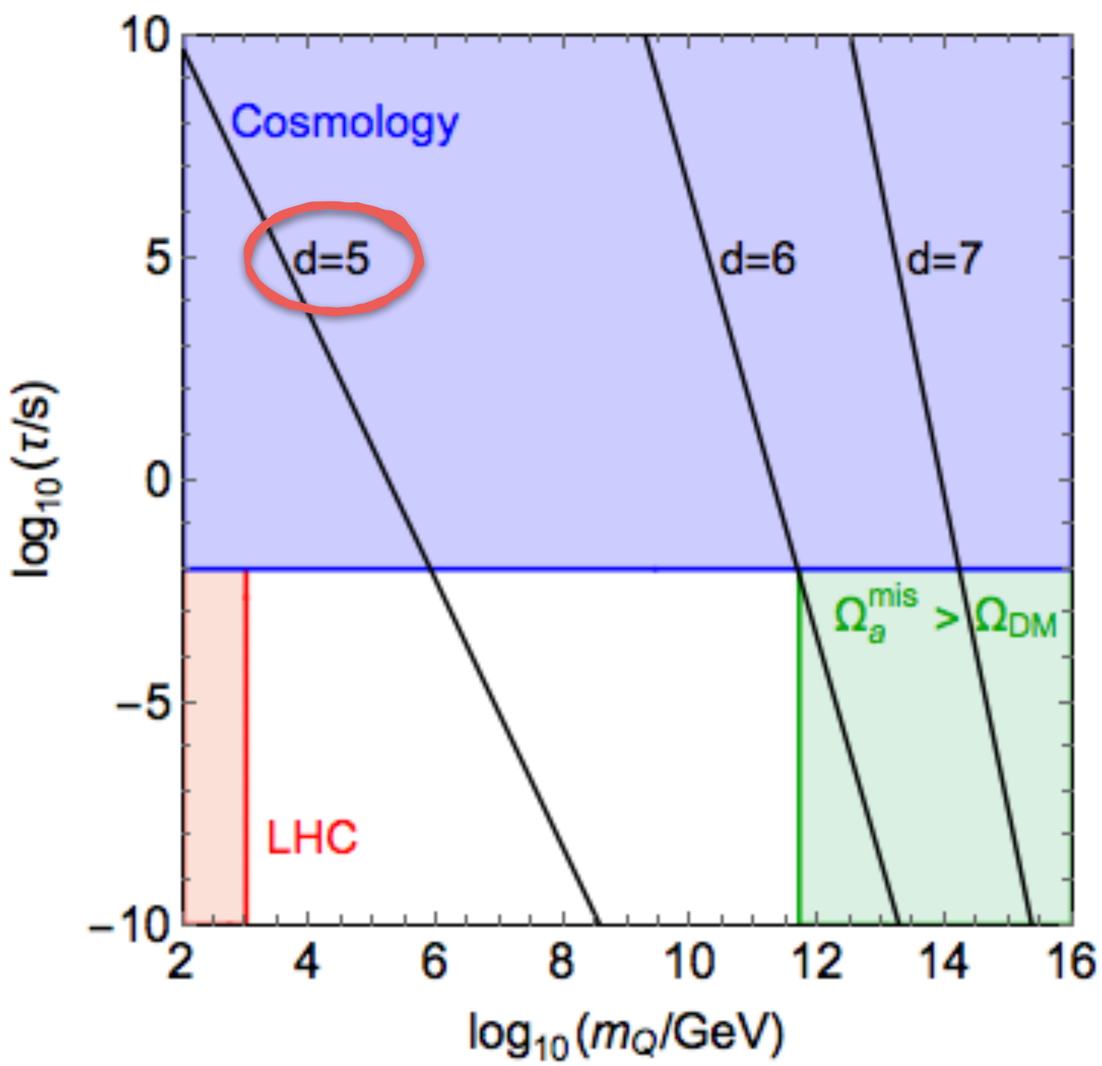
1. Q sufficiently short lived $\tau_Q \lesssim 10^{-2}$ s

- decays via d=4 operators are fast enough

- decays via effective operators

$$\mathcal{L}_{Qq}^{d>4} = \frac{1}{M_{\text{Planck}}^{(d-4)}} \mathcal{O}_{Qq}^{d>4} + \text{h.c.}$$

$$\Gamma_{\text{NDA}} = \frac{1}{4(4\pi)^{2n_f-3}(n_f-1)!(n_f-2)!} \frac{m_Q^{2d-7}}{M_{\text{Planck}}^{2(d-4)}}$$



→ "safe" Q must allow for d=4 or 5 decay op.

Selection criteria

- We require:

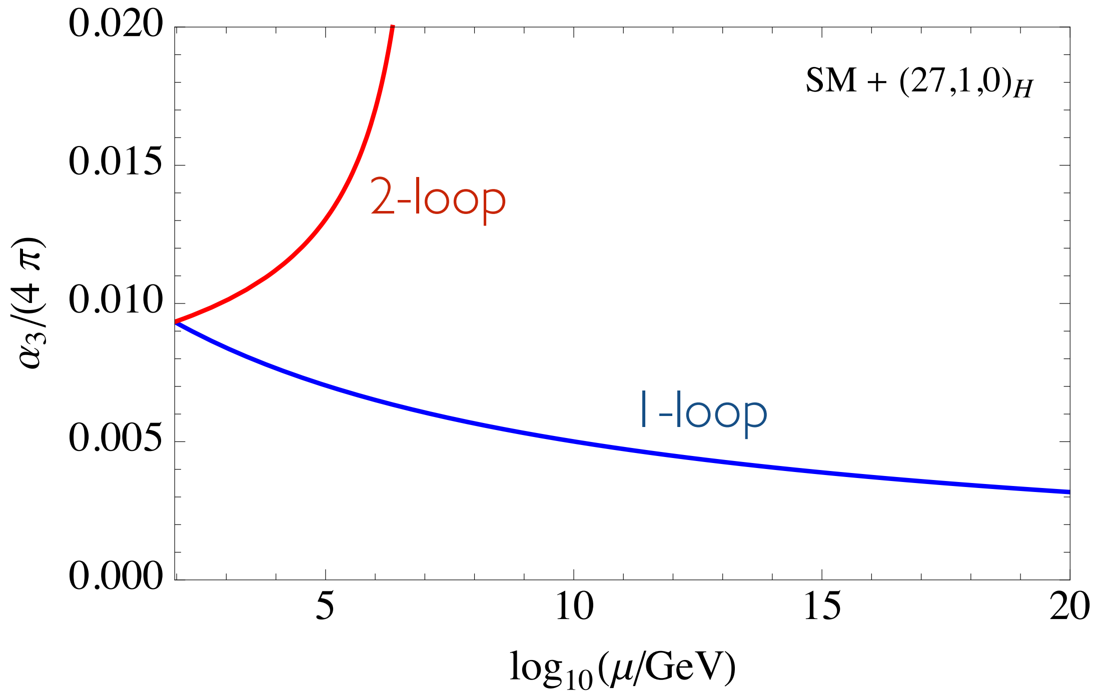
1. Q sufficiently short lived $\tau_Q \lesssim 10^{-2}$ s
2. No Landau poles below 10^{18} GeV

- bound on Q multiplet dimensionality

$$\mu \frac{d}{d\mu} g_i = -b_i \, g_i^3 \qquad b_i = \text{gauge -matter}$$

N.B. two-loop effects crucial if 1-loop b.f. is accidentally small

[LDL, Gröber, Kamenik, Nardecchia, 1504.00359]



Selection criteria

- We require:

1. Q sufficiently short lived $\tau_Q \lesssim 10^{-2}$ s
2. No Landau poles below 10^{18} GeV
3. Absence of domain walls *[see backup slides]*
4. Q-assisted unification *[see backup slides]*

Phenomenologically preferred Q's

- Only 15 Q's survive

$$g_{a\gamma\gamma} = \frac{m_a}{\text{eV}} \frac{2.0}{10^{10}\ \text{GeV}} \left(\frac{E}{N} - 1.92(4) \right)$$

$$\frac{E}{N} = \frac{\sum_Q \mathcal{Q}_Q^2}{\sum_Q T(\mathcal{C}_Q)}$$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{2-\text{loop}}_{\text{Landau}}[\text{GeV}]$	E/N
$(3,1,-1/3)$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$
$(3,1,2/3)$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$
$(3,2,1/6)$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$
$(3,2,-5/6)$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$
$(3,2,7/6)$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$
$(3,3,-1/3)$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$
$(3,3,2/3)$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$
$(3,3,-4/3)$	$\overline{Q}_L d_R H^{\dagger 2}$	$3.5 \cdot 10^{18}(g_1)$	$44/3$
$(\overline{6},1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$2.3 \cdot 10^{37}(g_1)$	$4/15$
$(\overline{6},1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$5.1 \cdot 10^{30}(g_1)$	$16/15$
$(\overline{6},2,1/6)$	$\overline{Q}_R \sigma_{\mu\nu} q_L G^{\mu\nu}$	$7.3 \cdot 10^{38}(g_1)$	$2/3$
$(8,1,-1)$	$\overline{Q}_L \sigma_{\mu\nu} e_R G^{\mu\nu}$	$7.6 \cdot 10^{22}(g_1)$	$8/3$
$(8,2,-1/2)$	$\overline{Q}_R \sigma_{\mu\nu} \ell_L G^{\mu\nu}$	$6.7 \cdot 10^{27}(g_1)$	$4/3$
$(15,1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$8.3 \cdot 10^{21}(g_3)$	$1/6$
$(15,1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$7.6 \cdot 10^{21}(g_3)$	$2/3$

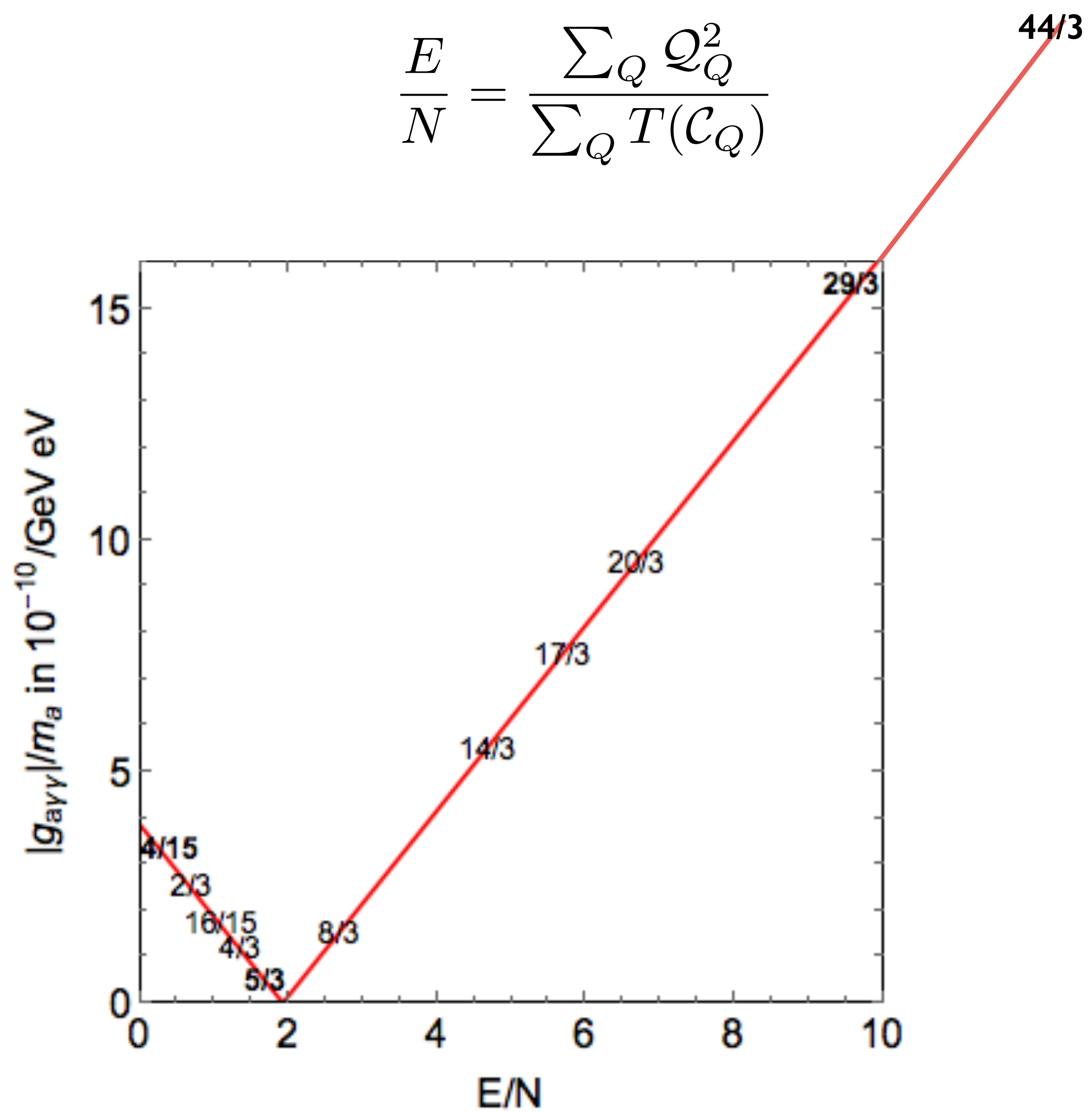
Phenomenologically preferred Q's

- Only 15 Q's survive

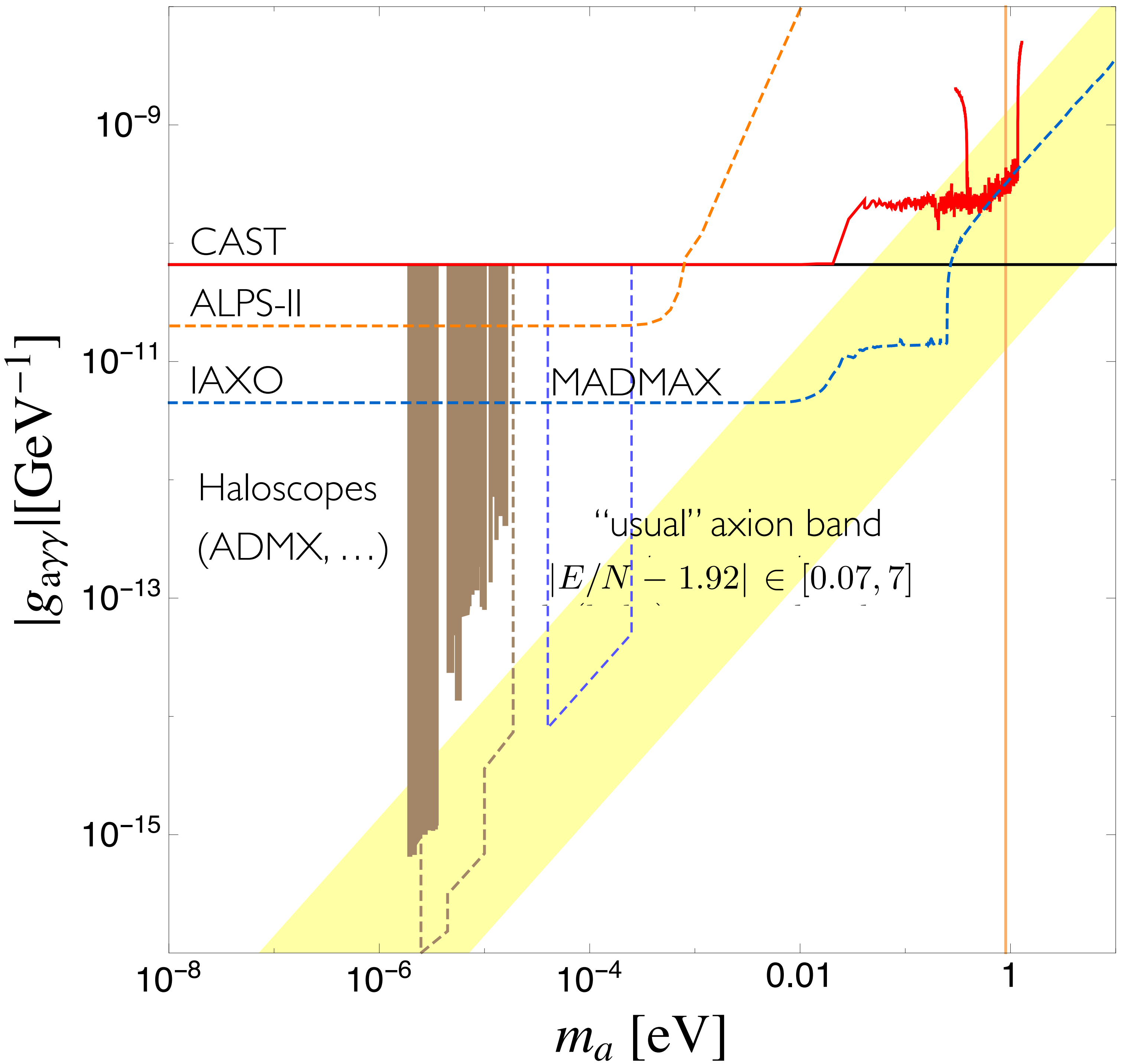
$$g_{a\gamma\gamma} = \frac{m_a}{\text{eV}} \frac{2.0}{10^{10}\ \text{GeV}} \left(\frac{E}{N} - 1.92(4) \right)$$

$$\frac{E}{N} = \frac{\sum_Q \mathcal{Q}_Q^2}{\sum_Q T(\mathcal{C}_Q)}$$

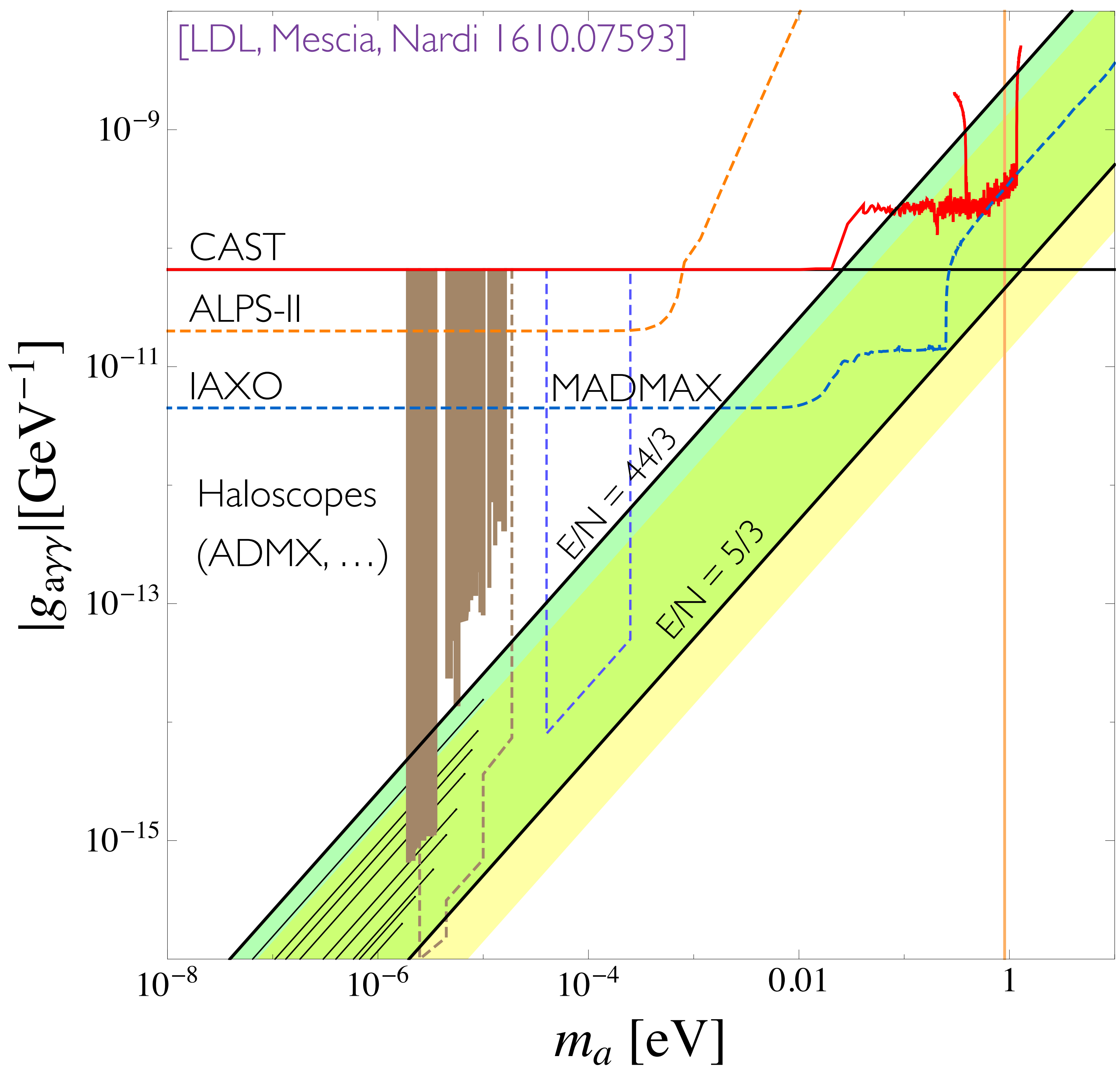
	R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{2-\text{loop}}_{\text{Landau}}[\text{GeV}]$	E/N
	$(3,1,-1/3)$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$
	$(3,1,2/3)$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$
R_Q^w	$(3,2,1/6)$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$
	$(3,2,-5/6)$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$
	$(3,2,7/6)$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$
	$(3,3,-1/3)$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$
	$(3,3,2/3)$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$
R_Q^s	$(3,3,-4/3)$	$\overline{Q}_L d_R H^{\dagger 2}$	$3.5 \cdot 10^{18}(g_1)$	$44/3$
	$(\overline{6},1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$2.3 \cdot 10^{37}(g_1)$	$4/15$
	$(\overline{6},1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$5.1 \cdot 10^{30}(g_1)$	$16/15$
	$(\overline{6},2,1/6)$	$\overline{Q}_R \sigma_{\mu\nu} q_L G^{\mu\nu}$	$7.3 \cdot 10^{38}(g_1)$	$2/3$
	$(8,1,-1)$	$\overline{Q}_L \sigma_{\mu\nu} e_R G^{\mu\nu}$	$7.6 \cdot 10^{22}(g_1)$	$8/3$
	$(8,2,-1/2)$	$\overline{Q}_R \sigma_{\mu\nu} \ell_L G^{\mu\nu}$	$6.7 \cdot 10^{27}(g_1)$	$4/3$
	$(15,1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$8.3 \cdot 10^{21}(g_3)$	$1/6$
	$(15,1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$7.6 \cdot 10^{21}(g_3)$	$2/3$



Redefining the axion window



Redefining the axion window



More Q's

- What about $N_Q > 1$?

  - combined anomaly factor for $R_Q^1 + R_Q^2 + \dots$: $$\frac{E_c}{N_c} = \frac{E_1 + E_2 + \dots}{N_1 + N_2 + \dots}$$

- Strongest coupling (compatible with LP criterium) is given by

$$(3,3,-4/3) \oplus (3,3,-1/3) \ominus (\overline{6},1,-1/3) \longrightarrow E_c/N_c = 170/3$$

- Complete decoupling within theoretical error is possible as well:

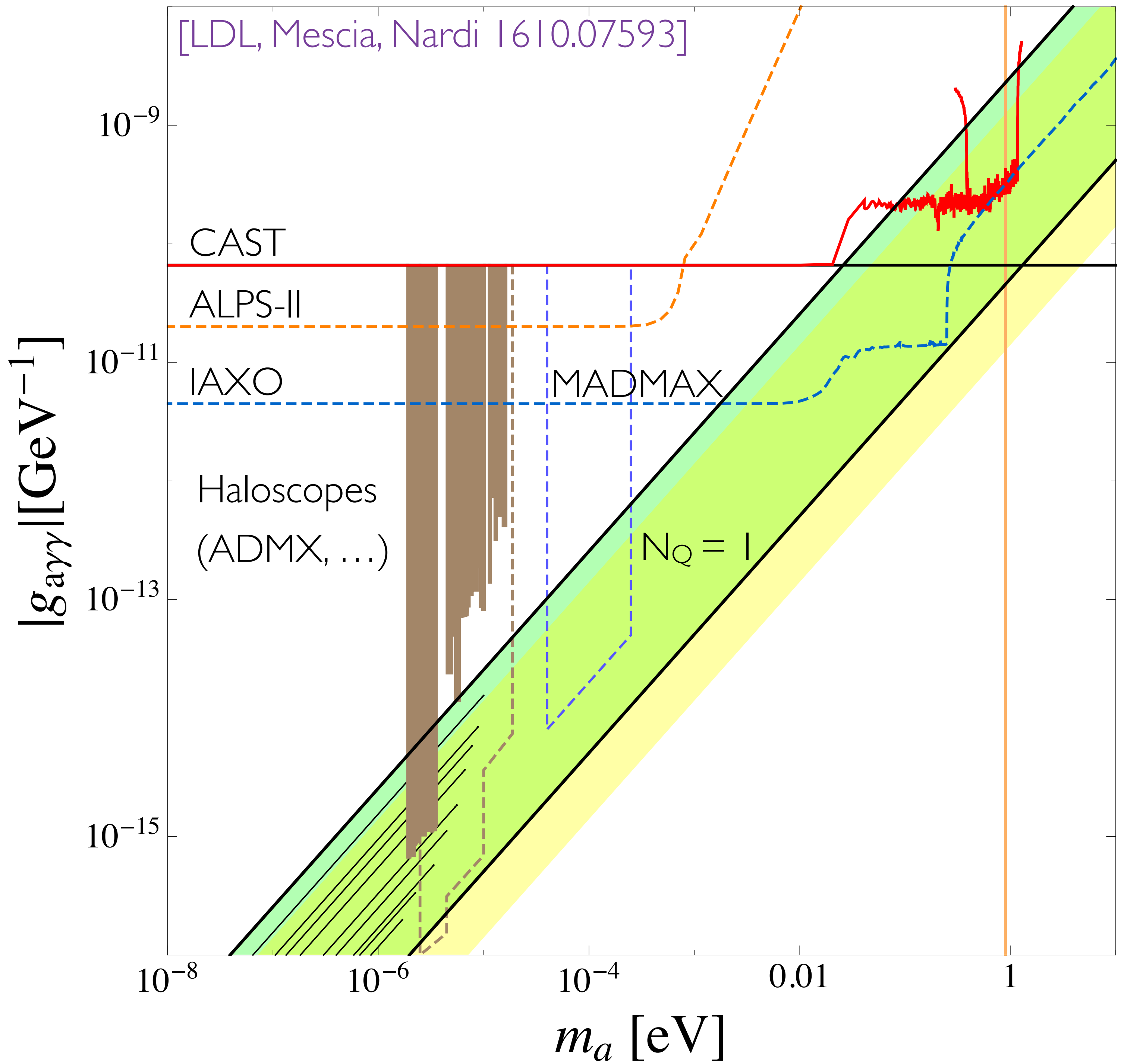
$$\left.\begin{array}{c}(3,3,-1/3) \oplus (\overline{6},1,-1/3) \\ (\overline{6},1,2/3) \oplus (8,1,-1) \\ (3,2,-5/6) \oplus (8,2,-1/2)\end{array}\right\} \quad E_c/N_c = (23/12, 64/33, 41/21) \approx (1.92, 1.94, 1.95)$$

$$g_{a\gamma\gamma} = \frac{m_a}{\text{eV}} \frac{2.0}{10^{10}\ \text{GeV}} \left(\frac{E_c}{N_c} - 1.92(4)\right)$$

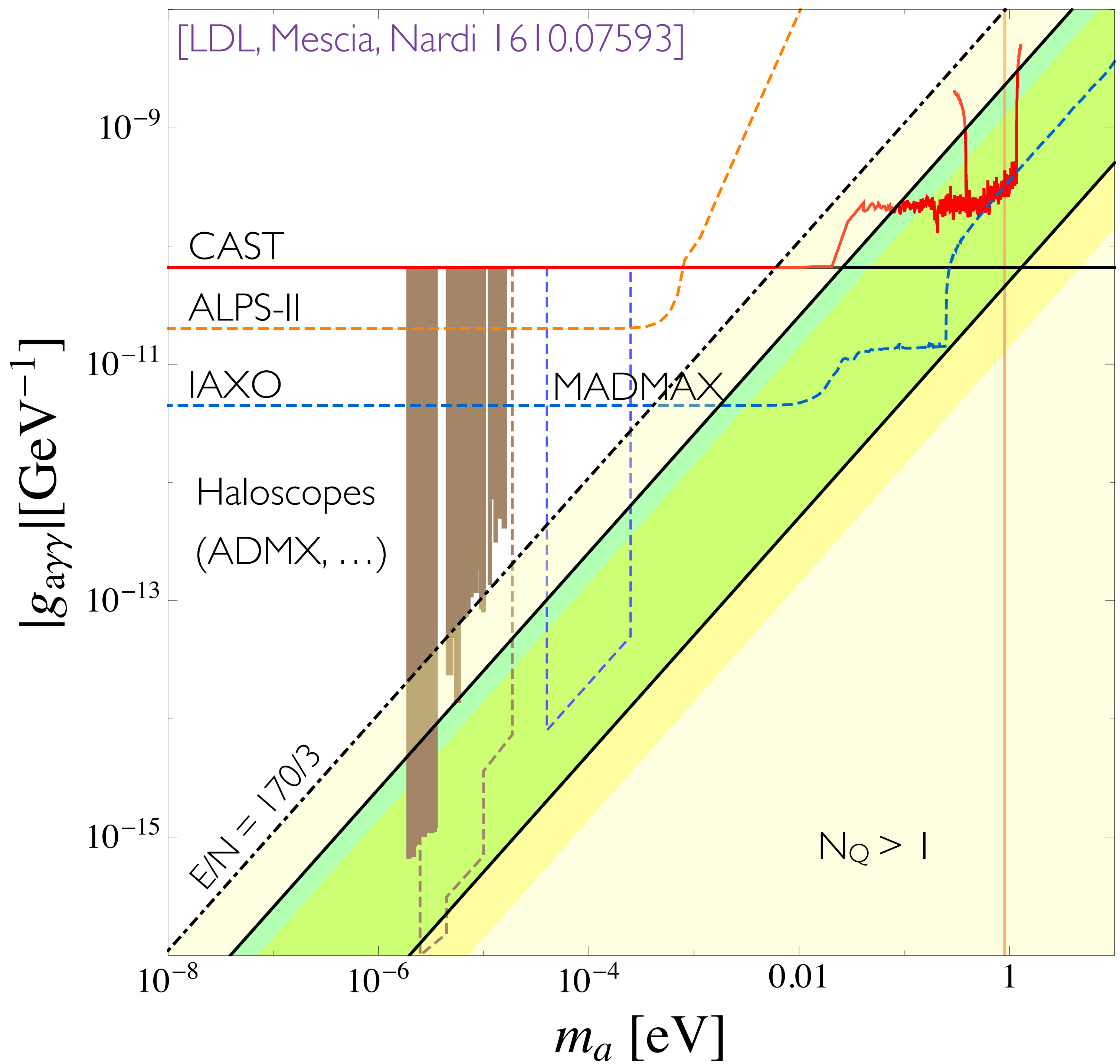
[Theoretical error from NLO χPT Grilli di Cortona et al., 1511.02867]

"such a cancellation is immoral, but not unnatural" [D. B. Kaplan, (1985)]

More Q's


[LDL, Mescia, Nardi 1610.07593]
CAST
ALPS-II
IAXO
MADMAX
Haloscopes
(ADMX, ...)
$N_Q = 1$
10^{-9}
10^{-11}
10^{-13}
10^{-15}
$|g_{a\gamma\gamma}|[\mathrm{GeV}^{-1}]$
10^{-8}
10^{-6}
10^{-4}
0.01
1
m_a [eV]

More Q's



[LDL, Mescia, Nardi 1610.07593]
CAST
ALPS-II
IAXO
MADMAX
Haloscopes
(ADMX, ...)
E/N = 170/3
N_Q > 1
10^{-9}
10^{-11}
10^{-13}
10^{-15}
$|g_{a\gamma\gamma}|[\mathrm{GeV}^{-1}]$
10^{-8}
10^{-6}
10^{-4}
0.01
1
m_a [eV]

KSVZ in pre-inflationary scenarios

- What about $T_{reheating} < m_Q$? [LDL, Mescia, Nardi 1705.05370]

  - condition on Q decay is relaxed, but Landau pole still applies

- $m_Q \sim y_Q f_a < 5 \cdot 10^{11}$ GeV

  - $N_Q = 1$: $(E/N)_{max(pre)} = 2.5\ (E/N)_{max(post)}$

  - $N_Q > 1$: $(E/N)_{max(pre)} = 1.2\ (E/N)_{max(post)}$

  → *axion-photon coupling well-described by post-inflationary axion window*

- $f_a \gg 5 \cdot 10^{11}$ GeV (requires $\theta_0 \ll 1$) softens Landau pole condition

  → *arbitrarily large axion-photon coupling at the cost of tuning initial mis. condition*

DFSZ-like axions

- Potentially large E/N due to electron PQ charge

$$\frac{E}{N} = \frac{\sum_j \left(\frac{4}{3}X_u^j + \frac{1}{3}X_d^j + X_e^j\right)}{\sum_j \left(\frac{1}{2}X_u^j + \frac{1}{2}X_d^j\right)}$$

- with n_H Higgs doublets and a SM singlet ϕ, enhanced global symmetry

$$U(1)^{n_H+1} \to U(1)_{\mathrm{PQ}} \times U(1)_Y$$

must be explicitly broken in the scalar potential via non-trivial invariants (e.g. $H_u H_d \Phi^2$)

→ *non-trivial constraints on PQ charges of SM fermions*

DFSZ-like axions

- Potentially large E/N due to electron PQ charge

$$\frac{E}{N} = \frac{\sum_j \left(\frac{4}{3}X_u^j + \frac{1}{3}X_d^j + X_e^j\right)}{\sum_j \left(\frac{1}{2}X_u^j + \frac{1}{2}X_d^j\right)}$$

$$\begin{aligned}\mathcal{L}_Y &= Y_u\,\overline{Q}_L u_R H_u + Y_d\,\overline{Q}_L d_R H_d \\ &+ Y_e\,\overline{L}_L e_R H_e + \text{h.c.}\end{aligned}$$

- With 2 or 3 Higgs doublets, DFSZ remains within N_Q = 1 KSVZ window

- n_H = 2

$$\begin{array}{lll}\text{DFSZ-I}: & X_e = X_d & E/N = 8/3 \\ \text{DFSZ-II}: & X_e = -X_u & E/N = 2/3\end{array}$$

- n_H = 3

$$\text{DFSZ-III}: \quad X_e \neq X_{u,d} \quad E/N_{(\max)} = -4/3$$

DFSZ-like axions

- Potentially large E/N due to electron PQ charge

$$\frac{E}{N} = \frac{\sum_j \left(\frac{4}{3}X_u^j + \frac{1}{3}X_d^j + X_e^j\right)}{\sum_j \left(\frac{1}{2}X_u^j + \frac{1}{2}X_d^j\right)}$$

$$\mathcal{L}_Y = Y_u \overline{Q}_L u_R H_u + Y_d \overline{Q}_L d_R H_d + Y_e \overline{L}_L e_R H_e + \text{h.c.}$$

- With 2 or 3 Higgs doublets, DFSZ remains within N_Q = 1 KSVZ window

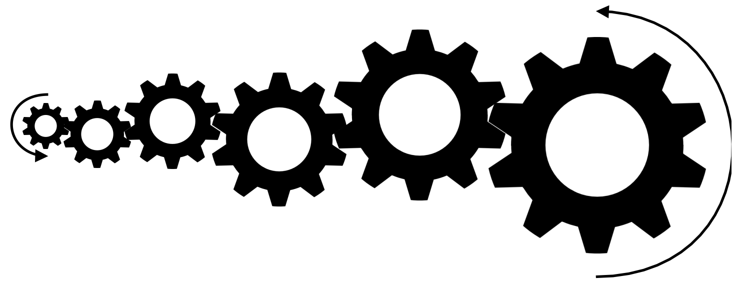
- Clockwork-like scenarios allow to boost E/N [LDL, Mescia, Nardi 1705.05370]

- n up-type doublets which *do not couple* to SM fermions (n ≲ 50 from LP condition)

$$(H_u H_d \Phi^2)$$

$$(H_k H_{k-1}^*)(H_{k-1}^* H_d^*)$$

$$(H_e H_n)(H_n H_d)$$



[Giudice, McCullough]

$$E/N \sim 2^n$$

[See also Farina et al. 1611.09855, for KSVZ clockwork]

Summary axion-photon

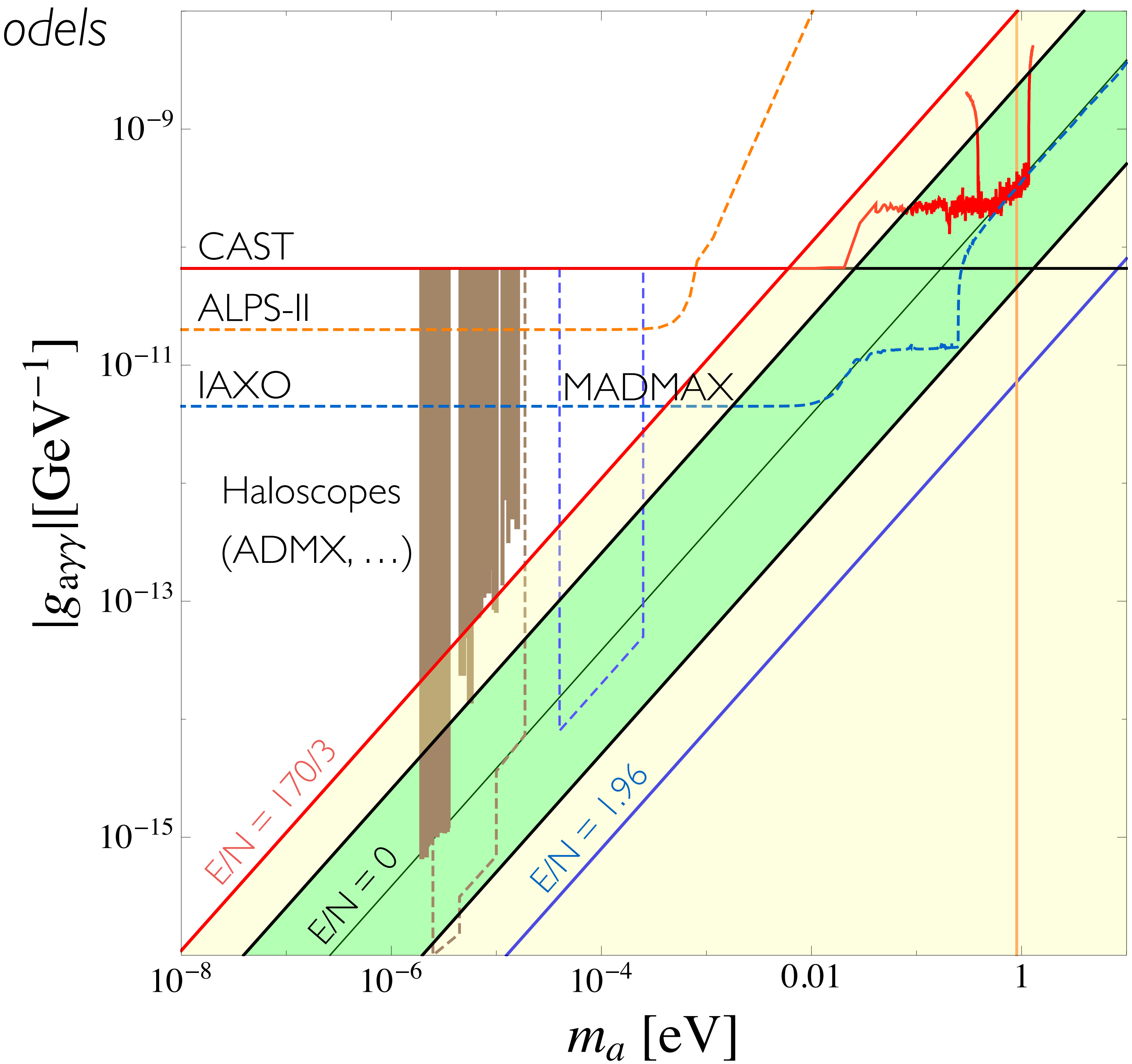
Region of 'realistic' KSVZ/DFSZ axion models

Going above red line requires either:

i) very exotic constructions

ii) tuning $\theta_0 \ll 1$ (KSVZ pre-inflat.)

Going below blue line requires a 'tuning in theory space' < 2%



Conclusions

- The QCD axion is a well-motivated BSM scenario

  - solves the strong CP problem

  - provides an excellent DM candidate

- Healthy experimental program

  - experiments are entering now the preferred window for the QCD axion

→ Take home message: *axion couplings might sizeably deviate from the standard DFSZ/KSVZ benchmarks (relevant when confronting exp. sensitivities and bounds)*

Backup slides

Axion coupling to photons

- Axion effective Lagrangian

[See e.g. Grilli di Cortona et al., 1511.02867]

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\, a\, g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad g^0_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\frac{E}{N}$$

field-depended chiral transformation to eliminate aGGtilde:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{2f_a}Q_a} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\mathrm{tr}\, Q_a = 1$$

Axion coupling to photons

- Axion effective Lagrangian

[See e.g. Grilli di Cortona et al., 1511.02867]

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\,a\,g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$g^0_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\frac{E}{N}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{2f_a}Q_a}\begin{pmatrix} u \\ d \end{pmatrix}$$

$$\mathrm{tr}\, Q_a = 1$$

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{4}\,a\,g_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 6\,\mathrm{tr}\left(Q_a Q^2\right)\right] = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - \frac{2}{3}\,\frac{4m_d + m_u}{m_d + m_u}\right] = \frac{m_a}{\mathrm{eV}}\;\frac{2.0}{10^{10}\ \mathrm{GeV}}\;\left(\frac{E}{N} - 1.92(4)\right)$$

$$Q_a = \frac{M_q^{-1}}{\langle M_q^{-1}\rangle}$$ (no axion-pion mixing)

model independent

depends on UV completion

Boosting E/N in DFSZ

1. Consider $(H_u H_d \Phi^2)$ and normalize $\mathcal{X}_\Phi \equiv q; \implies \mathcal{X}_u = -2q; \ \mathcal{X}_d = 0$

2. Define $H_1 = H_u$. Add n up-type doublets:

$$(H_k H^*_{k-1})(H^*_{k-1} H^*_d) \longrightarrow \mathcal{X}_k = -2^k q$$

3. Finally, couple also the "lepton" Higgs H_e

$$(H_e H_n)(H_n H_d) \longrightarrow \mathcal{X}_e = 2^{n+1} q$$

One can obtain in this way:

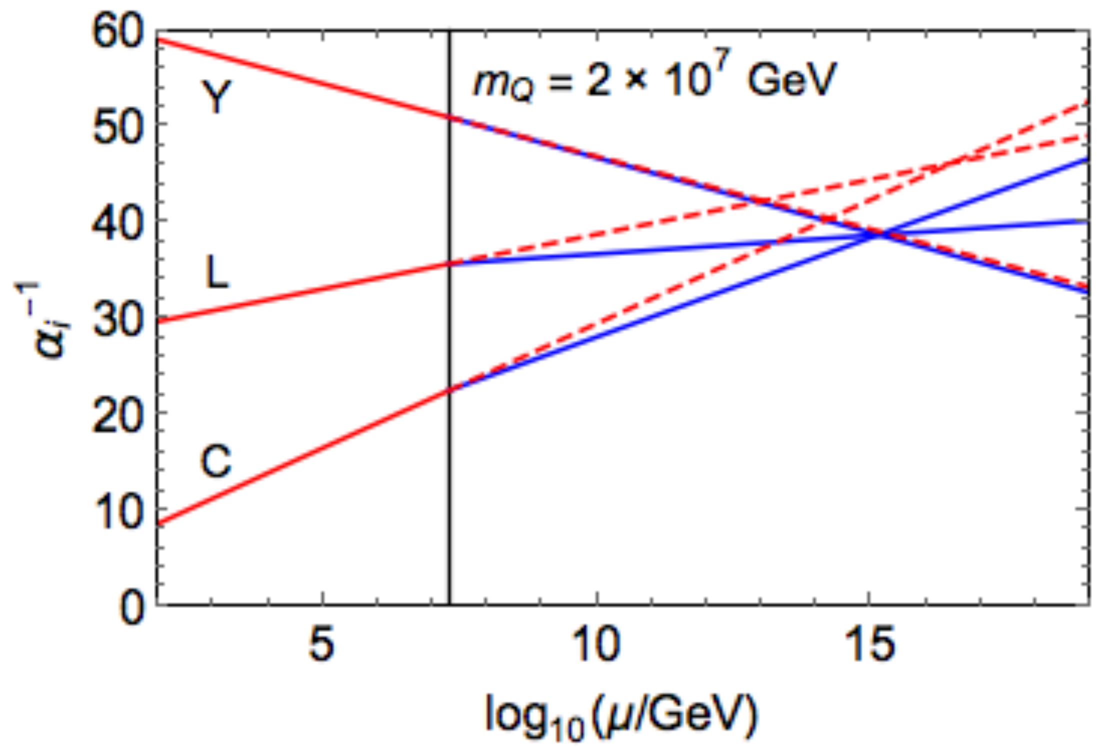
$$\frac{E}{N} = \frac{2}{3} + 2\frac{\mathcal{X}_u + \mathcal{X}_e}{\mathcal{X}_u + \mathcal{X}_d} \sim 2^{n+1}$$

- n ≲ 50 from Landau pole condition

Unificaxion

- Some Q's might improve gauge coupling unification [Giudice, Rattazzi, Strumia, 1204.5465]

  - out of all our 15 cases, just one works well: Q ~ (3, 2, 1/6)



Unificaxion

- Some Q's might improve gauge coupling unification [Giudice, Rattazzi, Strumia, 1204.5465]

  - out of all our 15 cases, just one works well: Q ~ (3, 2, 1/6)

- Conceiving a UV model remains, however, a non-trivial challenge

  - $Q \in \psi_{\text{GUT}}$

  - $m_Q \lesssim f_a \ll M_{\text{GUT}}$

$$[\text{PQ}, \text{GUT}] = 0 \quad \longrightarrow \quad m_{\psi_{\text{GUT}}} = \mathcal{O}(f_a)$$

  - a complete GUT multiplet doesn't help !

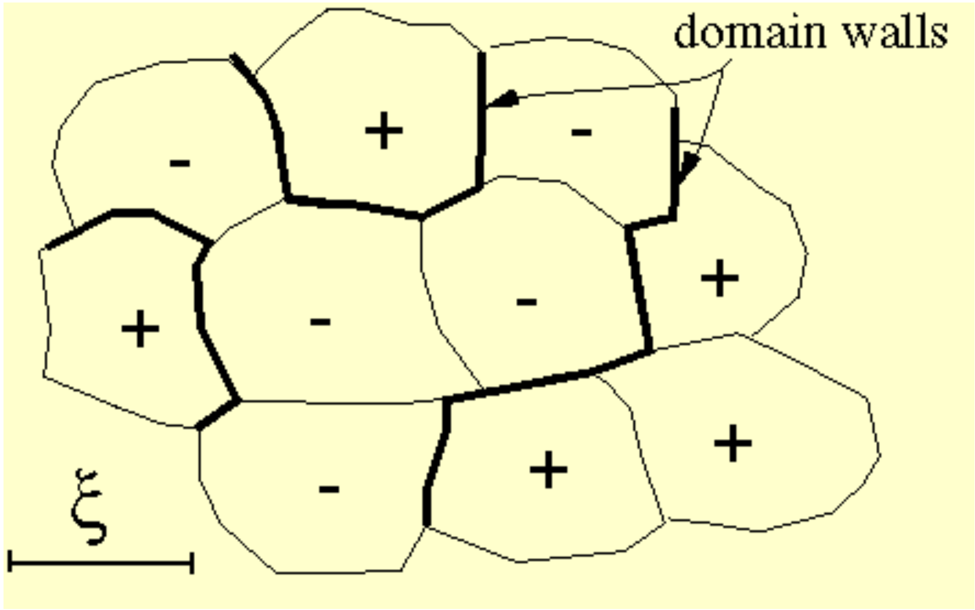
DW problem

- $U(1)_{\rm PQ} \longrightarrow Z_{N_{\rm DW}}$ explicitly broken by QCD effects

  - a(x) defined in [0, 2πv$_a$] $\langle \Phi(x) \rangle = \frac{1}{\sqrt{2}} v_a e^{i a(x)/v_a}$

  - axion potential periodic in $\Delta a = \dfrac{2\pi v_a}{N_{\rm DW}}$ $(N_{\rm DW} = 2N)$ → $N_{\rm DW}$ degenerate vacua

- SSB of a discrete symmetry leads to (stable) DW configurations, whose energy density can easily overclose the Universe



DW problem

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{\rm LP}$ [GeV]	E/N	$N_{\rm DW}$
$(3, 1, -1/3)$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38} (g_1)$	$2/3$	1
$(3, 1, 2/3)$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34} (g_1)$	$8/3$	1
$(3, 2, 1/6)$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39} (g_1)$	$5/3$	2
$(3, 2, -5/6)$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27} (g_1)$	$17/3$	2
$(3, 2, 7/6)$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22} (g_1)$	$29/3$	2
$(3, 3, -1/3)$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30} (g_2)$	$14/3$	3
$(3, 3, 2/3)$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27} (g_2)$	$20/3$	3
$(3, 3, -4/3)$	$\overline{Q}_L d_R H^{\dagger 2}$	$3.5 \cdot 10^{18} (g_1)$	$44/3$	3
$(\overline{6}, 1, -1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$2.3 \cdot 10^{37} (g_1)$	$4/15$	5
$(\overline{6}, 1, 2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$5.1 \cdot 10^{30} (g_1)$	$16/15$	5
$(\overline{6}, 2, 1/6)$	$\overline{Q}_R \sigma_{\mu\nu} q_L G^{\mu\nu}$	$7.3 \cdot 10^{38} (g_1)$	$2/3$	10
$(8, 1, -1)$	$\overline{Q}_L \sigma_{\mu\nu} e_R G^{\mu\nu}$	$7.6 \cdot 10^{22} (g_1)$	$8/3$	6
$(8, 2, -1/2)$	$\overline{Q}_R \sigma_{\mu\nu} \ell_L G^{\mu\nu}$	$6.7 \cdot 10^{27} (g_1)$	$4/3$	12
$(15, 1, -1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$8.3 \cdot 10^{21} (g_3)$	$1/6$	20
$(15, 1, 2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$7.6 \cdot 10^{21} (g_3)$	$2/3$	20

DW problem - solutions

- Inflation can dilute them away (pre-inflationary PQ breaking)
- $N_{DW} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{\rm LP}$[GeV]	E/N	$N_{\rm DW}$
$(3,1,-1/3)$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$	1
$(3,1,2/3)$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$	1
$(3,2,1/6)$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$	2
$(3,2,-5/6)$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$	2
$(3,2,7/6)$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$	2
$(3,3,-1/3)$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$	3
$(3,3,2/3)$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3
$(3,3,-4/3)$	$\overline{Q}_L d_R H^{\dagger 2}$	$3.5 \cdot 10^{18}(g_1)$	$44/3$	3
$(\overline{6},1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$2.3 \cdot 10^{37}(g_1)$	$4/15$	5
$(\overline{6},1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$5.1 \cdot 10^{30}(g_1)$	$16/15$	5
$(\overline{6},2,1/6)$	$\overline{Q}_R \sigma_{\mu\nu} q_L G^{\mu\nu}$	$7.3 \cdot 10^{38}(g_1)$	$2/3$	10
$(8,1,-1)$	$\overline{Q}_L \sigma_{\mu\nu} e_R G^{\mu\nu}$	$7.6 \cdot 10^{22}(g_1)$	$8/3$	6
$(8,2,-1/2)$	$\overline{Q}_R \sigma_{\mu\nu} \ell_L G^{\mu\nu}$	$6.7 \cdot 10^{27}(g_1)$	$4/3$	12
$(15,1,-1/3)$	$\overline{Q}_L \sigma_{\mu\nu} d_R G^{\mu\nu}$	$8.3 \cdot 10^{21}(g_3)$	$1/6$	20
$(15,1,2/3)$	$\overline{Q}_L \sigma_{\mu\nu} u_R G^{\mu\nu}$	$7.6 \cdot 10^{21}(g_3)$	$2/3$	20

+ new solutions with 2 Q's by combining 8 and 6 with opposite PQ charge

$$T(8) = 3$$

$$T(6) = \frac{5}{2}$$

$$N_{\rm DW}(6 \oplus 8) = 2\,(T(8) - T(6)) = 1$$

DW problem - solutions

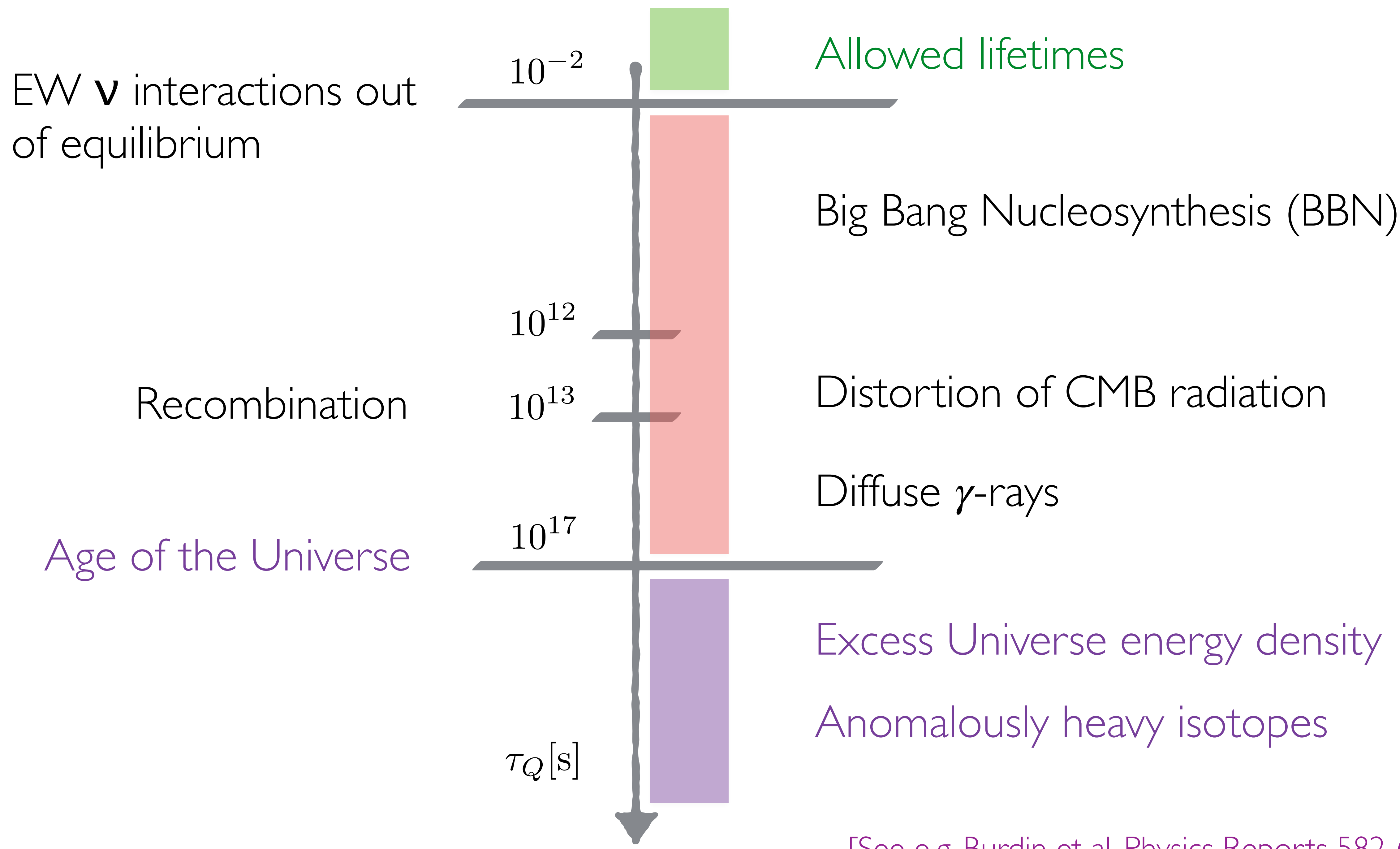
- Inflation can dilute them away (pre-inflationary PQ breaking)
- $N_{DW} = 1$
- Explicit PQ breaking [Sikivie (1982)]

$$\delta V = -\xi(\phi e^{-i\delta} + h.c.) \longrightarrow \delta V_a = -2 v_a \xi \, \cos\left(\frac{a}{v_a} - \delta\right) \longrightarrow \bar{\theta} \sim \frac{\xi}{m_a^2 f_a}$$

- large enough so that a unique vacuum takes over before DWs dominate energy density

- small enough so that PQ solution is not ruined

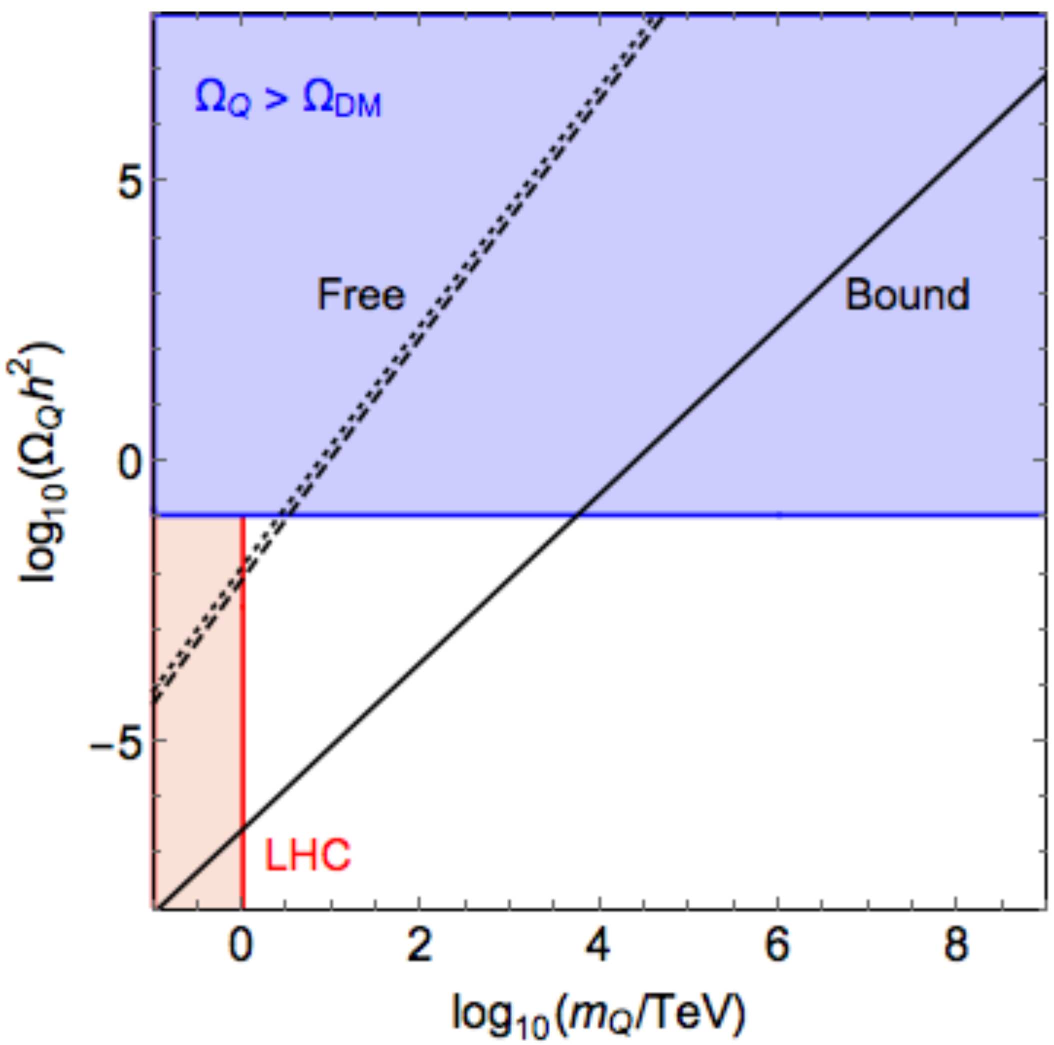
Cosmological constraints on τ_Q



[See e.g. Burdin et al. Physics Reports 582 (2015) 1–52]

Heavy Q's relic density (1)

- $T_{reheating} > m_Q$ (thermal distribution of Q's as initial condition)
- Reliable estimates on Ω_Q remain an open issue, but Q abundances still too high



[Rich literature: e.g.
Dover, Gaisser, Steigman PRL 42 (1979),
Nardi, Roulet PLB 245 (1990),
Arvanitaki et al., hep-ph/0504210,
Kang, Luty, Nasri, hep-ph/0611322,
Jacob, Nussinov, 0712.2681
Kusakabe, Takesako, 1112.0860]

Heavy Q's relic density (2)

- above $T_C \sim$ 180 MeV: perturbative annihilation

$$\left(\Omega_Q h^2\right)^{\text{Free}} = 2.0 \left(\frac{x_{fo}}{25}\right) \left(\frac{g_*}{106.75}\right)^{-1/2} \left(\frac{\langle \sigma v \rangle_{Q\overline{Q}}}{10^{-10}\ \text{GeV}^{-2}}\right)^{-1} \approx 8 \cdot 10^{-3} \left(\frac{m_Q}{\text{TeV}}\right)^2$$

- below $T_C \sim$ 180 MeV: heavy Q's get confined in color singlets and annihilation may restart via the formation of intermediate bound states (e.g. $\overline{Q}q + Qqq \to \overline{Q}Q + qqq$)

[Kang, Luty, Nasri, hep-ph/0611322]

$$\left(\Omega_Q h^2\right)^{\text{Bound}} = 8.7 \cdot 10^{-12} \left(\frac{R_{\text{had}}}{\text{GeV}^{-1}}\right)^{-2} \left(\frac{T_C}{180\ \text{MeV}}\right)^{-3/2} \left(\frac{m_Q}{\text{GeV}}\right)^{3/2}$$

- however QQ, QQQ, … bound states (so far not taken into account) would hinder it

[Kusakabe, Takesako, 1112.0860]

Q stability & PQ quality

- Symmetry of the kinetic term

$$U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_{\Phi} \xrightarrow{y_Q \neq 0} U(1)_{\rm PQ} \times U(1)_{\rm Q}$$

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c.})$$

- $U(1)_Q$ is the Q-baryon number. If exact, Q would be stable

- if $\mathcal{L}_{Qq} \neq 0$ $U(1)_Q$ is further broken and Q-decay is possible

- Global symmetries expected to be broken by Planck-scale physics

- $U(1)_{PQ}$ and $U(1)_Q$ breaking effective operators (can consistently coexist)

$V_\Phi^{d>4} \ni \dfrac{\Phi^N}{M_{\rm Planck}^{N-4}}$ → N > 10 not to ruin the PQ solution

$\mathcal{L}_{Qq}^{d>4}$ → responsible for Q decay (even for $\mathcal{L}_{Qq} = 0$)

Accidental symmetries

- Assume a suitable discrete (gauge) symmetry $\mathbb{Z}_\mathbb{N}$ ensuring

  1. U(1)$_{PQ}$ arises accidentally and is of the required *high quality*
  2. U(1)$_{Q}$ is either broken at the ren. level, or it can be of sufficient *bad quality*

- An example with Q ~ d$_R$. Under $\mathbb{Z}_\mathbb{N}$ (with $\omega \equiv e^{i2\pi/\mathbb{N}}$)

$$Q_L \to Q_L \,, \quad Q_R \to \omega^{\mathbb{N}-1} Q_R \,, \quad \Phi \to \omega\, \Phi \,,$$

$\mathbb{Z}_\mathbb{N}(q)$	$d \leq 4$	$d = 5$	$(\mathcal{X}_L, \mathcal{X}_R)$
1	$\overline{Q}_L d_R$	$\overline{Q}_L \gamma_\mu q_L \,(D^\mu H)^\dagger$	$(0, -1)$
ω	$\overline{Q}_L d_R \Phi^\dagger$		$(-1, -2)$
$\omega^{\mathbb{N}-2}$	–	$\overline{Q}_L d_R \Phi^2,\ \overline{Q}_R q_L H^\dagger \Phi$	$(2, 1)$
$\omega^{\mathbb{N}-1}$	$\overline{q}_L Q_R H,\ \overline{Q}_L d_R \Phi$	–	$(1, 0)$

→ ensures that the minimum dimension of the U(1)$_{PQ}$ breaking operators in $V_\Phi^{d>4}$ is $\mathbb{N}$, while the dim of the U(1)$_Q$ breaking operators depends on $\mathbb{Z}_\mathbb{N}(q)$