Axion predictions in SO(10) x $U(1)_{PQ}$ models

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The aim:

Study the properties of the axion –mass, couplings to photons, fermions, nucleons, domain wall number– in grand unified theories with SO(10) gauge groups, and furnished with a global U(1) symmetry so as to solve the strong CP problem. Account for constraints from unification, proton decay, star cooling, black hole superradiance, fermion mass fits.

The novelty:

Properties of SO(10) axion had not been studied systematically

Our formalism bridges the gap between the simple UV SO(10) x U(1) symmetries and the low-energy description (e.g. couplings to nucleons, domain wall number).

The plan:

Motivation of SO(10) theories and the axion solution to the strong CP problem

The guts of the axion solution to the strong CP problem

The GUTs of the axion solution to the strong CP problem

Motivation

Why unification?

Group structure

Matter content in each generation

Anomaly cancellation, charge quantization

Hierarchies of masses and mixing angles

Can the SM model be a low energy effective description of a more predictive theory with a simple gauge group, and fewer representations?

B and L are accidental symmetries in the SM: expect B violation, proton decay!

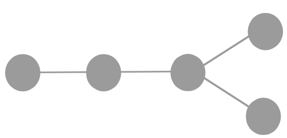
Bird's eye view of GUT models

SM group is of rank 4: Embed into simple groups of rank 4 or more.

SU(5)

Each generation a 5 and $\overline{10}$

Non-SUSY ruled out by $\sin^2\theta_W$ and proton lifetime

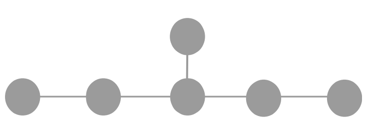


SO(10)

Each generation a single 16 irrep! (with RH neutrino)

Anomaly cancellation automatic

Larger rank allows for **multi-step symmetry breaking** with different chains



E6

A single generation plus Higgses fits in 27 irrep

Anomaly cancellation automatic

Multi-step breaking, multiple chains

Why SO(10) x $U(1)_{PQ}$?

Minimality and anomaly freedom of matter representations

RH neutrinos automatically included, allowing for leptogenesis

Rich Higgs sector can accommodate axion, inflaton

Multi-step breaking suggests intermediate mass scales which could be tied to leptogenesis and the axion, while playing a role in unification.

Can we build a grand-unified version of SMASH,
the model proposed by Ballesteros, Ringwald, Redondo,
CT, to account for:

Predictive inflation

Neutrino masses

CP problem

Dark matter

Baryogenesis?

Why the axion?

Solves the strong CP problem by making the θ angle dynamical [Peccei & Quinn, Weinberg, Wilczek].

The axion can be dark matter! [Preskill et all, Abbott and Sikivie, Dine and Fischler]

Need: Spontaneously broken U(1) symmetry, anomalous under QCD: scalar fields giving masses to strongly interacting fermions.

The axion is a combination of scalar phases. Its effective action has a finite symmetry group whose dimension is the domain wall number. Domain walls can overclose the universe it, but if $N_{DW}=1$ they can decay. Otherwise one needs to break U(1) (e.g. discrete symmetry)

Axion models classified by

- Coupling of axion to gauge bosons (nonderivative, from anomalies)
- Axion mass (from QCD effects, fixed by coupling to gluons)
- Coupling of axion to fermions, nucleons (derivative)
- Domain wall number

The axion's ID card

$$\mathcal{L}[A]_{\text{eff}} = \frac{1}{2}\partial_\mu A \partial^\mu A + \partial_\mu A \sum_a c_{A\psi_a} (\psi_a^\dagger \bar{\sigma}^\mu \psi_a) + A \sum_{k=1}^{N_g} \frac{1}{f_{A,k}} \frac{g_k^2}{16\pi^2} \, \bar{\mathrm{T}}\mathrm{r}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu}$$

$f_{A,k}$ Coupling of axion to gauge bosons (nonderivative, from anomalies)

m_A Axion mass (from QCD effects, fixed by coupling to gluons)

$$m_A = 57.0(7) \left(\frac{10^{11}\mathrm{GeV}}{f_{A,3}} \right) \mu\mathrm{eV}.$$

[Borsanyi et al, di Cortona et al]

$c_{A\psi_a}$ Coupling of axion to fermions, nucleons (derivative)

N_{DW} Domain wall number

Axion constraints: dark matter, cooling, superradiance

Dark matter:

Oscillating axion field behaves as pressureless dark matter. If U(1) restored after inflation:

$$\Omega_A h^2 \approx 0.12 \Rightarrow 3 \times 10^{10}\,\mathrm{GeV} \lesssim f_A \lesssim 1.2 \times 10^{11}\,\mathrm{GeV} \quad ; \quad 50\,\mu\mathrm{eV} \lesssim m_A \lesssim 200\,\mu\mathrm{eV}$$

Star cooling:

Axion coupling to photons allows photon-axion conversion in stars, enhancing energy loss and cooling rate. This gives bounds based on e.g. cooling of helium burning stars in globular clusters, duration of neutrino burst in supernovae explosions [Raffelt,...]

$$g_{aa\gamma} < 0.66 \times 10^{-10}$$ [Ayala et al]

Superradiance:

An axion scalar field interacting with a rotating black hole has unstable localized modes which extract energy and spin from the black hole, if the black hole radius is comparable to the scalar's Compton wavelength [Arvanitaki et al]. This would lead to reduced black hole populations for particular values of spin and mass. Leads to

$$f_A < 1 \times 10^{17}\mathrm{GeV}$$

What has been done for the axion in GUTs

$U(1)_{PQ}$ extensions of SO(10) GUTs have been studied before [Lazarides, Kim, Bacj et al, Babu et al, Bertolini et al, Altarelli et al].

A global U(1) can be motivated so as to make the Yukawa sector more predictive
It is automatically anomalous and provides an axion solution to the strong CP problem.

The need for intermediate scales in SO(10) unification implies that the axion can be dark matter (otherwise absent in non-SUSY SU(5) GUTs).

Axion field constructed in very few cases

Models have been proposed with $N_{DW}=1$, arguing in terms of the UV symmetries.

… and what was missing

A systematic identification of axion field and axion decay constant/mass in relation to thresholds/VEVs in the theory

A systematic calculation of couplings to gauge bosons, fermions/nucleons, including low energy effects

An identification of the global symmetry corresponding to the physical axion, which is orthogonal to the transformations associated with the massive gauge bosons

A direct calculation of domain wall number for the above symmetry

Studies of constraints from unification, proton decay, fermion mass fits, stellar cooling, superradiance.

Which experiments can probe GUT axions?

The guts of the strong CP problem

Ingredients: a global U(1), anomalous, broken

Weyl fermions ψ_a

Complex scalars ϕ_j

Nonabelian gauge fields

$$A_\mu^k = A_\mu^{k,a} T^a, \;\; k = 1, \ldots, N_g$$

Global U(1):

$$\psi_a \to e^{i q_a \alpha} \psi_a$$
$$\phi_j \to e^{i q_j \alpha} \phi_j$$

Current

$$J^\mu = \sum_a q_a \psi_a^\dagger \bar{\sigma}^\mu \psi_a + i \sum_j q_j (\partial_\mu \phi_j^\dagger \phi_j - \phi_j^\dagger \partial_\mu \phi_j),$$

Anomaly

$$\langle \partial_\mu J^\mu \rangle = \sum_{k=1}^{N_g} \frac{g_k^2 \hat{N}_k}{16\pi^2} \, \bar{\mathrm{T}}\mathrm{r} \, \tilde{F}_{\mu\nu}^k F^{k,\mu\nu},$$

$$\hat{N}_k = 2 \sum_a q_a T_k(\rho_a)$$

Symmetry breaking:

$$\phi_j = \frac{1}{\sqrt{2}} (v_j + \rho_j) e^{i A_j / v_j}.$$

The axion/Goldstone:

$$A_i = \frac{q_i v_i}{f_{\mathrm{PQ}}} A + \text{ orthogonal excitations}, \;\; f_{\mathrm{PQ}} = \sqrt{\sum_j q_j^2 v_j^2}$$

$$A = \frac{1}{f_{\mathrm{PQ}}} \sum_i q_i v_i A_i$$

From the anomaly to the effective Lagrangian

$$\langle \partial_\mu J^\mu \rangle = \sum_{k=1}^{N_g} \frac{g_k^2 \hat{N}_k}{16\pi^2} \,\bar{\mathrm{T}}\mathrm{r}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu} = f_{\mathrm{PQ}} \Box A + \sum_a q_a \partial_\mu (\psi_a^\dagger \bar{\sigma}^\mu \psi_a)$$

Equivalent to Euler-Lagrange equations from the following effective action [Srednicki]

$$\mathcal{L}[A]_{\mathrm{eff}} = \frac{1}{2} \partial_\mu A \partial^\mu A + \partial_\mu A \sum_a \frac{q_a}{f_{\mathrm{PQ}}} (\psi_a^\dagger \bar{\sigma}^\mu \psi_a) + A \sum_{k=1}^{N_g} \frac{1}{f_{A,k}} \frac{g_k^2}{16\pi^2} \,\bar{\mathrm{T}}\mathrm{r}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu}$$

$$f_{A,k} = \frac{f_{\mathrm{PQ}}}{\hat{N}_k},$$

Alternatively, one can obtain Lagrangian by redefining fermion fields [Kim, Dias et al]
Redefinition not unique: Physically equivalent Lagrangians that differ by field redefinitions.

Axion couplings to neutral gauge bosons is not affected by this ambiguity (EM unbroken).
Therefore f_{A3}, f_{AEM} only depend on the PQ charges of the scalars

Fermion-axion couplings reflect the PQ charges only in a particular basis; info on PQ charges can become hidden in axion coupling to massive bosons.

Nonperturbative hadronic/QCD effects

Axion coupling to nucleons:

Recovered in a **fermionic axial basis** in which interactions involve axial currents

$$\partial_\mu A \left[\frac{q_1}{f_{\rm PQ}} \psi^\dagger \bar\sigma^\mu \psi + \frac{q_2}{f_{\rm PQ}} \tilde\psi^\dagger \bar\sigma^\mu \tilde\psi \right] \to -\partial_\mu A \, \frac{q_1 + q_2}{2 f_{\rm PQ}} \, \overline{\Psi} \gamma^\mu \gamma_5 \Psi .$$

The Dirac fermions contain Weyl spinors linked by a PQ invariant Yukawa couplings, so $q_1 + q_2$ is equal to a scalar PQ charge. From chiral perturbation theory at NLO and lattice results [Villadoro et al]

$$\delta\mathcal{L}_{\rm eff} = -\,\partial_\mu A \, \frac{C_{AN}}{2 f_A} \, \overline{N} \gamma^\mu \gamma_5 N - \partial_\mu A \, \frac{C_{AP}}{2 f_A} \, \overline{P} \gamma^\mu \gamma_5 P,$$

$$C_{AN} = -\,0.02(3) + 0.41(2) \frac{q_{H_u} f_A}{f_{\rm PQ}} - 0.83(3) \frac{q_{H_d} f_A}{f_{\rm PQ}},$$

$$C_{AP} = -\,0.47(3) - 0.86(3) \frac{q_{H_u} f_A}{f_{\rm PQ}} + 0.44(2) \frac{q_{H_d} f_A}{f_{\rm PQ}}.$$

Fermion couplings in axial basis only depend on scalar PQ charges q_i / f_{PQ} .

Axion coupling to photons:

At low energy, axion field can mix with other pseudo-Goldstones, like the neutral pion. An appropriate field redefiniton removes the mixing and gives for the physical axion

$$\delta\mathcal{L}_{\rm eff} = \frac{\alpha}{8\pi f_A} \, \delta C_{A\gamma} A \tilde F_{\mu\nu} F^{\mu\nu}, \; \delta C_{A\gamma} = -\frac{2}{3} \left(\frac{4 m_u + m_d}{m_u + m_d} \right) + \text{higher order} = -1.92(4).$$

Wherefore art thou a physical Goldstone?

Axion field must be orthogonal to massive gauge bosons. Its associated **physical PQ symmetry will not be the simple naive one.**

PQ_{phys} is generated by a combination of original PQ and other symmetries S_j:

$$PQ_{\rm phys} = PQ + \sum_j \lambda_j S_j \to A = \sum_i c_i A_i, \quad c_i = \frac{q_i^{\rm phys} v_i}{\sqrt{(q^{\rm phys})_i^2 v_i^2}}$$

Unknown c_i giving charges of PQ_{phys} can be obtained by solving masslessness and orthogonality conditions

Art thou massless? $$\mathcal{L} \supset m \left(\sum_m d_m A_m \right)^2 \Rightarrow \sum_m c_m d_m = 0$$

Art thou orthogonal to massive gauge bosons?

Avoidance of kinetic mixing with gauge bosons implies $\sum_{mn} v_m T^a_{mn} c_n = 0.$

For a massive U(1) boson with associated scalar charges $\tilde{q}_m$: $\sum_m v_m \tilde{q}_m c_m = 0.$

Need at least 2 scalars charged under massive U(1) for them to contain the axion.
f_A of the order of the smallest VEV of fields charged under massive U(1).

Domain wall number

$$\mathcal{L}_{\rm eff}[A] \supset A \sum_{k=1}^{N_g} \left(\theta_k + \frac{A \hat{N}_k}{f_{\rm PQ}} \right) \frac{g_k^2}{16\pi^2} \,\bar{\rm Tr}\, \tilde{F}^k_{\mu\nu} F^{k,\mu\nu} + \text{derivatives}$$

Axion-Goldstone shifts under PQ_{phys}, $A \to A + \alpha f_{\rm PQ}$. The anomalies break shift symmetry in the SU(3) sector to a discrete subset (equivalent to translations $\theta_3 \to \theta_3 + 2\pi$).

$$S_{\rm phys}(n) : A \to A + \frac{2\pi n}{\hat{N}} f_{\rm PQ}, \quad n \in \mathbb{Z}.$$

Some of this translations correspond to unphysical rotations of phases A_i by 2π

$$P_{\rm phys}(n_i) : A \to A + \sum_i \frac{2\pi n_i q_i v_i^2}{f_{\rm PQ}}, \quad n_i \in \mathbb{Z}.$$

$$N_{\rm DW} = \dim \left[\frac{S_{\rm phys}}{P_{\rm phys}} \right] = \text{ min. integer} \left\{ \frac{1}{f_A} \sum_i n_i c_i v_i,\, n_i \in \mathbb{Z} \right\} \left(\to \frac{\hat{N}}{\text{MCD}[q_i]}, \text{ rational } q_i \right)$$

Relation to naive UV PQ symmetry (without imposing orthogonality conditions): Have to mod out by the discrete transformations in the center Z of the gauge group!

$$N_{\rm DW} = \dim \left[\frac{S_{\rm phys}}{P_{\rm phys}} \right] = \dim \left[\frac{S}{ZP} \right]$$

Fun fact. DFSZ models have $N_{\rm DW}=3$, not the usually quoted 6, because $Z_{SU(2)}=Z_2$.

The axion hunting flow

$$A = \sum_i c_i A_i$$

Orthogonality
Masslessness
Canonical normalization

scalar PQ_{phys} charges

$$\frac{q_i}{f_{PQ}} = \frac{c_i}{v_i}$$

f_A, $f_{A,EM}$, axion mass
Fermion/nucleon couplings in axial basis
N_{DW}

Axial basis can hide symmetries of the theory

Solve for q_i in terms of symmetries of the theory

fermion PQ_{phys} charges in "symmetry basis"

$$\frac{q_a}{f_{PQ}}$$

The GUTs of the strong CP problem

What to expect from GUT symmetry

$$\mathcal{L}_{\rm GUT} \supset \bar{\rm Tr}_{\rm GUT} \frac{g^2\theta_{\rm GUT}}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^{\rm GUT}_{\mu\nu}F^{\rm GUT}_{\rho\sigma} = \bar{\rm Tr}_3 \frac{g^2\theta_{\rm GUT}}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^3_{\mu\nu}F^3_{\rho\sigma} + \dots$$

Only one fundamental θ angle. Only one $\theta_{\rm phys}$.

The GUT axion must solve the CP problem in all subgroups!

$$\mathcal{L}_{\rm GUT}[A] \supset \frac{A}{f_A}\frac{g^2}{16\pi^2}\bar{\rm Tr}_{\rm GUT}\tilde{F}^{\rm GUT}_{\mu\nu}F^{{\rm GUT},\mu\nu} = \frac{A}{f_A}\frac{g^2}{16\pi^2}\bar{\rm Tr}_3\tilde{F}^3_{\mu\nu}F^{3,\mu\nu} + \dots$$

GUT symmetry explicit ⟶ all θ_k and all f_{Ak} will be the same [modulo normalization factors]

Field redefinitions can break the GUT symmetry and give different θ_k , f_{Ak} , yet with unique $\theta_{\rm phys}$

Eg: axial basis where the axion couplings to nucleons are calculated. In such basis f_{A2} deviates from f_{A3} and the GUT symmetry is hidden, but could in principle be uncovered if all couplings could be measured...

$PQ_{\rm phys}$ should be a combination of symmetries of GUT theory! Possibly involving Cartan of SO(10)

Relevant SO(10) representations

Each generation comes come in spinorial 16 representation

$SO(10)$	$3_C 2_L 1_Y$
16_F	$(3,2,\frac{1}{6}) := q$
	$(1,2,-\frac{1}{2}) := l$
	$(\bar{3},1,\frac{1}{3}) := d$
	$(1,1,1) := e$
	$(\bar{3},1,-\frac{2}{3}) := u$
	$(1,1,0) := n$

$$16_F \times 16_F = 10_H + 120_H + 126_H$$

Scalar VEVs can break the rank 5 group SO(10). Which scalar representations?

$$SO(10) \xrightarrow{M_U - 210_H} 4_C\, 2_L\, 2_R \xrightarrow{M_{BL} - \overline{126}_H} 3_C\, 2_L\, 1_Y \xrightarrow{M_Z - 10_H, 126_H} 3_C\, 1_{\text{em}}$$

$$SO(10) \xrightarrow{M_{\text{U}} - 210_H} 4_C\, 2_L\, 2_R \xrightarrow{M_{\text{PQ}} - 45_H} 4_C\, 2_L\, 1_R \longrightarrow 3_C\, 2_L\, 1_Y \xrightarrow{M_Z - 10_H, 126_H} 3_C\, 1_{\text{em}}$$

Why a U(1) symmetry? $\mathcal{L}_Y = 16_F \left(Y_{10} 10_H + \tilde{Y}_{10} 10_H^* + Y_{126} \overline{126}_H \right) 16_F + \text{h.c.}$

Can make it more predictive by imposing global U(1) which forbids $\tilde{Y}_{10}$. Anomalous, chiral

$$16_F \to 16_F e^{i\alpha}, \quad 10_H \to 10_H e^{-2i\alpha}, \quad \overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$

PQ symmetry of axion!

SO(10) models

Simplest model: (ruled out) electroweak axion

Model with above PQ charges has f_A at the electroweak scale due to orthogonality conditions!

Way out: need more fields with large VEVs charged under PQ!

$$N_{DW} = \frac{\hat{N}}{\mathrm{MCD}[q_i]}, \text{ rational } q_i$$

GUT scale axion (no axion DM in post-inflationary scenarios)

Charge 210_H under PQ ⟶ GUT scale f_{Ak}

Model 1 :

$$16_F \to 16_F e^{i\alpha},$$
$$10_H \to 10_H e^{-2i\alpha},$$
$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$
$$210_H \to 210_H e^{4i\alpha}$$

$\hat{N}_{\mathrm{GUT}} = 2 \times 3 \text{ gens} \times (\text{ index of } 16_F = 2) = 12$

Rational scalar PQ charges with MCD 2.

Center of SO(10): Z_2

Expect $N_{DW} = 3$

Intermediate scale (additional 45_H): Avoid GUT sale f_{Ak} by not charging 210 under PQ.

Model 2.1 :

$$16_F \to 16_F e^{i\alpha},$$
$$10_H \to 10_H e^{-2i\alpha},$$
$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$
$$210_H \to 210_H,$$
$$45_H \to 45_H e^{4i\alpha}$$

$\hat{N}_{\mathrm{GUT}} = 2 \times 3 \text{ gens} \times (\text{ index of } 16_F = 2) = 12$

Rational scalar PQ charges with MCD 2.

Center of SO(10): Z_2

Expect $N_{DW} = 3$

SO(10) models

Additional 45_H with 2 heavy fermion multiplets in 10_F: Add extra heavy fermions in 10_F to change GUT anomaly coefficient and get $N_{DW}=1$! [Lazarides].

Model 2.2 :

$$16_F \to 16_F e^{i\alpha},$$
$$10_H \to 10_H e^{-2i\alpha},$$
$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$
$$210_H \to 210_H,$$
$$45_H \to 45_H e^{4i\alpha},$$
$$10_F \to 10_F e^{-2i\alpha}$$

$$\hat{N}_{\rm GUT} = 2 \times (3 \text{ gens}) \times (1\text{PQ}) \times (\text{ ind of } 16_F = 2)$$
$$+ 2 \times (2 \text{ heavy F}) \times (-2\text{PQ}) \times (\text{ ind of } 10_F = 1) = 4$$

Rational scalar PQ charges with MCD 2.

Center of SO(10): Z_2

Expect N_{DW}= 1

Models with decay constants independent of gauge symmetry breaking

Model 3.1 :

$$16_F \to 16_F e^{i\alpha},$$
$$10_H \to 10_H e^{-2i\alpha},$$
$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$
$$210_H \to 210_H ,$$
$$S \to S e^{4i\alpha}$$

Model 3.2 :

$$16_F \to 16_F e^{i\alpha},$$
$$10_H \to 10_H e^{-2i\alpha},$$
$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha},$$
$$210_H \to 210_H ,$$
$$S \to S e^{4i\alpha},$$
$$10_F \to 10_F e^{-2i\alpha},$$

Example case: axion and couplings in 2.2

Scalar components getting VEVS, in terms of decompositions under PS=$SU(4)$x$SU(2)_L$x$SU(2)_R$.
SM arises from PS as follows:

$$SU(4) \supset SU(3) \times U(1)_{B-L} \qquad Y = U(1)_R + \frac{1}{2}U(1)_{B-L}$$
$$SU(2)_R \supset U(1)_R$$

$SO(10)$	$4_C 2_L 2_R$	$3_C 2_L 1_Y$	$3_C 1_{\text{em}}$	scale	VEV
10_H	$(1,2,2)$	$(1,2,\frac{1}{2})$	$(1,0) =: H_u$	M_Z	v_u^{10}
		$(1,2,-\frac{1}{2})$	$(1,0) =: H_d$	M_Z	v_d^{10}
45_H	$(1,1,3)$	$(1,1,0)$	$(1,0) := \sigma$	M_{PQ}	v_{PQ}
$\overline{126}_H$	$(10,1,3)$	$(1,1,0)$	$(1,0) := \Delta_R$	M_{BL}	v_{BL}
	$(15,2,2)$	$(1,2,\frac{1}{2})$	$(1,0) := \Sigma_u$	M_Z	v_u^{126}
		$(1,2,-\frac{1}{2})$	$(1,0) := \Sigma_d$	M_Z	v_d^{126}
210_H	$(1,1,1)$	$(1,1,0)$	$(1,0) := \phi$	M_{U}	v_{U}

Axion is a combination of phases of the first 6 scalars (because 210 has no PQ charge):

$$\phi_1 \equiv \Sigma_u, \phi_2 \equiv \Sigma_d, \phi_3 \equiv H_u, \phi_4 \equiv H_d, \phi_5 \equiv \Delta_R, \phi_6 \equiv \sigma, \qquad \phi_j = \frac{1}{\sqrt{2}}(v_j + \rho_j)e^{iA_j/v_j}.$$

$$A = \sum_i c_i A_i$$

Example case: axion and couplings in 2.2

All equations from orthogonality/constraints compatible! (need 5 eqs. and normalization condition)

$$A = -\frac{(A_4 v_4 + A_2 v_2)(v_3^2 + v_1^2) + (A_3 v_3 + A_1 v_1)(v_4^2 + v_2^2) - A_6 v_6 v^2}{\sqrt{v^2((v_4^2 + v_2^2)(v_3^2 + v_1^2) + v_6^2 v^2)}}, \quad v^2 \equiv \sum_{i=1}^{4} v_i^2.$$

From c_i to scalar PQ_{phys} charges: $\frac{q_i}{f_{PQ}} = \frac{c_i}{v_i}$

$$\mathrm{PQ}_{\mathrm{phys}} = s_1\,\mathrm{PQ} + s_2\,U(1)_R + s_3\,U(1)_{B-L}.$$

$$\frac{s_1}{f_{PQ}} = \frac{v}{4\sqrt{(v_1^2 + v_3^2)(v_2^2 + v_4^2) + v^2 v_6^2}}, \quad \frac{s_2}{f_{PQ}} = \frac{v_1^2 - v_2^2 + v_3^2 - v_4^2}{v\sqrt{(v_1^2 + v_3^2)(v_2^2 + v_4^2) + v^2 v_6^2}},$$

$$\frac{s_3}{f_{PQ}} = \frac{v_1^2 - 3v_2^2 + v_3^2 - 3v_4^2}{4v\sqrt{(v_1^2 + v_3^2)(v_2^2 + v_4^2) + v^2 v_6^2}}$$

From this we can get PQ_{phys} of Weyl fermions and from the latter f_{Ak}. All f_{Ak} are equal modulo hypercharge normalization, as follows from GUT symmetry!

$$f_{A,3c} = f_{A,2L} = \frac{5}{3} f_{A,Y} = \sqrt{\frac{v_6^2 v^2 + (v_4^2 + v_2^2)(v_3^2 + v_1^2)}{v^2}} \sim v_\sigma.$$

We also get $N_{DW}=1$ from the scalar PQ_{phys} charges! $N_{\mathrm{DW}} = \text{ min. integer}\left\{\frac{1}{f_A}\sum_i n_i c_i v_i,\ n_i \in \mathbb{Z}\right\}$

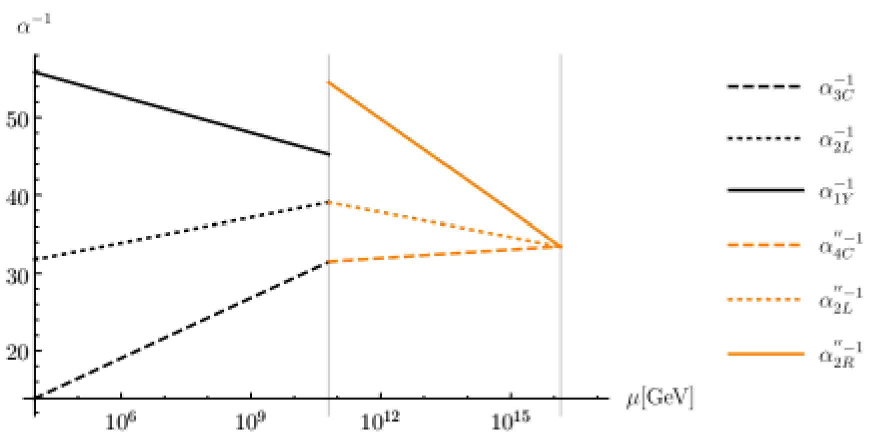
Unification constraints

2 loop RG equations, ignoring Yukawas. 3 thresholds scales $\mathbf{M_{PQ}, M_{BL}, M_U}$

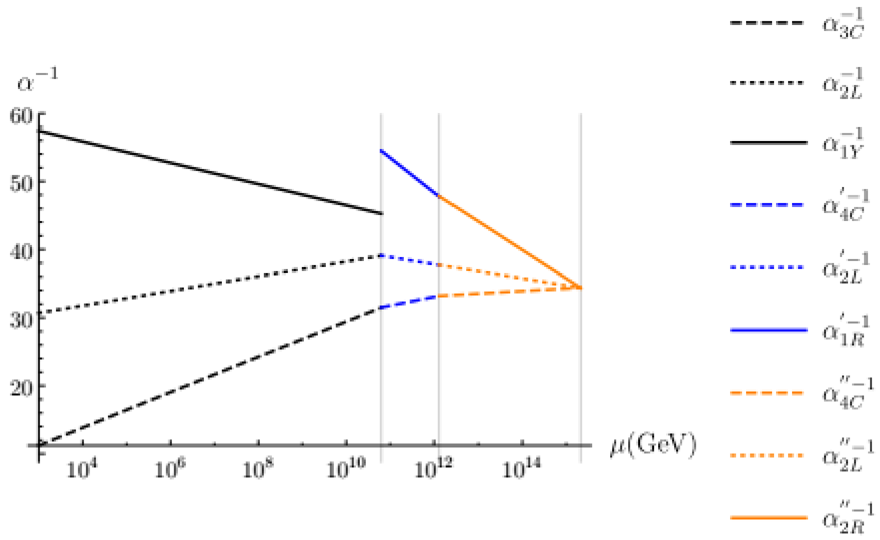
Full 1 loop threshold corrections

Extended survival hypothesis [del Aguila & Ibañez]: RG at a given scale only includes scalar multiplets which acquire a VEV at lower scales, with the exception of Σ_u , Σ_d, which are assumed to decouple at a scale M_{BL} in order to give rise to a low-energy 2HDM limit [Babu & Khan]

At thresholds we assume that the scalars and fermions which decouple take masses in the range of 1/10-10 times the threshold mass scale.

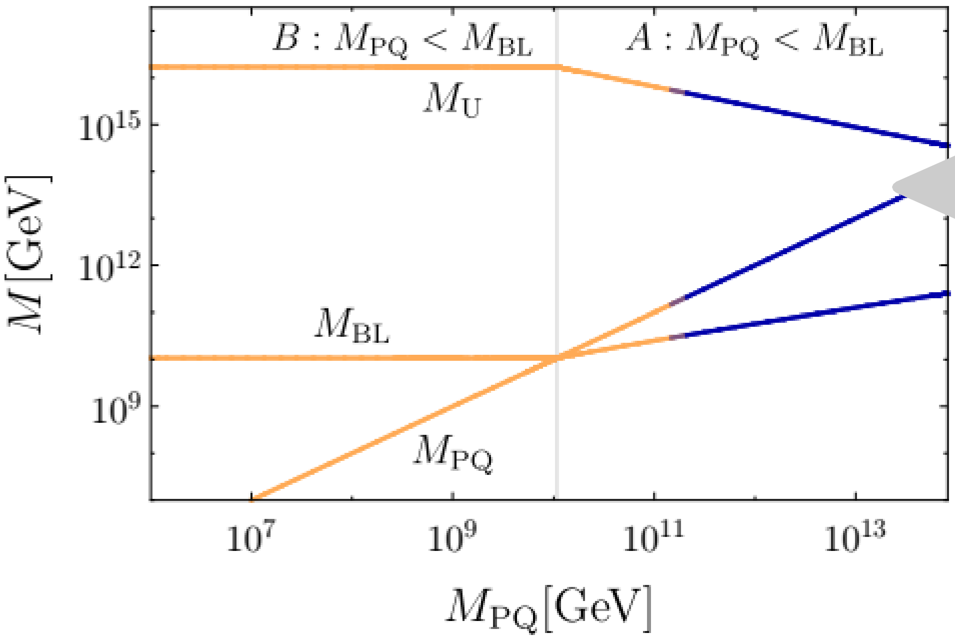


Model 1. 2 step. No loop thresholds



Model 2.1. 3 step. No loop thresholds

Unification constraints: impact of 1 loop thresholds

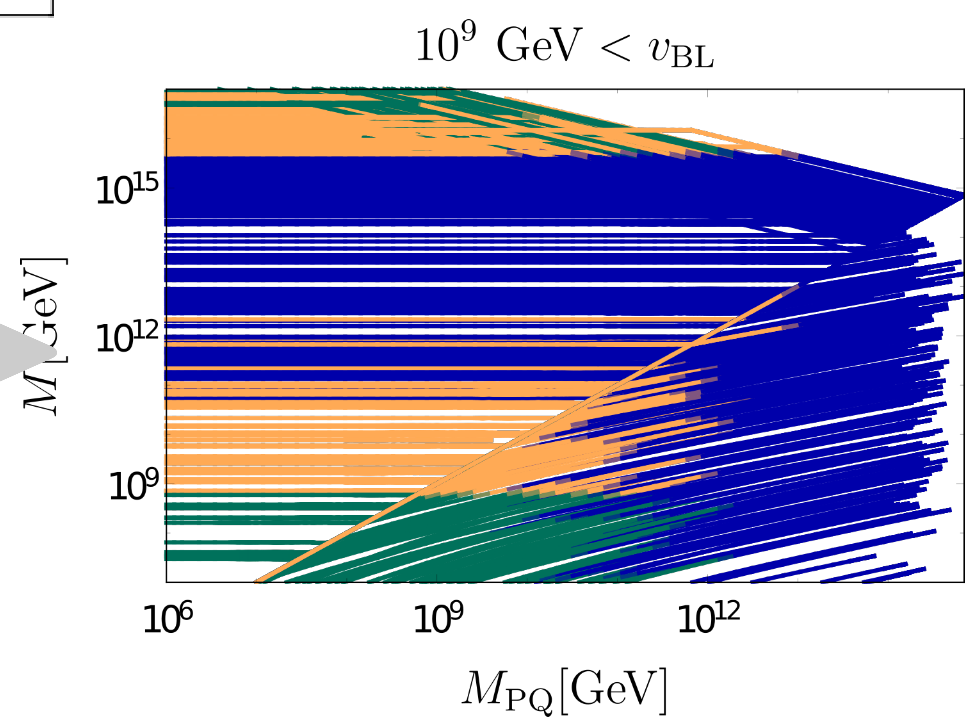


Model 2.1. 3 scales: $\mathrm{M_{PQ}, M_{BL}, M_U}$

No one-loop thresholds

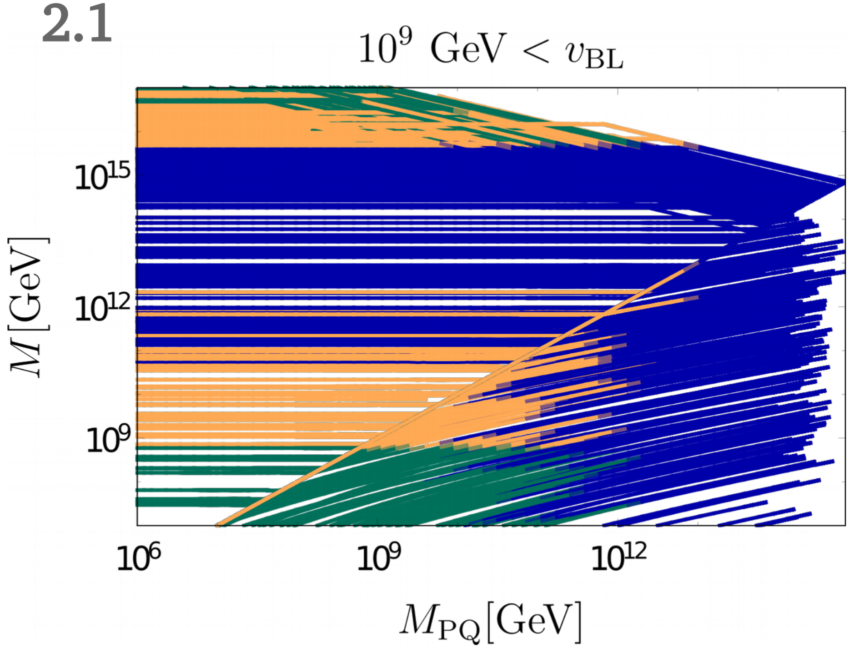
Random one-loop thresholds

Yellow: Allowed
Blue: Discarded by proton decay
Green: Discarded by fermion mass fits
[Joshipura & Patel, Dueck & Rodejohann]

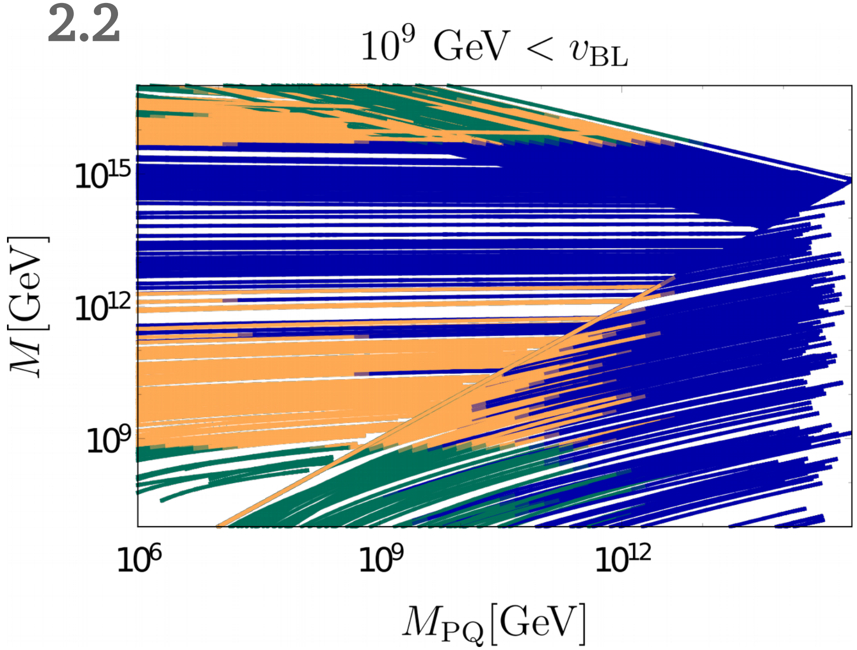


Unification constraints: impact of 1 loop thresholds

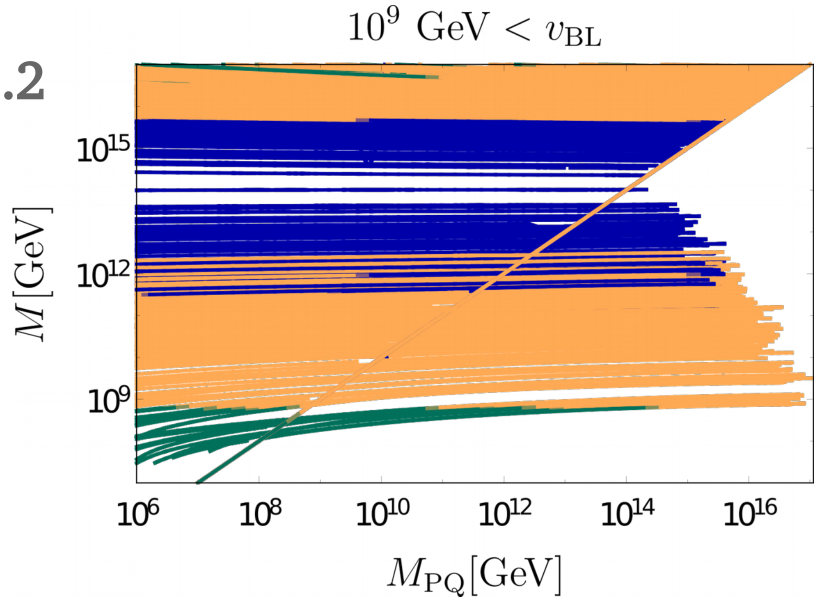
2.1



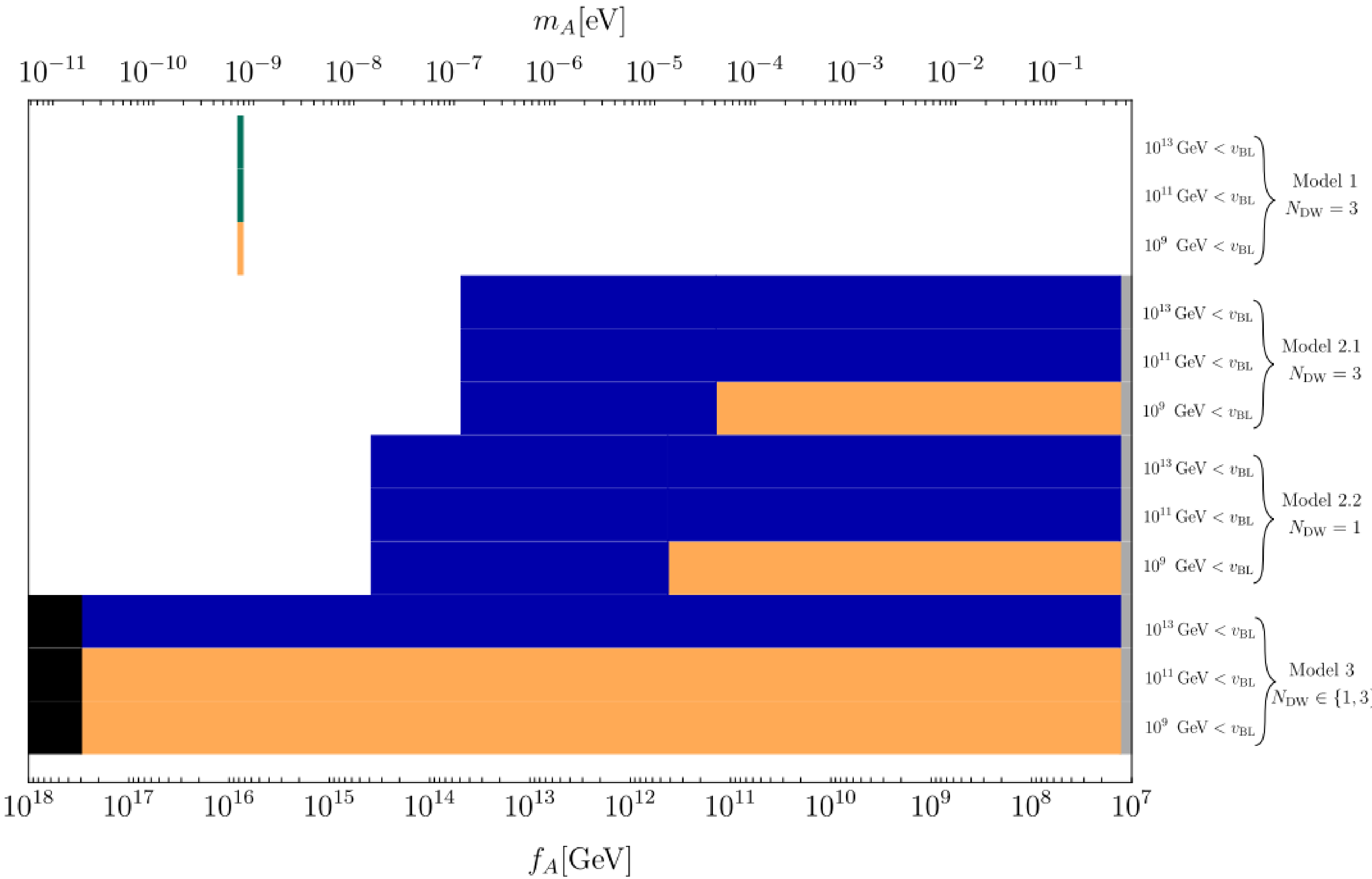
2.2



3.2



Unification constraints across models

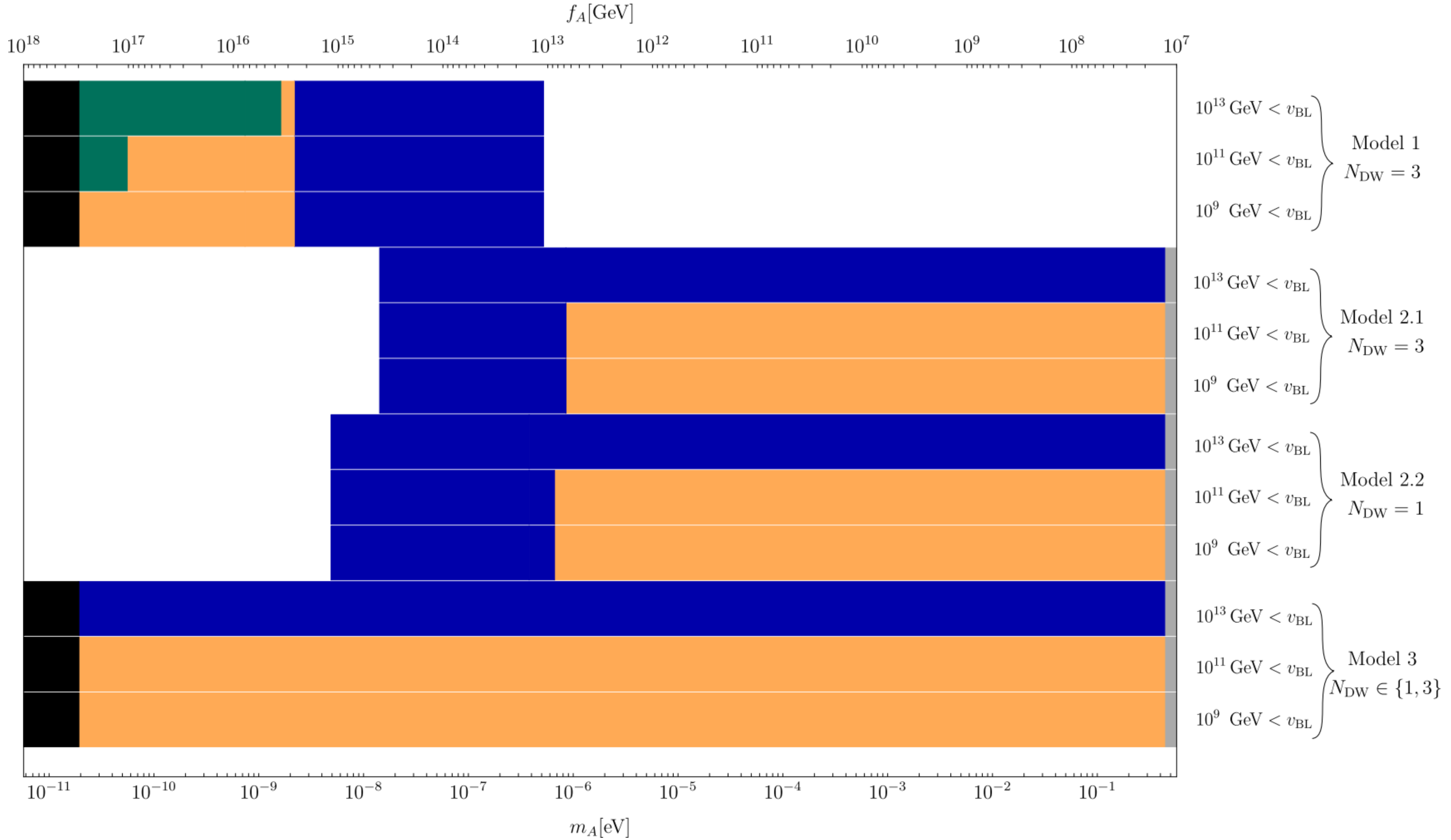


Yellow: Allowed
Blue: Discarded by proton decay

Black: Discarded by superradiance
Green: Discarded, fermion mass fits
Gray: Stelar cooling constraint

No one-loop thresholds

Unification constraints across models



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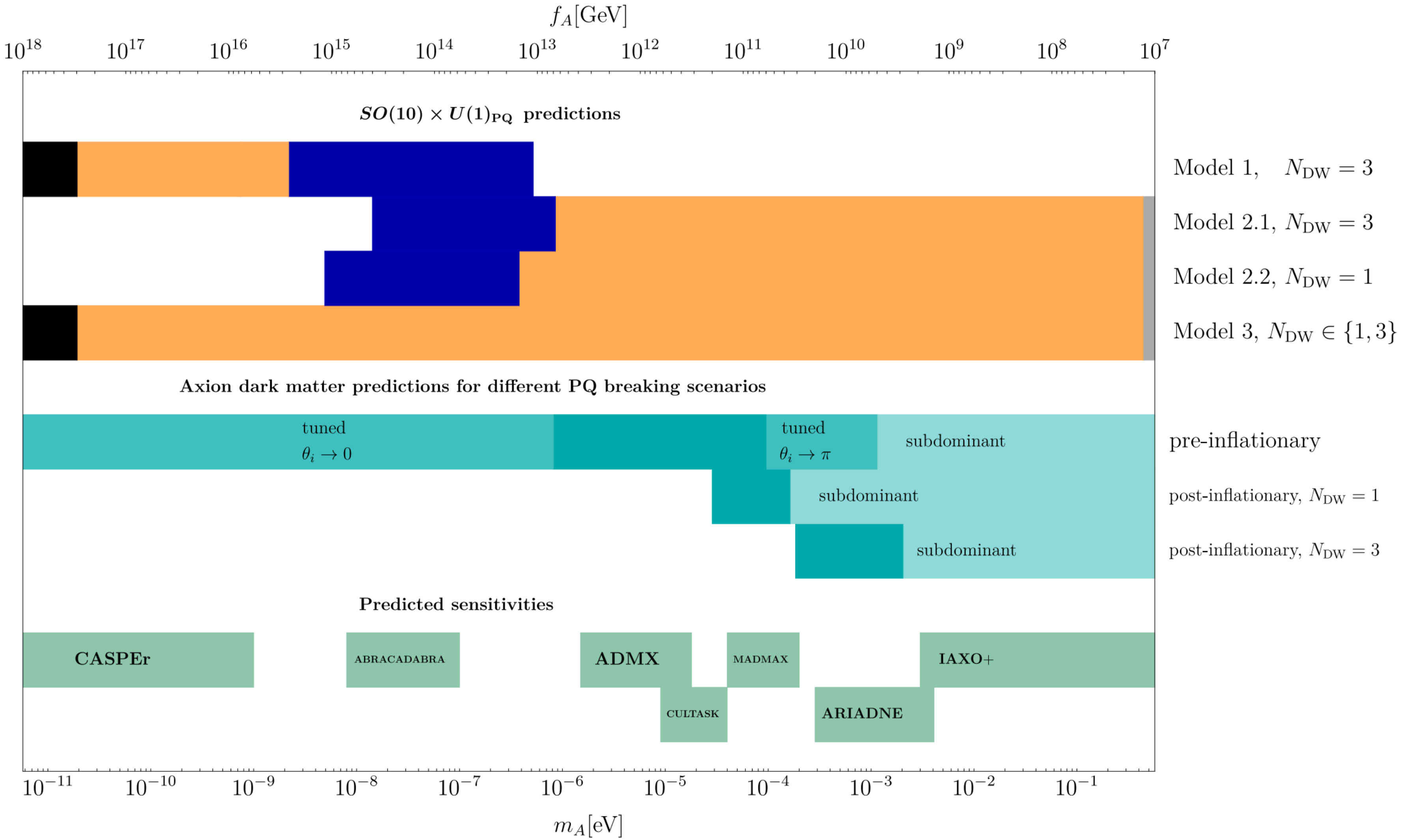
Random 1-loop thresholds

Summary: Unification constraints



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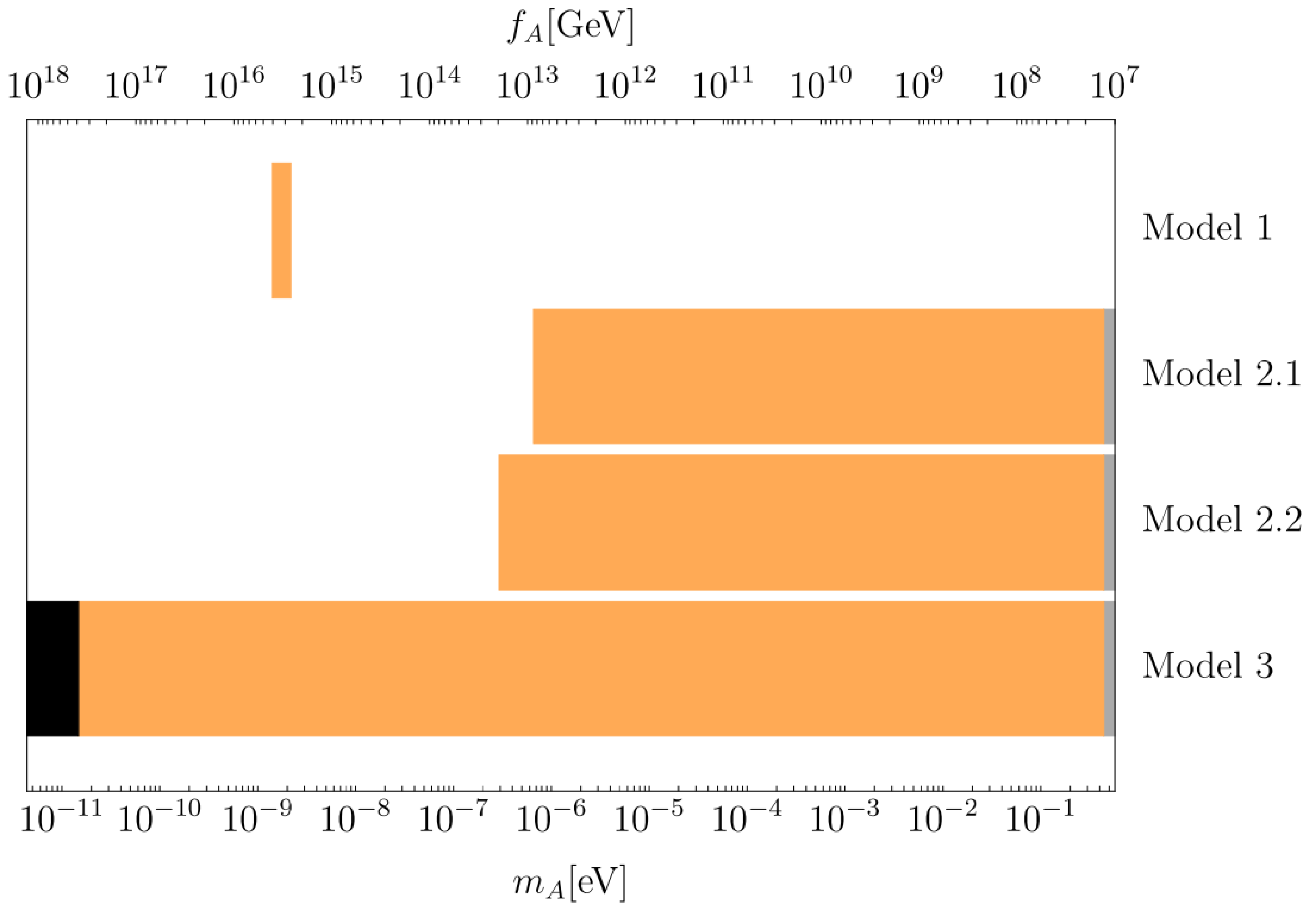
Random 1-loop thresholds

..If Hyperkamiokande saw proton decay in first 10 years



Yellow: Allowed

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Gray: Stellar cooling constraint

Random 1-loop thresholds

Summary of axion couplings

In axial basis (in which GUT symmetry is not manifest)

$$\mathcal{L} = \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{1}{2} m_A^2 A^2 + \frac{\alpha}{8\pi}\frac{C_{A\gamma}}{f_A} A\, F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}\frac{C_{Af}}{f_A}\partial_\mu A\, \overline{\Psi}_f \gamma^\mu \gamma_5 \Psi_f \,,$$

$$C_{A\gamma} = \frac{8}{3} - 1.92(4)\,, \qquad C_{Ae} = \frac{1}{N_{\rm DW}}\sin^2\beta\,,$$

$$C_{Ap} = -0.47(3) + \frac{3}{N_{\rm DW}}[0.29\cos^2\beta - 0.15\sin^2\beta \pm 0.02]\,,$$

$$C_{An} = -0.02(3) + \frac{3}{N_{\rm DW}}[-0.14\cos^2\beta + 0.28\sin^2\beta \pm 0.02]\,,$$

$$\tan^2\beta = ((v_u^{10})^2 + (v_u^{126})^2)/((v_d^{10})^2 + (v_d^{126})^2)$$

DFSZ recovered for $N_{\rm DW}$=3

Conclusions

We identified the axion field, obtained its couplings to gauge bosons, elementary fermions and nucleons, and computed the domain wall number corresponding to the physical PQ symmetry, in several SO(10) models.

Our formalism bridges the gap between UV and IR symmetries. Accounting for orthogonality with respect to massive gauge bosons, we identified the physical PQ symmetry as a combination of UV symmetries.

We clarified issues pertaining to fermion field redefinitions.

We studied in detail constraints from unification, superradiance, cooling, and fermion masses.

The axion in these models can be probed by upcoming experiments!