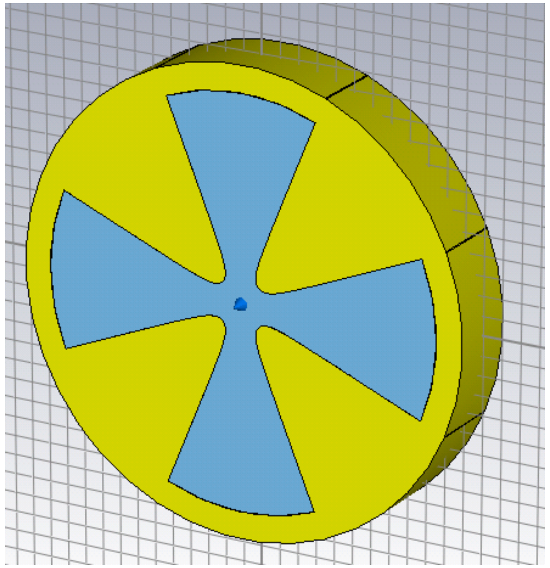
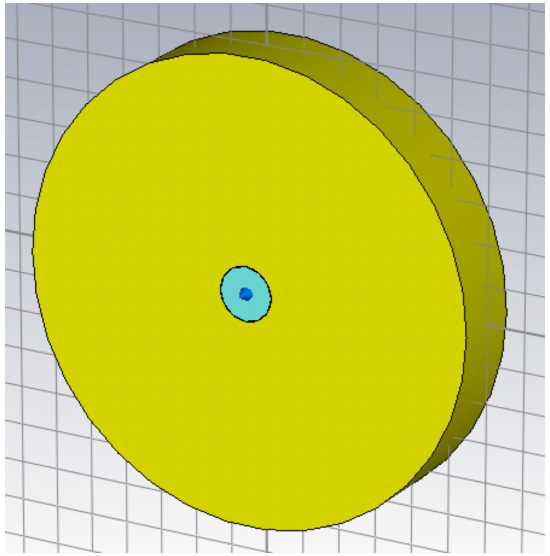
New simulation with PBCI by Erion Gjonaj (TU Darmstadt) confirms that the dipolar impedance of 90 degrees symmetric structures is the same in all directions (as is the case for a circular beam pipe)

O.Berrig HSC section meeting

Comparison of a 4-pole and a round structure





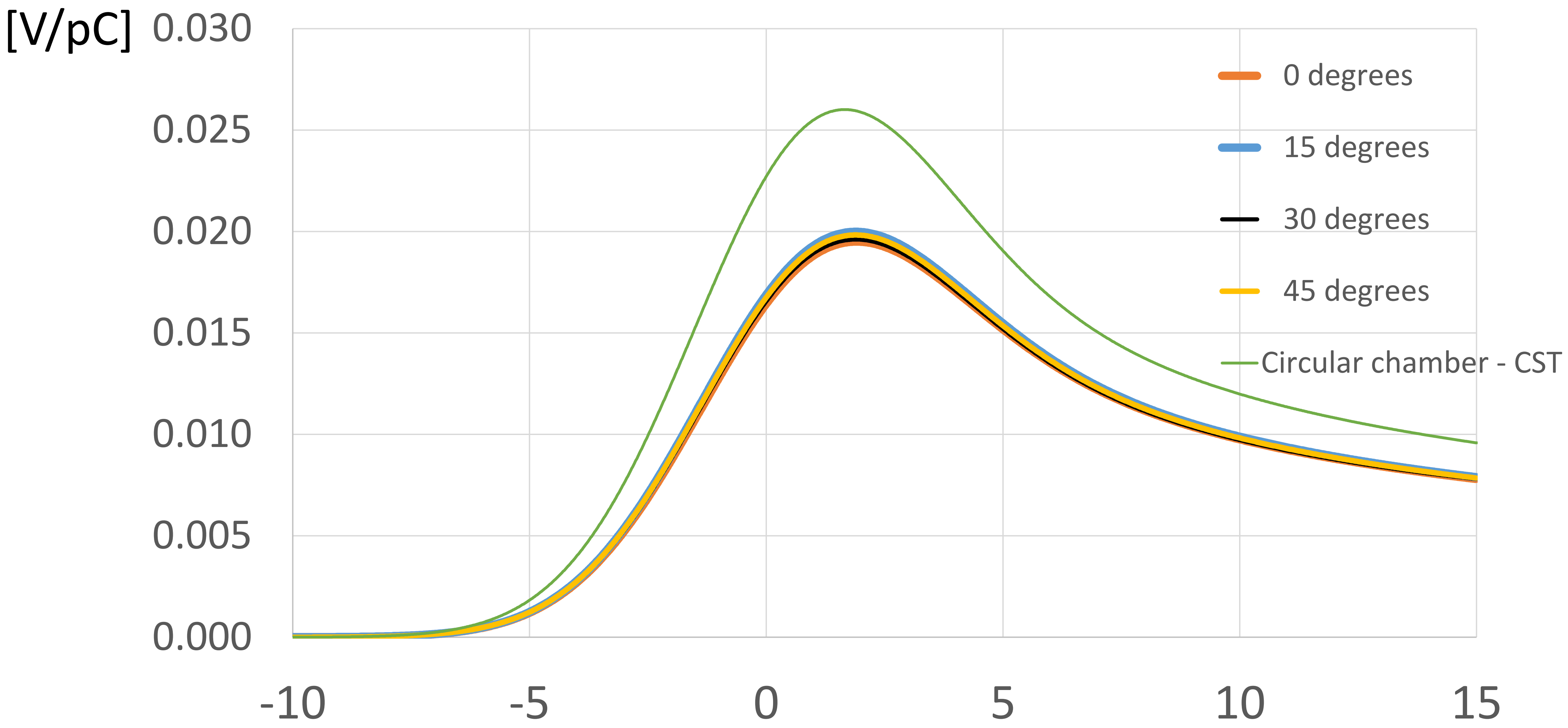
In terms of impedance (up to second order), these two structures are **identical** in that they have only dipolar impedance. Their only difference is the value of the dipolar impedance.

$$Z_{||}[xd, xt, yd, yt] =$$

$$Z0 + Z1X(xd+xt) + Z1Y(yd+yt) + Z2A(xd^2+xt^2-yd^2-yt^2)$$

$$+ Z2B(xd\,yd+xt\,yt) + Z2C(xd\,yt+xt\,yd) + Z2D\,xd\,xt + Z2E\,yd\,yt$$

Wakefields are independent of rotation angle



Simulation curtesy of E.Gjonaj - TU Darmstadt.
(L=400 mm, r=7 mm, conductivity=10^7 S/m)

Calculation of theoretical wakefield – problems!

- **Why is the beam impedance quoted as both R+jwL and R-iwL ?**

- **Is the wake potential seen in the rest frame or in the laboratory frame?**

- **What is the exact transformation from beam impedance to wakefield?**

- **With a Gaussian charge distribution of the particle bunch, then any test particle will be at the same place as a bunch particle which results in infinite forces between them, which gives infinite wakepotential. How to solve this?**

Beam impedance: R + j w L versus R – i w L

Circuit analysis
R + j w L

$$\text{Impedance:}\quad Z(w) = R + j\omega L$$
$$\text{Voltage:}\quad V(t) = V_0 \cdot Cos(w_0\, t)$$
$$\text{Add imaginary part:}\quad V(t) = V_0 \cdot (Cos(w_0\, t) + j \cdot Sin(w_0\, t)) = V_0 \cdot e^{jw_0 t}$$
$$\text{Voltage for analysis:}\quad V(\omega) = V_0$$
$$\text{Current for analysis:}\quad I(\omega) = I_0$$
$$\text{Circuit equation:}\quad V(\omega) = (R + jw_0 L) \cdot I(\omega)$$
$$\text{Solution for I:}\quad I(\omega) = \frac{V(\omega)}{R + jw_0 L} = \frac{e^{-j\phi}}{\sqrt{R^2 + (w_0 L)^2}} \cdot V(\omega)$$
$$\text{Convert to time domain:}\quad I(t) = \frac{e^{-j\phi}}{\sqrt{R^2 + (w_0 L)^2}} \cdot V_0 \cdot e^{jw_0 t}$$
$$\text{Remove imaginary part:}\quad I(t) = \frac{V_0}{\sqrt{R^2 + (w_0 L)^2}} \cdot Cos(w_0 t - \phi)$$
$$\text{where } \phi = ArcTan(\tfrac{w_0 L}{R})$$

“American” Fourier
R + j w L

$$\text{Voltage:}\quad V(t) = V_0 \cdot Cos(w_0\, t)$$
$$\text{Circuit equation:}\quad V(t) = R + L\frac{dI(t)}{dt}$$
$$\text{Fourier Transform:}\quad V(\omega) = R \cdot I(\omega) + j\omega \cdot I(\omega)$$
$$V(\omega) = (R + j\omega L) \cdot I(\omega)$$
$$\text{Solution for I:}\quad I(\omega) = \frac{V(\omega)}{R + j\omega L}$$
$$\text{Inv.Fourier Transform:}\quad I(t) = \frac{V_0(R\, Cos(w_0\, t) + w_0 L\, Sin(w_0\, t))}{R^2 + w_0^2 L^2}$$
$$I(t) = \frac{V_0}{\sqrt{R^2 + (w_0 L)^2}} \cdot Cos(w_0 t - \phi)$$
$$\text{where } \phi = ArcTan(\tfrac{w_0 L}{R})$$

European Fourier
R – i w L

$$\text{Voltage:}\quad V(t) = V_0 \cdot Cos(w_0\, t)$$
$$\text{Circuit equation:}\quad V(t) = R + L\frac{dI(t)}{dt}$$
$$\text{Fourier Transform:}\quad V(\omega) = R \cdot I(\omega) - i\omega \cdot I(\omega)$$
$$V(\omega) = (R - i\omega L) \cdot I(\omega)$$
$$\text{Solution for I:}\quad I(\omega) = \frac{V(\omega)}{R - i\omega L}$$
$$\text{Inv.Fourier Transform:}\quad I(t) = \frac{V_0(R\, Cos(w_0\, t) + w_0 L\, Sin(w_0\, t))}{R^2 + w_0^2 L^2}$$
$$I(t) = \frac{V_0}{\sqrt{R^2 + (w_0 L)^2}} \cdot Cos(w_0 t - \phi)$$
$$\text{where } \phi = ArcTan(\tfrac{w_0 L}{R})$$

Concluding remarks

- **The symmetry analysis** (see https://indico.cern.ch/event/677919/) show that 90 degree symmetric structures have only dipolar impedance and that this dipolar impedance does not change when the structure is rotated.

- **This property has been confirmed by simulations.** The simulations were done by E.Gjonaj (TU Darmstadt) with the program PBCI.

- **Still problems calculating the theoretical wake potential**