Choosing the On-Shell RS in the Complex MSSM

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Outline: On-shell RS in the Complex MSSM

- Introduction
- Fully Automated Calculations in the cMSSM:
  Implement Renormalization in FeynArts/FormCalc
- RS choice in the chargino/neutralino sector
- Conclusions

Introduction

Low Energy Supersymmetry (here MSSM)

- Solves hierarchy/naturalness problem:
  Higgs mass stable against rad.corr. $m_H \sim \mathcal{O}(M_Z)$
- Provides a natural candidate for CDM:
  here the neutralino $\tilde{\chi}_1^0$ $m_{\tilde{\chi}^0/\tilde{\chi}^\pm} < \mathcal{O}(\mathrm{TeV})$
- Unification of gauge couplings @ M_{GUT}: GUT relations?

CP-violation

- Baryon asymmetry: CP-violation in the SM not large enough
  MSSM with complex couplings (cMSSM)
  $\Rightarrow$ new sources of CP-violation

Motivation

Why precisition calculations for DM & why in the MSSM?

thermal relic DM

$\Rightarrow$

most allowed scenarios coannihilation channels

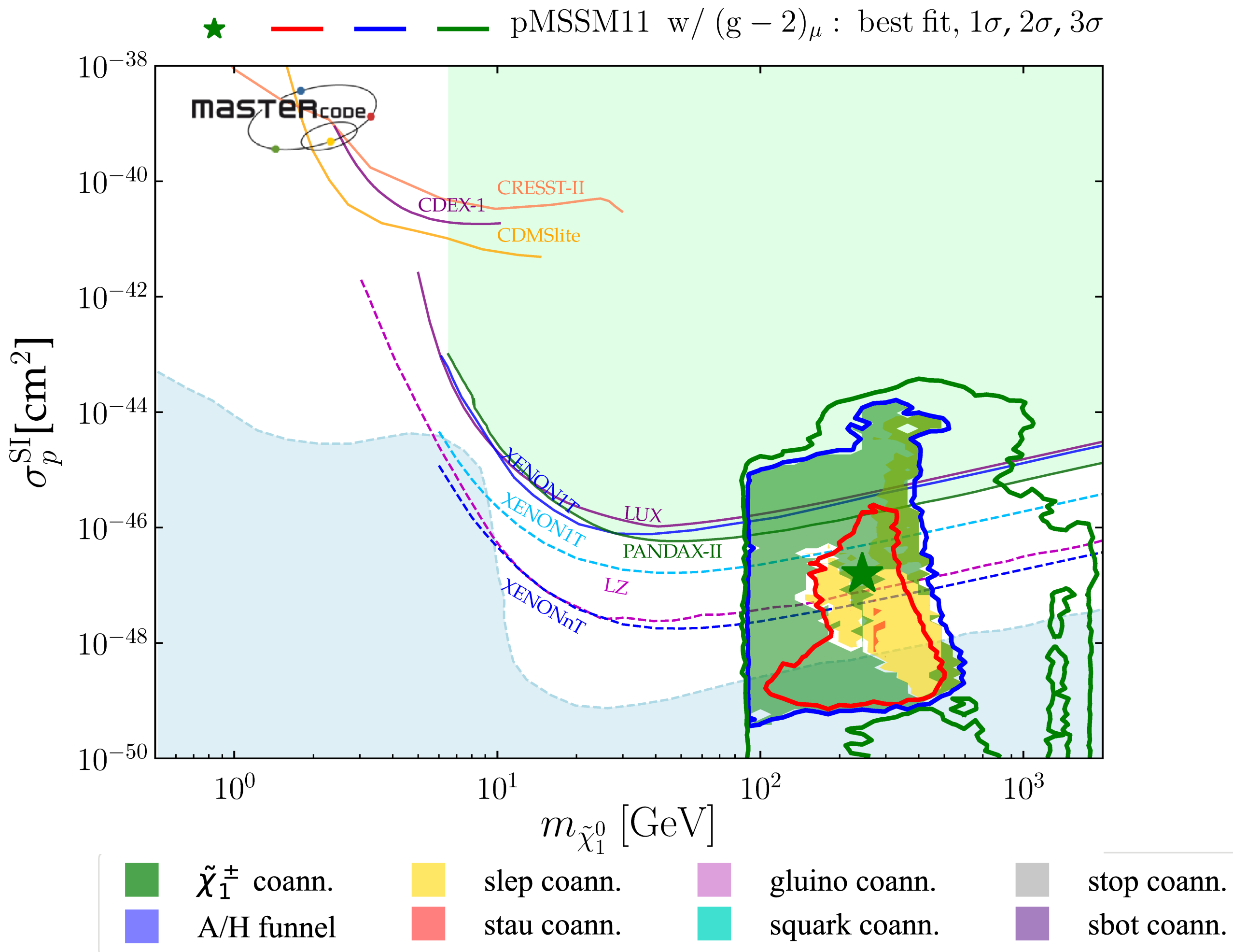
$\Rightarrow$

very sensitive to mass differences

$\Rightarrow$

need on-shell masses!

Δm needs to be precisely computed to determine model parameters!



[Mastercode Colab., 1710.11091]

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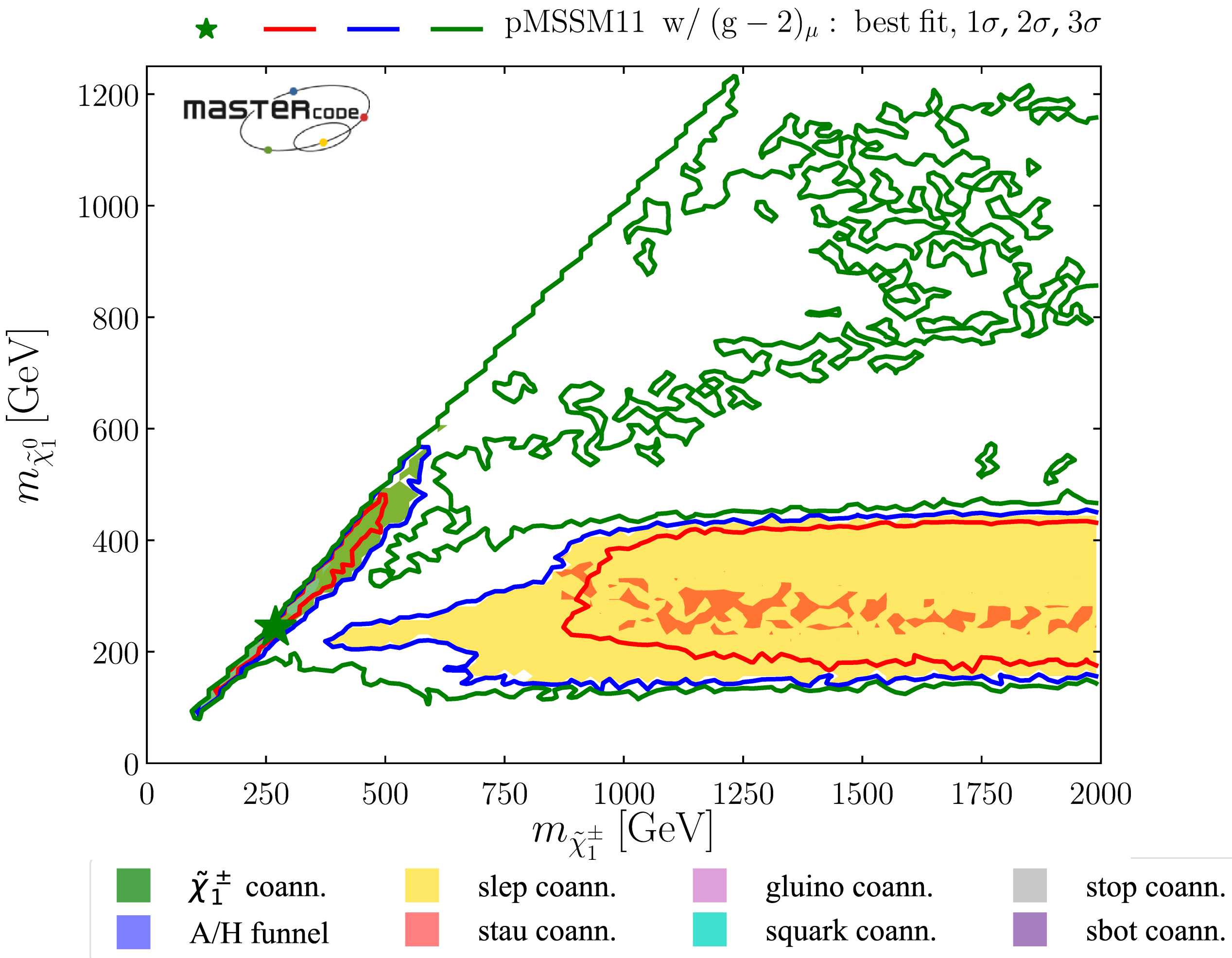
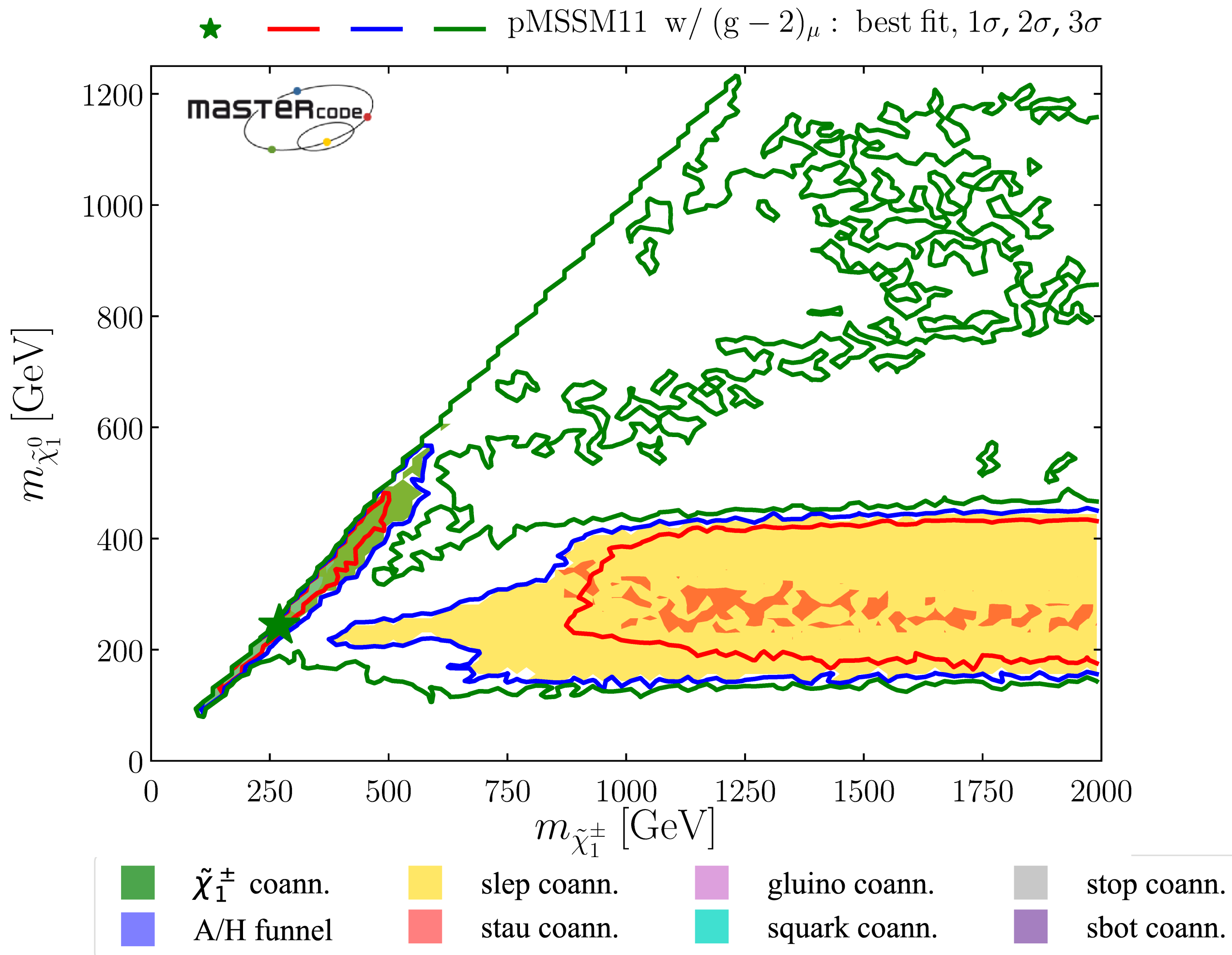


[Mastercode Colab., 1710.11091]

Motivation

Why precisition calculations for DM & why in the MSSM?

compressed spectra typical for other WIMP DM models: small DM couplings, enhanced @freeze-out
$\Rightarrow$
avoid constraints (DD,ID,LHC)



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Motivation

Precision calculations must match experiemental precision

Examples:

- m_h @ 2-Loop requires 1-Loop subrenormalization
- 1-Loop Branching ratios: renormalization of all sectors

We need:

- Consistent renormalization of the full cMSSM
- Implementation in FA/FC for fully automated calculations

Renormalization Scheme:

- Mostly on-shell: all processes with on-shell external particles
- Need criteria to decide between possible RS choices

Aim

- What is the best choice of On-Shell RS?
- Need quantitative prescription
  - automate this choice!
  - implement in code
- Switch between RS:
  needed to scan over full parameter space
- Compare different RS:
  - estimate 1-loop uncertainties
  - analyze full set of observables in different RS

The Grand Scheme

The Big Question: which Lagrangian describes the world?

- The LHC may discover BSM physics soon
- $\Rightarrow$ precise measurements at the ILC

- Theory calculations must match experiemental precision:
  —masses
  —cross sections
  —branching ratios
  —angular distributions
  —etc.

We focus on the MSSM

- Enlarged Higgs sector: two Higgs doublets
- Many scales
- complex phases

Fully Automated cMSSM Calculations

Generic problems for SUSY loop calculations:

- SUSY has to be preserved in the calculation
- Many different mass scales
- Many more mass scales than free parameters
- Even more parameters: mixing angles, complex phases
- Renormalization is much more involved than in the SM
  –much less explored than in the SM
  –has to preserve/respect mass relations
  –depend on mass scales realized in Nature
  –sometimes no really good solution exists (e.g. $\tan\beta$)
  –many sectors enter at the same time

  $\Rightarrow$ this was the biggest issue!

Recap: chargino and neutralino sectors

argino and neutralino mass matrices:

$$\text{ass} = \left(\tilde{W}^\pm \;\; \tilde{H}^\pm\right) \cdot \begin{pmatrix} M_2 & \sqrt{2}\sin\beta\, M_W \\ \sqrt{2}\cos\beta\, M_W & \mu \end{pmatrix} \cdot \begin{pmatrix} \tilde{W}^\pm \\ \tilde{H}^\pm \end{pmatrix}$$

$$\left(\tilde{B}^0 \tilde{W}^0 \tilde{H}_1^0 \tilde{H}_2^0\right) \cdot \begin{pmatrix} M_1 & 0 & -M_Z\, s_W \cos\beta & M_Z\, s_W \sin\beta \\ 0 & M_2 & M_Z\, c_W \cos\beta & -M_Z\, c_W \sin\beta \\ -M_Z\, s_W \cos\beta & M_Z\, c_W \cos\beta & 0 & -\mu \\ M_Z\, s_W \sin\beta & -M_Z\, c_W \sin\beta & -\mu & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{B}^0, \\ \tilde{W}^0, \\ \tilde{H}_1^0, \\ \tilde{H}_2^0 \end{pmatrix}$$

agonalization $\Rightarrow$ Higgsinos and gauginos mix:

$\tilde{V}^\pm, \tilde{H}^\pm \to \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$: chargino mass eigenstates

$\tilde{B}^0, \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$: neutralino mass eigenstates

mmon parameters $\Rightarrow$ relations between masses and couplings

Renormalization of the cMSSM

Example: Chargino and neutralino sector

On-shell renormalization

- renormalize 3 (complex) parameters: $M_1,\ M_2,\ \mu$
- chargino-neutralino sector $\Rightarrow$ 6 mass paramters:

$$m_{\tilde{\chi}_i^\pm},\ i = 1,2,\ m_{\tilde{\chi}_j^0},\ j = 1,\ldots,4$$

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$$m_{\tilde{\chi}_i^\pm},\ i = 1, 2,\ m_{\tilde{\chi}_j^0},\ j = 1, \ldots, 4$$

CCN_j scheme: choose $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, m_{\tilde{\chi}_j^0}$ on-shell $\Rightarrow$ $\delta M_1,\ \delta M_2,\ \delta\mu$

remaining masses receive finite mass shifts

Q: why $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, m_{\tilde{\chi}_j^0}$?

Chargino and neutralino sectors: renormalization

CCN$_j$ scheme: choose $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, m_{\tilde{\chi}_j^0}$ on-shell $\Rightarrow$ $\delta M_1,\ \delta M_2,\ \delta\mu$

$$\left[\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{\chi}_i^\pm}(p)\right]_{ii}\tilde{\chi}_i^\pm(p)\Big|_{p^2=m^2_{\tilde{\chi}_i^\pm}} = 0, \qquad (i=1,2),$$

$$\left[\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{\chi}_j^0}(p)\right]_{jj}\tilde{\chi}_j^0(p)\Big|_{p^2=m^2_{\tilde{\chi}_j^0}} = 0,$$

3 eqs. define 3 complex parameters & field renormalization const.

Mass shifts

$$m_{\tilde{\chi}_k^0} = m^{(0)}_{\tilde{\chi}_k^0} + \Delta m_{\tilde{\chi}_k^0}, \qquad (k=1,\ldots,4;\ k\neq j)$$

$$\Delta m_{\tilde{\chi}_k^0} = -\tfrac{1}{2}\text{Re}\Big\{\widetilde{\text{Re}}\Big[m_{\tilde{\chi}_k^0}\hat{\Sigma}^L_{\tilde{\chi}_k^0}(m^2_{\tilde{\chi}_k^0}) + \hat{\Sigma}^{SL}_{\tilde{\chi}_k^0}(m^2_{\tilde{\chi}_k^0}) + (L\leftrightarrow R)\Big]\Big\}$$

Choose masses of charged particles as input to avoid IR divergencies

Chargino and neutralino sectors: renormalization

On-Shell schemes have numerical instabilities for some parameters!
$\to$ f.i. $|\mu| = M_2$, example in next slides
$\Rightarrow$ no fundamental problem, need alternative RS conditions

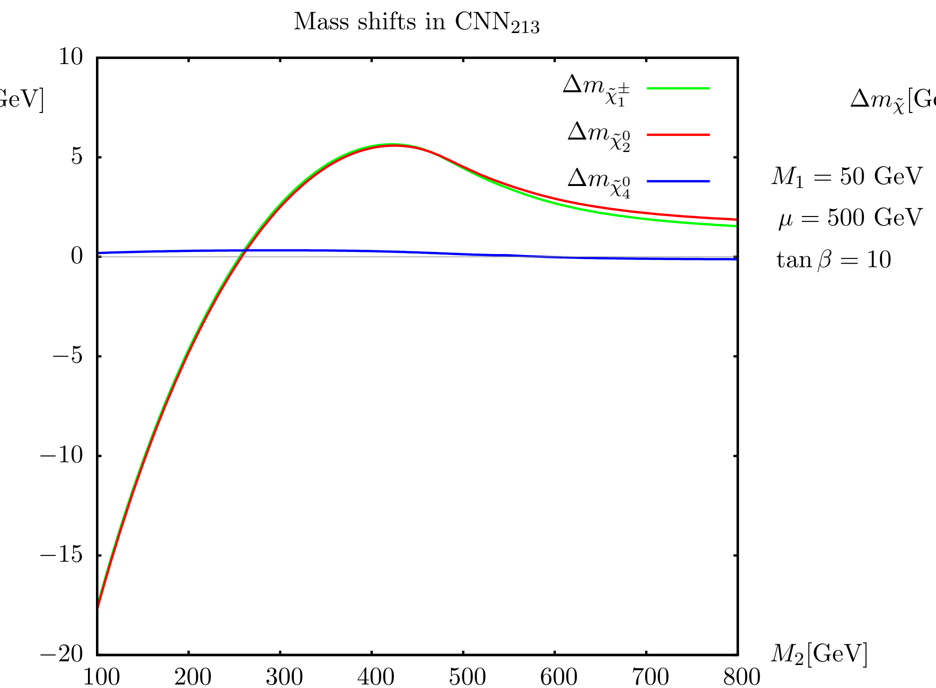
CCN$_j$ scheme: ($m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, m_{\tilde{\chi}_j^0}$ on-shell)

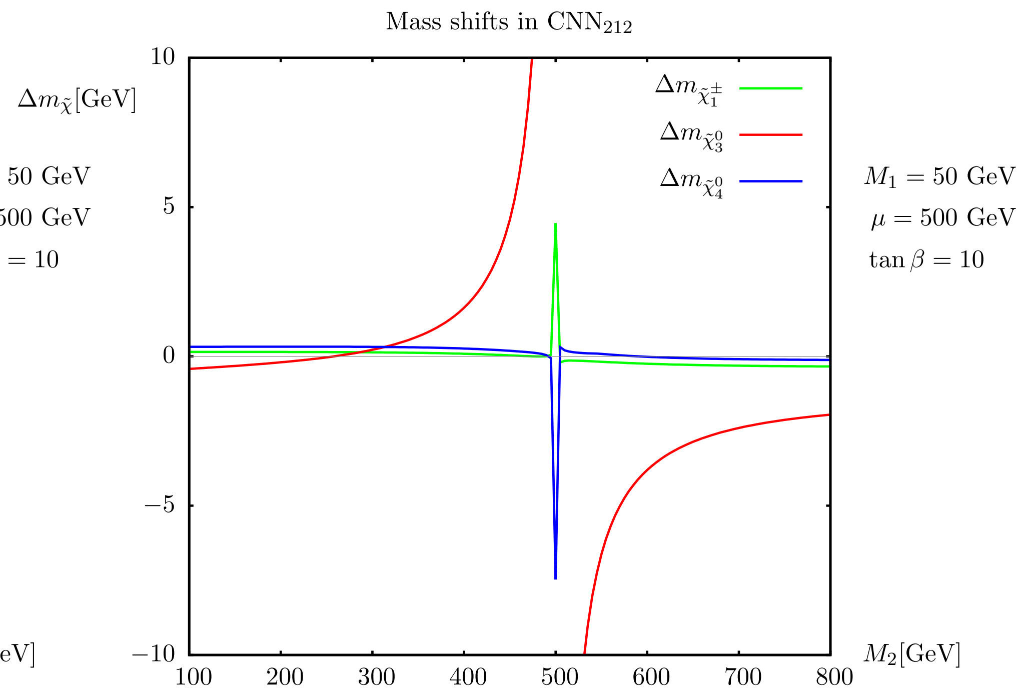
CNN$_{i,j,k}$ scheme: ($m_{\tilde{\chi}_i^\pm}, m_{\tilde{\chi}_j^0}, m_{\tilde{\chi}_k^0}$ on-shell)

NNN$_{i,j,k}$ scheme: ($m_{\tilde{\chi}_i^0}, m_{\tilde{\chi}_j^0}, m_{\tilde{\chi}_k^0}$ on-shell)

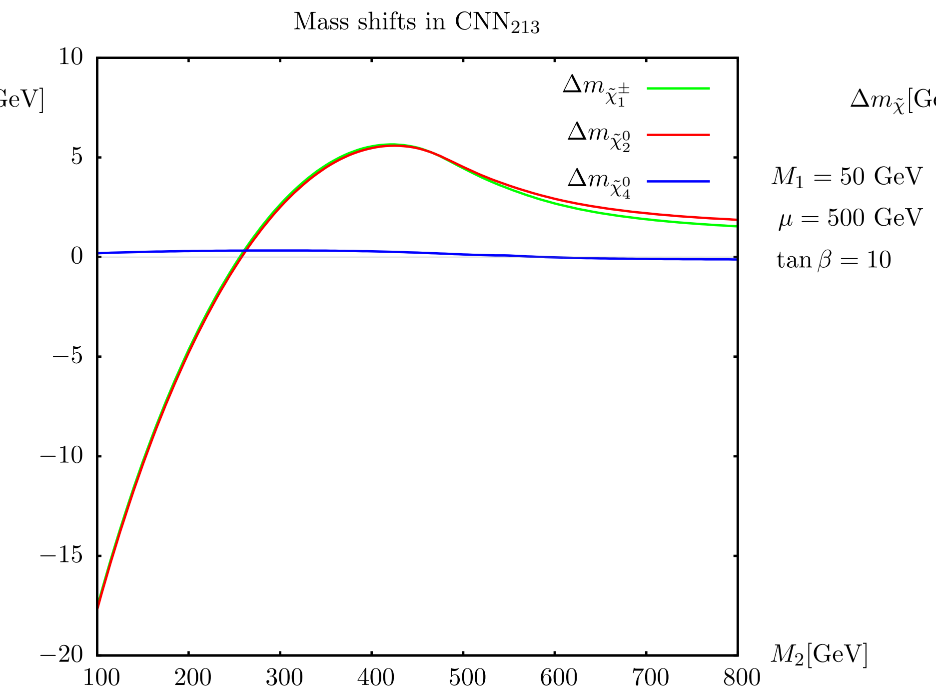
Shift mass of chargino only if not an external particle!
to avoid introducing higher order IR singluarities

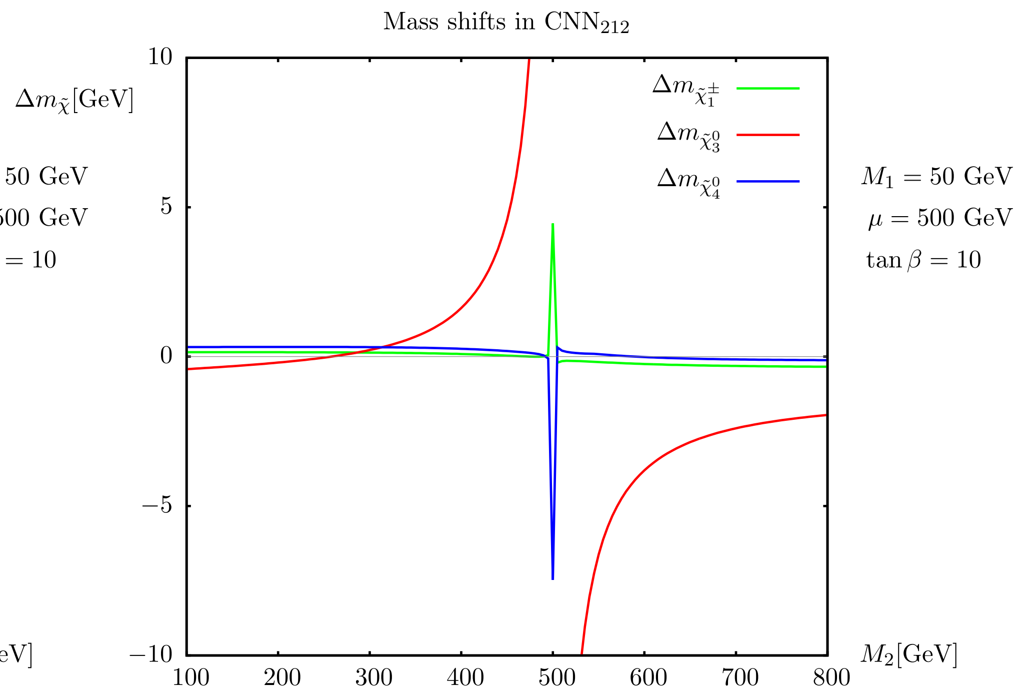
Example: mass shifts in CNN_{213} & CNN_{212} Ren.Schemes





Example: mass shifts in CNN_{213} & CNN_{212} Ren.Schemes

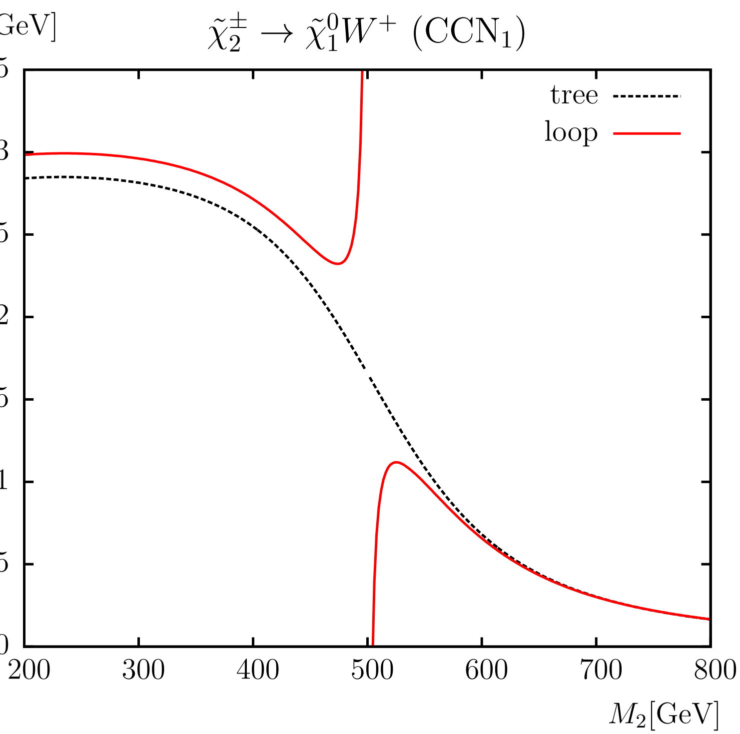


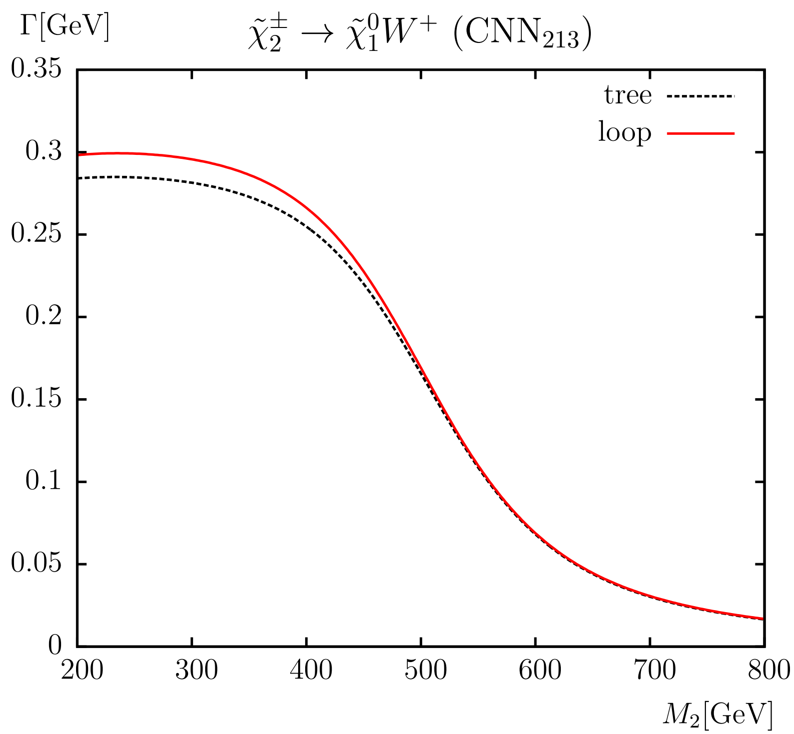


- CNN212 unstable at $\mu = M_2$
- CNN213 unstable at $M_2 \approx M_1$

Renormalization schemes: matching

No on-shell scheme works everywhere: here $|\mu| \simeq |M_2|$ region





$M_1 = 50\text{GeV}$

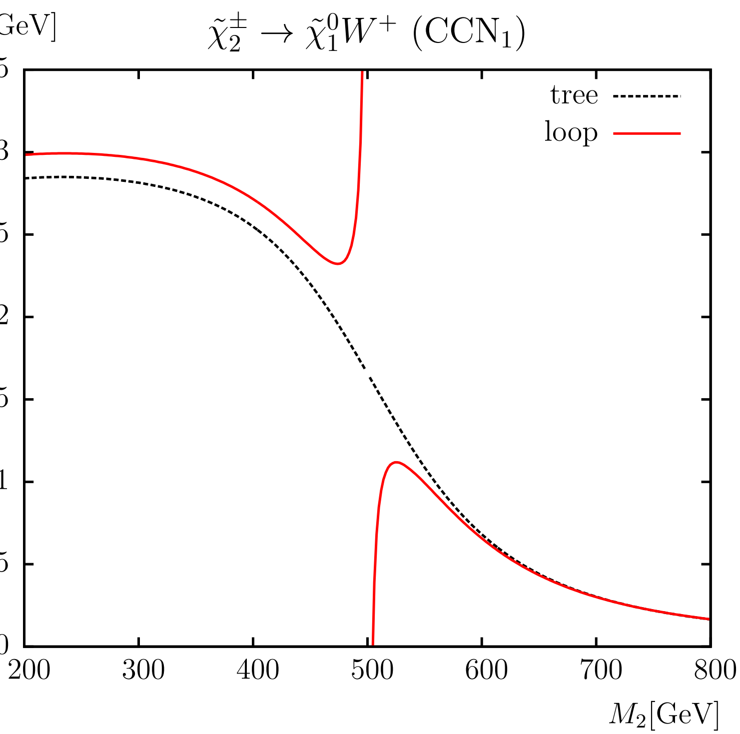
$\mu = 500\text{GeV}$

Beware: M_1, M_2, μ renormalized in each RS

$\Rightarrow$ should compare schemes as function of same $\overline{\text{DR}}$ parameters!

Renormalization schemes: matching

No on-shell scheme works everywhere: here $|\mu| \simeq |M_2|$ region



- "good RS"
  $\Rightarrow$ smaller rad. corrections
- different schemes
  $\Rightarrow$ theory uncertainty
- implement results in FA/FC:
  matching of RSs

Chargino and neutralino sectors: renormalization

Analysis of Δm in CCN,CNN,NNN [Chatterjee,Drees,Kulkarni,Xu 2011]

Conclude that best choice is:

CNN$_{i,j,k}$ scheme:

- $m_{\tilde{\chi}_i^\pm}$ wino-like
- $m_{\tilde{\chi}_j^0}$ bino-like
- $m_{\tilde{\chi}_k^0}$ higgsino-like

but:

- what about mixed scenarios?
- which higgsino?
  (pseudo-Dirac system $\Rightarrow$ opposite relat. intrinsic CP)
- need quantitative prescription!

Chargino and neutralino sectors: renormalization

Dark Matter relic density scans in the MSSM very sensitive to Δm

[Beneke,Bharucha et al '16] choose as "best":

- $M_2 < |M_1| < |\mu|$: CNN[1,2,3] ($\tilde{\chi}^0_3$; $m < 0$ higgsino)
- $M_2 < |\mu| < |M_1|$: CNN[1,2,4] ($\tilde{\chi}^0_2$; $m < 0$ higgsino)

- Scan choice needs too much human input and insight (error prone, slow, . . .)!
- Interesting DM scenarios: coannihilation $\Rightarrow$ mixed scenarios! (see e.g. [Beneke,Hellmann,Ruiz-Femenía '14][Harz,Herrmann,Klasen'16])
  $\rightarrow$ implementation in spectrum generators!
  FeynArts/FormCalc/LoopTools $\Rightarrow$ FeynHiggs

Chargino and neutralino sectors: renormalization

(notation: $\mathbf{M}_{\tilde{\chi}^\pm} = V^* \mathbf{X}^T U^\dagger$, $\quad \mathbf{M}_{\tilde{\chi}^0} = N^* \mathbf{Y} N^\dagger$)

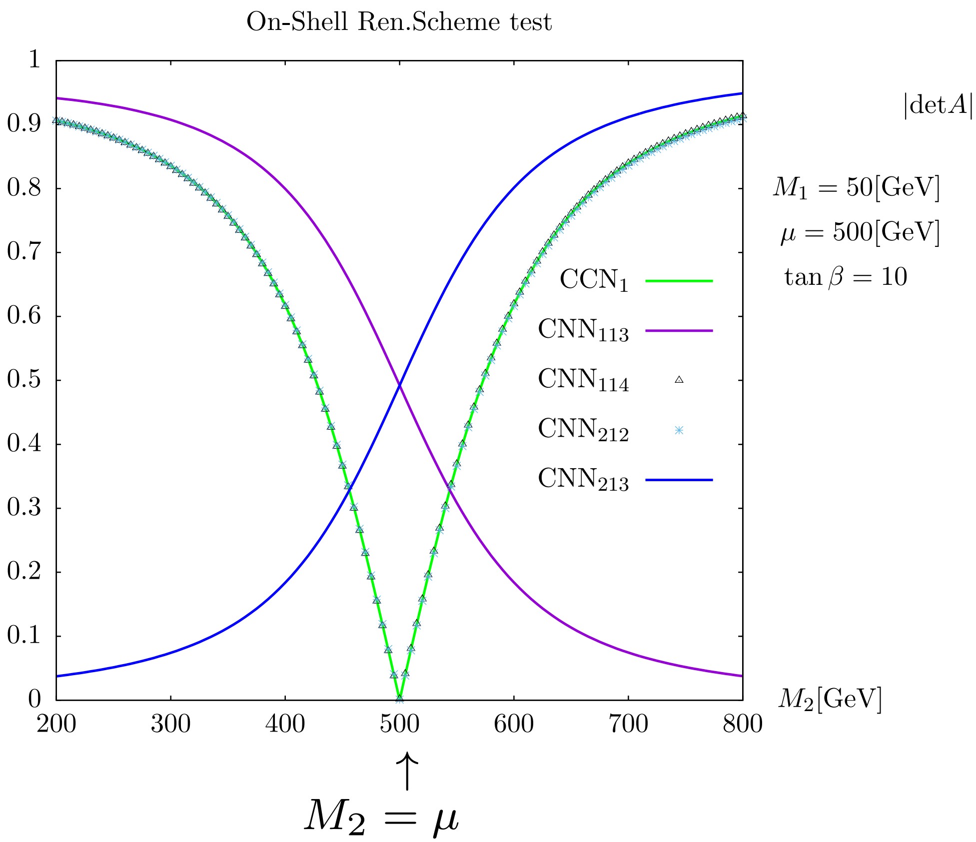
- Starting point
  $[\delta \mathbf{M}_{\tilde{\chi}^\pm}]_{ii} = [V^* \delta \mathbf{X}^T U^\dagger]_{ii}$
  $[\delta \mathbf{M}_{\tilde{\chi}^0}]_{ii} = [N^* \delta \mathbf{Y} N^\dagger]_{jj}$ & On-Shell Ren.Cond.
- leads to linear equations:

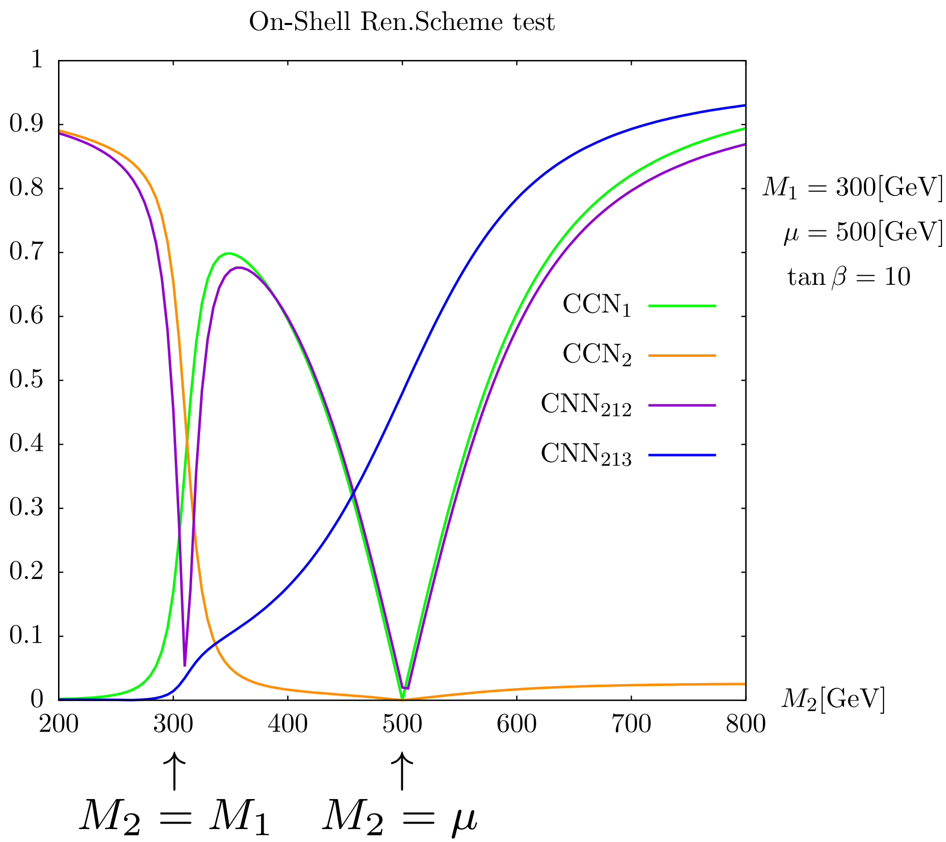
$$f_i(\Sigma) + g_i(\delta M_W, \delta M_Z, \delta \tan\beta, \delta \sin\theta_W) := \delta \tilde{m}_{\tilde{\chi}_i^\pm}$$
$$= a_{i1} \delta M_1 + a_{i2} \delta M_2 + a_{i3} \delta \mu$$

$\Rightarrow$ solve for $\delta M_1 \delta M_1, \delta M_2, \delta \mu$: need to invert $A \equiv \{a_{ij}\}$
$\Rightarrow$ only possible if A can be inverted

measure of good RS: $|\det A| \gg 0$ $\quad (\max|\det A|$ normalized to $1)$

Ren.Scheme tests: (example)





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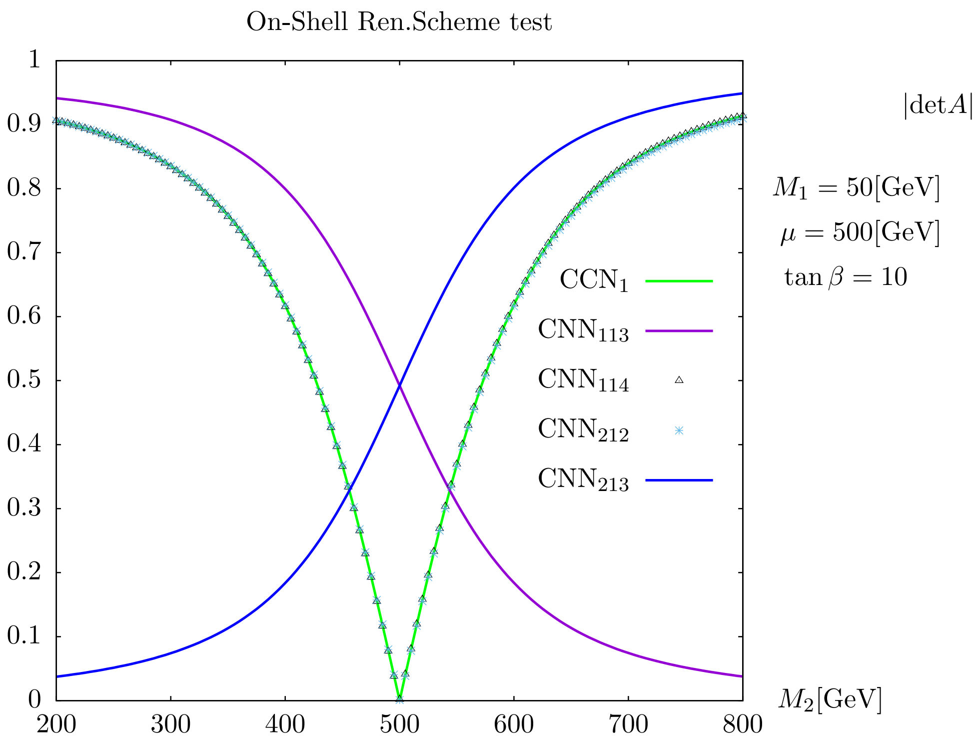
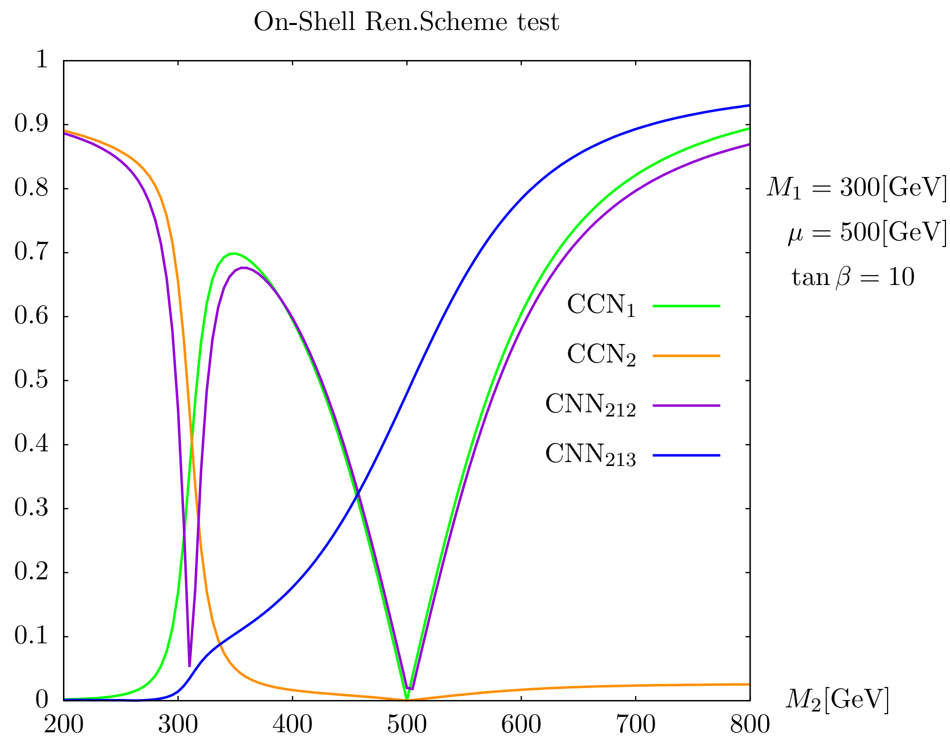




$\uparrow$ $M_2 = \mu$ $\qquad$ $\uparrow$ $M_2 = M_1$ $\quad$ $\uparrow$ $M_2 = \mu$

- need to switch On-Shell Ren.Schemes in any scan
- choice should be automized

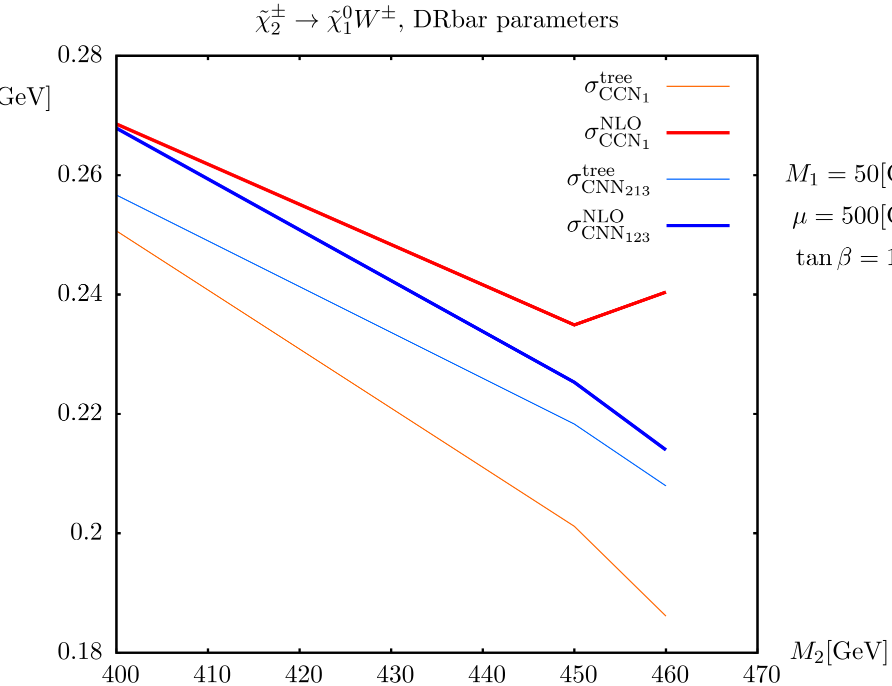
Ren.Scheme selection

- (a) RS w/ no mass-shifts on external charged particles necessary to avoid IR singularities (loops w/same internal & external masses)
- (b) RS with $|\mathrm{detA}| \gtrsim 0.3$? Analysis in progress.
- (c) FormCalc allows run-time selection of Ren.Schemes $\Rightarrow$ code only needs selection condition: work in progress

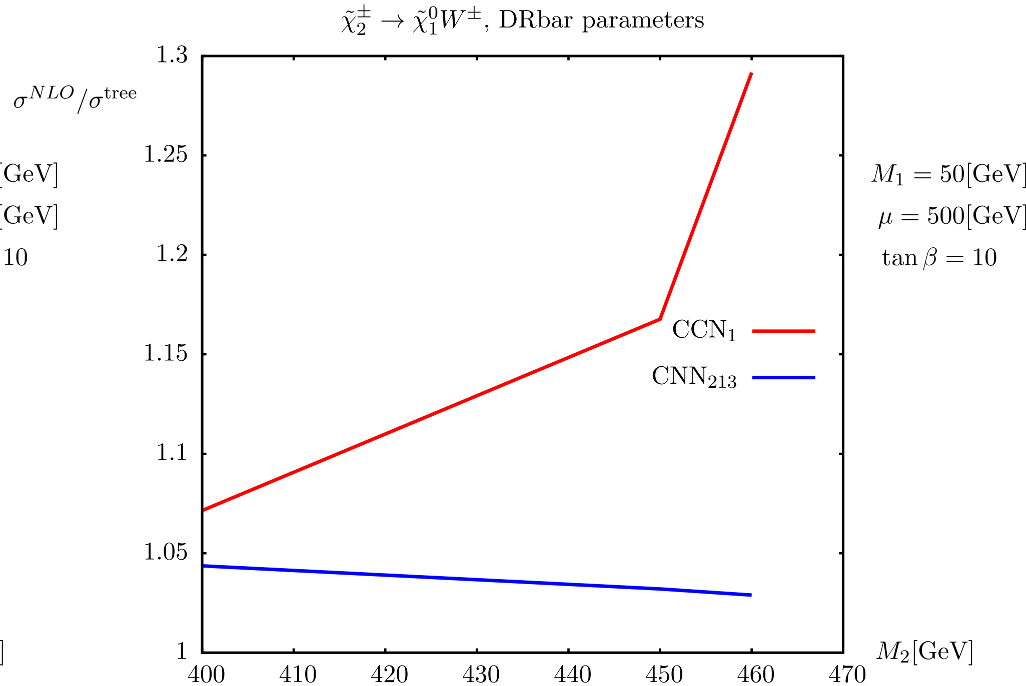
Renormalization scheme comparison (preliminary)

$\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm$ in CCN$_1$ & CNN$_{213}$, $\overline{\mathrm{DR}}$ paramters

LO & NLO cross-sections (prelim.)



NLO/LO cross-sections (prelim.)



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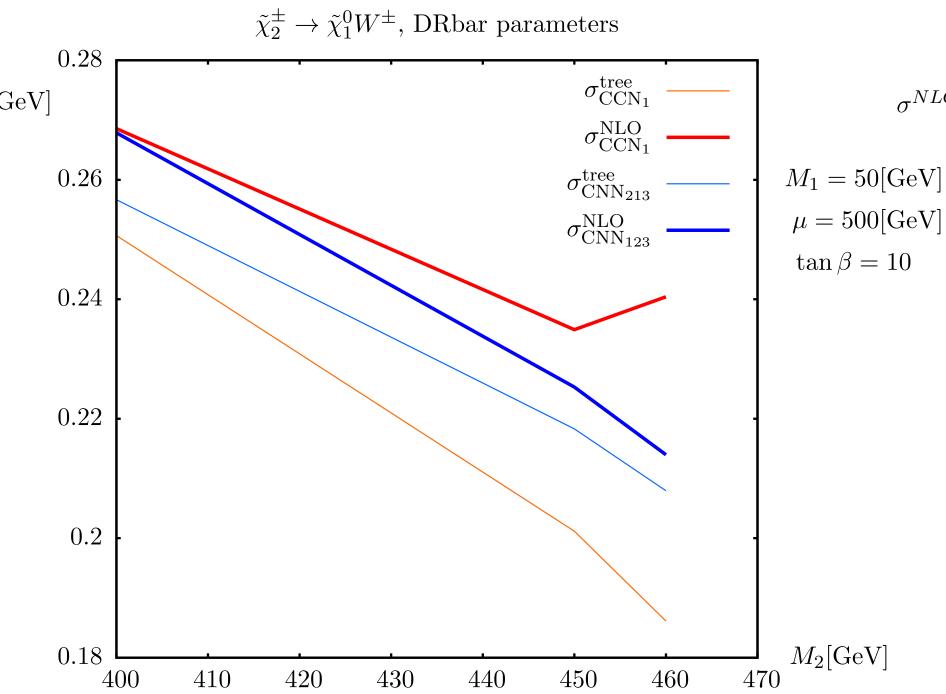

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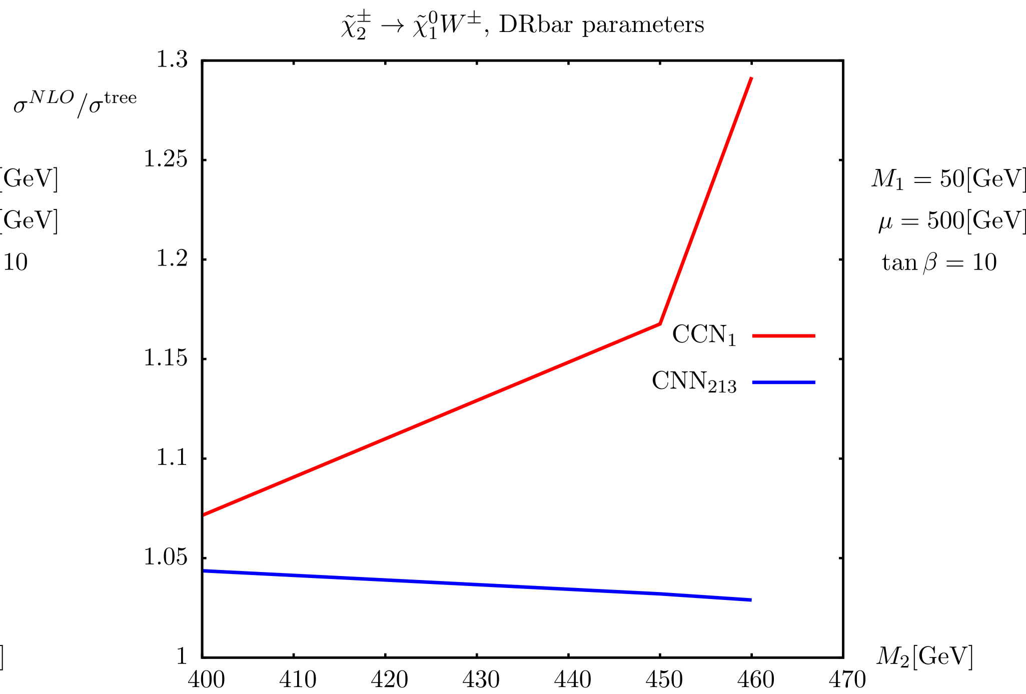
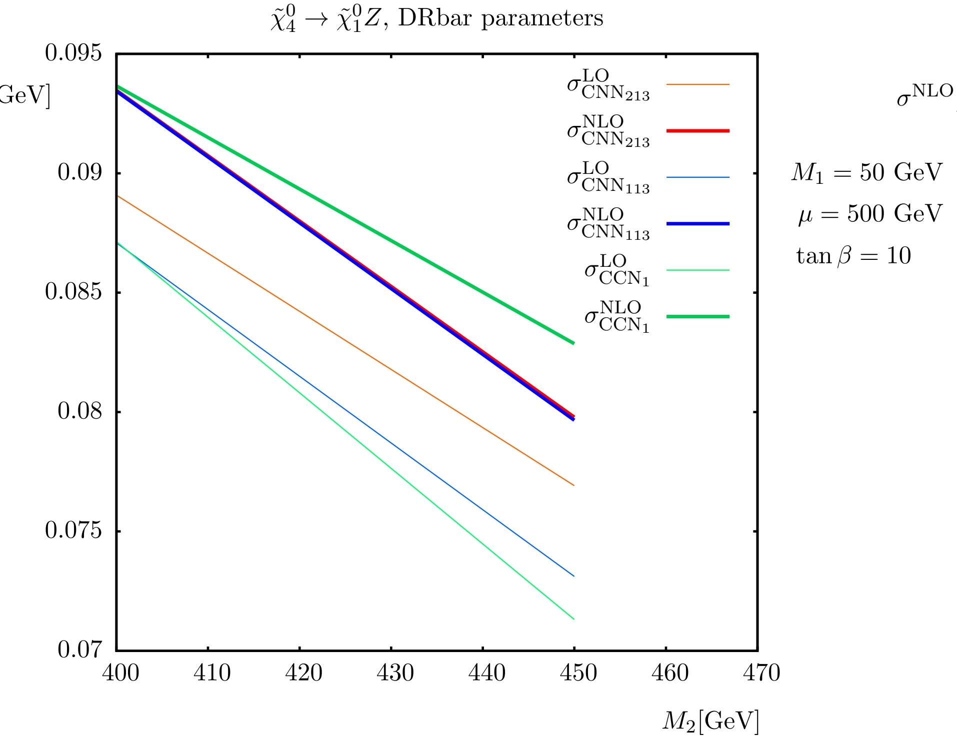


- tree-level differ due to $\overline{\mathrm{DR}} \to \mathrm{OS}$ parameter-shift
- NLO results converge at low M_2 (@ larger $|\mu - M_2|$)

Renormalization scheme comparison (preliminary)

$\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 Z$ in CCN$_1$, CNN$_{113}$ & CNN$_{213}$, $\overline{\mathrm{DR}}$ paramters

LO & NLO cross-sections (prelim.)



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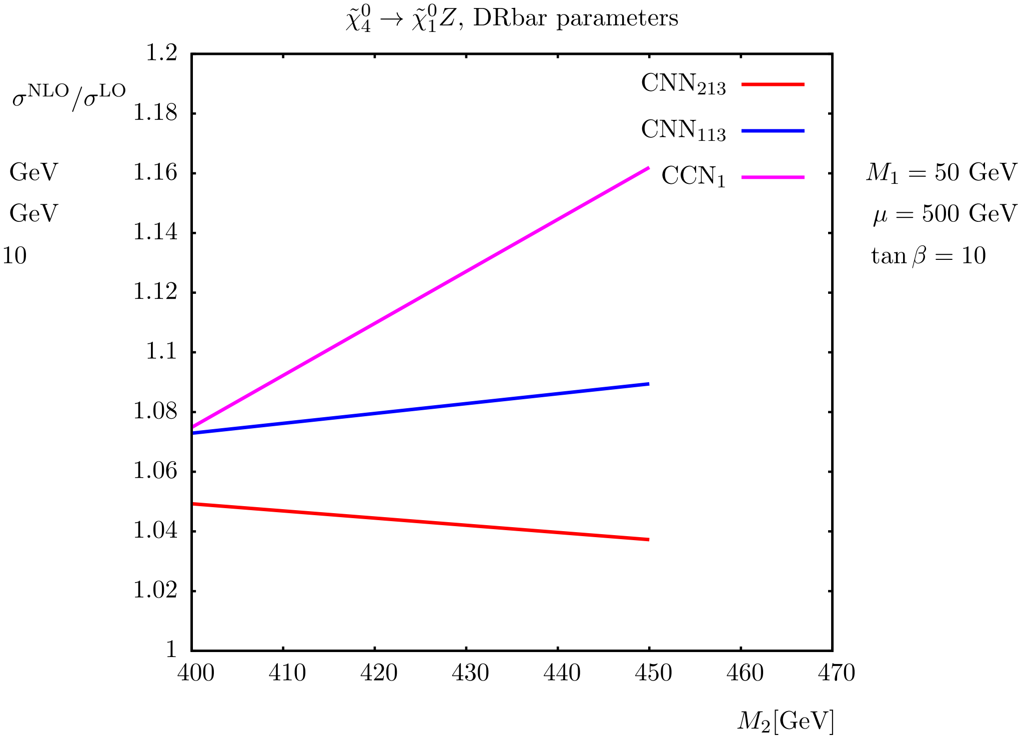

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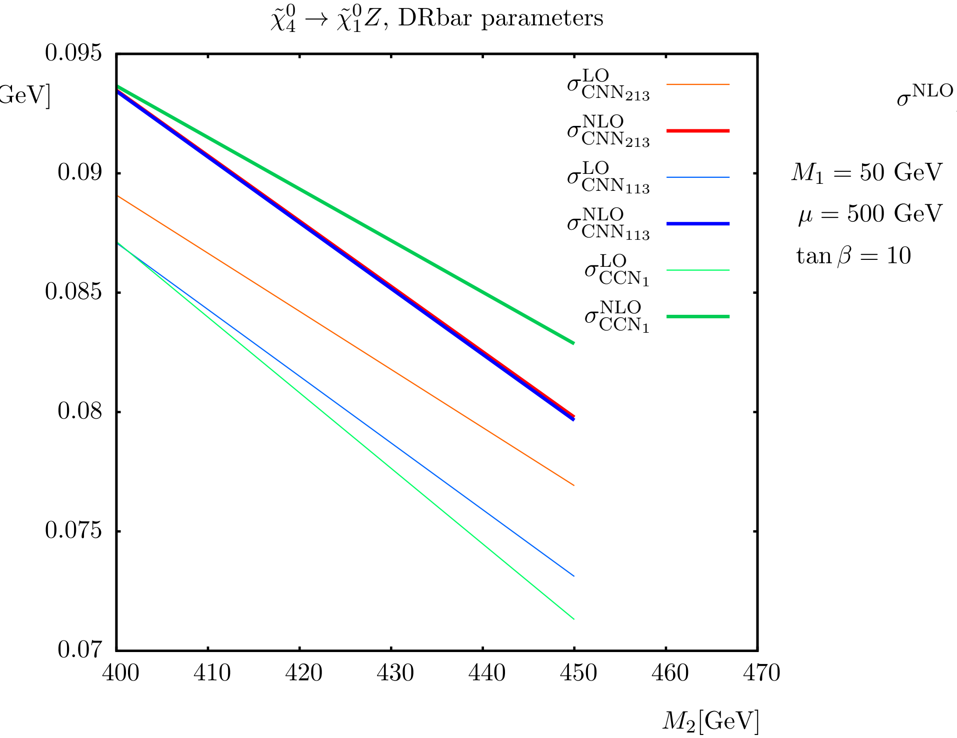

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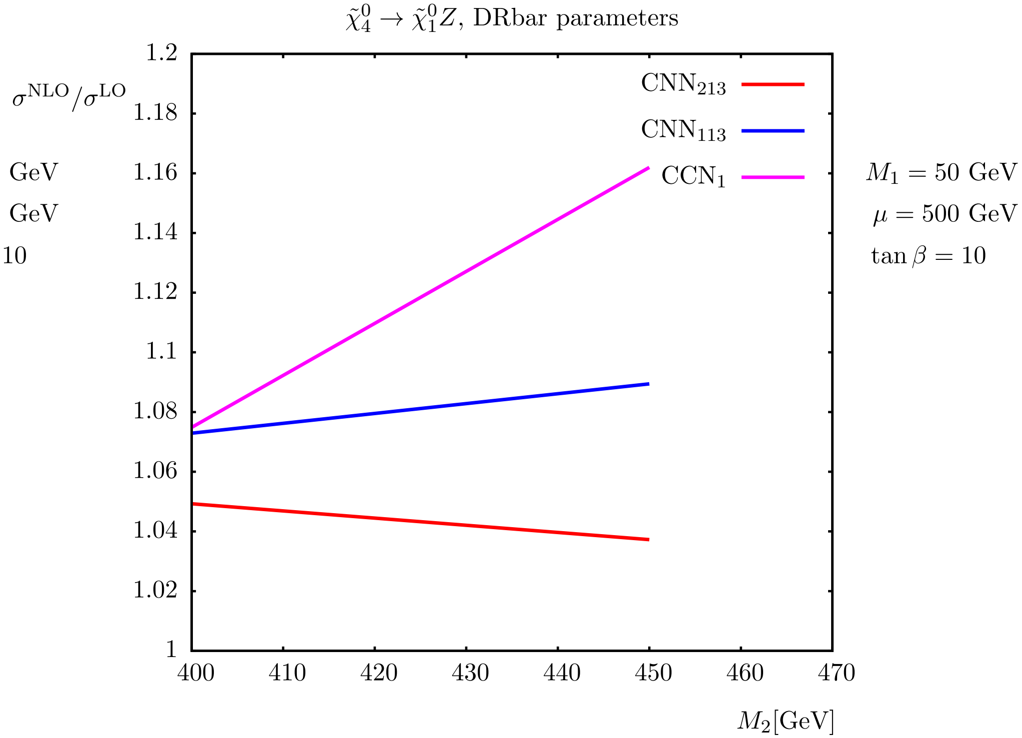

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Summary

- Renormalization of the full complex MSSM under control: FeynArts 3.9 MSSMCT model file including complete 1-loop renormalization
- FormCalc (≥ 8.4): run-time Ren.Scheme selection
  $\rightarrow$ implement automatic selection of on-shell ren.scheme in the chargino/neutralino sector
- evaluate scheme dependence and uncertanties for masses, $1 \rightarrow 2$ and $2 \rightarrow 2$ processes.
- procedure extendable to other sectors & other models

backup transparencies

cMSSM @ one-loop: overview

- Higgs wave function renormalization and $\tan\beta$: $\overline{\mathrm{DR}}$
- Higgs masses: on-shell.
  Z_H-matrix: $h, H, A \to h_1, h_2, h_3$ [FeynHiggs]
- electroweak gauge bosons: on-shell
- quark sector: internal m_b $\overline{\mathrm{DR}}$, external m_b on-shell,
  other quarks on-shell
- squark sector: A_b $\overline{\mathrm{DR}}$, squarks on-shell
- lepton/slepton sector: on-shell
- chargino-neutralino sector: on-shell

Fully Automated cMSSM Calculations: Higgs sector

- Higher-order corrections phenomenologically very important
- But: including these corrections on propagators and vertices mixes orders of perturbartion theory
- $\Rightarrow$ no cancellation of UV and IR divergencies
- Masses of Higgs propagators should be consistent with mixing angle α parametrizing the vertices

Fully Automated cMSSM Calculations: Higgs sector

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- Recipe:
  - Vertices with tree-level α
  - Loop propagators with tree-level Higgs masses
  - tree propagators with loop-correctied masses

- FeynArts: Diagram generation: $\Rightarrow$ Amplitudes
  - create topologies
  - insert fields
  - apply Feynman rules
  - paint diagrams
- FormCalc: Algebraic simplification
  - contract indices
  - calculate traces
  - reduce tensor integrals
  - introduce abbrevations
- FormCalc : numerical evaluation
  - convert Mathematica output to Fortran code
  - supply driver routines
  - link LoopTools: implement integrals
- $\to$ Squared amplitudes

Fully Automated cMSSM Calculations Modelfile

- Modelfile `MSSMCT` :
  complex MSSM including all one-loop counterterms

Fully Automated cMSSM Calculations: Higgs sector

- Recipe:
  - Vertices with tree-level α
  - Loop propagators with tree-level Higgs masses
  - tree propagators with loop-correctied masses
- Implementation in FeynArts:

```
S[1] == {
 Mass -> Mh0,
 Mass[Loop] -> Mh0tree, ...  }
```

Fully Automated cMSSM Calculations: CKM and NMFV

- MSSMCT presently limited to minimal flavor violation (MFV) in the Sfermion Sector (Only mixing within each generation)
- for non-trivial CKM matrix: $\Rightarrow$ imbalance between fermions and sfermions
- CKM mixing turned off by default may be switched on: `$CKM == True`

Run-time Renormalization Scheme selection

- Choice of RS conditions dependent on parameter: (e.g. Chargino/Neutralino sector)
- Schemes require different computation of a set of Renormalization Constant, e.g. `dMino11, dMino21, dMUE1`
- Solution1: `dMUE1 = IndexIf[`$cond$`, `$\delta\mu^A, \delta\mu^B$`]`
- However: dependences cannot be resolved with individual `IndexIfs` f.i.:
  Scheme A: $\delta\mu = f(\delta M_1)$
  Scheme B: $\delta M_1 = f(\delta\mu)$
- Solution2: One-pass ordering collects & recurses on `IndexIfs`

Run-time RS selection: One-pass ordering

dMUE1 = IndexIf[$cond$, $\delta\mu^A(\delta M_1^A), \delta\mu^B$];
dMino11 = IndexIf[$cond$, δM_1^A, $\delta M_1^B(\delta\mu^B)$];
dMino21 = IndexIf[$cond$, δM_2^A, δM_2^B];

$\rightarrow$

IndexIf[$cond$,
  dMino11 = $\delta\mu^A(\delta M_1^A)$;
  dMUE1 = $\delta\mu^A(\delta M_1^A)$;
  dMino21 = δM_2^B,
(* else *)
  dMUE1 = $\delta\mu^B$;
  dMino11 = $\delta M_1^B(\delta\mu^B)$;
  dMino21 = δM_2^B];

Run-time RS selection

Scheme switching selected as

- `$InoScheme = IndexIf[`*cond*`, CNN[2,1,3], CNN[1]]`
  where *cond* might be `Abs[Abs[MUE]] - Abs[Mino2]] < 50`
  to chose most stable RS for $|\mu| \approx |M_2|$
- `$InoScheme = CCN[1]`
  where `nbino` is determined at run-time to be the most bino-like neutralino

Warning:
note that a renormalization-scheme switch in principle requires a corresponding transition of the affected parameters from one scheme to the other for a fully consistent interpretation of the results

Where are we? (a selection!)

1. **Neutral Higgs boson masses**
   - $\mathcal{O}\left(\alpha_t \alpha_s\right)$ in the cMSSM [Heinemeyer, Hollik, Rzehak, Weiglein '07]
   - $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$, $\mathcal{O}\left(\alpha_t^2 \alpha_s\right)$, rMSSM [Martin '07]
   - $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$, rMSSM (incl. fin. terms) [Harlander, Kant, Mihaila, Steinhauser '08]
   - FD $\oplus$ log resummation [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '13]
2. **Charged Higgs mass**
   - full 1-loop [M. Frank et al. '06]
   - $\mathcal{O}(\alpha_t \alpha_s)$ [Frank et al. '13]

Where are we? (a selection!) II

3. Production cross sections at the LHC

- $gg \to h$ at 2-loop [Anastasiou et al. '08] [Mühlleitner et al. '08] [Slavich et al. '11]
  [Harlander et al. '12 (SusHi)] [Bagnaschi et al. '14]
- WBF at 1-loop [Ciccolini et al. '07] [Hollik et al. '08] [Palmer, Weiglein '11]
- $bb \to h$: 4FS vs. 5FS, Santander matching
  [Dittmaier et al. '06] [Dawson et al. '06] [Harlander et al. '11] [Maltoni et al. '12]
- Z-factors at 2-loop [Frank, Hahn,Heinemeyer, Hollik, Rzehak, G. Weiglein '06]

Where are we? (a selection!) III

4. Higgs decays to SM
   - full 1-loop, leading 2-loop, . . . (depending on final state) [. . .]
   - Z-factors at 2-loop [Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '06]
5. Higgs decays to SUSY
   - full 1-loop (depending on final state) [. . .]
   - Z-factors at 2-loop [Frank, Hahn, Heinemeyer, Hollik, Rzehak, G. Weiglein '06]
6. SUSY decays to Higgs bosons
   - (partial) 1-loop, rMSSM [. . .]
   - (partial) 1-loop, cMSSM [Rzehak, Weiglein, Williams] [Bharucha, Fritzsche, Heinemeyer, FP, Rzehak, Schappacher]

What is missing? (a selection!)

1. Neutral Higgs boson masses
   - full 2-loop
   - more 3-loop (and in "easier accessible" scheme?)
   - leading 4-loop
   - improved log resummations
2. Charged Higgs boson mass
   - leading 2-loop
3. Higgs decays
   - full 1-loop in the r/cMSSM (some final states)
   - leading 2-loop
4. Decays to Higgs bosons
   - full 1-loop in the rMSSM
   - full 1-loop in the cMSSM

$\Rightarrow$ provide corresponding codes!