The breakings of "simple" N=2 theories

Jean-Pierre Derendinger

Albert Einstein Center for Fundamental Physics, ITP, University of Bern

Rencontre de Physique des Particules 2018

11 – 13 April 2018, Paris

With I. Antoniadis, C. Markou,
P. M. Petropoulos, K. Siampos, . . .

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Simple $N = 2$ theories ?

- "Simple": $N_V = 0, 1, (2)$ abelian vector multiplets, $N_H = 0, 1$ hypermultiplets.
  For which an almost complete treatment is feasible.
- Large classes of (factorized) geometries for vector and hypermultiplets.

= (Special) Kähler for vector (global and local): in terms of an arbitrary holomorphic prepotential.

= Geometries with (anti-)self-dual curvature for $N_H = 1$ hypermultiplet.

= Riemann curvature (Ricci-flat) for the global hypermultiplet (gravitational instantons). Hyperkähler in general.

= Weyl curvature for the local hypermultiplet: Quaternion-Kähler geometries in general (Einstein).

- Literature exists.
  Explicit metrics, especially for the hypermultiplet with one isometry or more.

Simple $N = 2$ theories ?

- Take advantage of off-shell $N = 2$ supermultiplets:
  - Maxwell supermultiplet A_μ, λ^i, z, Y_{ij}
    (triplet of real electric auxiliary fields), $8_B + 8_F$
    $\mathcal{N} = 1$ superfields: X, W_α.
  - Single-tensor supermultiplet $B_{\mu\nu}$, C, x, χ^i
    complex auxiliary field f, $8_B + 8_F$
    $\mathcal{N} = 1$ superfields: Φ, L (real linear)

    Dual to the hypermultiplet with a (translational) isometry.

    Hyperkähler potential $\sim$ (Legendre)
    A solution of $3d$ Laplace for L, Φ, $\overline{\Phi}$. (Lindström, Roček)

Laplace (global) $\Longrightarrow$ Toda (local): “Przanovski-Tod" metric, . . .

Summary

$\mathcal{N}=2$ a unique playground: matter and gauge multiplets, large classes of couplings (special Kähler, hyper-Kähler, quaternion-Kähler), many breaking phases: $\mathcal{N}=2$, partial $\mathcal{N}=1$, $\mathcal{N}=0$ single-scale, $\mathcal{N}=0$ two-scale.

All spontaneously-broken phases need gauge field(s): scalar potentials follow from gauging isometry(ies), but the graviphoton may suffice for some breaking phases of local $\mathcal{N}=2$.

Local and global are very different.

Global $\mathcal{N}=2$:

- Partial breaking with one Maxwell multiplet (Antoniadis, Partouche, Taylor, 1995), partial breaking with one hypermultiplet, new (IA, Markou, JPD).
- Nonlinear realizations with both multiplets, partial breaking by deformations, infinite-mass limit leading to constrained multiplets, (AdS ?), $\mathcal{N}=2$ DBI, Chern-Simons interactions between constrained multiplets . . .

Summary

Global $\mathcal{N} = 2$:

- Two-scale breaking, $U(1) \times U(1)$ coupled to a hypermultiplet, $N_V = 2$, $N_H = 1$ (IA, Jacot,JPD).
- $\mathcal{N} = 2$ DBI coupled to linear (dilaton) hypermultiplet (IA, JPD with Maillard and Ambrosetti-Tziveloglou) ($4d$ “brane lagrangians")
- Heisenberg symmetry (string universal dilaton).
- Earlier work: Ivanov-Zupnik, Bagger-Galperin, Kuzenko, Lindström-Roček, and many others.

Local $\mathcal{N} = 2$:

- Spontaneous partial breaking to $\mathcal{N} = 1$ surprisingly subtle, complicated and rare.
- A simple example based on $SO(4,1)/SO(4)$ known since 1995 (Ferrara, Girardello, Porrati), $N_V = N_H = 1$.
  Actually a two scale (two gauge coupling) model to $\mathcal{N} = 0$ with $\mathcal{N} = 1$ at a tuned value of the couplings.

Summary

Local $\mathcal{N} = 2$:

- Contrary to the vector multiplet sector (kählerian), or to $N_H > 1$ quaternion-Kähler, there are generic expressions for $N_H = 1$ metrics in the math literature (Przadnowski-Tod (PT), Calderbank- Pedersen (CP)) . . . for the hypermultiplet with one or two isometries.
- Allows a general study of partial breaking to $\mathcal{N} = 1$ (in Minkowski space) in the $N_V = N_H = 1$.
- Result: . . . a unique solution, the $SO(4, 1)/SO(4)$ model with two gauged "translations" (Antoniadis, Petropoulos, Siampos, JPD)
- Its existence actually uses an interesting loophole in the math literature (in the CP metric) . . .
- Exotic: partial breaking to AdS ??

Partial breaking: spontanous or not spontaneous

- Standard definition: spontaneous $\Longleftrightarrow$ vacuum degeneracy
  Induced by the ground state: scalar vev's.

- Not the most practical for supersymmetry

$$\delta\,\phi_G = C\alpha + \text{linear} \quad \Longleftrightarrow \quad \delta\,\psi_G = C\epsilon + \textbf{linear}$$

  $\delta\langle\phi_G\rangle$ induces the vacuum degeneracy (orbit of $\langle\phi_G\rangle$).

- A more meaningful definition for susy is the existence of one or several goldstinos in the ground state:

$$\delta\,\psi_G = C\epsilon + \textbf{linear}$$

  Leads to nonlinear realization obtained by simple **??** of the linear theory (same degrees of freedom at least if off-shell representations exist).

- As usual, algebra, currents, . . . preserved
  Not *induced* by a ground state, *defines* the states.

Global $\mathcal{N} = 2 \to \mathcal{N} = 1$ with one hypermultiplet

Antoniadis, Markou, JPD (1703.08806, JHEP 1706 (2017) 052)

Use a hypermultiplet with a translational isometry: dual to a single-tensor multiplet which admits, as the Maxwell multiplet, an off-shell realization.

Hypermultiplet $\mathcal{N} = 2$ $\Leftrightarrow$ Two chiral $\mathcal{N} = 1$

Four scalars T, Φ, two two-component spinors

Assume: (Translational) isometry: say $\delta\,\mathrm{Im}\,T =$ constant

Poincaré duality: $\partial_\mu \,\mathrm{Im}\, T \Leftrightarrow H_{\mu\nu\rho} = 3\,\partial_{[\mu} B_{\nu\rho]}$

- $\mathcal{N} = 1$: chiral $T \Leftrightarrow$ linear L $\qquad \overline{DD}\,L = 0$, $L = L^\dagger$
  ($B_{\mu\nu}$, real scalar C, spinor, no auxiliary field)
- $\mathcal{N} = 2$: hypermultiplet $T, \Phi \Leftrightarrow$ single-tensor multiplet L, Φ

Hypermultiplet with isometry / Single-tensor multiplet

Off-shell representation, second susy:

$$\delta^* L = -\frac{i}{\sqrt{2}}(\eta D\Phi + \overline{\eta D\Phi}) \qquad \delta^*\Phi = \sqrt{2}i\,\overline{\eta D}L$$

Kinetic lagrangian:

$$\int d^2\theta d^2\overline{\theta}\,\mathcal{H}(L,\Phi,\overline{\Phi}) \qquad \mathcal{H}_{LL} + 2\mathcal{H}_{\Phi\overline{\Phi}} = 0$$

Laplace for second supersymmetry [Lindström, Roček]

Superpotential allowed is

$$W(X) = m^2\,X$$

Similar to the Maxwell case ...

And partial breaking occurs with a deformation of the chiral $\delta^*\overline{D}_{\dot\alpha}L$

Hypermultiplet with isometry / Single-tensor multiplet

Choose an arbitrary $W(\Phi)$ and lagrangian

$$\mathcal{L} = \int d^2\theta \left[\frac{i}{2} W_\Phi (\overline{D}L)(\overline{D}L) - \frac{i}{4} W\, \overline{DD}\, \overline{\Phi} + \widetilde{m}^2 \Phi + \widetilde{M}^2\, W\right] + \text{h.c.}$$

$$= i \int d^2\theta d^2\overline{\theta} \left[-L^2 (W_\Phi - \overline{W}_{\overline{\Phi}}) + \overline{\Phi} W - \Phi \overline{W}\right] \qquad \Longleftarrow \text{Laplace}$$

$$+ \int d^2\theta \left[\widetilde{m}^2 \Phi + \widetilde{M}^2\, W\right] + \text{h.c.}$$

The second supersymmetry is deformed:

$$\delta^* L = \delta^*_{nl} L - \frac{i}{\sqrt{2}} (\eta D \Phi + \overline{\eta D \Phi}) \qquad \delta^*_{nl} L = \sqrt{2}\, \widetilde{M}^2\, (\overline{\theta \eta} + \theta \eta)$$

$$\delta^*_{nl} \overline{D}_{\dot\alpha} L = -\sqrt{2}\, \widetilde{M}^2\, \overline{\eta}_{\dot\alpha}$$

with $\delta^* \Phi$ unchanged, and the fermion $\overline{D}_{\dot\alpha} L|_{\theta=0}$ is the Goldstino.

Hypermultiplet with isometry / Single-tensor multiplet

L massless ($B_{\mu\nu}$ gauge field, dual scalar axion-like)
Φ has mass controlled by $\langle W_{\Phi\Phi}\rangle$
$\langle W_{\Phi\Phi}\rangle \rightarrow \infty$: constrained single-tensor multiplet (Bagger-Galperin)

Analogy with Maxwell (and APT): (Note change of chirality)

$\overline{D}_{\dot\alpha} L$	$\Longleftrightarrow$	W_α	Φ	$\Longleftrightarrow$	X	chiral
$W(\Phi)$	$\Longleftrightarrow$	$F_X(X)$	F(X) prepotential			
$\widetilde{M}^2\, W$	$\Longleftrightarrow$	$M^2\, F_X$	magnetic prepotential / FI term			

In both cases, the deformation parameter $\widetilde{M}^2$ or M^2 CANNOT be produced by the shift of an auxiliary field: this would destroy the partial breaking.
An intrinsic deformation of the linear representation.

Hence, this mechanism of partial breaking is not a “pure" spontaneous breaking, induced by scalar vev's. It is induced by the deformation.

Partial breaking: a network of conditions

"Pure spontaneous breaking": (induced by scalar vev's)

- Global $\mathcal{N} = 2$:
  partial breaking needs complete breaking of $SU(2) \times U(1)_R$.
  Maxwell: $SU(2)$ unbroken.
  Hypermultiplets $U(1)_R$ unbroken: potential too restrictive.
- To break $SU(2)$, introduce Fayet-Iliopoulos term to switch on the auxiliary fields Y_{ij}: but $U(1)$ unbroken, breaking to $\mathcal{N} = 0$ only.
- More needed: magnetic and electric Fayet-Iliopoulos terms (not in off-shell multiplet), actually a deformation of the multiplet, the APT model.
- Local $\mathcal{N} = 2$:
  partial breaking of supergravity involves a massive $\mathcal{N} = 1$ gravitino multiplet, with $6_B + 6_F$ on-shell fields (spins 3/2, 1, 1, 1/2), and $SU(2)$ broken obviously: at least one vector and one hypermultiplet.
- To get a potential: needs to gauge two commuting isometries of the hypermultiplet quaternion-Kähler manifold

$\mathcal{N} = 2$ local, partial breaking with $N_V = N_H = 1$

Needed: a metric for a generic $4d$ Weyl self-dual Einstein (quaternion-Kähler) space with two commuting isometries. Two possibilities:

- The Przanowski-Tod metric, for one isometry (or more), using a function solving Toda equation (not great. . .)
- The Calderbank-Pedersen metric for a space with two commuting isometries, using a function solving a two-dimensional Laplace-like linear equation (much better. . .)

They are related (if a pair of isometries exists) by a complicated change of coordinates involving a Legendre-like transformation. Often hard to solve.

Some hypermultiplet spaces have more that one or two isometries. Examples are $SO(4,1)/SO(4) \sim Sp(2,2)/Sp(2) \times Sp(2)$ and $SU(2,1)/SU(2) \times U(1)$ (the second only is relevant to strings)

$SO(4,1)$ has a *three*-dimensional abelian subalgebra (acting as translations in standard coordinates).
$SU(2,1)$ and $SO(4,1)$ have several inequivalent pairs of commuting isometries (and then different PT or CP coordinates).

Przanowski-Tod

$N_H = 1$ $(4d)$ quaternion-Kähler metric in coordinates (X, Y, Z, ψ) and shift isometry $\delta\psi = C$.

- For a solution $\Psi(X, Y, Z)$ of Toda equation, define U:

$$\Psi_{XX} + \Psi_{YY} + \left(e^{\Psi}\right)_{ZZ} = 0 \quad \text{(Toda)} \qquad 2U = 2 - Z\,\Psi_Z$$

- The Przanowski-Tod quaternion-Kähler metric is

$$ds^2 = \frac{1}{Z^2}\left[\frac{1}{U}(d\psi + \omega)^2 + U\left(dZ^2 + e^{\Psi}\left(dX^2 + dY^2\right)\right)\right]$$

- The one-form ω is defined by

$$d\omega = U_X\, dY \wedge dZ + U_Y\, dZ \wedge dX + \left(U\, e^{\Psi}\right)_Z\, dX \wedge dY$$

Not the most convenient for a general treatment. For each space and pair of isometries (for partial breaking), find the Toda solution in the appropriate coordinates . . .

But it can be done for each pair of isometries of $SO(4,1)/SO(4)$.

Calderbank-Pedersen

Coordinates: ρ, η, ψ, ϕ, two commuting shift isometries $\delta\psi = c$, $\delta\phi = d$.

For $F(\rho, \eta)$ such that $\frac{\partial^2 F}{\partial \rho^2} + \frac{\partial^2 F}{\partial \eta^2} = \frac{3F}{4\rho^2}$ quaternion-Kähler metric:

$$ds^2 = \frac{4\rho^2(F_\rho^2 + F_\eta^2) - F^2}{4F^2} d\ell^2 + \frac{((F - 2\rho F_\rho)\alpha - 2\rho F_\eta \beta)^2 + ((F + 2\rho F_\rho)\beta - 2\rho F_\eta \alpha)^2}{F^2(4\rho^2(F_\rho^2 + F_\eta^2) - F^2)}$$

$$\alpha = \sqrt{\rho}\, d\varphi \qquad \beta = (d\psi + \eta d\varphi)/\sqrt{\rho} \qquad d\ell^2 = \rho^{-2}(d\rho^2 + d\eta^2)$$

Calderbank-Pedersen: (arXiv:math/0105253)

“Any selfdual Einstein metric of nonzero scalar curvature with two linearly independent commuting Killing fields is arises locally in this way (i.e., in a neighbourhood of any point, it is of the form (1.1) up to a constant multiple)."

$\mathcal{N} = 2$ local, partial breaking with $N_V = N_H = 1$

(Antoniadis, Petropoulos, Siampos, JPD)

A general analysis of partial breaking in Minkowski space using the Calderbank-Pedersen metric leads to surprising conclusions:

- For all spaces and isometries which admit a set of Calderbank-Pedersen coordinates, partial breaking does not occur.
- A phenomenon similar to a run-away is found and requiring partial breaking is not compatible with the existence of a ground state.
- An apparent contradiction with the $SO(4,1)/SO(4)$ example of Ferrara, Girardello, Porrati (FGP).
- It turns out that the pair of isometries used by FGP can be easily written in a PT metric, but the change of coordinates to CP coordinates is singular: CP coordinates do not exist for this case (contrary to the claim of CP).
- The *unique* pair of isometries for which CP coordinates do not exist.
- The FGP model is then unique in this class of $\mathcal{N} = 2$ supergravities.

END

The case of Maxwell $\mathcal{N} = 2$ APT model

In terms of chiral superfields X (complex, scalar, gaugino, complex auxiliary scalar) and W_α (gaugino, $F_{\mu\nu}$, real auxiliary scalar).

Second supersymmetry:

$$\begin{aligned} \delta^* W_\alpha &= \sqrt{2}\, i \left[\frac{1}{4} \eta_\alpha \overline{DD}\, \overline{X} + i(\sigma^\mu \overline{\eta})_\alpha \, \partial_\mu X \right] \\ \delta^* X &= \sqrt{2}\, i\, \eta^\alpha W_\alpha , \end{aligned}$$

Lagrangian: $\mathcal{F}(X)$: holomorphic prepotential, $\mathcal{F}_X = \frac{d\mathcal{F}}{dX} \ldots$

$$\mathcal{L}_{Max.} = \frac{1}{2} \int d^2\theta \, \left[\frac{1}{2} \mathcal{F}_{XX} W W - \frac{1}{4} \mathcal{F}_X \overline{DD}\, \overline{X} + m^2 X - i M^2 \mathcal{F}_X \right] + \text{h.c.}$$

Invariant under linear $\mathcal{N} = 2$ if $M^2 = 0$.

Invariant under deformed second supersymmetry if $M^2 \neq 0$:

$$\delta^* W_\alpha = -\sqrt{2} M^2 \, \eta_\alpha + \sqrt{2}\, i \left[\frac{1}{4} \eta_\alpha \overline{DD}\, \overline{X} + i(\sigma^\mu \overline{\eta})_\alpha \, \partial_\mu X \right]$$

The case of Maxwell $\mathcal{N} = 2$ APT model

This theory has superpotential $W(X) = m^2 X - iM^2 \mathcal{F}_X$

Ground state: auxiliary field $\langle f_X \rangle = 0$: $m^2 - iM^2 \langle \mathcal{F}_{XX} \rangle = 0$

Requires non canonical coupling, $\mathcal{F}_{XXX} \neq 0$. Then $N = 1$ unbroken, partial breaking controlled by scale M and $\mathcal{F}_{XXX} \neq 0$.

Spectrum:

Massless W_α: gauge field and goldstino
Massive X, mass proportional to $M \langle \mathcal{F}_{XXX} \rangle$

M^2 is not equivalent to $\langle f_X \rangle$: would require the magnetic $\widetilde{f}_X$

In general, $\mathcal{F}(X)$ breaks $U(1)_R$ (in non canonical cases).
In any case, M^2 breaks $SU(2)_R \times U(1)_R$, no goldstone boson.