The SM Evaluation of the Muon $g - 2$
a status report

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OUTLINE

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- II. Status of QED and EW contributions
- III. Status of HVP contribution
- IV. Status of HLxL contribution
- V. Summary - Conclusion

I. Experimental situation and perspectives

At present, there is a discrepancy, of $\gtrsim 3.5\sigma$, between the measured value of the anomalous magnetic moment of the muon

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \;\; [0.54\text{ppm}] \qquad\qquad \text{BNL} - \text{E821}$$

G. W. Bennett et al, Phys Rev D 73, 072003 (2006)

and its value as evaluated within the standard model

This situation results, on the one hand, from the outcome of the BNL-E821 experiment and, on the other hand, from the evaluation of rather subtle quantum effects in the SM

On the experimental side, the confirmation (or not!) of the Brookhaven measurement by the FNAL E989 experiment is around the corner [expect impactful results in about 2 years (M. Smith, Moriond 2018)]...

...and should also be checked later on by the E34 experiment at J-PARC

If the BNL-E821 central value is confirmed with reduced error bars (up to a factor of four), a_μ may well become the only single observable of the standard model [besides neutrino masses] showing a deviation from its measured value by more than 5σ in a few years from now!

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- provide, whenever possible, cross-checks for existing calculations

- improve precision, especially in the hadronic contributions, which are the most delicate to evaluate, and which at present dominate the theoretical uncertainties

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- improve precision, especially in the hadronic contributions, which are the most delicate to evaluate, and which at present dominate the theoretical uncertainties

Fortunately, quite some progress has been made since the release of the final BNL result

II. Status of QED and EW contributions

QED provides more than 99.99% of the standard-model value of a_μ

General structure of the perturbative series

$$a_\ell{}^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

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Well-defined theoretical framework

Issue: computing high perturbative orders

Situation in ~ 2016

- Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)
C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)
A. Petermann, Helv. Phys. Acta 30, 407 (1957)
H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)
S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)
S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision on $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$ (not relevant for a_μ at present and future precisions)

- Values of $C_\ell^{(8)}$ and $C_\ell^{(10)}$ computed numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); D 85, 093013 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012); Phys. Rev. D 91, 033006 (2015) [Err. D 96, 019901 (2017)]; Phys. Rev. D 97, 036001 (2018)

$\longrightarrow$ for all practical purposes, $a_\mu^{\rm QED}$ has no uncertainties

Situation in ~ 2016

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.9096(20)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots \cdot 10^{-3}$	$5.39\ldots \cdot 10^{-6}$	$1.25\ldots \cdot 10^{-8}$	$2.91\ldots \cdot 10^{-11}$	$6.76\ldots \cdot 10^{-14}$

order $(\alpha/\pi)^4$: 891 diagrams

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373)\cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118)\cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28)\cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

order $(\alpha/\pi)^5$: 12672 diagrams

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How reliable are these results?

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Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz et al., Phys. Rev. D 92, 073019 (2015); Phys. Rev. D 93, 053017 (2016)

Agreement at the level of accuracy required by present (and future) experiments for a_μ

Important cross-check, since $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim a_\mu^{\text{QED}}(\alpha^4)$

Cross-check for $C_\mu^{(10)}$? $C_\mu^{(10)}(\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$ $\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \longrightarrow \sim 1.6 \cdot 10^{-10}$

order $(\alpha/\pi)^4$: 891 diagrams

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A. Kurz et al., Phys. Rev. D 92, 073019 (2015); Phys. Rev. D 93, 053017 (2016)

semi-analytic computation of $A_1^{(8)}$! $A_1^{(8)} = -1.912\,245\,764\,926\,4\ldots$

S. Laporta, Phys. Lett. B 772, 232 (2017) [arXiv:1704.06996 [hep-ph]]

Main order α^4 contribution to a_e!

Cross-check for $C_e^{(10)}$? $C_e^{(10)}(\alpha/\pi)^5 \sim 5\cdot 10^{-13}$ $\Delta a_e^{\rm exp} = 2.8\cdot 10^{-13} \longrightarrow$?

Present situation

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

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$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.595(223)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots \cdot 10^{-3}$	$5.39\ldots \cdot 10^{-6}$	$1.25\ldots \cdot 10^{-8}$	$2.91\ldots \cdot 10^{-11}$	$6.76\ldots \cdot 10^{-14}$

A few comments about the QED contributions

- $a_e^{\text{QED}} = 1\,159\,652\,180.277(00)_{\alpha^4}(15)_{\alpha^5}(720)_{\alpha(Rb11)} \cdot 10^{-12}$

$$a_e^{\text{exp}} - a_e^{\text{QED}} = 0.434(772) \cdot 10^{-12}$$

T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)
S. Laporta, Phys. Lett. B 772, 232 (2017)

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

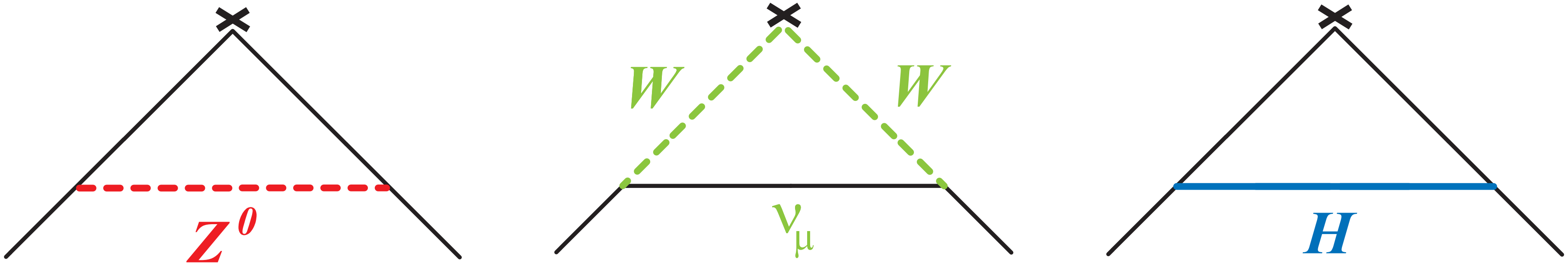
$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \text{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}$$

- No sign of substantial contribution to a_μ from higher order QED

- Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak}(1)} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$
$$= 19.48\times 10^{-10}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)
G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)
R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)
I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)
M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)

$$\begin{aligned} a_\mu^{\text{weak}} &= (154 \pm 1) \cdot 10^{-11} \\ a_e^{\text{weak}} &= (0.0297 \pm 0.0005) \cdot 10^{-12} \end{aligned}$$

Recent update: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathsf{weak}} = 721.65(6.38) \cdot 10^{-10}$$

III. Status of HVP contribution

HVP from $e^+e^- \to \text{hadrons}$

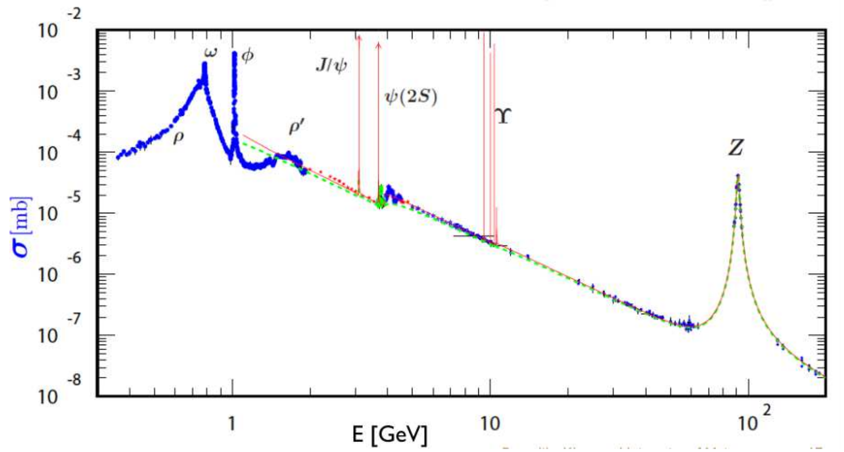
Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

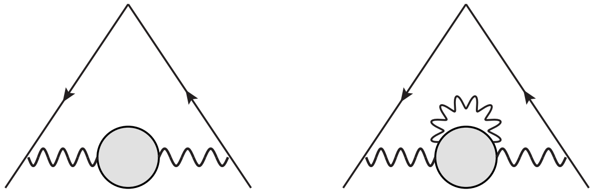
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)



$\longrightarrow$



Note that some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons
- some radiative exclusive modes, e.g. $\pi^0\gamma$, $\eta\gamma$

$$a_\mu^{\pi^0\gamma}(600\,\text{MeV} - 1030\,\text{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

New determinations from $e^+e^- \to \mathrm{hadrons}$

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$ situation in 2011

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)	
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)	$\sim 0.6\%$
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)	

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$\Downarrow$

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$ situation today

$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)	
$693.27(2.46)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]	$\sim 0.4\%$
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]	

Some tension remains

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600 - 900\ \mathrm{MeV})$
BaBar(09)	376.7(2.7)
KLOE(comb)	366.7(2.2)
BESIII(15)	368.2(4.2)
SND(04)	371.7(5.0)
CMD-2(comb)	372.4 (3.0)

A. Anastasi et al. [KLOE-2], arXiv:1711.03085 [hep-ex]

New determinations from $e^+e^- \to \mathrm{hadrons}$

Higher-order corrections

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]
$-9.82(4)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

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$1.22(1)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

Possibilities to cross-check these results ?

HVP from the space-like region (from Bhabha or μe scattering)

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)
G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\rm HVP} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\rm had}(-\frac{x^2}{1-x} m_\mu^2)$

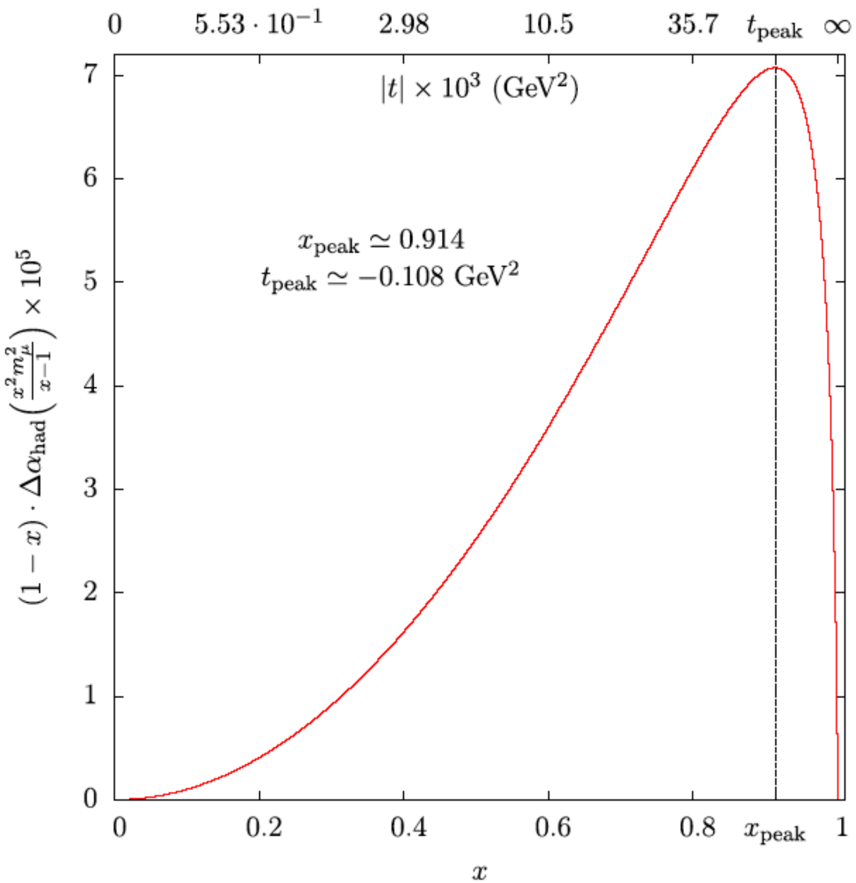
$t = \frac{x^2 m_\mu^2}{x-1}, \; 0 \le -t < +\infty, \; 0 \le x < 1$

- $a_\mu^{\rm HVP}$ given by the integral

- measurement of $\Delta\alpha_{\rm had}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 $\mu/{\rm s}$ (CERN)



- real challenge: getting the systematics below 10ppm (higher-order corrections,...)

would provide an interesting cross-check of the time-like determinations of HVP

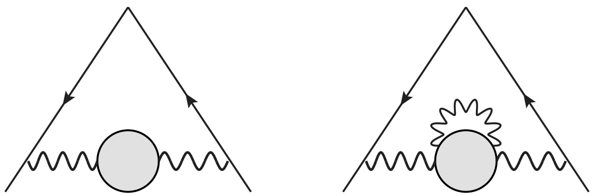
HVP from lattice QCD

Several groups are producing results, e.g.

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$	Recent lattice results
$654(32)^{+21}_{-23}$	M. Della Morte et al., JHEP 1710, 020 (2017)
$667(6)(12)$	B. Chakraborty et al. [HPQCD], Phys. Rev. D 96, 034516 (2017)
$711.0(7.5)(17.3)$	S. Borsanyi et al. [BMWc], arXiv:1711.04980 [hep-lat]
$715.4(16.3)(9.2)$	T. Blum et al., arXiv:1801.07224 [hep-lat]

Still quite large uncertainties, but steady progress

Comparison with data at the sub-percent level requires to deal with isospin breaking effects (radiative corrections)



(Experimentalists don't live in the theoretician's paradise)

Interesting perspective: combining data and lattice

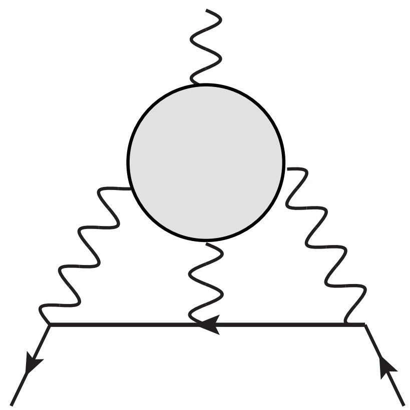
$$a_\mu^{\text{HVP-LO}} \cdot 10^{10} = 692.5(1.4)(0.5)(0.7)(2.1)$$

T. Blum et al., arXiv:1801.07224 [hep-lat]

IV. Status of HLxL contribution

Not related, as a whole, to an experimental observable...

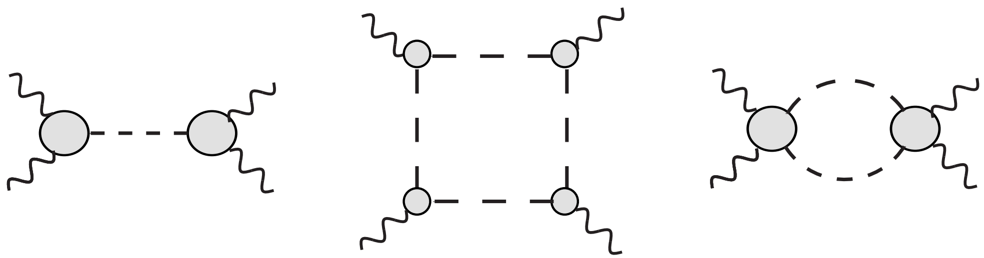
? $\longrightarrow$



Involves a much more complicated object, the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

Many individual contributions have been identified...



$+\cdots$

$$\Pi = \Pi^{\pi^0,\eta,\eta'\ \text{poles}} + \Pi^{\pi^\pm,K^\pm\ \text{loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

Π^{residual}: other intermediate states (3π...), high-energy part,...

Present estimates still rely on two model-dependent calculation

$$a_\mu^{\text{HLxL}} = +(8.3 \pm 3.2) \cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\text{HLxL}} = +(89.6 \pm 15.4) \cdot 10^{-11}$$

M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)
M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

Provide useful hints on the expected sizes of various contributions

Recent overview

J. Bijnens, arXiv:1712.09787 [hep-ph]

units: 10^{-11}

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	–	–	–	0 ± 10	–	–	–
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Erratum-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Erratum-ibid. 626 (2002) 410]

HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137

KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034

MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006

BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; Phys. Proc. Suppl. 181-182 (2008) 15; Mod. Phys. Lett. A 22 (2007) 767

BdRV: J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph]

N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

units: 10^{-11}

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N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

Recent reevaluation of single meson exchanges...

$$a_\mu^{\rm HLxL}(f_1, f_1') = 6.4(2.0) \cdot 10^{-11},\ a_\mu^{\rm HLxL}(f_0, f_0', a_0) = (-1 \text{ to } -4) \cdot 10^{-11},\ a_\mu^{\rm HLxL}(f_2, f_2', a_2, a_2') = 1.1(0.1) \cdot 10^{-11}$$

V. Pauk, M. Vanderhaeghen, Eur. Phys. J C 74, 3008 (2014)

$$a_\mu^{\rm HLxL}(a_1, f_1, f_1') = 7.51(2.71) \cdot 10^{-11}$$ F. Jegerlehner, EPJ Web Conf. 118 (2016)

...and of pion box and $\pi\pi$: $a_\mu^{\pi \text{ box}} + a_\mu^{\pi\pi;\pi\rm LHC} = -24(1) \cdot 10^{-11}$

G. Colangelo et al., Phys. Rev. Lett. 118, 232001 (2017)

See also J. Bijnens, J. Relefors, JHEP 1609, 113 (2016)

HLxL within a dispersive framework

- decompose $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$ into a set of independent invariant functions, free of kinematic singularities and zeros
- write dispersion relations for these functions
- saturate the dispersion relations by lowest one-, two- or more meson states

G. Colangelo et al., JHEP 1409, 091 (2014); JHEP 1509, 074 (2015)

DRs require input for form factors...

G. Colangelo et al., Phys. Lett. B 738, 6 (2014)

either from experiment, or from lattice

A. Nyffeler, Phys. Rev. D 94, 074507 (2016)

A. Feng et al.,, Phys. Rev. D 91, 054504 (2015)

A. Gerardin, H. B. Meyer, A. Nyffeler, Phys. Rev. D 94, 074507 (2016)

Alternative dispersive approaches:
- write dispersion relation directly for the Pauli form factor

V. Pauk, M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014)

- use the relation between a_μ and the L/T photo-absorption cross-section (Schwinger sum rule)

F. Hagelstein, V. Pascalutsa, arXiv:1710.04571 [hep-ph]

HLxL from lattice QCD

- Several groups involves in computation of HLxL on the lattice

J. Green et al., arXiv:1510.08384 [hep-lat]

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1801.04238 [hep-lat]

$$a_\mu^{\mathrm{HLxL}} = 5.35(1.35) \cdot 10^{-10}$$

Summary: updated estimate

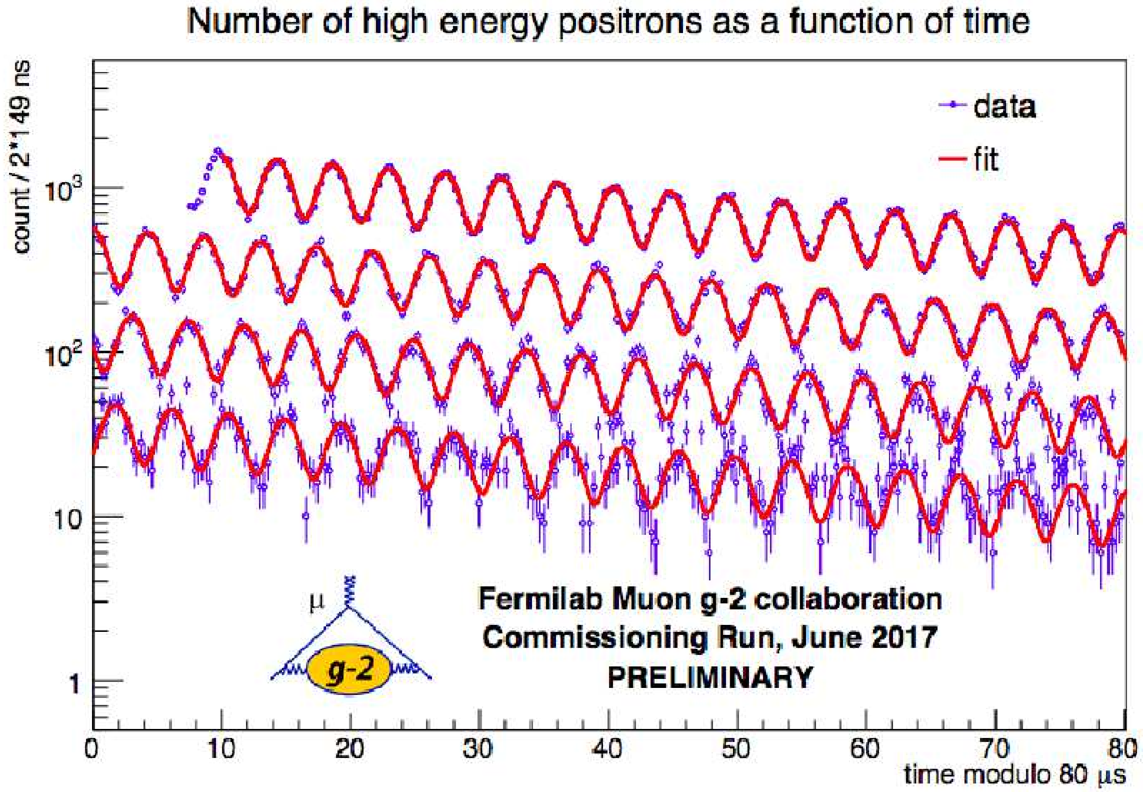
$$a_\mu^{\mathrm{HLxL}} \quad = \quad +(10.3 \pm 2.9) \cdot 10^{-10}$$

F. Jegerlehner, arXiv:1705.00263 [hep-ph]

$\longrightarrow$ Goal: determination of HLxL at ~ 10% with controlled systematics

V. Summary - Conclusion

June 2017 ~ 700,000 positrons (~2 weeks)



Successfully commissioning run of FNAL-E989 last Summer

Data taking started in February this year

Impactful results expected in about 2 years

If central value of BNL-E821 confirmed, then a_μ might well be the only single observable showing clear deviation from SM prediction

Experiment E34 at J-PARC in progress (full approval and funding still pending)

Operates under completely different experimental conditions

Important to provide independent cross-check of BNL E821 and FNAL E989 results (even if only at a later stage)

QED

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

QED

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

Next steps?

- $A_2^{(10)}(m_\mu/m_e)$ for a_μ
- $A_1^{(10)}$ for a_e

HVP

New estimates based on 39 measured exclusive channels

Precision below the 0.5% level in relative terms

Some tensions between data remain

$\longrightarrow$ would be interesting to see analysis for the $\pi^+\pi^-$ channel of the data collected at VEPP...)

Possibilities for cross-checks, either from

- Bhabha or $e\mu$ scattering
- Lattice QCD

interesting results from several groups
statistical error still large; all systematics not yet under full control
interesting perspective: combine data-driven evaluations with lattice evaluations

HLxL

Dispersive evaluation $\longrightarrow \pi$-box, $\pi\pi$ (π-LHC)

Still missing:

- implementation of short-distance constraints
- estimate for Π^{residual}? Cf. axial vectors (leading in large-N_c) $\rightarrow 3\pi$ channel
- form factors to be provided from data and/or lattice QCD

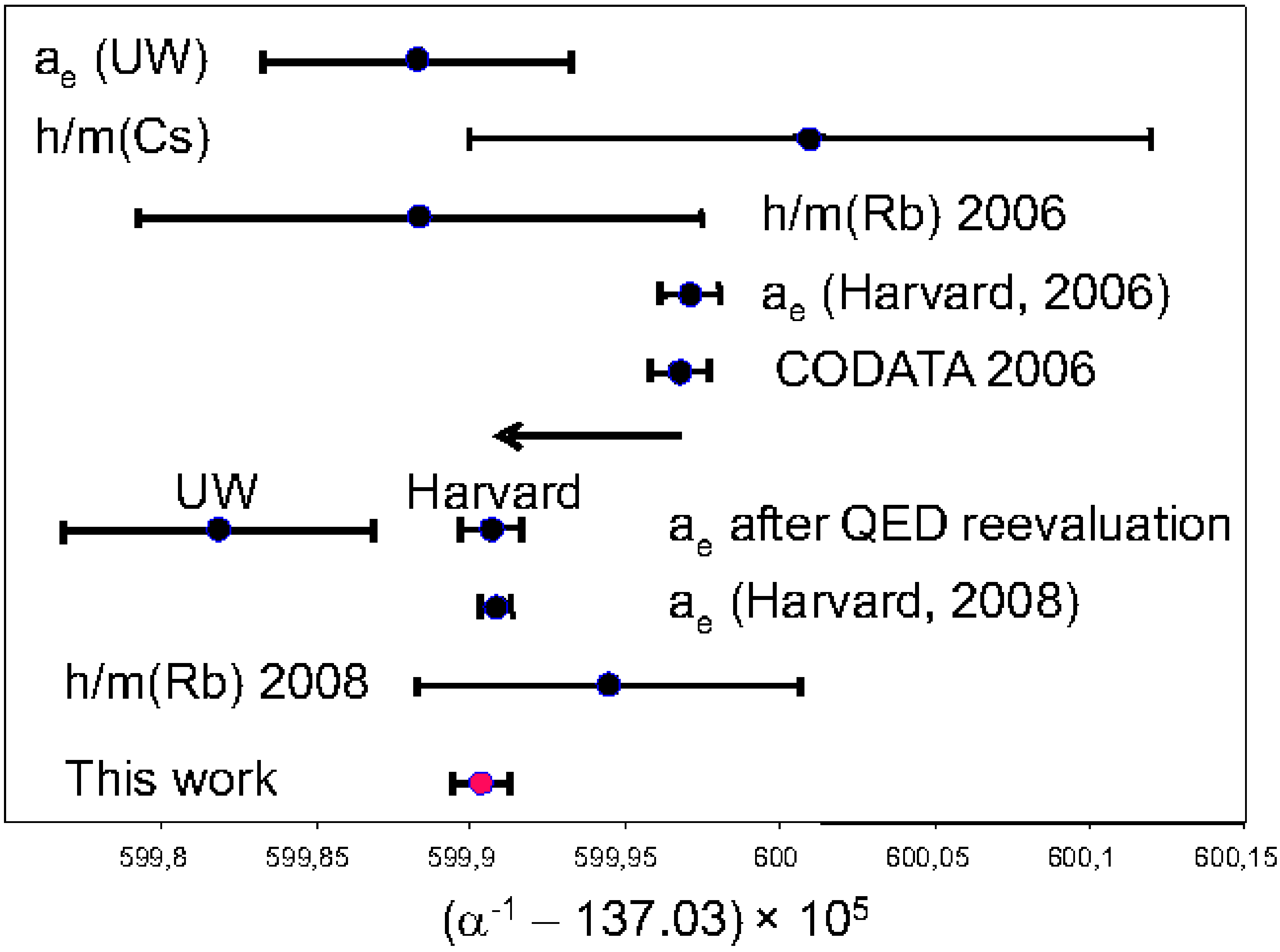
- Lattice QCD
(several groups, different strategies to overcome challenging difficulties)

$\longrightarrow$ Goal: determination of HLxL at ~ 10% with controlled systematics

Crucial to exploit all possibilities to perform necessary cross-checks

Thanks for your attention!

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]} \\
&= -1.557(70) && \text{[Kinoshita (1995)]} \\
&= -1.4092(384) && \text{[Kinoshita (1997)]} \\
&= -1.5098(384) && \text{[Kinoshita (2001)]} \\
&= -1.7366(60) && \text{[Kinoshita (2005)]} \\
&= -1.7260(50) && \text{[Kinoshita (2005)]} \\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]} \\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \leftarrow \\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]} \\
&= -1.91298(84) && \text{[Aoyama et al., Phys. Rev. D 91, 033006 (2015)]}
\end{aligned}
$$



a_e (UW)
h/m(Cs)
h/m(Rb) 2006
a_e (Harvard, 2006)
CODATA 2006
UW
Harvard
a_e after QED reevaluation
a_e (Harvard, 2008)
h/m(Rb) 2008
This work
599,8
599,85
599,9
599,95
600
600,05
600,1
600,15
$(\alpha^{-1} - 137.03) \times 10^5$