Minimal constrained superfields and the Fayet-Iliopoulos model

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Outlines

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Motivation

- Supersymmetry is spontaneously broken.
- Below the soft mass scale, the theory is described by non-linear realizations.
- Constrained superfields are a powerful tool to write the effective lagrange in superspace.

Volkov-Akulov action

- Spontaneous breaking of SUSY leads to a Goldstone fermion: the goldstino.
- One of the first written SUSY lagrangian: Volkov-Akulov action (VA, 1973)

$$\mathcal{L}_{AV} = det(E^a_\mu), \qquad where \qquad E^a_\mu = e^a_\mu + (\frac{i}{2f^2} G\sigma^a \partial_\mu \overline{G} + h.c.).$$

- VA action is equivalent to the general form of a nilpotent superfield X_{NL} (Rocek,78):

$$\mathcal{L}_{X_{NL}} = \int d^4\theta \overline{X_{NL}} X_{NL} + (\int d^2\theta f X_{NL} + h.c.), \qquad where \qquad X^2_{NL} = 0.$$

Why $X_{NL}^2 = 0$?

- For a chiral superfield X:

$$X = \phi + \sqrt{2}\theta\psi + \theta\theta F,$$

- To decouple the scalar ϕ, write it in terms of ψ and F. The supersymmetry transformation reads then:

$$\begin{aligned} \delta_\epsilon \phi(\psi, F) &= \frac{\partial \phi}{\partial \psi_\alpha}\delta_\epsilon \psi_\alpha + \frac{\partial \phi}{\partial F}\delta_\epsilon F \\ \epsilon\psi &= \frac{\partial \phi}{\partial \psi_\alpha}[-i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \phi + \epsilon_\alpha F] - \frac{\partial \phi}{\partial F}(i\overline{\epsilon\sigma}^\mu \partial_\mu \psi). \end{aligned}$$

- The solution is $\phi = \frac{\psi\psi}{2F}$ and X becomes nilpotent:

$$X_{NL}^2 = (\frac{\psi\psi}{2F} + \sqrt{2}\theta\psi + \theta\theta F)^2 = 0.$$

Origin of $X_{NL}^2 = 0$?

Add the mass term for scalar ϕ:

$$\mathcal{L}_X = \int d^4\theta(\overline{X}X - \frac{m_\phi^2}{4f^2}\overline{X}X\overline{X}X) + (\int d^2\theta f X + h.c.),$$

- When $E \ll m_\phi$, the equation of motion leads to a non-zero vev for the auxiliary field and the nilpotency

$$\begin{aligned} X &\to X_{NL}; \\ \mathcal{L}_X &\to \mathcal{L}_{X_{NL}}. \end{aligned}$$

Matter Sector

One can use X_{NL} to impose constraint on superfields in order to (Komargodski-Seiberg, 09):

- Project out the complex scalar of a chiral multiplet Q

$$X_{NL} Q = 0;$$

- Project out the fermion of a matter multiplet $\mathcal{H}$ or a gauge multiplet W_α

$$\begin{aligned} X_{NL} \overline{D}_{\dot{\alpha}} \overline{\mathcal{H}} &= 0; \\ X_{NL} W_\alpha &= 0; \end{aligned}$$

- Project out the fermion, one real scalar and the auxiliary field of a chiral multiplet $\mathcal{A}$

$$X_{NL}(\mathcal{A} + \overline{\mathcal{A}}) = 0.$$

Which microscopic theory gives rise to these constraints?

Constrained Superfields for a Real Scalar

- The KS constraint for real scalar is equivalent to (Dall'Agata-Dudas-Farakos, 16):

$$\begin{aligned} \overline{X_{NL}} X_{NL} (\mathcal{A} + \overline{\mathcal{A}}) &= 0; \\ \overline{X_{NL}} X_{NL} D_\alpha \mathcal{A} &= 0; \\ \overline{X_{NL}} X_{NL} D^2 \mathcal{A} &= 0. \end{aligned}$$

- The microscopic theory DDF proposed contains three independent operators.

Minimal Constrained Superfields

If $\mathcal{A}^a$ is in an adjoint representation:

$$-\frac{m_D}{4\sqrt{2}f^2}\int d^2\theta \overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})W^a_\alpha\mathcal{A}^a.$$

Equation of motion to the $\mathcal{A}^a$:

$$\overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})W^a_\alpha = 0.$$

- Acting by $\overline{X_{NL}}X_{NL}D_\beta$ to the left hand side gives

$$\overline{X_{NL}}X_{NL}W^a_\alpha = 0.$$

(uses the non-zero property of the $D\overline{D}^2 D(\overline{X_{NL}}X_{NL})$ and the nilpotency $X_{NL}D^\alpha X_{NL} = 0$).

Minimal Constrained Superfields

If $\mathcal{A}^a$ is in an adjoint representation:

$$-\frac{m_D}{4\sqrt{2}f^2}\int d^2\theta\overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})W_\alpha^a\mathcal{A}^a.$$

Equation of motion to the W_α^a:

$$D_\alpha\overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})(\mathcal{A}^a+\overline{\mathcal{A}^a})-[\overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})D_\alpha\mathcal{A}^a+h.c.]=0.$$

- Acting by $\overline{X_{NL}}X_{NL}$ leads to $\overline{X_{NL}}X_{NL}(\mathcal{A}^a+\overline{\mathcal{A}^a})=0$;
- Acting by $\overline{X_{NL}}X_{NL}D_\beta$ leads to $\overline{X_{NL}}X_{NL}D_\alpha\mathcal{A}^a=0$;
- Acting by $\overline{X_{NL}}X_{NL}D^2$ leads to $\overline{X_{NL}}X_{NL}D^2\mathcal{A}^a=0$;

Constrained superfield for Higgs sector

For two chiral (doublets) $H_{1,2}$ carrying opposite U(1) charges, one can write the operators

$$\begin{aligned}\mathcal{O}_{H_{22}} &= \frac{a_{22} m_H}{8\sqrt{2}f} \int d^2\theta \overline{D}^2 (D^\alpha (\overline{X_{NL}} X_{NL}) D_\alpha H_1) H_2 \\ &= \frac{a_{22} m_H}{16 f} \int d^2\theta \overline{D}^2 D^\alpha (\overline{X_{NL}} X_{NL}) D_\alpha H_1 H_2 + \ldots,\end{aligned}$$

This leads to the constraints for two fermions and the complex scalar in H_2:

$$\begin{aligned}\overline{X_{NL}} X_{NL} D_\alpha H_1 &= 0; \\ \overline{X_{NL}} X_{NL} D_\alpha H_2 &= 0; \\ \overline{X_{NL}} X_{NL} H_2 &= 0.\end{aligned}$$

Constrained superfield for Higgs sector

In order to give the correct Yukawa couplings and avoid the too large $tan\beta$, we need to supplement $\mathcal{O}_{H_{11}}$ as well as the additional operator:

$$\mathcal{O}_{H_{12}} = -\frac{a_{12}^2 m_H^2}{64 f^4} \int d^2\theta \overline{D}^2 D^\alpha (\overline{X_{NL}} X_{NL}) \overline{D}^2 D_\alpha (\overline{X_{NL}} X_{NL}) H_1 H_2$$

Together one gets the constraints for the Higgsinos:

$$\overline{X_{NL}} X_{NL} D_\alpha H_1 = \overline{X_{NL}} X_{NL} D_\alpha H_2 = 0,$$

as well as the constraint for the linear combination of higgs:

$$\overline{X_{NL}} X_{NL} (a_{12}^2 H_j + a_{ii}^2 \overline{H_i}) = 0.$$

In the limit $a_{11} a_{22} - a_{12}^2 \simeq 0$, one gets a light complex scalar and the constraint for the heavy one:

$$\overline{X_{NL}} X_{NL} (a_{11} H_1 + a_{22} \overline{H_2}) = 0.$$

D-term SUSY breaking

- The previous operators all use $\overline{D}^2 D_\alpha(\overline{X_{NL}}X_{NL})$, which looks similar to the form of the field strength W_α^a of a gauge field. Can we generate it from D-term SUSY breaking?
- Consider Fayet-Iliopoulos model in the regime where both gauge symmetry and supersymmetry are spontaneously broken.

The goldstino is the only light mode in this sector.

Fayet-Iliopoulos model

In the case where both gauge symmetry and supersymmetry are spontaneously broken, we can rotate to the superunitary gauge where the chiral superfield with non-zero vev in the scalar component is eaten by the gauge field:

$$\begin{aligned}\mathcal{L}_{SU} &= \int d^2\theta(\frac{1}{4}W^\alpha W_\alpha + \frac{1}{\sqrt{2}}mv\Phi_+) + h.c. \\ &+ \int d^4\theta(\overline{\Phi_+}e^{2gV}\Phi_+ + \frac{1}{2}v^2e^{-2gV} + 2\xi V)\end{aligned}$$

The equation of motion to V gives the leading order solution proportional to $\overline{\Phi_+}\Phi_+$, a quartic term in Φ_+ is also generated leading to nilpotency of Φ_+.

Ferrara-Zumino supercurrent

- KS conjectured that the superfield X which controls the violation of the Ferrara-Zumino supercurrent $\mathcal{J}_{\alpha\dot{\alpha}}$ conservation equation:

$$\overline{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

  flows in the infrared to the superfield X_{NL}, i.e. $X \to X_{NL}$.
- In the presence of a FI term, X can be formally obtained in a gauge invariant form as [Arnold-Derendinger-Hartong, 12]:

$$X = 4W - \frac{1}{3}\overline{D}^2 \left[K + 2\xi(V + i\Lambda - i\Lambda^\dagger) \right].$$

- By matching the Fayet-Iliopoulos model to X_{NL}, we obtain:

$$\begin{aligned} \Phi_+ &\to \frac{gv}{\sqrt{m^2 + g^2 v^2}} X_{NL} \\ V &\to -\frac{g}{m^2 + g^2 v^2} \overline{X_{NL}} X_{NL} \end{aligned}$$

(Fixed by the θ component (goldstino) and the vev of the auxiliary fields)

Minimal Constrained Superfields

Now the origin for the minimal constrained superfields

$$-\frac{m_D}{4\sqrt{2}f^2}\int d^2\theta\overline{D}^2 D^\alpha(\overline{X_{NL}}X_{NL})W^a_\alpha\mathcal{A}^a.$$

can be understood as the axion coupling to the kinetic mixing between two different $U(1)$ gauge fields.

$$-\frac{m_D}{f}\int d^2\theta W^\alpha_{NL}W^{U(1)}_\alpha\mathcal{A}.$$

This also leads to a relation between the supersymmetry breaking mediation scale and the axion symmetry breaking scale:

$$m_D\sim\frac{f}{\Lambda}\sim\frac{f}{f_A}\rightarrow\Lambda\sim f_A.$$

Conclusion

- Constrained superfields are a powerful tool to study the spontaneous breaking supersymmetry since some d.o.f are projected out.
- We show how the constraints to project out all the components of a chiral superfield except for some scalar degrees of freedom originate from simple operators in the microscopic theory.
- This is in particular useful in constructing the simplest models of a goldstone boson/inflaton; or extracting the Standard Model Higgs doublet from a supersymmetric electroweak sector.
- The operator we used can either come from F-term breaking or D-term breaking where both gauge symmetry and supersymmetry are spontaneously broken.
- Future: Implications for axion/inflation/higgs? Supergravity embedding?

Thanks!