R^2 Dark Energy in the Laboratory

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arXiv:1711.03356 to appear in PRD + work in progress

Gravity is described by General Relativity (GR):

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}R$$

Uniqueness theorem (Weinberg 1965):

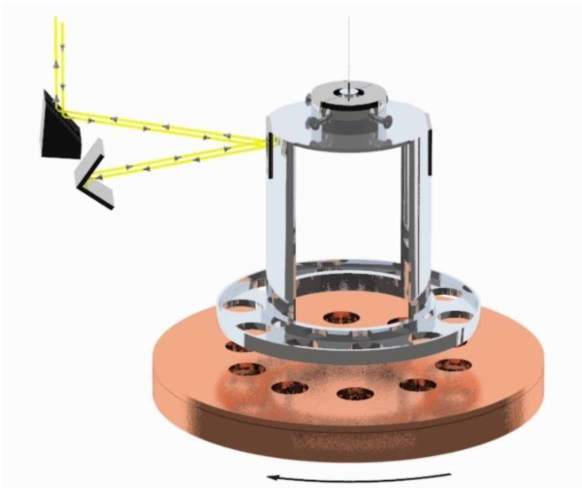
GR is the unique Lorentz invariant theory of massless helicity 2 fields

Lorentz invariance implies the weak equivalence principle (Weinberg 1965) for elementary particles.

$$S_m(\psi_i, g_{\mu\nu})$$

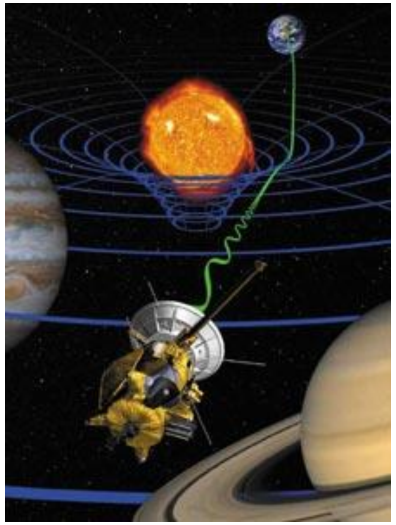
Particles couple to a unique metric.

GR has been wonderfully tested on many length scales:

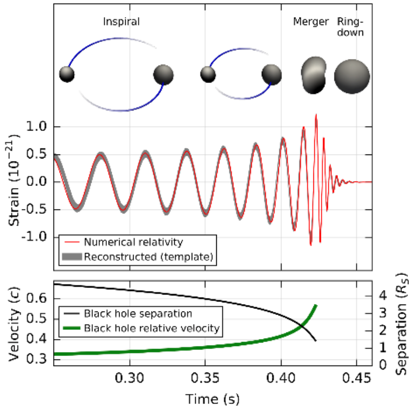


Laboratory experiments (Eotwash) tests of fifth forces and equivalence principle
0.1 mm

Cassini probe test of fifth forces
1 a.u., 150 million km.

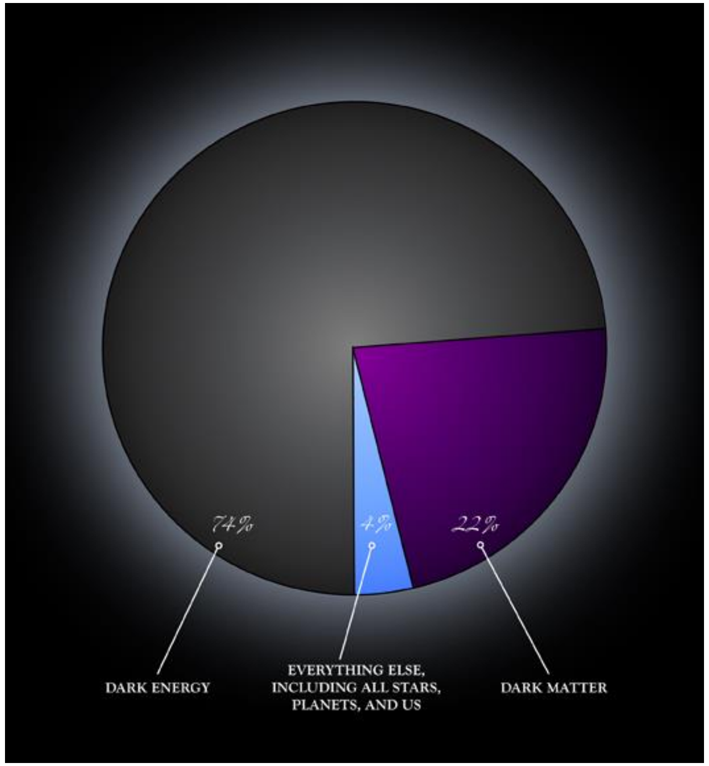


Lunar ranging tests of strong equivalence principle and time variation of Newton’s constant,
400 000 km



Gravitational wave emissions from black hole and neutron star mergers
50 Mpc

But GR fails miserably on cosmological scales where both dark matter and dark energy are necessary to explain Baryon Acoustic Oscillations (BAO) or the Cosmic Microwave Background (CMB).



$$S_{\Lambda\mathrm{CDM}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R-2\Lambda)$$

Cosmological constant

The cosmological constant behaves like a constant and uniform vacuum energy.

The problem of the vacuum energy was brought to the fore of cosmological research thanks to the discovery of the acceleration of the expansion of the Universe:

$$\rho_\Lambda = \Lambda m_{\rm Pl}^2 + \rho_{\rm transition} + \sum (2j+1)(-1)^{2j} \frac{m_j^4}{64\pi^2} \ln \frac{\mu^2}{m_j^2}$$

Cosmological constant

Phase transitions

Quantum fluctuations

See J. Martin's review « all you wanted to know… » (2012)

- ✓ The result takes into account all particles of any spin.
- ✓ The sole contribution from the top quark is larger than the energy at the formation of the elements (Big Bang Nucleosynthesis) preventing one from understanding the Universe's dynamics since then.

It is thus a sheer catastrophe.

There are three contribution to the vacuum energy: the “latent heat” from phase transitions, e.g. electroweak, the vacuum fluctuations and the cosmological constant. The latter plays the role of a counter term and the measured value of the vacuum energy is:

$$\rho_{\rm vac}^{1/4} = 2.4\ {\rm meV}$$

This scale is far lower than any of the scales in particle physics and the early Universe (apart from neutrinos ….) !

Hence the cosmological constant (counter term) must almost cancel all the disparate contributions from all the particles and the phase transitions of the Universe:

WHO ORDERED THAT ?

I.I. Rabi about the muon in 1936

AN EFFECTIVE FIELD THEORY APPROACH

The late time acceleration is a large distance phenomenon. It should be describable by a low energy field theory approach. Analogous to the Landau-Ginzburg theory of second order phase transitions (irrelevance of short distance details)

Two logical possibilities:

Fine-tuning:

all the contributions to the vacuum energy are tuned to the measured value cosmologically. GR + cosmological constant is the low energy effective field theory.

Dynamics:

The acceleration should be described by a theory where all particles have decoupled.

$\Downarrow$

EXTENSIONS of GR at LOW ENERGY

This requires to classify the violations of Weinberg's uniqueness theorem:

- ✓ Lorentz violating theories
- ✓ Massive spin 2 fields
- ✓ Fields of spin 0, 1 … : *scalars* or vectors

Massive graviton

vector

$5=2+2+1$

scalar

GR + scalar= **Scalar-tensor theories**

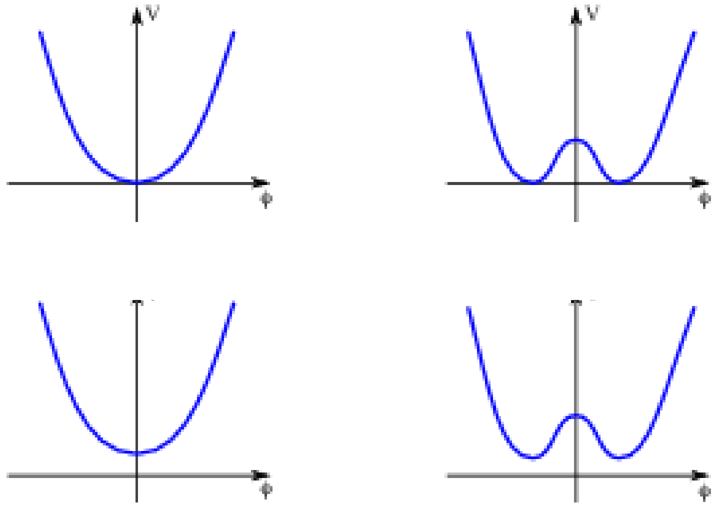
Self-acceleration

Dynamical (violation of Weinberg's no-go theorem: no Poincare invariant self-accelerating vacuum) evolution of the scalar leading to an effective vacuum energy.

Quantum effects

The latter when quantum corrections play a crucial role:

The vacuum energy is chosen to vanish here. It cannot be negative otherwise the universe has a big crunch singularity.



Low energy effective potential

After quantum corrections

The quantum corrections due to the massive scalar field, only field present at low energy together with gravity, lift the vacuum energy.

$$V(\phi) \quad \rightarrow \quad V(\phi) + \frac{m_\phi^4}{64\pi^2} \ln \frac{\mu^2}{m_\phi^2}$$

(see later for the RGE evolution in DS)

We would like to use something where the latter may occur: *the gravitational effective action*

$$S = \int d^4x\sqrt{-g}(\ m_{\rm Pl}^2\frac{R}{2} + c_0 R^2 + \dots)$$

The devil is the dots… more later

dimensionless

Similar approach Stelle(1978) and Starobinsky (1979)

We consider the physics at late times where the curvature is low, i.e. describing the physics from BBN till now:

$$R \sim H^2 \ll m_e^2$$

This theory is the simplest example of f(R) model, i.e. completely equivalent to a scalar-tensor theory with a constant coupling to matter. Moreover we assume that the correction to the Einstein-Hilbert term is small:

$$c_0 R \ll m_{\rm Pl}^2 \to \frac{\phi}{m_{\rm Pl}} \ll 1$$

This scalar is the ingredient which violates Weinberg's theorem

What is f(R) gravity?

$$S_{\rm MG} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R)$$

See Sotiriou-Faraoni (2010)

f(R) is totally equivalent to a field theory with gravity and a scalar

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\rm Pl}}g_{\mu\nu})\right)$$

Crucial coupling between matter and the scalar field

$$\beta = \frac{1}{\sqrt{6}}$$

The potential V is directly related to f(R)

$$V(\phi) = m_{\rm Pl}^2\frac{Rf' - f}{2f'^2}, \quad f' = e^{-2\phi/\sqrt{6}m_{\rm Pl}}$$

*Generically would be ruled out if no **chameleon effect***

The effect of the environment

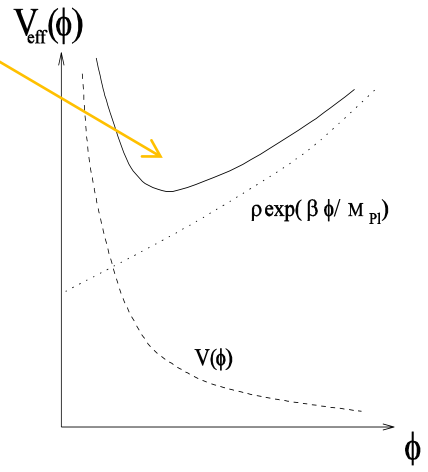
When conformally coupled to matter, scalar fields have a matter dependent effective potential:

$$V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1)$$

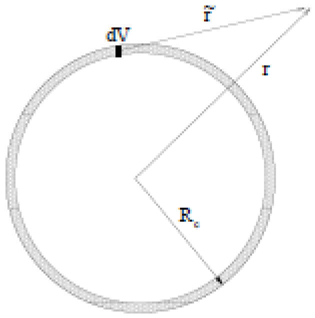
Khoury-Weltman (2003)

Environment
dependent
minimum

$$\beta(\phi) = m_{\rm Pl}\frac{d\ln A(\phi)}{d\phi}$$



Chameleon



The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

In our case the scalar field theory is extremely simple in the domain of validity of the effective field theory:

$$V(\phi) = \lambda^4 m_{\rm Pl}^4 e^{4\beta\phi/m_{\rm Pl}} + \frac{m_{\rm Pl}^4}{16 c_0}(e^{2\beta\phi/m_{\rm Pl}} - 1)^2$$

The coupling to matter complements this:

$$\delta V_{\rm matter} = \rho(e^{\beta\phi/m_{\rm Pl}} - 1)$$

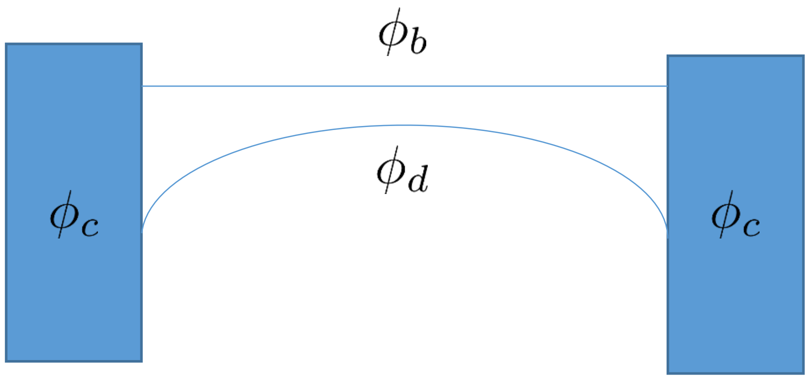
For vev's less than the Planck scale, this is nothing but a massive scalar field with a linear coupling to matter:

$$\boxed{V_{\rm eff}(\phi) = \lambda^4 m_{\rm Pl}^4 + 4\lambda^4 m_{\rm Pl}^3\phi + \frac{m^2}{2}\phi^2 + \frac{\beta}{m_{\rm Pl}}\phi\rho} \qquad 1 \ll c_0 \ll \lambda^{-4}$$

Mass almost independent of the density $\boxed{m^2 = \frac{\beta^2 m_{\rm Pl}^2}{2c_0}}$ Large coupling=small mass

This theory is best tested in the laboratory

$$\phi_b = -\frac{2c_0}{\beta}\frac{\rho}{m_{\rm pl}^3}$$



A bubble profile !

The scalar interaction between the plates depends on the difference of potential between the situation with and without boundary plates:

Brax-Davis (2014)

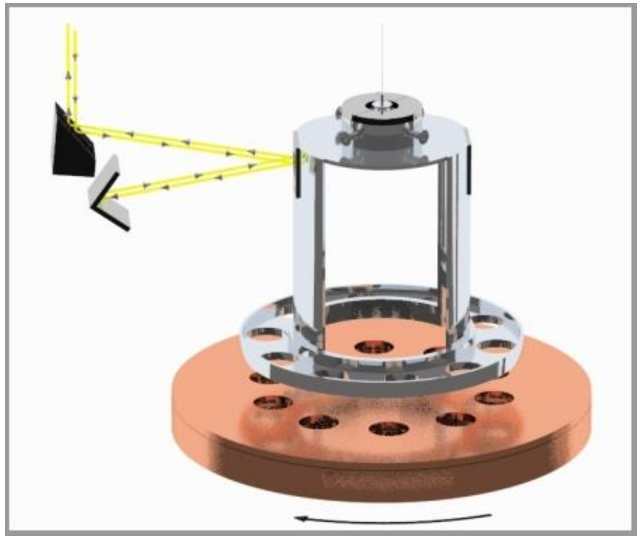
$$\frac{\Delta F_\phi}{A} = V_{\rm eff}(\phi_b) - V_{\rm eff}(\phi_d) \longrightarrow \frac{\Delta F_\phi}{A} = \frac{\beta^2\rho_c^2}{2m_{\rm pl}^2 m^2} e^{-md}$$

When the mass of the scalar field is larger than the inverse distance between the plates the scalar pressure is Yukawa-suppressed.

Eotwash experiment:

Measurement of the torque between two plaques with holes.
The potential energy of the system due to a chameleon force between the plates is simply the work:

$$W = A_\theta \int_d^\infty dx \frac{\Delta F_\phi}{A}(x)$$



The torque between the plates can be approximated:

$$T = a_T e^{-m_c d_s} \int_d^\infty dx \frac{\Delta F_\phi}{A}(x)$$

$$a_T = \frac{dA}{d\theta}$$

Electrostatic shielding

Brax et al. (2008)

The Eotwash experiments gives the strongest experimental constraint on the model:

$$T = a_\theta \frac{\beta^2 \rho_c^2}{2m_{\rm pl}^2 m^3} e^{-md}$$

$$T \le a_\theta \Lambda_T^3,$$

$$\Lambda_T \sim 0.35 \lambda m_{\rm Pl}$$

Dark energy scale

As the dark energy scale is of order 82 microns, this bound gives that the range of the scalar interaction must be larger than 0.1 mm:

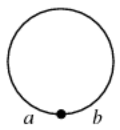
$$m \ge \lambda m_{\rm Pl} \;\; \to c_0 \;\; \le \lambda^{-2}$$

$$\lambda \sim 4 \times 10^{-31}$$

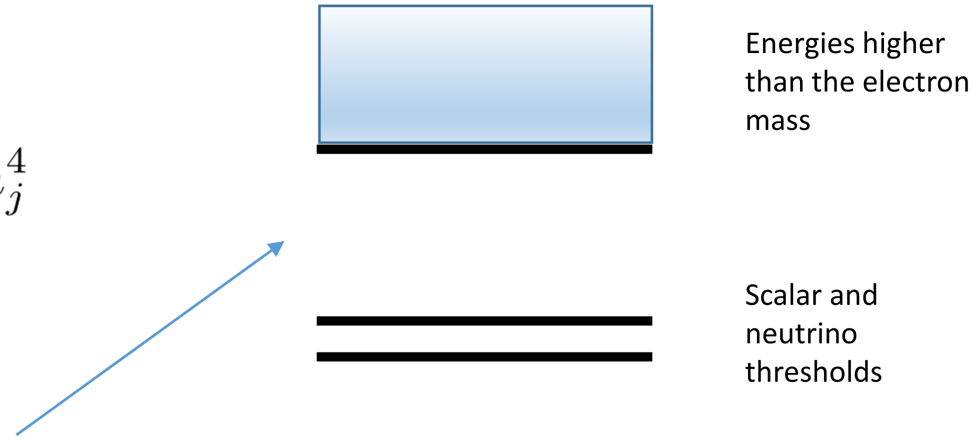
The coupling is also bounded from below by quantum stability.

Large upper bound on the coupling

In the Decoupling Substraction scheme (DS) the vacuum energy is affected by the quantum loops due to the particles which have not been integrated out..



$$\frac{d\rho_\Lambda}{d\ln\mu} = \frac{1}{32\pi^2}\sum_{m_j<\mu}(-1)^{2j+1}(2j+1)m_j^4$$



The vacuum energy when the electrons have decoupled. The scalar and neutrinos are in still the spectrum:

$$\rho_\Lambda(\mu) = \rho_\Lambda(m_e) + \frac{m^4}{64\pi^2}\ln\frac{m_e^2}{\mu^2} - \frac{1}{32\pi^2}\sum_{j=1}^{N_\nu} m_j^4 \ln\frac{m_e^2}{\mu^2}$$

The vacuum energy at low energy is fixed by observation at an energy which is of the order of the horizon inverse size, i.e. all quantum fluctuations up to the horizon size are taken into account (the ones outside the horizon are not causally connected and left out):

$$\rho_{\rm vac} = \rho_\Lambda(m_e) + \frac{m^4}{64\pi^2}\ln\frac{m_e^2}{m^2} - \frac{1}{32\pi^2}\sum_{j=1}^{N_\nu} m_j^4 \ln\frac{m_e^2}{m_j^2}$$

The positivity of $\rho_\Lambda(m_e)$ and the bounds on neutrino masses from oscillation data imply that:

$$c_0 \geq \frac{\beta^2 m_{\rm Pl}^2}{2\bar{m}_\nu^2} \qquad \bar{m}_\nu \leq 0.1 \text{ eV}$$

This leads to a tight interval for the range of the scalar interaction.

The coupling is extremely constrained:

$$\lambda^{-2} \geq c_0 \geq \frac{\beta^2 m_{\rm Pl}^2}{2\bar{m}_\nu^2}$$

implying that a signal could be around the corner! Measurable effects at around 50 microns.

$$2\mu\mathrm{m} \leq \mathrm{range} \leq 67\mu\mathrm{m}$$

As the range is tiny (less than 0.1 mm) these models are ultra-local with no effects on cosmological perturbations. Very small effects on astrophysical scales in galactic halos too.

Brax-Valageas (2015)

So far we have only considered the leading terms in a curvature expansion. Can we neglect the higher order terms? Not always!

$$\delta S_2 = -\int d^4x\sqrt{-g}c_2(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2)$$

This leads to higher order equations of motion and a propagator:

$$\frac{1}{k^2} - \frac{1}{k^2 + m_2^2}$$

This term adds a massive degree of freedom to the spectrum:

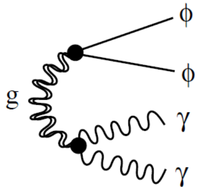
$$m_2^2 = \frac{m_{\rm Pl}^2}{c_2}$$

Stelle (1978)

Massive ghost cured when including higher derivative interactions.

Such a very massive (meV) spin 2 state would have escaped the LIGO window. On the other hand, as it is ghost-like this term only makes sense at very low energy making the decay rate low enough, typically a few MeV's.

Cline et al. (2003)



Emission of two ghosts and two photons.

This is a general phenomenon corresponding to Ostrogradski's theorem (1850):

Higher order derivatives in the action lead to

New degrees of freedom

Ghosts…

A consistent way of dealing with these unwanted degrees of freedom is to make them appear only at the scale where the derivative expansion breaks down.

For the massive graviton which is ghost-like this requires that c2<< c0 and the mass of the massive graviton is at least as large as the cut-off scale of the derivative expansion.

If the effective field theory had the expansion

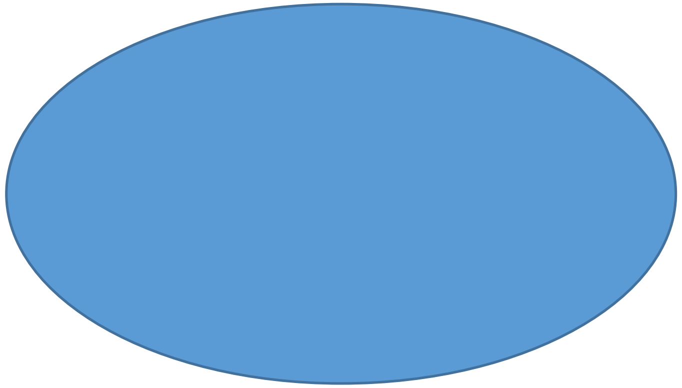
$$\cancel{S = \int d^4x \sqrt{-g} m_{\rm Pl}^2 \left(-\lambda^4 m_{\rm Pl}^2 + \frac{R}{2} + \frac{R^2}{m^2} \left(1 + \ldots \alpha_p \frac{R^p}{m^{2p}} + \ldots \right) \right)}$$

where there were only one scale in the problem… this would not make sense as m is the mass of the low energy scalar and in this approach the cut-off scale of the theory where another particle has been integrated out… The effective theory should behave like:

$$S = \int d^4x \sqrt{-g} m_{\rm Pl}^2 \left(-\lambda^4 m_{\rm Pl}^2 + \frac{R}{2} + \frac{R^2}{m^2} \left(1 + \ldots \alpha_p \frac{R^p}{M^{2p}} + \ldots \right) \right)$$

with a hierarchy **M>>m** to guarantee that the higher order terms can be dropped at low energy and the leading terms is the lowest one, i.e. there is one field of mass m much lower than the fields at the mass scale M which have been integrated out.

String theory is a natural setting to decouple higher order effects from the leading term:



Smooth 6d compactification

At low energy we can expect that the theory will be described by one scalar, the volume modulus, representing the « breathing » mode of the compactification manifold, coupled to gravity. We expect the dictionary:

Breathing mode $\longleftrightarrow$ R^2 term

String scale $\longleftrightarrow$ Cut-off scale M of the derivative expansion

Symbolically, all sorts of indices!

We consider the compactification of the 10d string action:

$$S_{10} = \int d^{10}x\sqrt{-g}\frac{1}{g_s^2 l_s^8}(R + \alpha_3 l_s^6 R^4 + \alpha_5 l_s^{10} R^6 + \dots)$$

$$V_6 = l_6^6$$

Compactification volume

Tree level in string theory, p loops g_s^{2p-2}

$$M_s = 1/l_s$$

String scale

The 4d action after dimensional reduction is of the form :

$$S = \int d^4x\sqrt{-g}m_{\rm Pl}^2\left(\frac{R}{2} + \frac{R^2}{M_s^2}\sum_{p\geq 0} d_p(Rl_s^2)^p\right)$$

The four dimensional quantities are derived:

$$m_{\rm Pl}^2 = \frac{l_6^6}{g_s^2 l_s^8}(1 + \sum_p \alpha_p (\frac{l_s}{l_6})^{2p})$$

$$d_p \sim \sum_{n \geq \max(p+1,3)} \alpha_n (\frac{l_s}{l_6})^{2(n-p-1)}$$

$$d_0 \sim \alpha_3 (\frac{l_s}{l_6})^4, \; d_1 \sim \alpha_3 (\frac{l_s}{l_6})^2, \; p \geq 2, \; d_p \sim \alpha_{p+1}$$

$$l_6 \gg l_s$$

In a **non-warped** compactification:

$$c_0 \sim c_2 \sim \frac{1}{g_s^2} \frac{l_6^2}{l_s^2}$$

$$m_0^2 \sim m_2^2 \sim \frac{l_6^4}{l_s^4} M_s^2$$

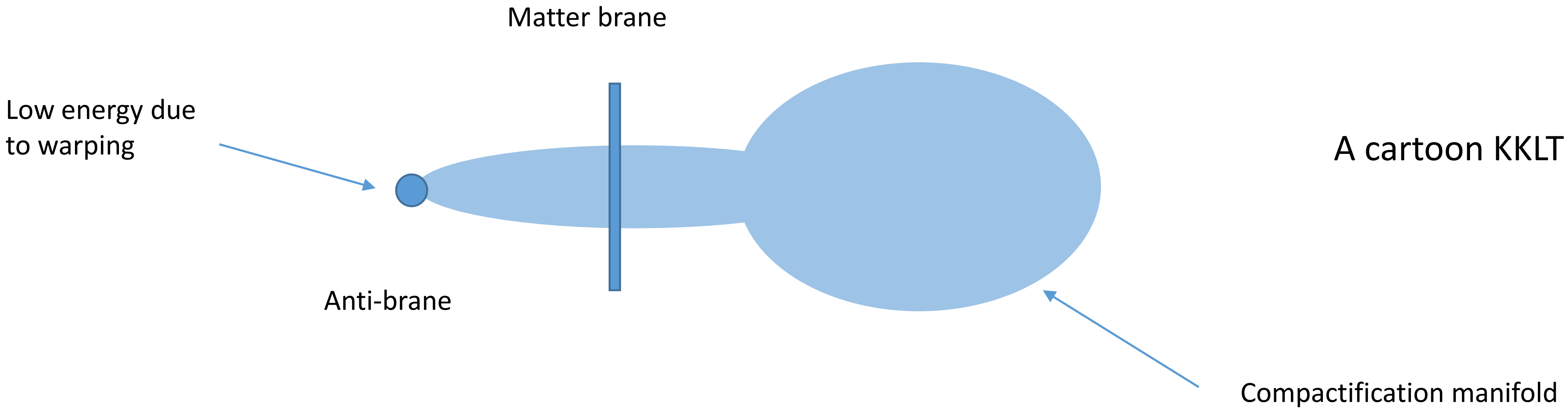
Hence the scalar and massive graviton have a mass larger than the string scale, and the string scale is the scale where the derivative expansion breaks down: GR at low energy.

NEED EXTRA INGREDIENTS

This can be remedied by introducing susy breaking by brane-antibrane. The Goldstino field of broken supersymmetry is captured by a nilpotent superfield S.

Working at low energy this can be captured in N=1 supergravity in 4d which works at the two derivative level with the dictionary. This does not change the coefficients of the higher derivative terms in the curvature expansion but will modify only the lowest order due to the dictionary:

$$\text{Breathing mode} \longleftrightarrow R^2$$



A cartoon KKLT compactification

The model is described by a T volume modulus and a nilpotent field S corresponding to the anti-brane:

$$K = -3m_{\rm Pl}^2 \ln(\frac{T+\bar{T}}{m_{\rm Pl}} - \frac{3S\bar{S}}{m_{\rm Pl}^2})$$

where S satifies the constraint: $S^2 = 0$

and the superpotential is: $W = M_W S(T - <T>)$

$$\frac{T+\bar{T}}{2} =< T > e^{-2\beta\phi/m_{\rm Pl}}$$

When M_W is much smaller than the Planck scale:

$$V(\phi) \sim \frac{M_W^2 m_{\rm Pl}^2}{4}(e^{2\beta\phi/m_{\rm Pl}} - 1)^2$$

The exact form of the potential is not relevant. What matters is that T is stabilised (moduli stabilisation) with a low enough mass.

Nothing but the scalar model!

The identification is even true for the matter coupling:

$$K \supset \frac{C\bar{C}}{T+\bar{T}}$$

$$\mathcal{L}_m \supset e^{K/2m_{\rm Pl}^2} m_\psi \bar{\psi}_C \psi_C$$

Normalised field

$$D = \sqrt{\frac{m_{\rm Pl}}{T+\bar{T}}}C$$

$$\mathcal{L}_m \supset (\frac{T+\bar{T}}{m_{\rm Pl}})^{-1/2} m_\psi \bar{\psi}_D \psi_D$$

This is the field dependence of f(R)

$$m_\psi(\phi) = e^{\beta\phi/m_{\rm Pl}} m_\psi$$

This is the crucial point as the identification with the quadratic correction to Einstein-Hilbert only requires a **massive scalar** coupled with this **particular coupling**:

$$\beta = \frac{1}{\sqrt{6}}$$

This completes the equivalence with the f(R) theory at low energy

We can now identify the low energy parameter of the quadratic term:

$$M_W \sim 1\ \mathrm{meV}$$

$$c_0 = \frac{m_{\mathrm{Pl}}^2}{4M_W^2}$$

$$< T > \sim m_{\mathrm{Pl}} (\frac{l_6}{l_s})^4, \quad l_6 \gg l_s$$

The low energy vacuum energy is then due to the quantum fluctuations of the volume modulus:

The massive spin 2 state has a mass larger the string scale implying that it becomes relevant at the scale where all the higher order corrections are relevant too, not at low energy.

$$(\lambda m_{\mathrm{Pl}})^4 \sim M_W^4$$

Vacuum energy

This may hint in favour of a fundamental origin for a scalar field whose mass and coupling to matter could be testable by laboratory experiments. A better understanding of the underlying microscopic theory is certainly needed.

SUMMARY

- ✓ We have argued that the effective field theory of gravity at the lowest order in curvature could have phenomenologically appealing properties such as a of scalar degree of freedom which could be just around the corner for laboratory experiments... *New bound on the range at the 40 micron level (Eotwash unpublished)*

- ✓ In particular, such a low energy effective field theory is only valid when there is a **hierarchy of scales**. If this is not the case then the effective field theory of gravity reduces to a pure cosmological constant at low energy: no dynamics …

- ✓ The non-trivial dynamical case may occur in string theory where the lowest order in the curvature expansion may dominate when SUSY is broken by low energy brane-antibrane effects. All this requires is **moduli stabilisation** with a large vev and a small mass for the compactification breathing mode.

- ✓ Building a precise string model which reproduces the indications of the supergravity models would be extremely interesting.

A more “debated” superpotential could be envisaged:

$$W = W_0 + BS + (P + CS)e^{-aT/m_{\rm Pl}}$$

where B is warped down and C not. Close to the minimum of the superpotential in S, this has a similar shape as the previous one with the identifications:

$$\frac{< T >}{m_{\rm Pl}} \sim \frac{2A_0}{a} \qquad\qquad M_W \sim \frac{\sqrt{T}_3}{m_{\rm Pl}} e^{-2A_0}$$

which can lead to a large volume and a small energy scale.