Heavy quark thresholds in PDFs

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Based on: V. Bertone, A. Glazov, A. Mitov, A. Papanastasiou, M. Ubiali , arXiv:1711.03355

What this work is/is not/ about?

- ✓ We do not propose new prescriptions etc.
- ✓ We aim at simplifying things, not complicating them
- ✓ We point out that the position of the heavy flavor threshold represents a genuine theoretical ambiguity. It needs to be addressed
- ✓ Will not speak of threshold (which is observable dependent statement)
- ✓ Will speak of Heavy Flavor Matching Point (HFMP)

The basics

- ✓ Consider bottom as the only heavy flavor (for simplicity)
- ✓ A Variable Flavor Number Scheme (VFNS) is better than a fixed FFNS
- ✓ How to construct a VFNS? Here are the solid basics:
  - ✓ Can't have massive initial partons (dictated by factorization)
  - ✓ When quarks are very heavy they decouple
  - ✓ When very light – should be taken massless
- ✓ However: the intermediate region -- where the theory provides no guidance -- is large
- ✓ In the literature the HFMP is taken to be equal to the mass of the heavy quark
- ✓ We question the wisdom for a fixed choice μ_b=m and explore beyond it
- ✓ HFMP (with/out mass) variation interpreted as uncertainty has been studied in:

Stirling et al '11
Bonvini, Papanastasiou, Tackmann '15
V. Bertone et al. [xFitter Developers Team], arXiv:1707.05343

A historic accident?

- ✓ Why in the past the threshold has been taken equal to the mass of the heavy quark?
- ✓ Short answer: an accident. Based on:
- ✓ Many years ago, when NLO was something to dream about, people wanted:
  - ✓ Continuity: it was feared that a discontinuity in the pdf at threshold may induce discontinuities in observables.
  - ✓ Of course, we still want continuity: we observe that at higher orders continuity in observables is restored for a range of thresholds! In other words the discontinuity in PDFs need not be feared.
  - ✓ At LO in QCD, the matching is continuous for any value of the threshold
  - ✓ At NLO the continuity of PDFs is only present if the threshold is chosen to be at the heavy quark mass
  - ✓ The mass m is a "natural" scale
- ✓ However, this continuity at NLO is just an accident and is not a feature that persists
  - ✓ In the space-like at higher orders (NNLO and N^3LO)
  - ✓ In the time-like
  - ✓ For α_S

What are the options when μ_b=m?

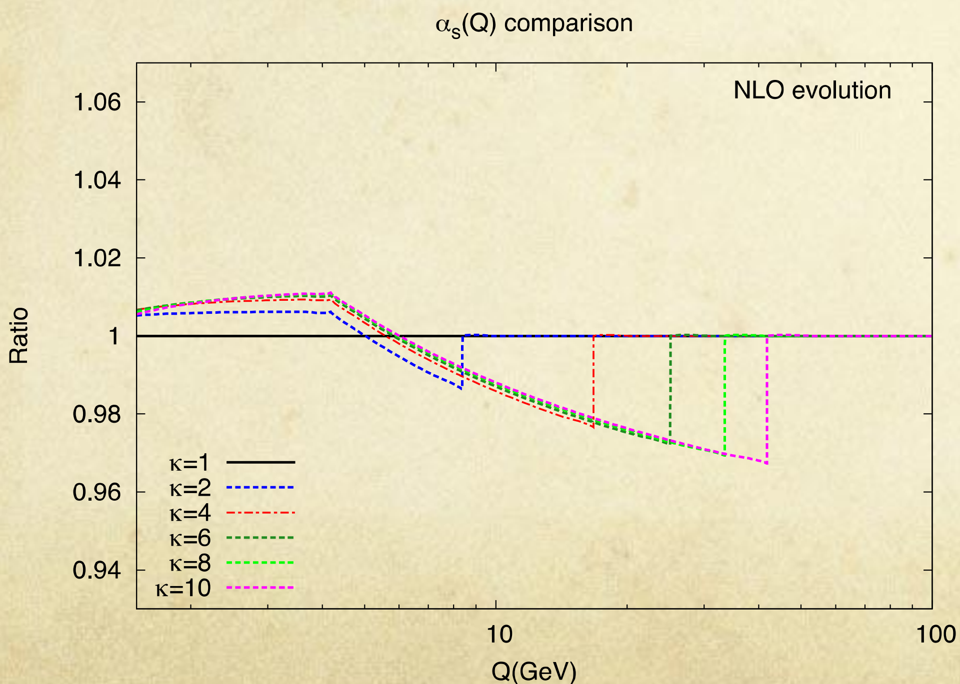
- ✓ The options are well known:
  - ✓ Work in FFNS: no resummation for large scales
  - ✓ Work in ZM-VFNS: easy to implement but inaccurate just above the HFMP μ_b due to missing power corrections O(m)
  - ✓ Work in GM-VFNS:
    - ✓ Some power corrections O(m) are restored
    - ✓ Could be cumbersome to implement. Many schemes proposed in the past
- ✓ <u>However, if one explores the fact that the HFMP need not be equal to the mass m</u>, then by switching to larger values of μ_b one can reduce the size of the missing power corrections.
  - ➢ In other words one can still work in ZM-VFNS without being penalized for missing power corrections (which are $O(m^2/\mu_b^2)$).

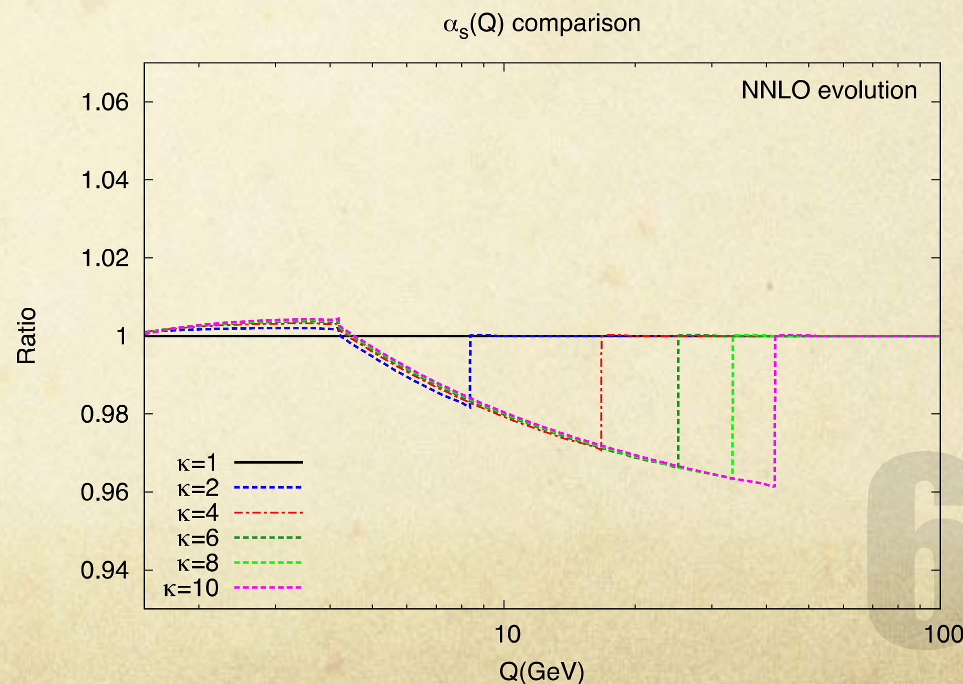
Our proposal

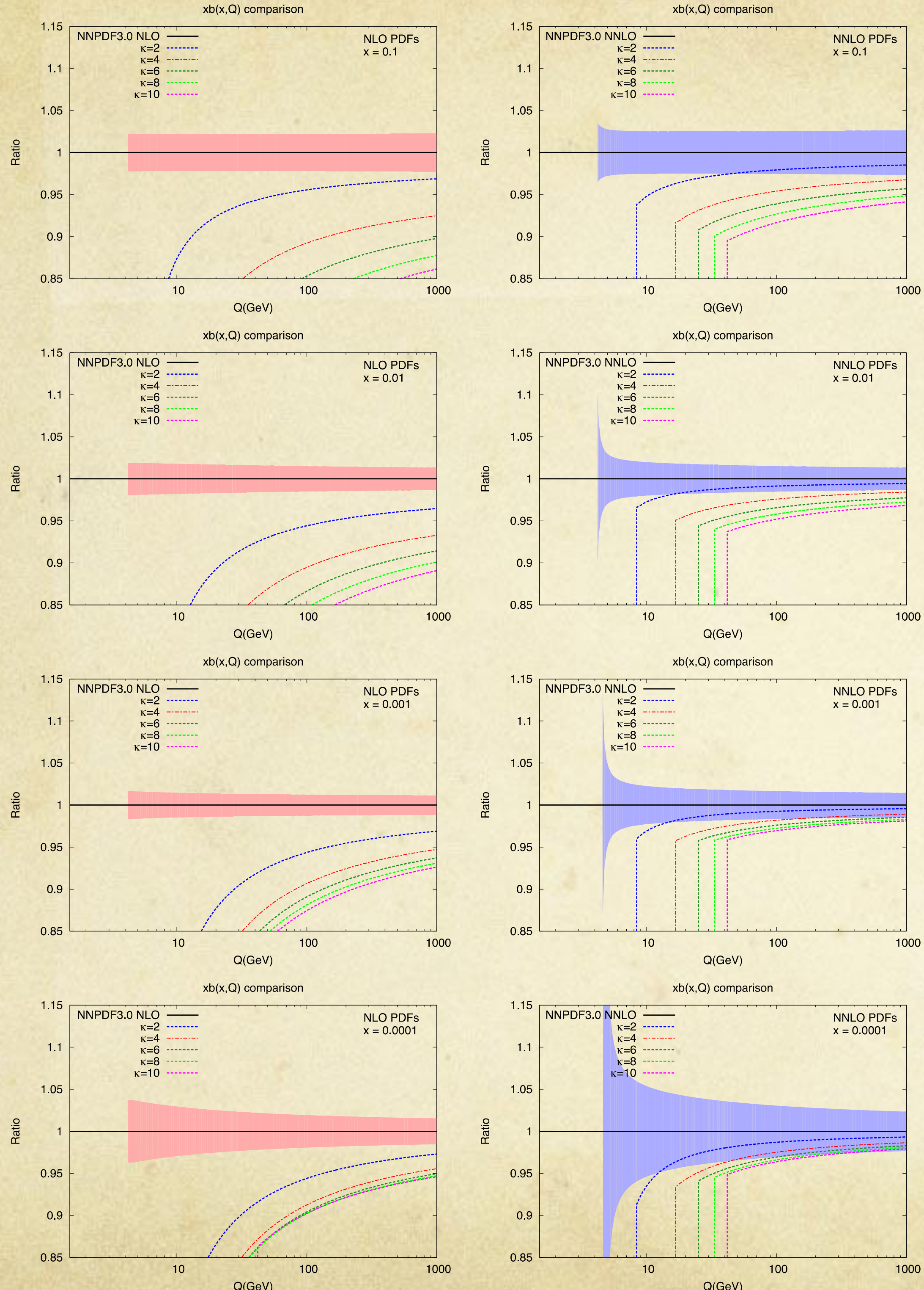
- ✓ Its formulation and implementation are very simple:
  - ✓ Take the HFMP to be significantly higher than the mass (x5 – x10)
  - ✓ Assume the functional form of μ_F and μ_R has been chosen. For simplicity we take $\mu_F=\mu_R$.
  - ✓ Below HFMP (i.e. $\mu_F < \mu_{THR}$) we work in 4FS, as usual
  - ✓ Above HFMP (i.e. $\mu_F > \mu_{THR}$) we work in 5FS with a massless heavy flavor
- ✓ Massless calculations account for all terms $\sim\ln^n(m)$ and m^0. They only miss terms of $O(m^2)$
- ✓ Our approach involves an approximation; however, unlike the standard ZM-VFNS we have a parameter that controls the error: $O(m^2/\mu_b^2)$
- ✓ If μ_b = (5–10) x m, then missing power corrections are completely negligible (1%-4%)
- ✓ Benefits:
  - ✓ No need for complicated and sometimes cumbersome to implement prescriptions
  - ✓ Use 'plain' calculations that are always used in their region of validity
  - ✓ Control over $O(m^2)$ type terms
  - ✓ Threshold should not be taken too high (more than around x10) in order not to spoil collinear resummation.

Effect of large HFMP on PDFs

- ✓ Consider the effect on α_S and PDFs
- ✓ To this end we have generated a family of PDFs, based on NNPDF3.0, with various thresholds spanning the range [1 – 10] x m.
- ✓ Evolution for each set is done consistently, at LO, NLO and NNLO.
- ✓ Same initial condition for any HFMP
- ✓ Charm and top are considered here as usual. In principle their thresholds should be increased.
- ✓ The scale dependence of α_S , shown relative to the standard case, looks like:







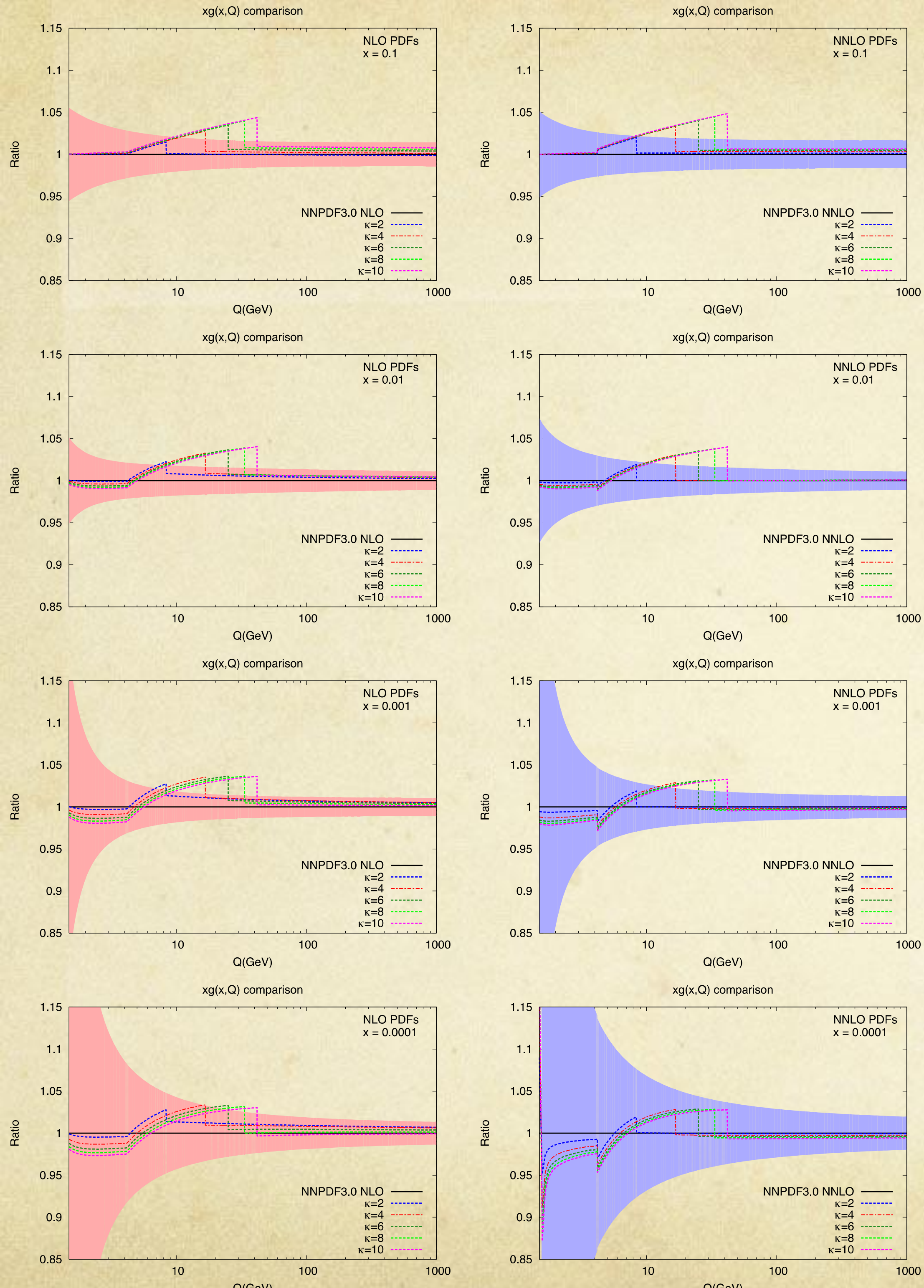
b - PDF

NLO (left) and NNLO (right)

for four values of x

Note the drastic improvement at NNLO

"Discontinuities are good"



g - PDF

NLO (left) and NNLO (right)

for four values of x

Note the agreement between all sets at large Q (as desired).

NNLO an improvement over NLO

"Discontinuities are good"

Similar story for all other PDFs

Effect of large HFMP on observables

- The discontinuities in α_S and PDFs are clearly visible. What happens with observables?

- We study the effect on:

  1. Standard precision LHC candles (tt, single top, Higgs, Z)
  2. Processes sensitive to b-PDF (single top, bbZ, b-jet P_T in b+Z)
  3. Discontinuities in 2) and in tt-like and Z-like processes that can be computed through NNLO in both 4FS and 5FS (Z-like means Z production but with modified m_Z; same for top)

- We observe the amazing self-consistency of the theory:

  - The large discontinuities in α_S and PDFs are nowhere to be found in observables.

  - In other words, an apparent cancellation takes place

    - A hint of a possible non-minimal formulation of the theory?

- It is absolutely essential to not only look at LO but always at NLO and whenever possible at NNLO.

- Huge improvement from the inclusion of higher orders which brings about continuity in observables.

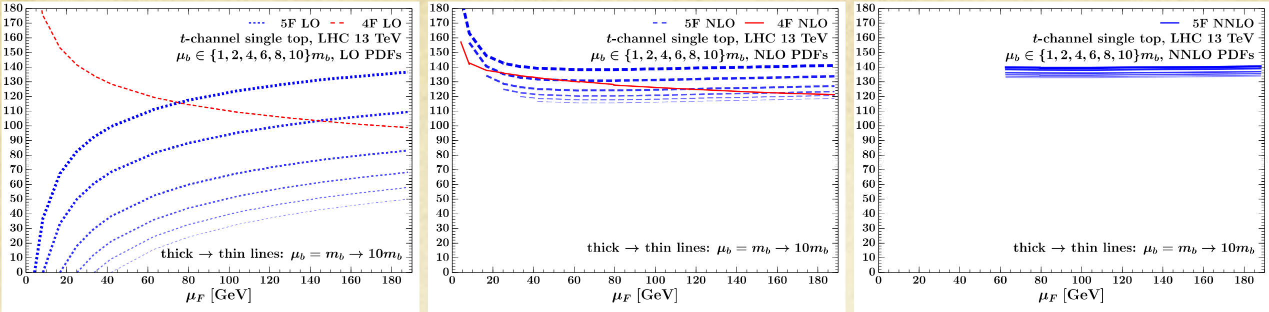


Figure 3. Single top t-channel cross-section at LHC 13 TeV at LO (left), NLO (center) and NNLO (right) as a function of μ_F for several values of μ_b.

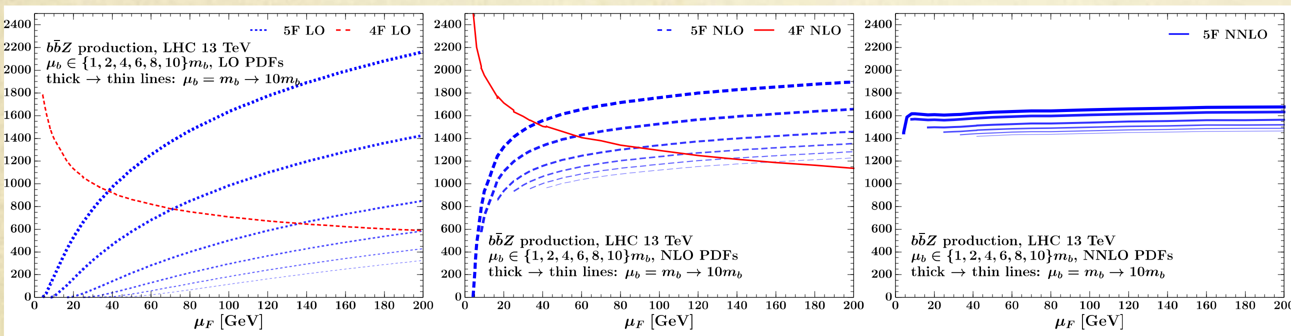


Figure 4. As in fig. 3 but for the total $b\bar{b}Z$ production cross-section.

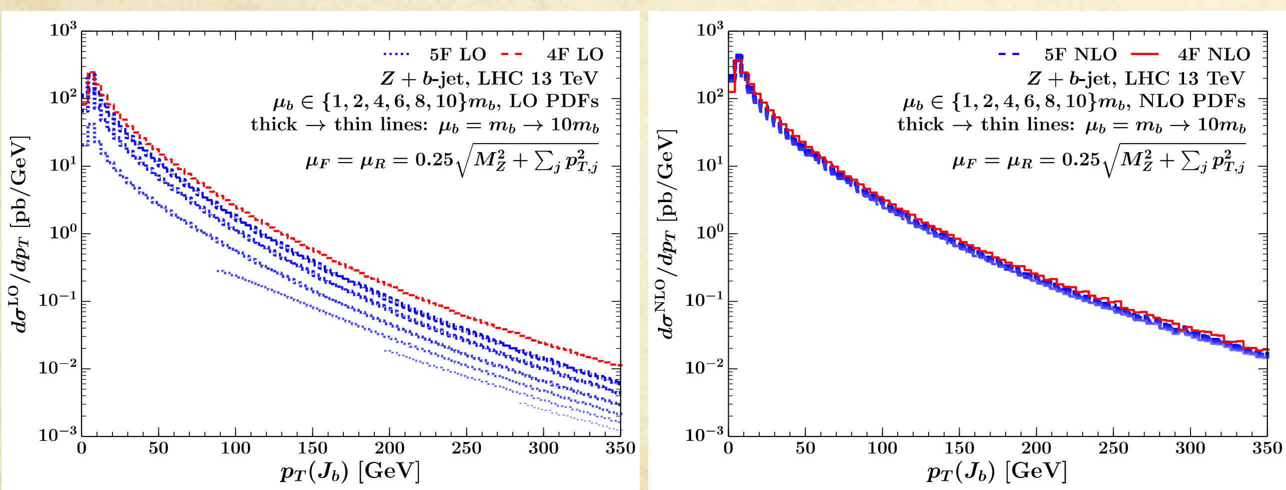


Figure 5. The differential cross-section of $Z + b$-jet at LHC 13 TeV as a function of $p_T(J_b)$ at LO (left) and NLO (right) for several values of μ_b.

Effect on observables

Drastic improvement from inclusion of higher orders

Discontinuities in observables

$$\text{Discontinuity} = 1 - \frac{\sigma^{5\text{F}}(M \text{ fixed}, \mu_{F,R} = \mu_b + \epsilon)}{\sigma^{4\text{F}}(M \text{ fixed}, \mu_{F,R} = \mu_b - \epsilon)}$$

✓ Drastic improvement from inclusion of higher orders

✓ Improvement with larger thresholds

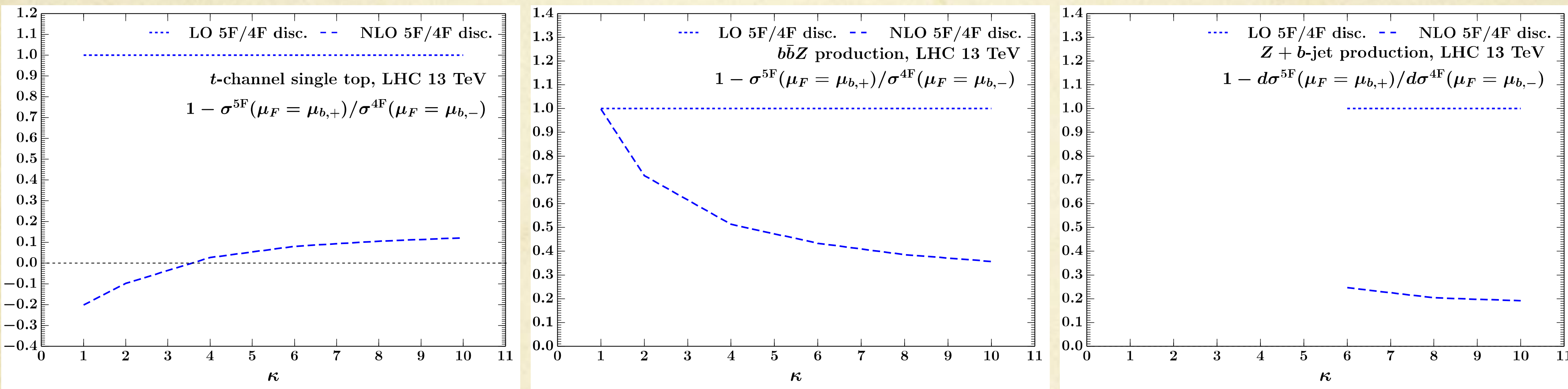


Figure 9. Discontinuity between 5FS cross-section above threshold and 4FS cross-section below threshold for LO (dotted) and NLO (dashed) for t-channel single top cross-section (left), $b\bar{b}Z$ production cross-section (center) and the $p_T(J_b)$ differential distribution in $Z + b$ (right). All are for LHC at 13 TeV as a function of κ. The discontinuity is defined in eq. (4.4) and, in the ideal case, it should be zero.

Standard LHC candles (Z and tt)

κ	σ_{LO} [pb$\times 10^4$]	σ_{NLO} [pb$\times 10^4$]	σ_{NNLO} [pb$\times 10^4$]
1	4.42964	5.42333	5.64074
2	4.46018 (+0.7%)	5.43158 (+0.1%)	5.62619 (-0.3%)
4	4.51340 (+1.9%)	5.40903 (-0.3%)	5.60047 (-0.7%)
6	4.55424 (+2.8%)	5.38918 (-0.6%)	5.58349 (-1.0%)
8	4.58731 (+3.6%)	5.37355 (-0.9%)	5.57117 (-1.2%)
10	4.61520 (+4.2%)	5.36088 (-1.2%)	5.56158 (-1.4%)

Table 3. Dependence of the total Z cross section at LHC 13 TeV on the threshold scale $\mu_b = \kappa \cdot m$ (recall that $\kappa = 1$ represents the standard choice in all publicly available pdf sets). Shown is the 5FS cross-section predicted at LO, NLO and NNLO for $m_Z = \mu_R = \mu_F = 91.1876$ GeV.

κ	σ_{LO} [pb]	σ_{NLO} [pb]	σ_{NNLO} [pb]
1	560.86	735.21	806.15
2	566.28 (+1.0%)	736.49 (+0.2%)	807.50 (+0.2%)
4	570.59 (+1.7%)	739.52 (+0.6%)	809.22 (+0.4%)
6	572.63 (+2.1%)	741.78 (+0.9%)	810.33 (+0.5%)
8	573.86 (+2.3%)	743.53 (+1.1%)	811.14 (+0.6%)
10	574.70 (+2.5%)	744.95 (+1.3%)	811.78 (+0.7%)

Table 4. As in table 3 but for the $t\bar{t}$ total cross-section with $m_t = \mu_R = \mu_F = 173.3$ GeV.

Standard LHC candles (ggH and t-ch top)

κ	$\sigma_{\rm LO}$ [pb]	$\sigma_{\rm NLO}$ [pb]	$\sigma_{\rm NNLO}$ [pb]
1	18.375	35.055	44.423
2	18.836 (+2.5%)	35.327 (+0.8%)	44.466 (+0.1%)
4	19.332 (+5.2%)	35.442 (+1.1%)	44.480 (+0.1%)
6	19.635 (+6.7%)	35.466 (+1.2%)	44.481 (+0.1%)
8	19.855 (+8.1%)	35.469 (+1.2%)	44.478 (+0.1%)
10	20.028 (+9.0%)	35.465 (+1.2%)	44.475 (+0.1%)

Table 5. As in table 3 but for the ggH total cross-section with $m_H = 2\mu_R = 2\mu_F = 125.0$ GeV.

κ	$\sigma_{\rm LO}$ [pb]	$\sigma_{\rm NLO}$ [pb]	$\sigma_{\rm NNLO}$ [pb]	$R^{\rm LO}_{t/\bar t}$	$R^{\rm NLO}_{t/\bar t}$	$R^{\rm NNLO}_{t/\bar t}$
1	119.19	138.28	139.90	1.654	1.660	1.638
2	90.26 (-24.2%)	130.78 (-5.4%)	138.48 (-1.0%)	1.668	1.658	1.641
4	62.22 (-47.8%)	124.10 (-10.2%)	136.30 (-2.6%)	1.680	1.662	1.644
6	46.30 (-61.2%)	120.34 (-13.0%)	134.90 (-3.6%)	1.687	1.666	1.645
8	35.23 (-70.4%)	117.69 (-14.9%)	133.86 (-4.3%)	1.691	1.670	1.647
10	26.78 (-77.5%)	115.63 (-16.4%)	133.03 (-4.9%)	1.694	1.673	1.649

Table 6. As in table 3 but for the t-channel single-top total cross-section and the ratio $R_{t/\bar t}$ of single top versus single antitop cross-sections with $m_t = 173.3$ GeV and $\mu_{F,R} = m_t/2$.

Comparison with FONLL

- ✓ We do not say one should not use GM-VFNS;
- ✓ We only say one does not have to if high HFMP is chosen

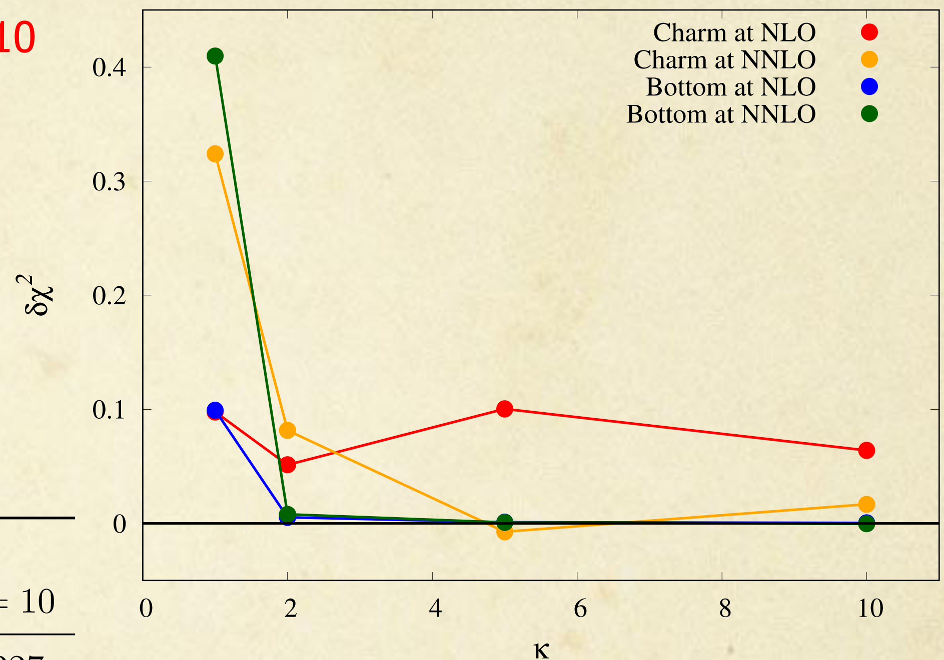
- ✓ In other words for large HFMP our predictions should be identical to the ones from GM-VFNS.

- ✓ We have checked with FONLL:

Forte, Laenen, Nason, Rojo '10

  - ✓ Charm and bottom fits from DIS data

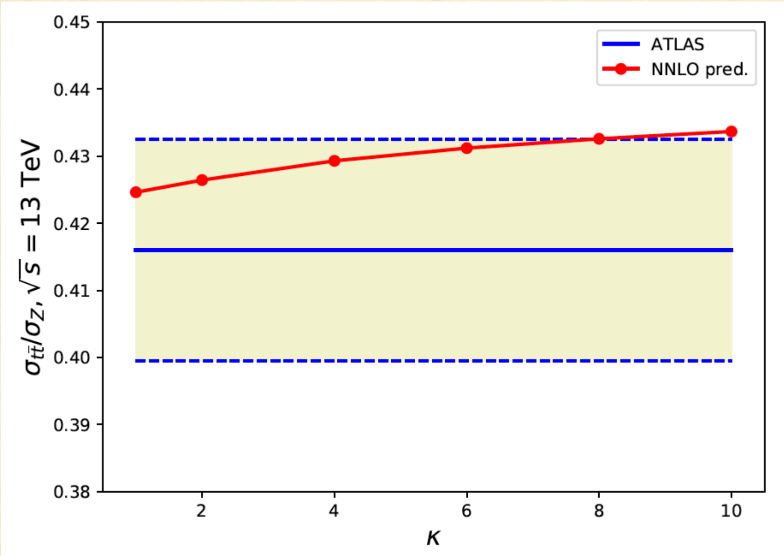
$$\delta\chi^2(\kappa) = \frac{\chi^2(\text{this work}) - \chi^2(\text{FONLL})}{\chi^2(\text{FONLL})}$$



Heavy quark	Pert. order	Scheme	χ^2 / d.o.f. $\kappa = 1$	$\kappa = 2$	$\kappa = 5$	$\kappa = 10$
Charm	NLO	FONLL	1.144	1.179	1.166	1.227
		This work	1.255	1.239	1.283	1.305
	NNLO	FONLL	1.207	1.194	1.214	1.226
		This work	1.598	1.292	1.205	1.246
Bottom	NLO	FONLL	1.148	1.143	1.144	1.146
		This work	1.262	1.149	1.145	1.146
	NNLO	FONLL	1.204	1.208	1.207	1.212
		This work	1.697	1.218	1.208	1.212

What still needs to be understood (I)

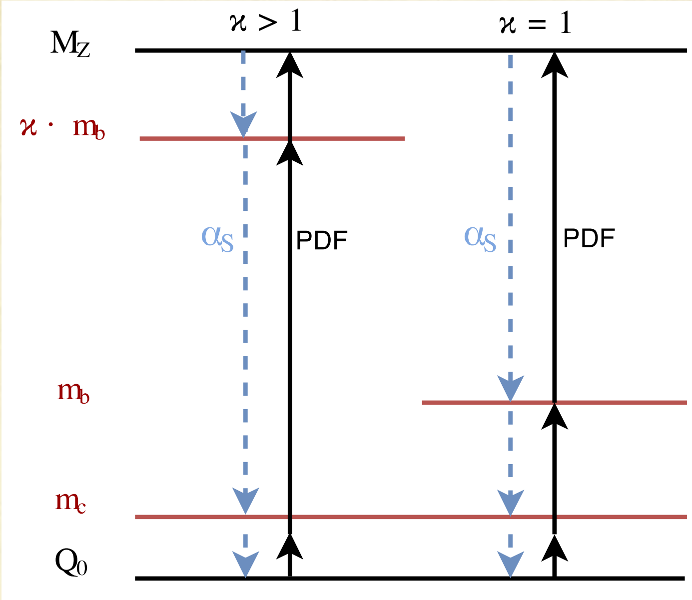
- ✓ Predictions for observables – when HFMP is changed – get modified.
- ✓ In the observables we have checked the changes are small. But there are outliers:
  - ✓ Single top: 5% shift for μ_b=10m (given NNLO scale variation is 1%)
  - ✓ tt/Z ratio



- ✓ Both are consistent within current measurements
- ✓ It will be good to keep an eye in the future

What still needs to be understood (II)

- ✓ What happens when all heavy flavors get shifted?



- ✓ A consistent treatment within this framework is needed. What we did is not fully consistent:
  - ✓ On one hand pdfs are evolved up from the same boundary condition (the standard NNPDF3.0)
    - ➢ Now pdf are given at low scale. But should they be
      - Fitted?
      - Fixed there? (lattice-based arguments)
  - ✓ On the other hand α_S is fixed at m_Z. If HFMP gets changed then $\alpha_S(m_Z)$ is not unique anymore. New convention needed. Perhaps, given most precise input is from lattice anyway, we should also fix α_S at low scale (1.5 GeV or so)?

Conclusions

- ✓ The position of the HFMP is not specified by theory (just like, for example, the factorization and renormalization scales)

    - ➢ This is a freedom which must be explored and understood

- ✓ Our impression is that higher order corrections to PDF matching conditions are very important for the continuity of pdf's. Looking forward to the complete N^3LO corrections!

- ✓ The approach is very simple to apply in practice for any process and to any fully differential observable

- ✓ Approach would be very appropriate for top production. So far all top calculations done in 5FS, not 6FS, despite the fact current kinematics is already in the TeV range.

- ✓ Full exploration will require a complete implementation in the context of global PDF fits. All tools needed for dealing with variable HFMP's are now publicly available.

xFitter, APFEL