Pion angular distribution in electroweak pion production off nucleons

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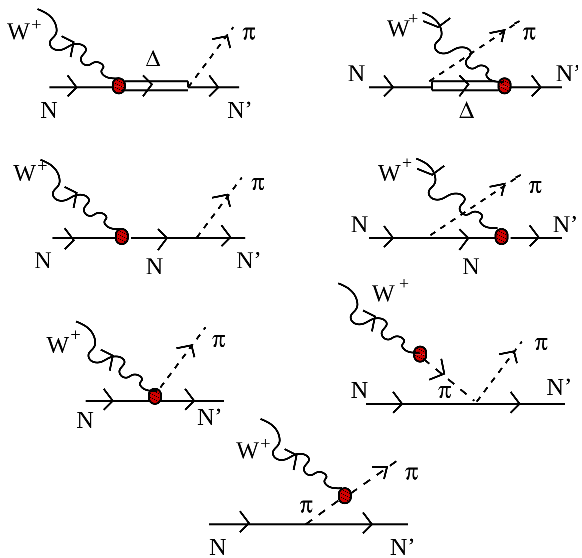
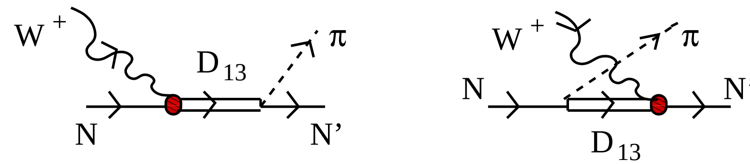
Phys. Rev. D 98, 073001 (2018)

Motivation

- We wanted to make a comparison of the model developed initially by Nieves, Valverde and mysel (HNV for short) and the DCC model of Sato, Nakamura and collaborators. We wanted a comparison at the nucleon level (to avoid the complication of medium effects and final state interacrtion effects) that went beyond comparing total cross sections.
- Pion angular distribution in the center of mass of the πN system is a very rich and non trivial observable that can prefectly serve that purpose.
  Caveat: Unfortunately, there is very little data available.
- Pion angular distribution in pion electroproduction is also a very strict test of the vector part of any model for pion production by neutrinos.

HNV pion production model in one slide





Phys. Rev. D87, 113009 (2013)

Phys. Rev. D76, 033005(2007)

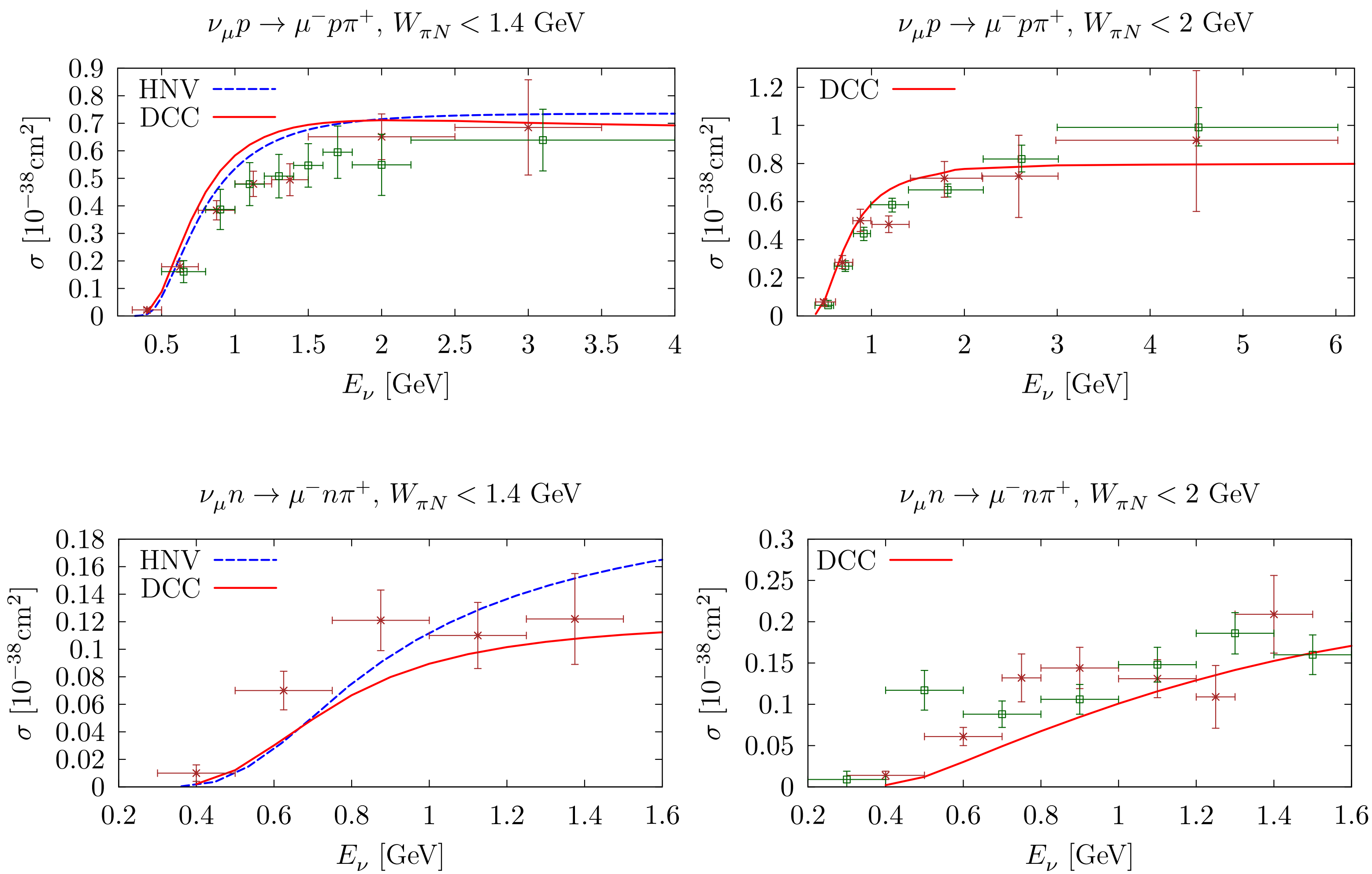
Partial unitarization through Olsson's method: $T_B + T_{\Delta P} \rightarrow T_B + e^{i\delta_V} T^V_{\Delta P} + e^{i\delta_A} T^A_{\Delta P}$
Phys. Rev. D93, 014016 (2016)

Modified Delta propagator:

$$\frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \rightarrow \frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c\,\delta P_{\mu\nu}, \quad \delta P_{\mu\nu} = \frac{P_{\mu\nu} - \frac{p_\Delta^2}{M_\Delta^2} P^{\frac{3}{2}}_{\mu\nu}}{p_\Delta^2 - M_\Delta^2}$$

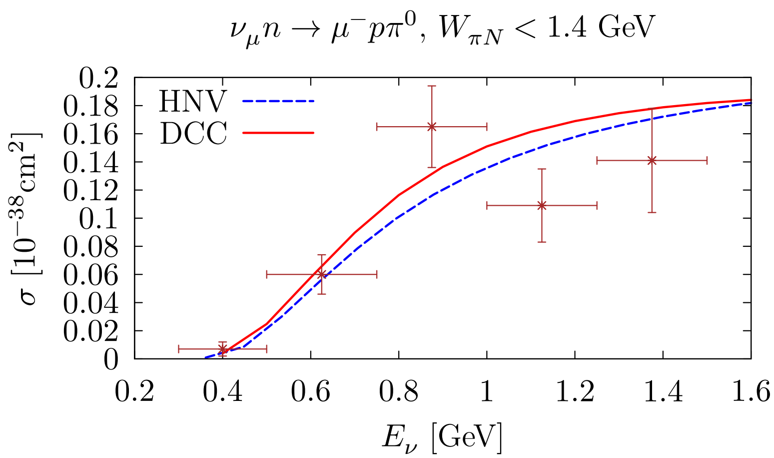
We get better results for $\nu_\mu n \rightarrow \mu^- n \pi^+$ by fitting c [Phys. Rev. D95, 053007 (2017)]

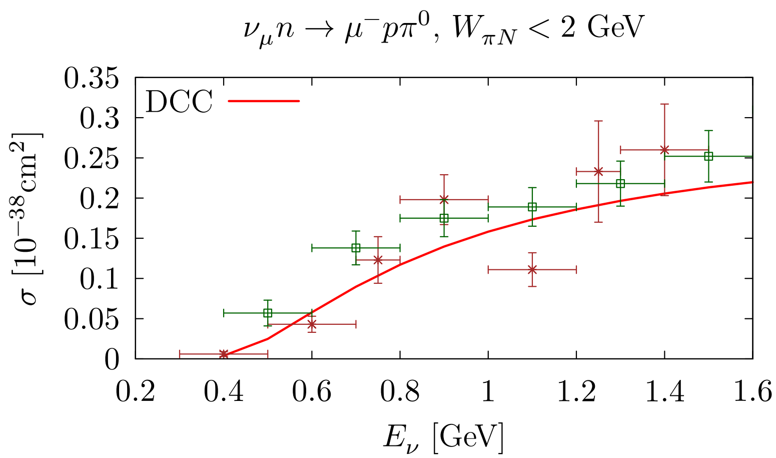
Total cross section comparison between the DCC and HNV models



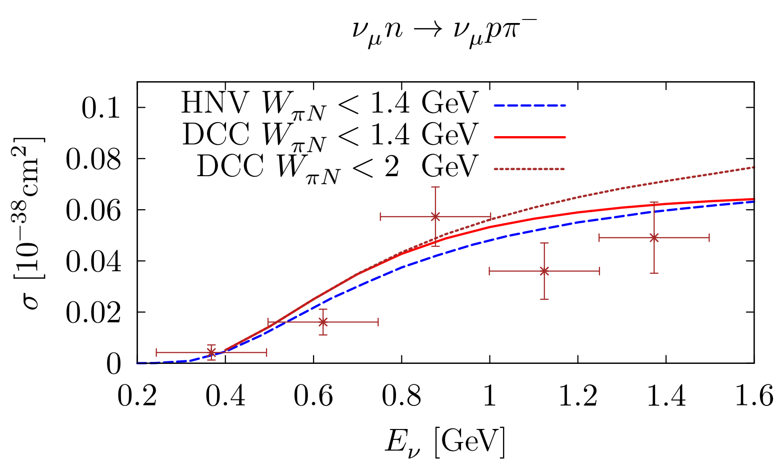
Data from P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016) and C. Wilkinson, P. Rodrigues, S. Cartwright, L. Thompson, and K. McFarland, Phys. Rev. D90, 112017 (2014).

Total cross section comparison between the DCC and HNV models II



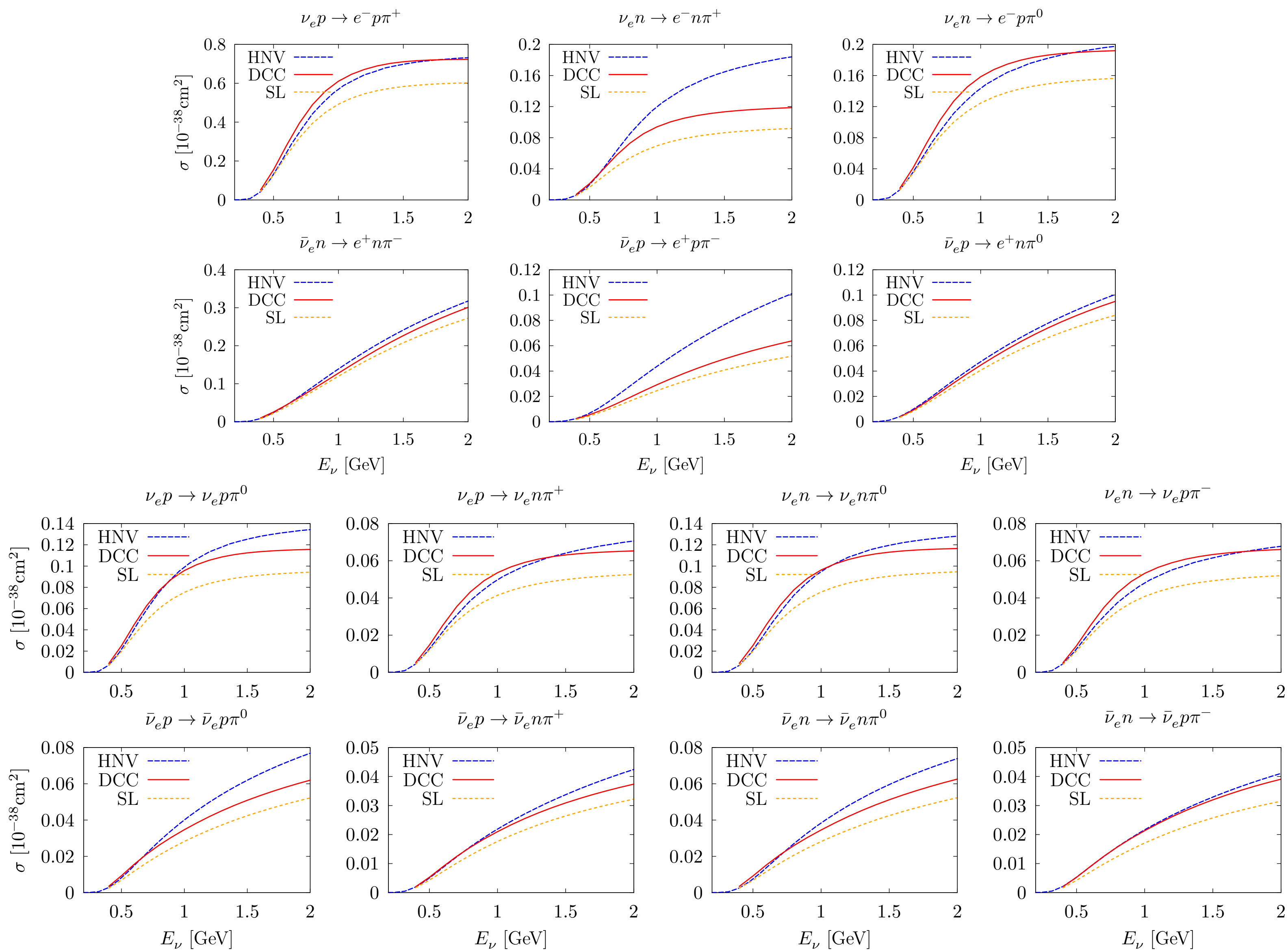


Data P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016).



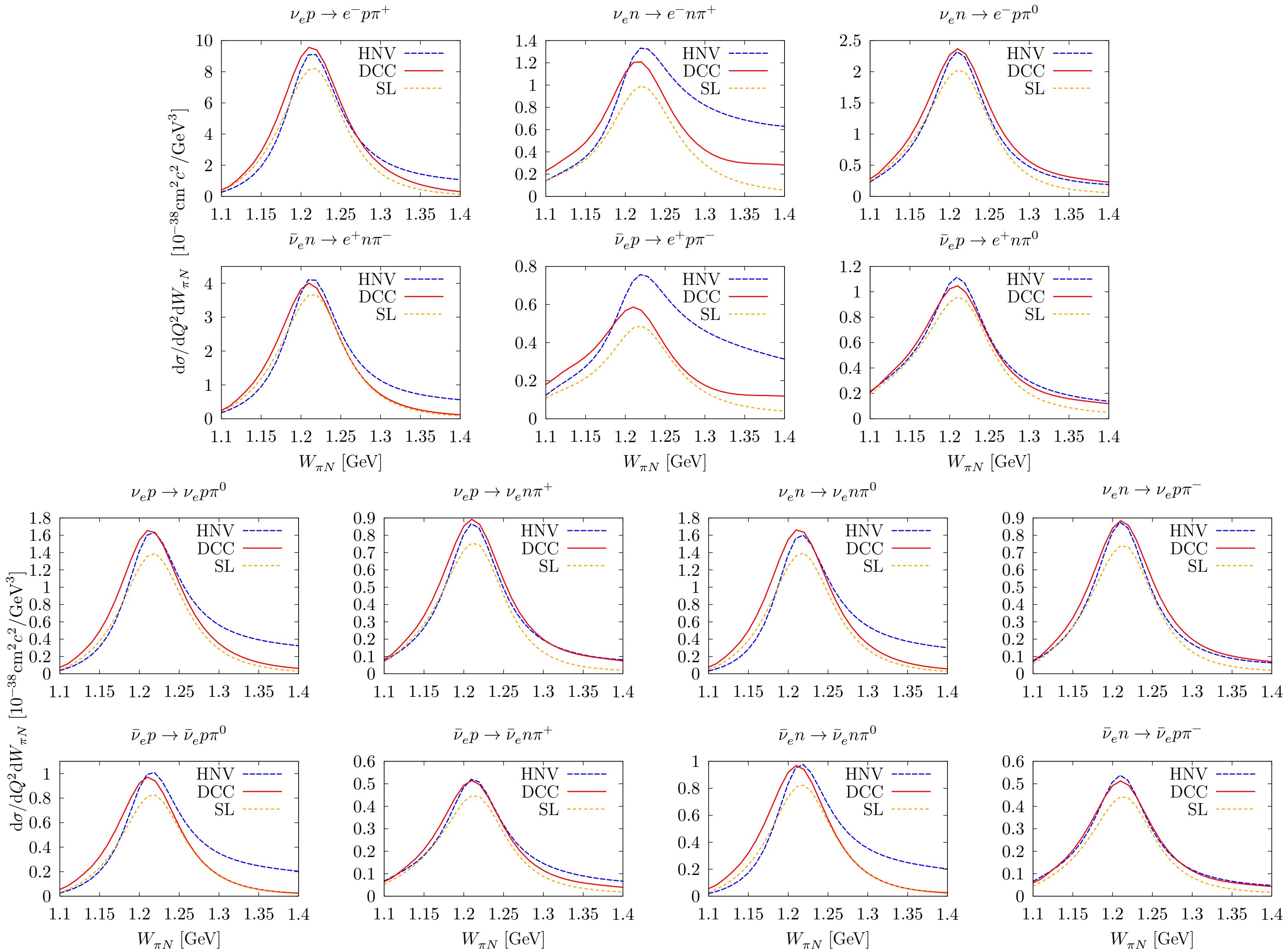
Data from M. Derrick et al., Phys. Lett. 92B, 363 (1980).

Cross section comparison between the DCC, SL and HNV models



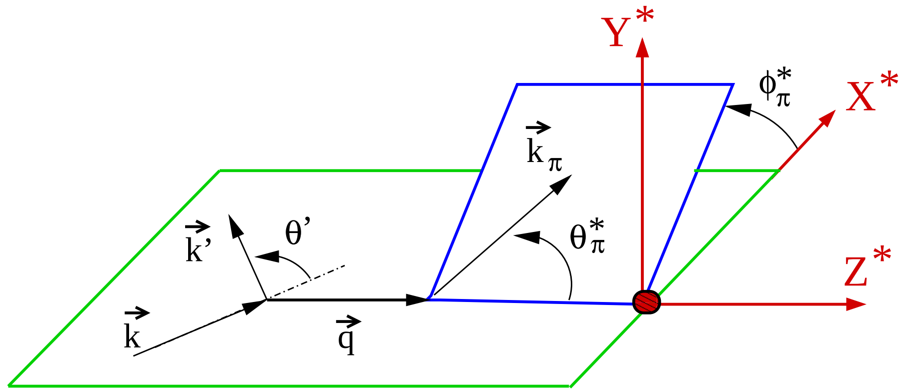
$$W_{\pi N} < 1.4\,\text{GeV}$$

Differential cross section comparison



$$E_\nu = 1.GeV,\ Q^2 = 0.1\mathrm{GeV}^2/c^2$$

Pion angular distribution



$$\frac{d\sigma_{\rm CC\pm}}{dQ^2 dW_{\pi N} d\Omega^*_\pi} = \frac{G_F^2 W_{\pi N}}{4\pi M |\vec{k}|^2} \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi} L^{\mu\nu}(k^*, k'^*)\, W_{\mu\nu}(q^*, p^*, k^*_\pi)$$

with

$$L^{\mu\nu}(k^*, k'^*) = k^{*\mu} k'^{*\nu} + k^{*\nu} k'^{*\mu} - g^{\mu\nu} k^* \cdot k'^* \pm i\epsilon^{\mu\nu\alpha\beta} k'^*_\alpha k^*_\beta$$

$$W^{\mu\nu}(q^*, p^*, k^*_\pi) = \frac{1}{4M} \int \frac{d^3 p'^*}{(2\pi)^3\, 2E'^*_N}\, \delta^4(q^* + p^* - p'^* - k^*_\pi) \mathcal{H}^{\mu\nu}(p^*, p'^*, k^*_\pi)$$

$$\mathcal{H}^{\mu\nu}(p^*, p'^*, k^*_\pi) = \frac{1}{2} \sum_{s,s'} \langle N'(p'^*, s')\, \pi(k^*_\pi) | J^\mu_{\rm CC\pm}(0) | N(p^*, s)\rangle \langle N'(p'^*, s')\, \pi(k^*_\pi) | J^\nu_{\rm CC\pm}(0) | N(p^*, s)\rangle^*,$$

Pion angular distribution II

Using Lorentz covariance one can write

$$\frac{d\sigma_{\mathrm{CC}\pm}}{dQ^2 dW_{\pi N} d\Omega^*_\pi} = \frac{G_F^2 W_{\pi N}}{4\pi M|\vec{k}|^2}\left(A^* \cdot 1 + B^* \cdot \cos\phi^*_\pi + C^* \cdot \cos 2\phi^*_\pi + D^* \cdot \sin\phi^*_\pi + E^* \cdot \sin 2\phi^*_\pi\right)$$

$$A^* = \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi}\left[L^{00}W^{(s)}_{00} + 2L^{03}W^{(s)}_{03} + L^{33}W^{(s)}_{33} + \frac{1}{2}(L^{11}+L^{22})\left(W^{(s)}_{11}+W^{(s)}_{22}\right) + 2iL^{12}W^{(a)}_{12}\right]_{\phi^*_\pi=0},$$

$$B^* = \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi}\, 2\left[L^{01}\, W^{(s)}_{01} + L^{13}\, W^{(s)}_{13} + iL^{02}\, W^{(a)}_{02} + iL^{23}\, W^{(a)}_{23}\right]_{\phi^*_\pi=0},$$

$$C^* = \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi}\, \frac{1}{2}\left[(L^{11} - L^{22})\left(W^{(s)}_{11} - W^{(s)}_{22}\right)\right]_{\phi^*_\pi=0},$$

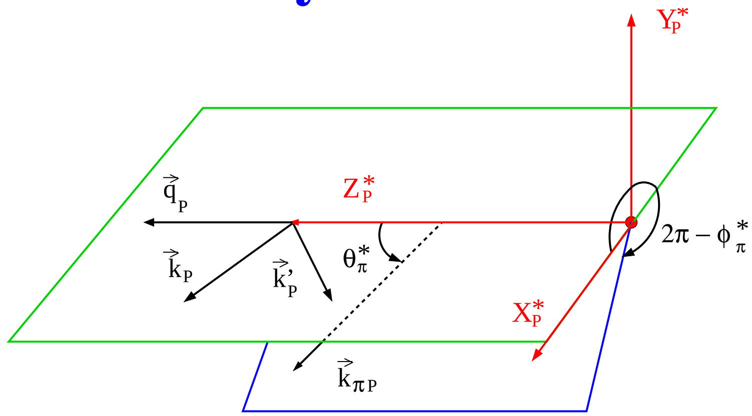
$$D^* = \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi}\, 2\left[-L^{01}\, W^{(s)}_{02} - L^{13}\, W^{(s)}_{23} + iL^{02}\, W^{(a)}_{01} + iL^{23}\, W^{(a)}_{13}\right]_{\phi^*_\pi=0},$$

$$E^* = \int \frac{|\vec{k}^*_\pi|^2 d|\vec{k}^*_\pi|}{E^*_\pi}\left[(L^{22} - L^{11})\, W^{(s)}_{12}\right]_{\phi^*_\pi=0},$$

s and a stand for symmetric and antisymmetric.

Besides, B^*, D^* have a multiplicative $\sin\theta^*_\pi$ factor and E^* a $\sin^2\theta^*_\pi$ one.

Parity violation

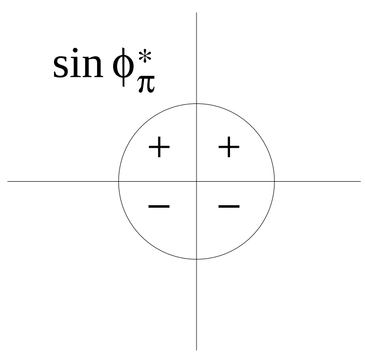


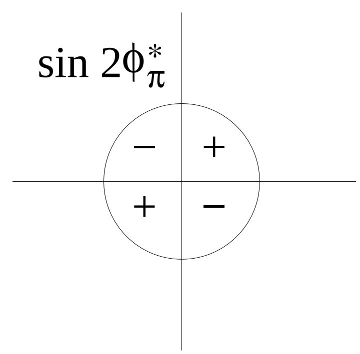
Under parity all three-momenta get reversed but so do the Z^* and X^* axes. As a result the components of $k_P^*, k_P'^*, q_P^*, p_P^*$ in the new system remain unchanged. Only $k^*_{\pi P}$ is modified

$$\theta^*_\pi \rightarrow \theta^*_\pi \quad , \quad \phi^*_\pi \rightarrow 2\pi - \phi^*_\pi .$$

Thus, the terms in $sin\phi^*_\pi, sin2\phi^*_\pi$ change sign.

Parity violation reflects itself in the fact that the pion distributions measured above and below the scattering plane are different.





Parity violating terms I

$$\widetilde{\mathcal{H}}^{\mu\nu} = \mathcal{H}^{\mu\nu}(p^*, p'^*, k_\pi^*)|_{\phi_\pi^*=0} = \frac{1}{2}\mathrm{Tr}[h^\mu(p^*, p'^*, k_\pi^*)h^{\nu\dagger}(p^*, p'^*, k_\pi^*)]_{\phi_\pi^*=0}$$

$$\langle N'(p'^*, s')\, \pi(k_\pi^*)|J^\mu_{\mathrm{CC}\pm}(0)|N(p^*, s)\rangle = \chi^\dagger_{s'} h^\mu(p^*, p'^*, k_\pi^*)\chi_s$$

h^μ can be expanded in multipoles

$$h^\mu = \sum_{j_1} e^{i\delta_{j_1}}\left[|M_{Vj_1}|\mathcal{O}^\mu_{Vj_1} - |M_{Aj_1}|\mathcal{O}^\mu_{Aj_1}\right]$$

with $\mathcal{O}^\mu_{V,A}$ operators built out of Pauli matrices and momenta. The $\mathcal{O}^\mu_V$ operators violate parity while the $\mathcal{O}^\mu_A$ ones do not.

Besides

$$M_{Vj_1} = e^{i\delta_{j_1}}|M_{Vj_1}| \ , \ \ M_{Aj_1} = e^{i\delta_{j_1}}|M_{Aj_1}|$$

All multipoles corresponding to given total angular momentum, parity and pion orbital angular momentum quantum numbers have the same phase. The phases are fixed by the Watson theorem below the two pion threshold.

Parity violating terms II

Then,

$$\widetilde{\mathcal{H}}^{\mu\nu} = \frac{1}{2}\sum_{j_1}\sum_{j_2} e^{i(\delta_{j_1}-\delta_{j_2})}\mathrm{Tr}\left[(|M_{Vj_1}|\mathcal{O}^{\mu}_{Vj_1} - |M_{Aj_1}|\mathcal{O}^{\mu}_{Aj_1})(|M_{Vj_2}|\mathcal{O}^{\nu\dagger}_{Vj_2} - |M_{Aj_2}|\mathcal{O}^{\nu\dagger}_{Aj_2})\right]$$

$$\mathrm{Tr}(\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}) \;,\; \mathrm{Tr}(\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}) \in \mathbb{R} \qquad \text{They are tensors}$$

$$i\mathrm{Tr}(\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}) \;,\; i\mathrm{Tr}(\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}) \in \mathbb{R} \qquad \text{They are pseudotensors}$$

One has the following contributions.

$$\underbrace{\widetilde{\mathcal{H}}^{\mu\nu\,(s)}_{VV+AA}}_{\mathrm{PC}} = \frac{1}{2}\sum_{j_1}\sum_{j_2}\cos(\delta_{j_1}-\delta_{j_2})\left\{|M_{Vj_1}||M_{Vj_2}|\mathrm{Tr}\left[\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}\right] + |M_{Aj_1}||M_{Aj_2}|\mathrm{Tr}\left[\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}\right]\right\},$$

which is real, symmetric and parity conserving since when contracted with the symmetric part of the lepton tensor it gives rise to a pure scalar, and

$$\underbrace{i\widetilde{\mathcal{H}}^{\mu\nu\,(a)}_{VV+AA}}_{\mathrm{PV}} = \frac{i}{2}\sum_{j_1\neq j_2}\sin(\delta_{j_1}-\delta_{j_2})\left\{|M_{Vj_1}||M_{Vj_2}|\mathrm{Tr}\left[\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}\right] + |M_{Aj_1}||M_{Aj_2}|\mathrm{Tr}\left[\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}\right]\right\}.$$

which is imaginary, antisymmetric and parity violating since when contracted with the antisymmetric part of the lepton tensor it gives rise to a pseudoscalar.

Parity violating terms II

Besides, we also have

$$\underbrace{i\widetilde{\mathcal{H}}^{\mu\nu\,(a)}_{VA+AV}}_{\rm PC} = -\frac{1}{2}\sum_{j_1}\sum_{j_2}\cos(\delta_{j_1}-\delta_{j_2})\left\{|M_{Vj_1}||M_{Aj_2}|{\rm Tr}\left[\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}\right]+|M_{Vj_2}||M_{Aj_1}|{\rm Tr}\left[\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}\right]\right\}.$$

which is imaginary, antisymmetric and parity conserving, since when contracted with the antisymmetric part of the leptonic tensor it produces a scalar, and

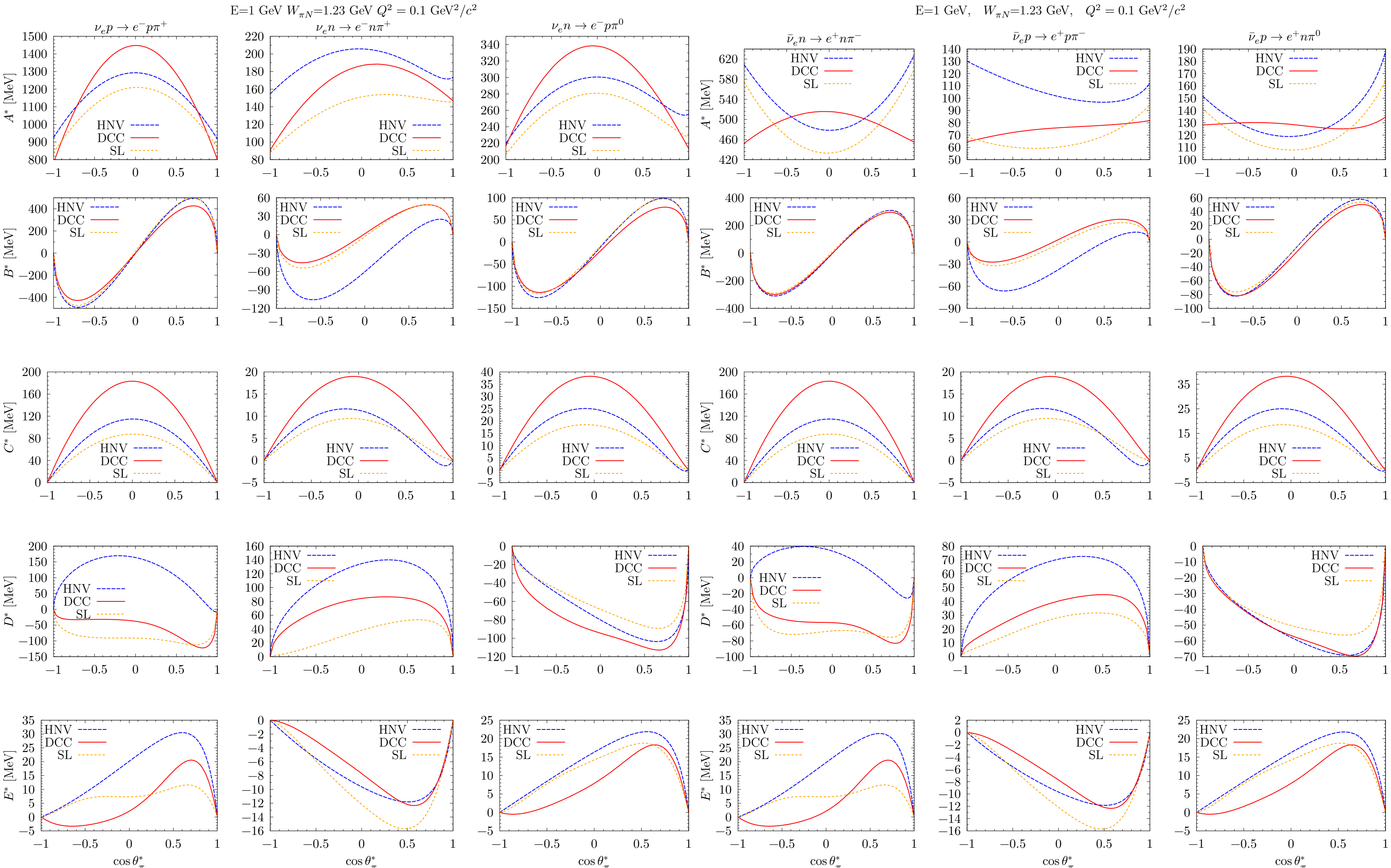
$$\underbrace{\widetilde{\mathcal{H}}^{\mu\nu\,(s)}_{VA+AV}}_{\rm PV} = -\frac{i}{2}\sum_{j_1}\sum_{j_2}\sin(\delta_{j_1}-\delta_{j_2})\left\{|M_{Vj_1}||M_{Aj_2}|{\rm Tr}\left[\mathcal{O}^{\mu}_{Vj_1}\mathcal{O}^{\nu\dagger}_{Aj_2}\right]-|M_{Vj_2}||M_{Aj_1}|{\rm Tr}\left[\mathcal{O}^{\mu}_{Aj_1}\mathcal{O}^{\nu\dagger}_{Vj_2}\right]\right\},$$

which is real, symmetric and parity violating, since when contracted with the symmetric part of the leptonic tensor it produces a pseudoscalar

Parity violating terms are proportional to $\sin(\delta-\delta')$ and they are present whenever one has contributions from different multipoles that are not relatively real.

Pion angular distribution III

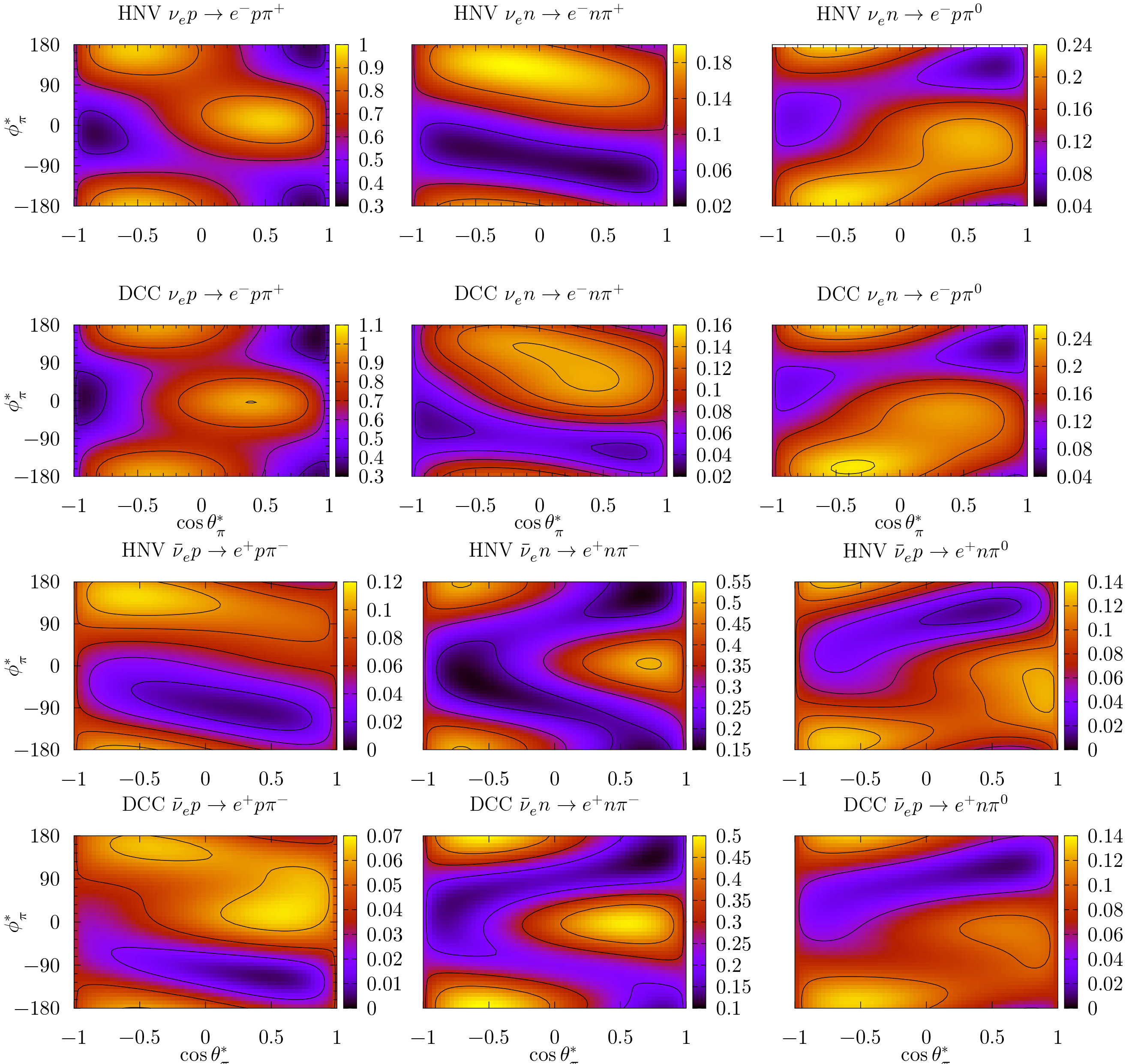
Comparison of the A^*, B^*, C^*, D^*, E^* functions



$E_\nu = 1.$ GeV, $Q^2 = 0.1\mathrm{GeV}^2/c^2$, $W_{\pi N} = 1.23$ GeV.

Pion angular distribution IV

CC processes



$d\sigma/dQ^2 dW_{\pi N} d\Omega^*_\pi$

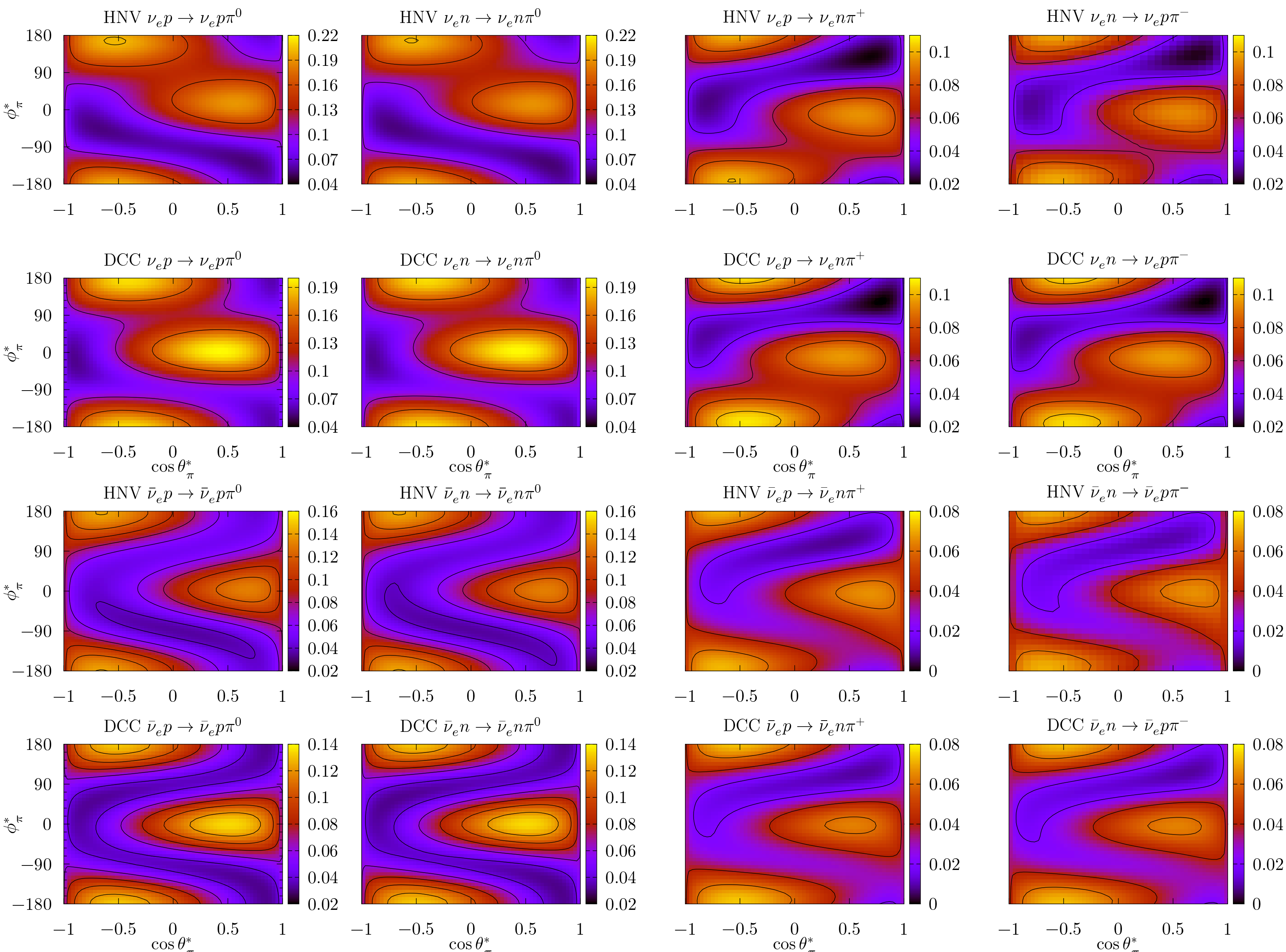
$[10^{-38}\mathrm{cm}^2 c^2/\mathrm{GeV}^2]$

$E_\nu = 1\,\mathrm{GeV},\ Q^2 = 0.1\,\mathrm{GeV}^2/c^2$

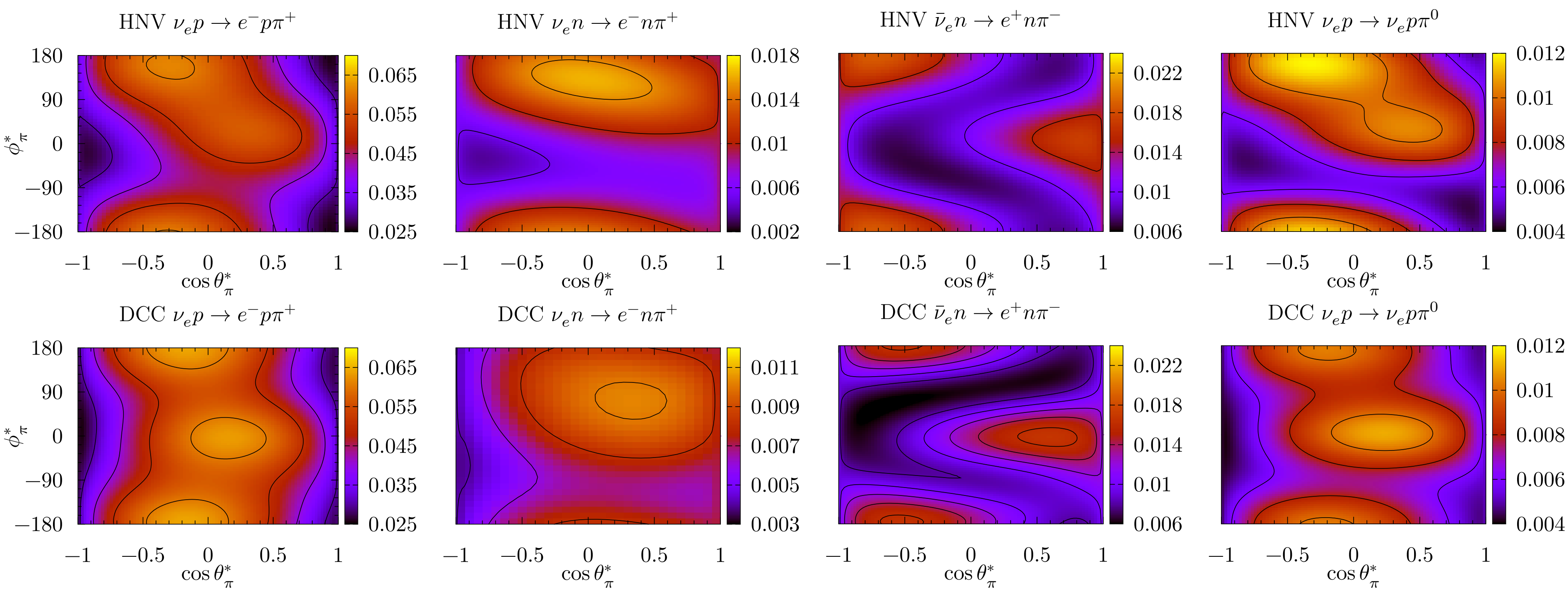
$W_{\pi N} = 1.23$ GeV

Pion angular distribution V

Same as before for NC processes



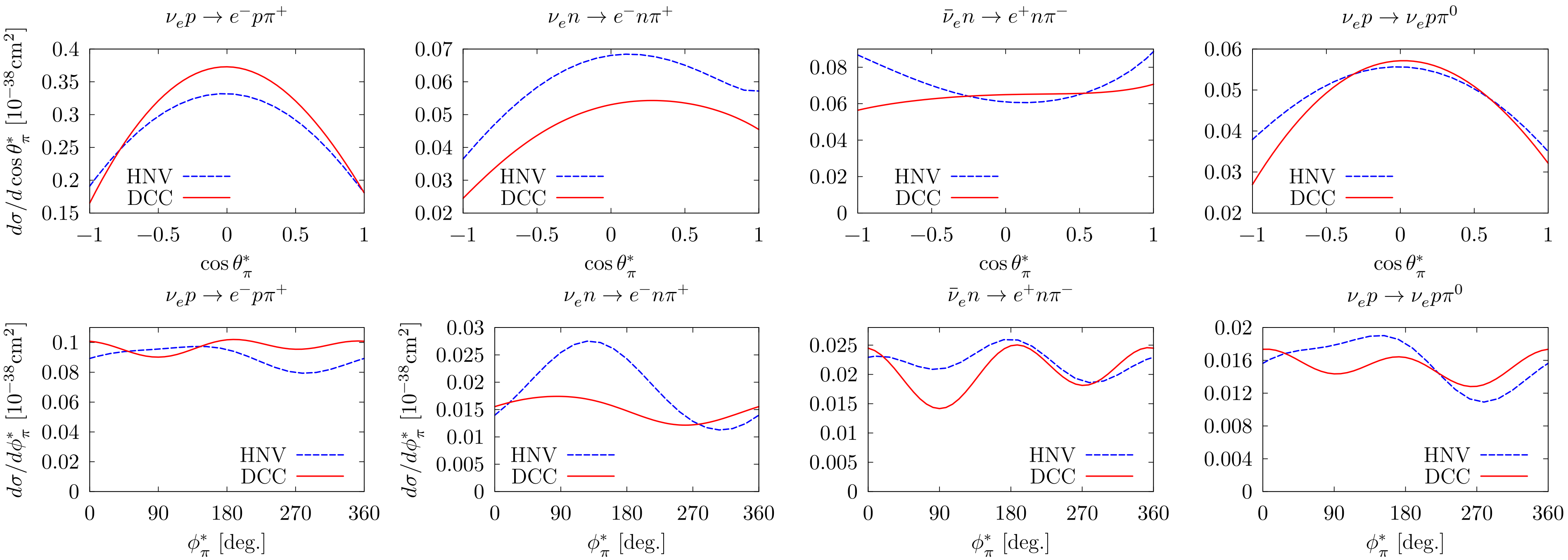
Pion angular distribution VI



$d\sigma/d\Omega^*_\pi [10^{-38}\mathrm{cm}^2]$, evaluated at $E_\nu = 1\,\mathrm{GeV}$ and with a $W_{\pi N} < 1.4$ GeV cut.

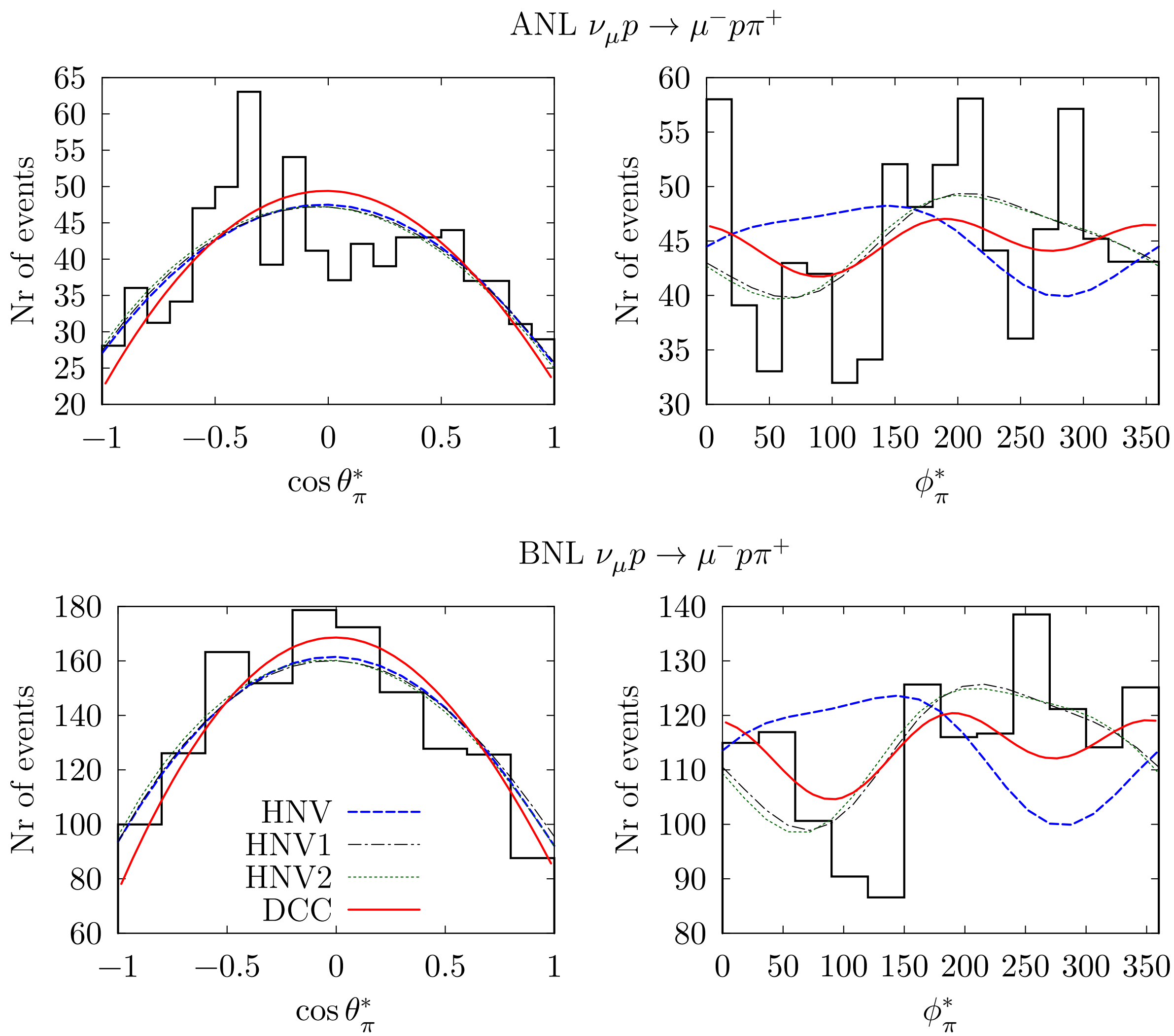
A clear anisotropy is still seen and the distribution is channel dependent

Pion angular distribution VII



$d\sigma/d\cos\theta^*_\pi$ and $d\sigma/d\phi^*_\pi$ in units of $10^{-38}\mathrm{cm}^2$ at $E_\nu = 1$ GeV and with $W_{\pi N} < 1.4$ GeV

Pion angular distribution VIII



Unnormalized, flux-averaged $d\sigma/d\cos\theta^*_\pi$ and $d\sigma/d\phi^*_\pi$. $W_{\pi N} < 1.4$ GeV.

ANL data: G. M. Radecky et al, Phys. Rev. D 25, 1161 (1982).

BNL data:T. Kitagaki, Phys. Rev. D 34, 2554 (1986).

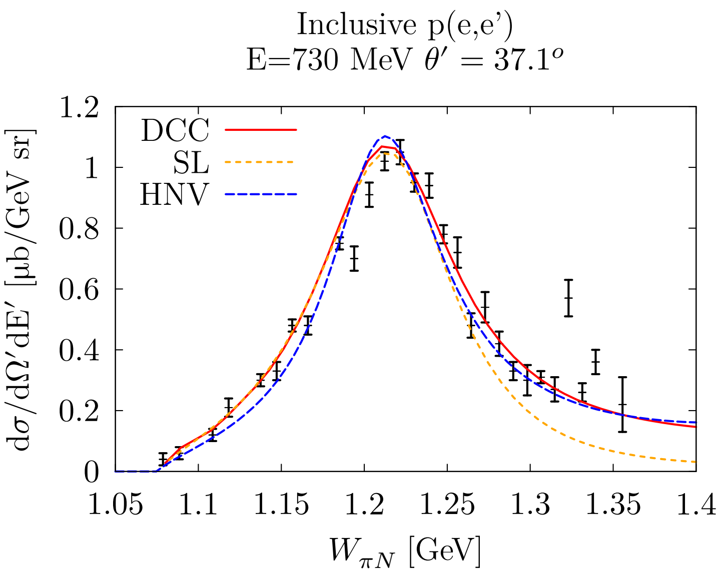
Pion electroproduction

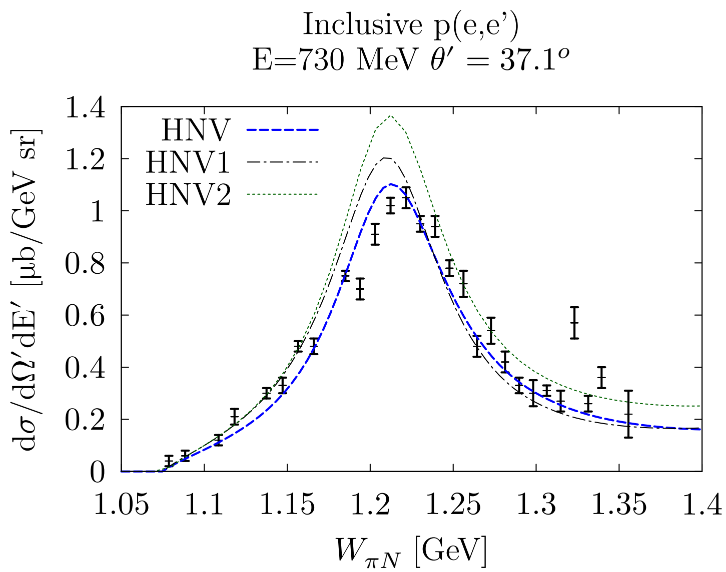
- Allows for a direct comparison of the vector part of the different models [the HNV model por pion photo or electroproduction can be found inthe appendix of PRD95,053007 (2017)].
- There is precise data to compare with.

For unpolarized nucleons and initial electrons with well defined helicity one can write (in the zero lepton mass limit)

$$\frac{d\sigma_{em}}{d\Omega' dE' d\Omega^*_\pi} = \Gamma_{em}\left\{ \underbrace{(\sigma_T + \varepsilon\,\sigma_L)}_{A^*}\, 1 + \underbrace{\sqrt{2\varepsilon(1+\varepsilon)}\,\sigma_{LT}}_{B^*} \cos\phi^*_\pi + \underbrace{\varepsilon\,\sigma_{TT}}_{C^*} \cos 2\phi^*_\pi + \underbrace{h\sqrt{2\varepsilon(1-\varepsilon)}\,\sigma_{LT'}}_{D^*} \sin\phi^*_\pi \right\}$$

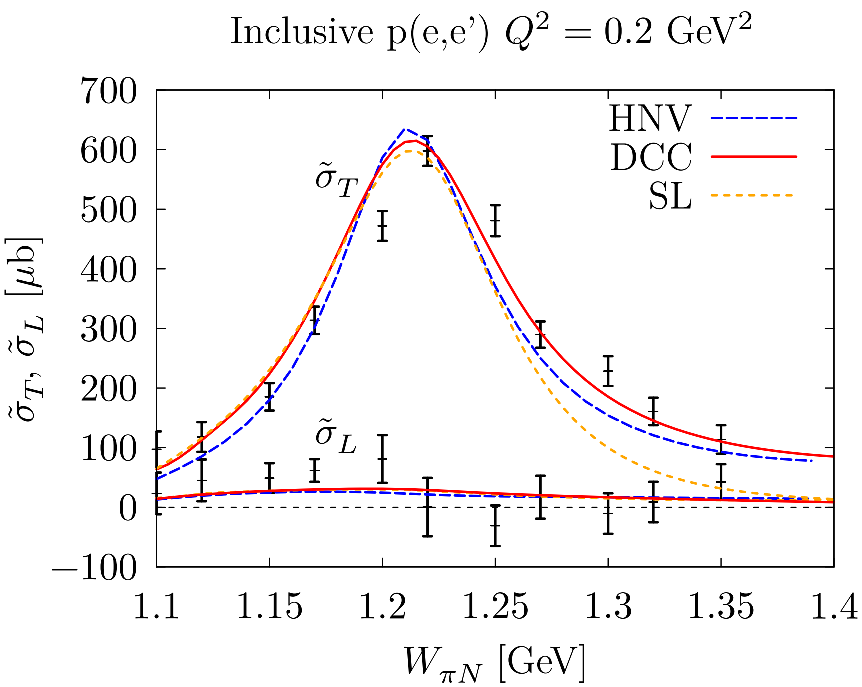
with $\epsilon = Q^2/(Q^2 + 2|\vec{q}\,|^2 \tan^2\theta'/2)$.

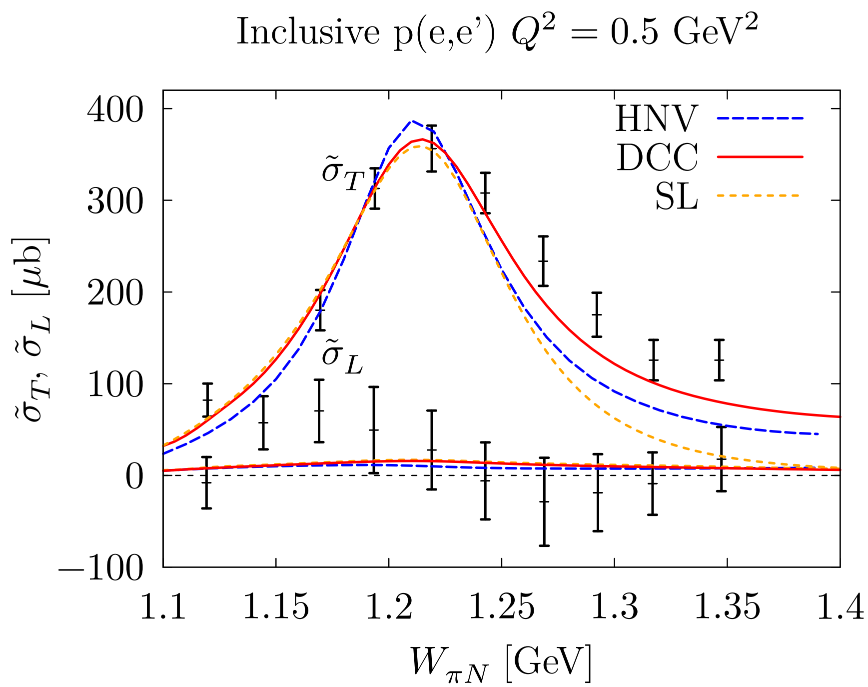




Data from J. S. O'Connell et al., Phys. Rev. Lett. 53, 1627 (1984).

Pion electroproduction II

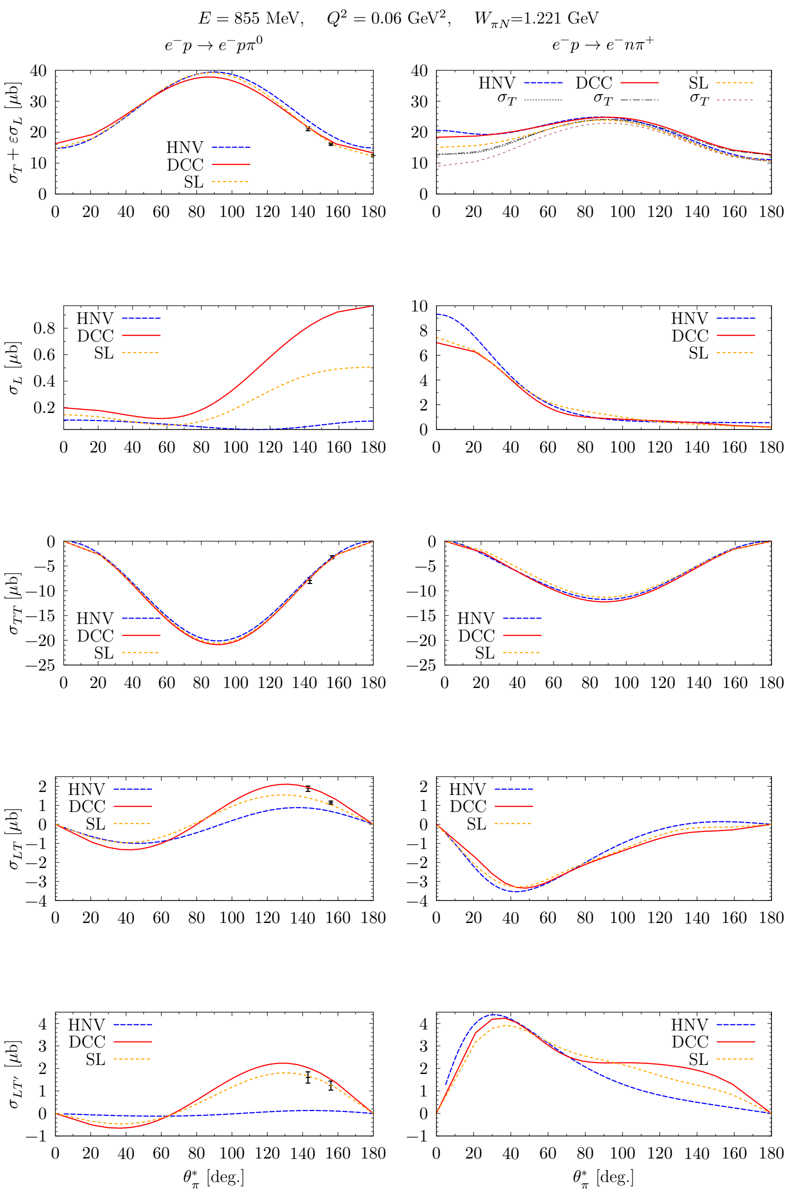


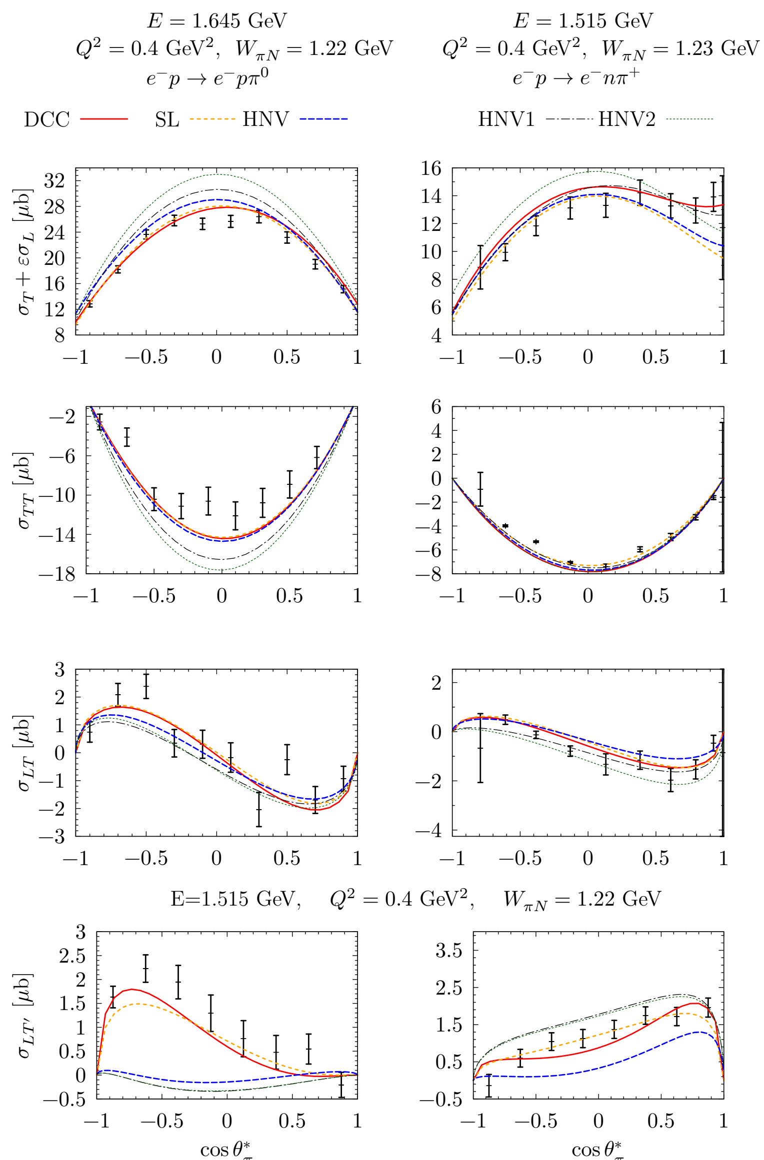


$$\tilde{\sigma}_T = \int \sigma_T d\Omega^*_\pi \quad , \quad \tilde{\sigma}_L = \int \sigma_L d\Omega^*_\pi$$

Data from K. Baetzner et al., Phys. Lett. 39B, 575 (1972).

Pion electroproduction III

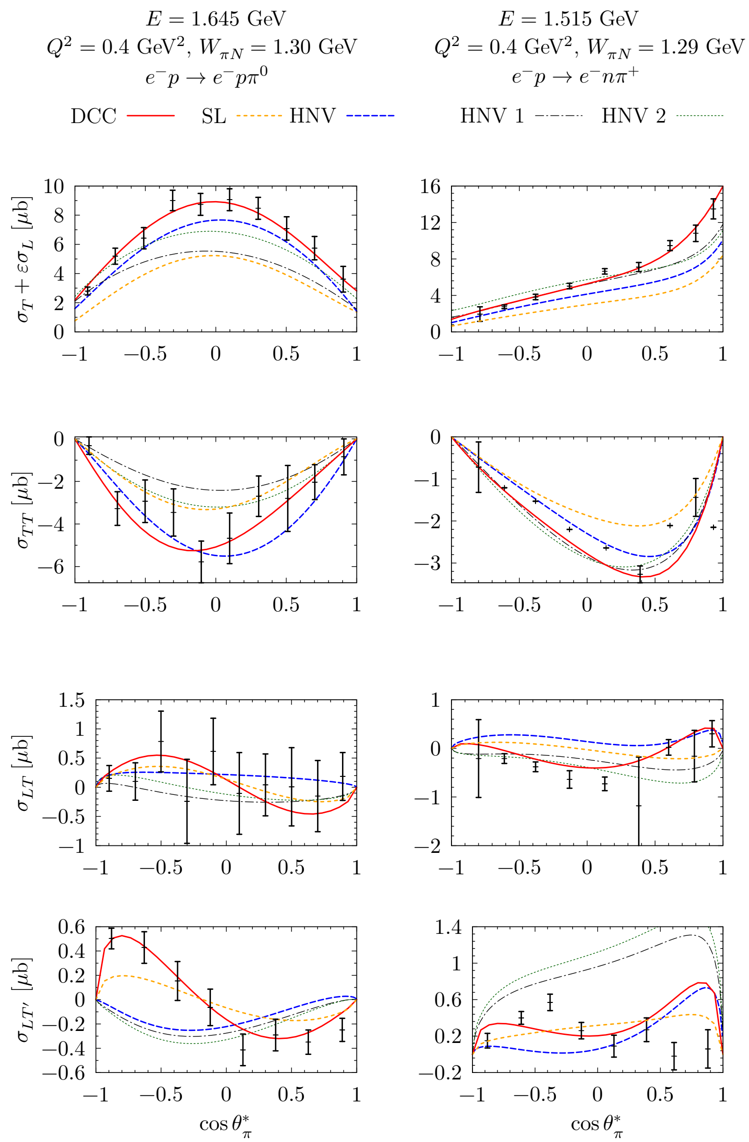




Left Figure: Data from S. Stave et al., Eur. Phys. J. A30, 471 (2006)

Right Figure: Data from CLAS Collaboration, PRL 88, 122001 (2002); PRC 68, 032201 (2003) PRC72, 058202 (2005); PRC73, 025204 (2006)

Pion electroproduction IV



Same as before at a higher invariant mass

Summary and conclusions

- We have shown an analysis of the pion angular distribution in the πN CM system for neutrino reactions.
  - Possible azimuthal angle dependencies are $1, \cos\phi^*_\pi, \cos 2\phi^*_\pi, \sin\phi^*_\pi, \sin 2\phi^*_\pi$.

    The latter two give rise to parity violation that originates from the interference between multipoles corresponding to different quantum numbers (multipoles that are not relatively real).

    Parity violation is less prominent for antineutrino NC processes.
  - Pion angular isotropic distributions are not supported by this study. We have found clear anisotropies instead. Besides, angular distributions are channel dependent.
- As a test for the vector part of the models, we have also analyzed the pion angular distribution in pion electroproduction.
- If you want to reproduce the pion angular distribution in detail, a fully unitarized model, like the DCC model, seems to be needed.
- The HNV model has the advantage of simplicity and still is able to give a fair global description of the data.
- To develop a good model at the nucleon level we need new data.