

Pion Production At MINERvA:

Overview of Results from the Low Energy NuMI Beam Configuration

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On Behalf of the MINERvA Collaboration

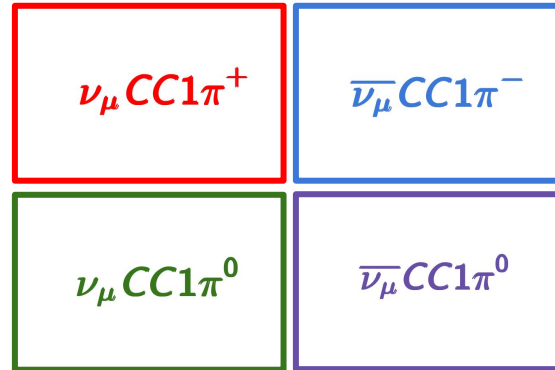
NuINT 2018

October 16, 2018



Pions in the Low Energy Beam

- With our latest measurement of $\bar{\nu}_\mu CC1\pi^-$, MINERvA has measured all single pion production channels on scintillator
 - Discrepancies between data and GENIE in all channels
- Quick overview for this talk:
 1. Single pion production channels and how MINERvA measures them
 2. Comparisons between kinematic distributions of each channel
 3. NUISANCE Pion Tune with MINERvA Data



The four incoherent single pion production channels

$$\nu_{\mu}CC1\pi^{+}, \nu_{\mu}CCN\pi^{+}$$

$$\nu_{\mu} + CH \rightarrow \mu^{-} + 1\pi^{+} + X(\text{nucleons})$$

$$\nu_{\mu} + CH \rightarrow \mu^{-} + N\pi^{+} + X'(\text{baryons/mesons})$$

Phys.Rev.D **92** (2015) 092008

Phys.Rev.D **94** (2016) 052005

$$\bar{\nu}_{\mu}CC1\pi^{-}$$

$$\bar{\nu}_{\mu} + CH \rightarrow \mu^{+} + 1\pi^{-} + X(\text{nucleons})$$

In preparation

$$\nu_{\mu}CC1\pi^{0}$$

$$\nu_{\mu} + CH \rightarrow \mu^{-} + 1\pi^{0} + X(\text{nucleons})$$

Phys.Rev.D **96** (2017) 072003

$$\bar{\nu}_{\mu}CC1\pi^{0}$$

$$\bar{\nu}_{\mu} + CH \rightarrow \mu^{+} + 1\pi^{0} + X(\text{nucleons})$$

Phys.Lett.**B749** (2015) 130-136

Phys.Rev.D **94** (2016) 052005

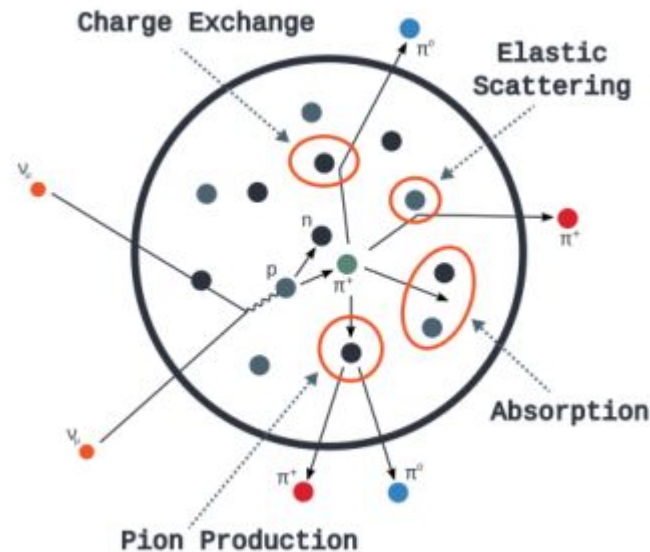
- These results were taken using the low energy beam configuration
 - All plots for the $N\pi^{+}$ results are in the backup slides

Why measure all four incoherent single pion production channels

- Difficult to disentangle nucleon and nuclear effects
- For example, final state interactions (FSI) can migrate between charged pion and neutral pion events

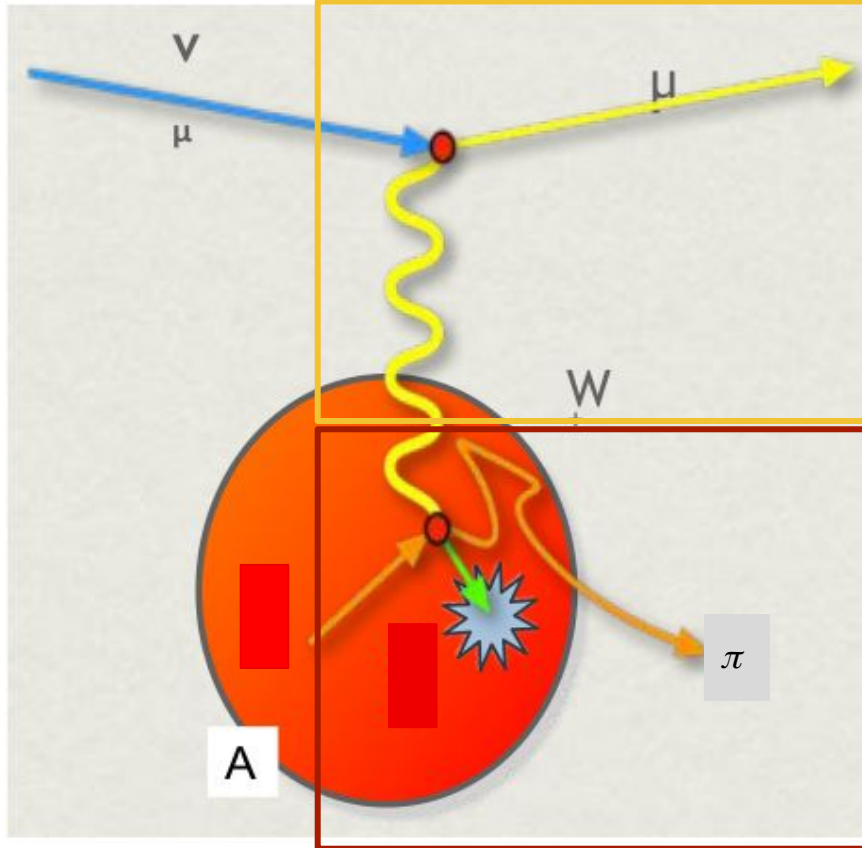


- Discrepancies in one channel may show up in the other
- Initial nuclear state cares about ν_{μ} vs $\overline{\nu}_{\mu}$ modes



from T. Golan

Looking at muons and pions



Muon

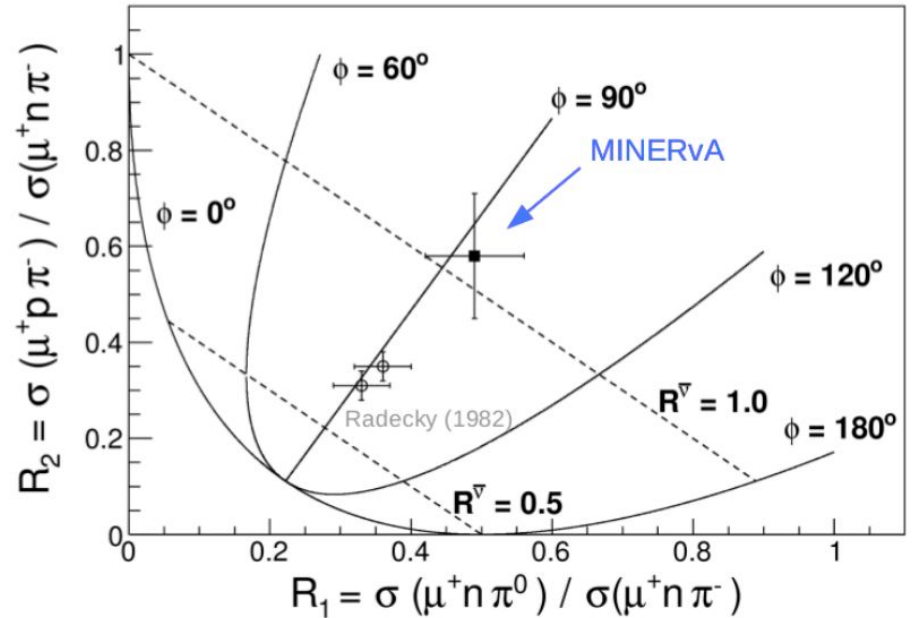
- Provides information on the initial state of the nucleons within the nucleus

Pion

- Undergoes final state interactions within the nucleus
- By comparing to muon kinematics, allows isolation of final state effects

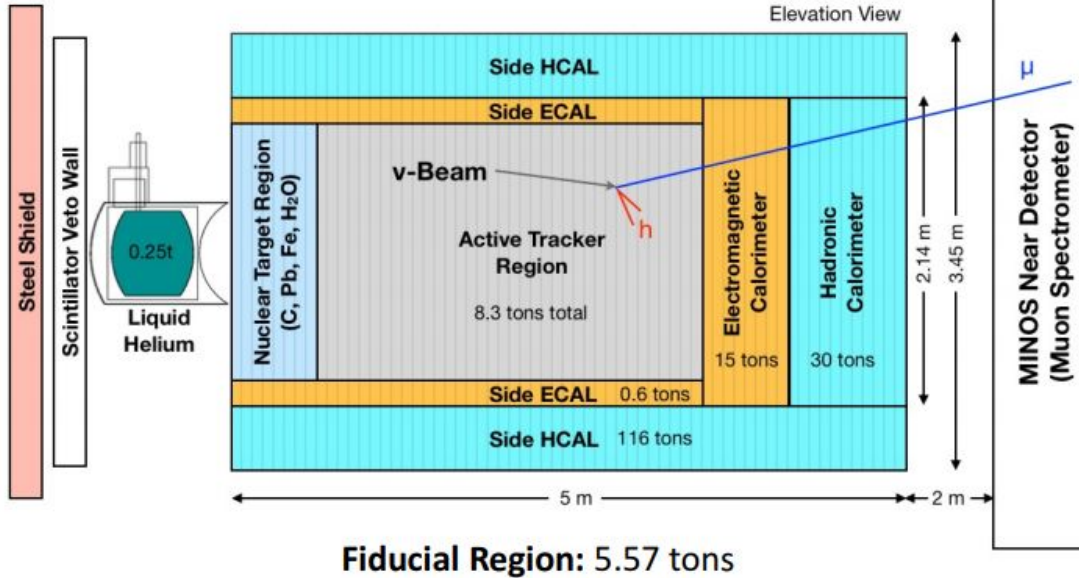
Measurement of isospin amplitudes

- Can decompose different channels into nucleon level cross sections
- Allows us to find the relative strength and phase between the amplitudes of isospin $I = 1/2, 3/2$

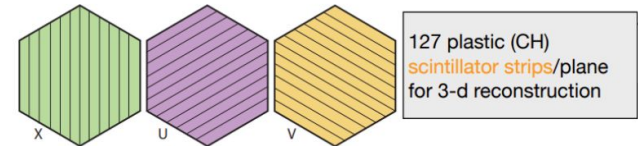
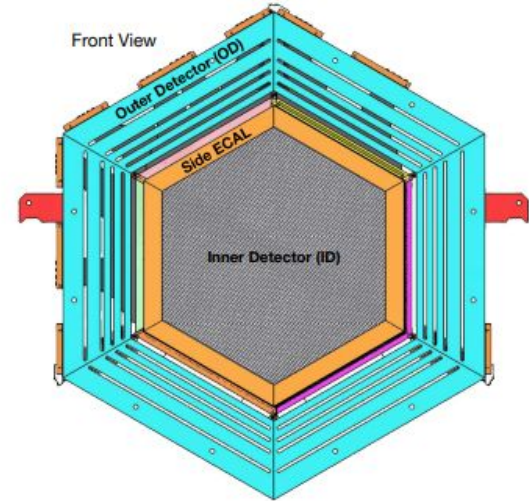


MINERvA Detector

Side View
Complete Detector



Front View
Single Module

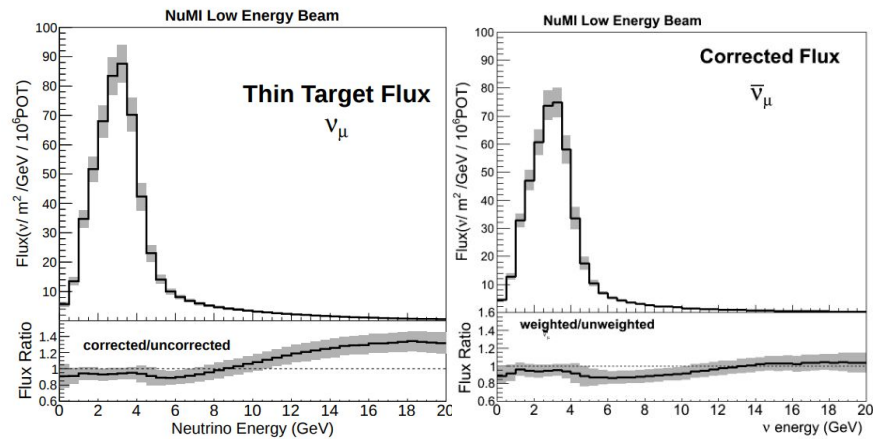
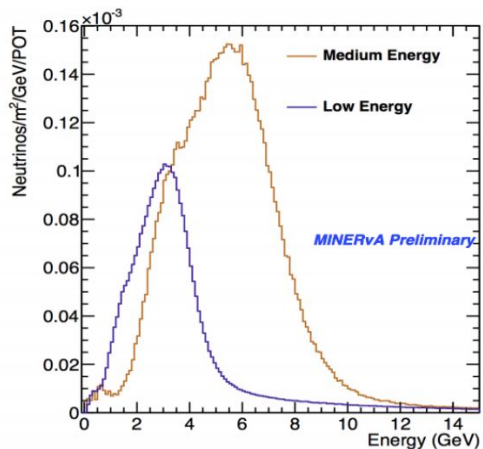


Nucl. Instrum. Methods Phys. Res., Sect. A 743, 130 (2014).

Nucl. Instrum. Methods Phys. Res., Sect. A 789, 28 (2015).

MINERvA Low Energy Flux

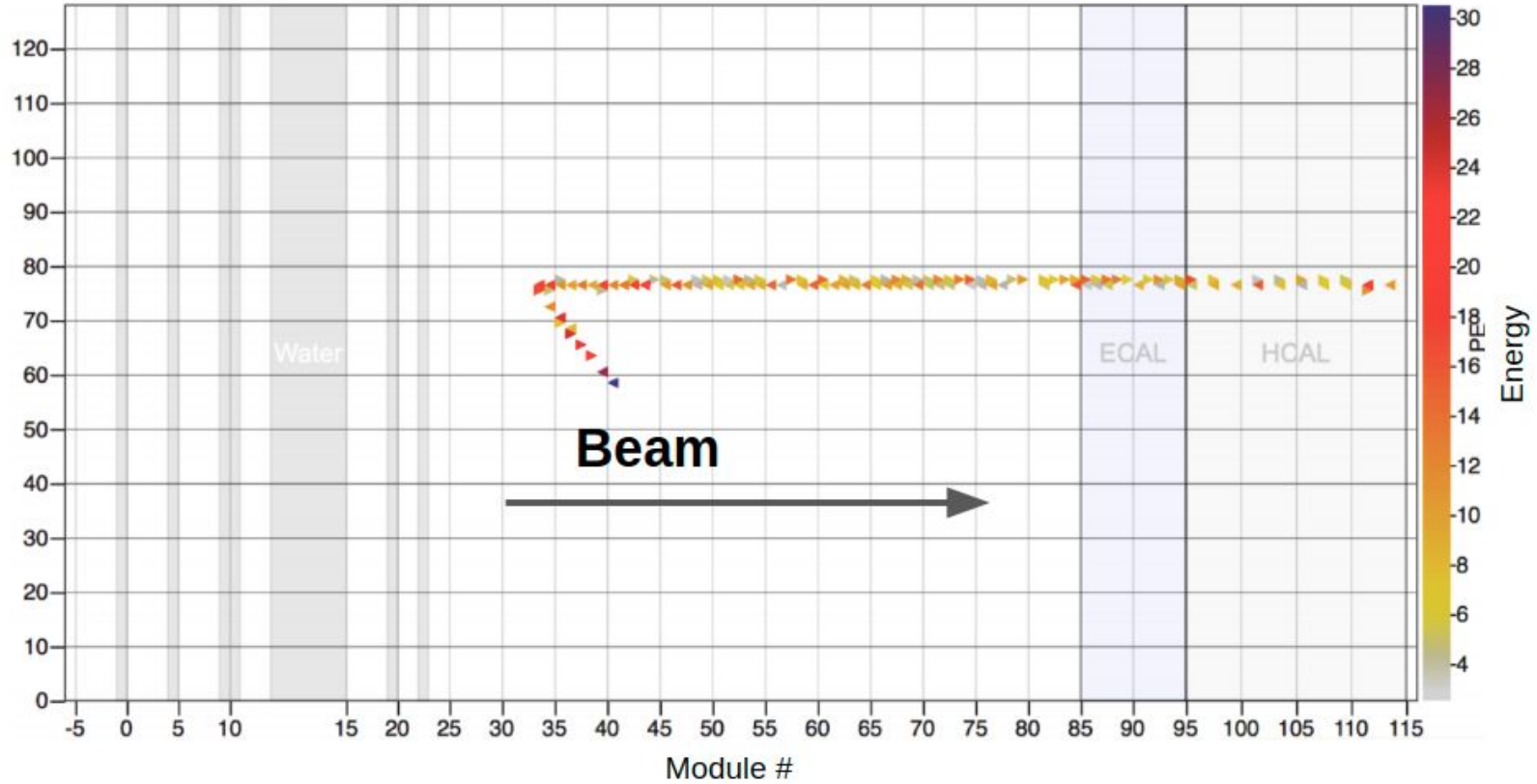
- From 2005-2012 (2010-2012 full detector), MINERvA received both neutrino and antineutrinos from the NuMI beamline in low energy mode
 - 4e20 protons on target
- Improved our prediction of flux using simulation tuned to world hadron production data as well as in situ measurements (low nu method, neutrino electron scattering)
 - Deepika Jena will be giving a talk about the MINERvA flux in the medium energy era



L.Aliaga et al., PRD **94** 092005 (2016)

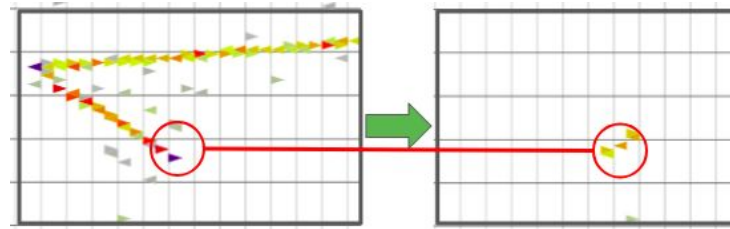
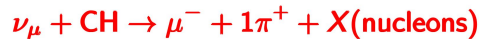
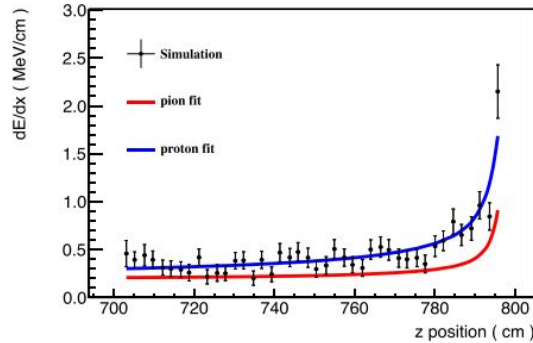


Arachne Display of MINERvA Event



π^\pm Pion Reconstruction

- dE/dX profile of track consistent with π^\pm
- Momentum found from dE/dX fit (by range)



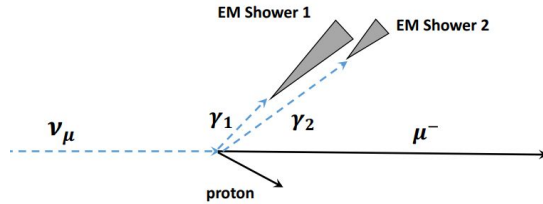
- Michel electron matched to the end of track
- No Michel electron



- Ability to see charged pions in MINERvA has energy thresholds $\sim[50,300]$ MeV
 - Tracking requirements, pion interactions
- π^- typically absorbed into nucleus
 - Hard to tag with Michel electron, difficult to reconstruct momentum

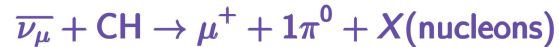
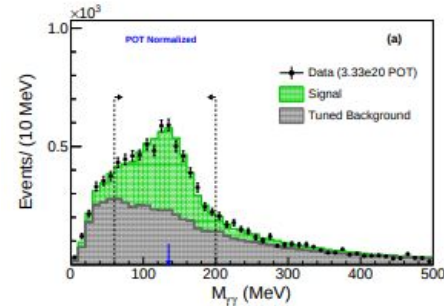
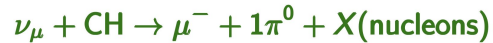
π^0 Pion Reconstruction

- Has two photon candidates
- Photon conversion distance from interaction vertex > 15 cm (> 14 cm for most energetic γ)
- Energy found calorimetrically, energy distribution fitted to invariant mass peak



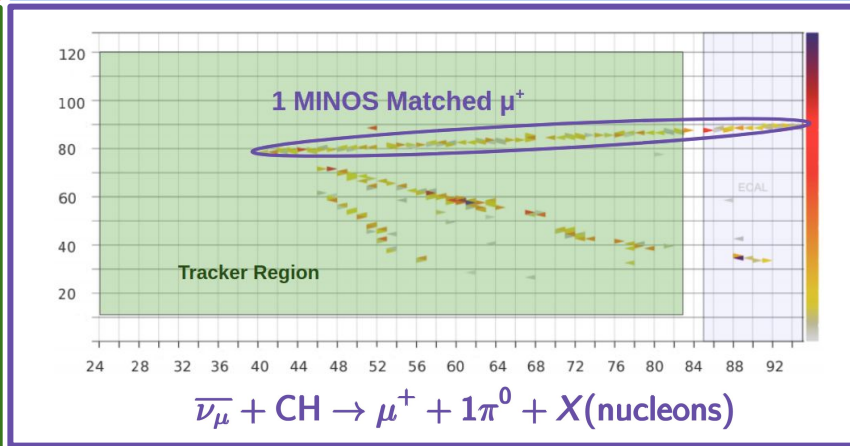
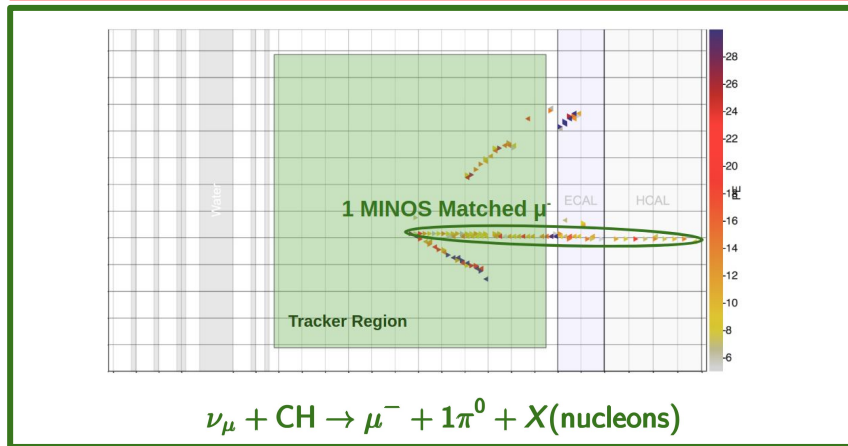
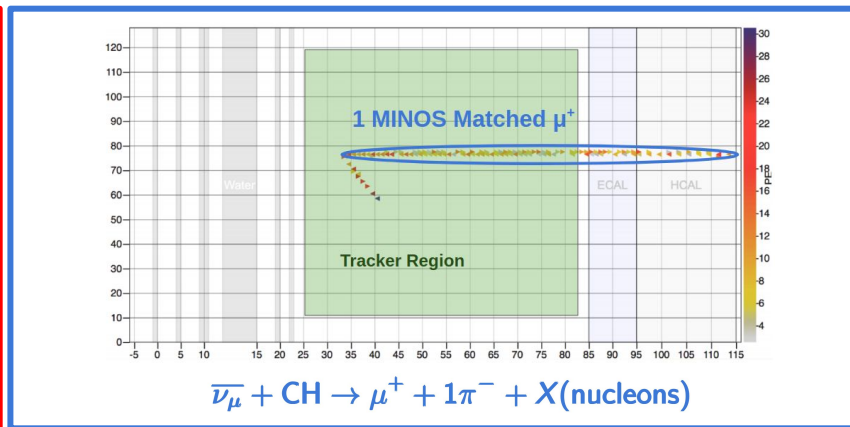
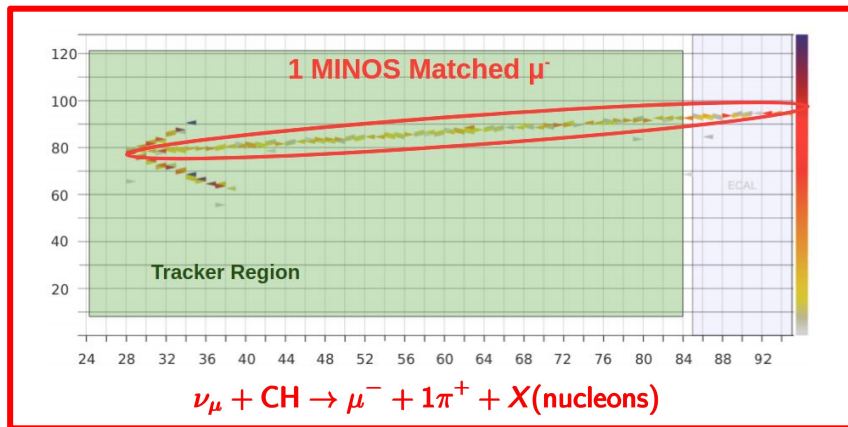
Invariant π^0 Mass

$$M_{\gamma\gamma}^2 = 2E_{\gamma 1}E_{\gamma 2}(1 - \cos(\theta_{\gamma\gamma}))$$

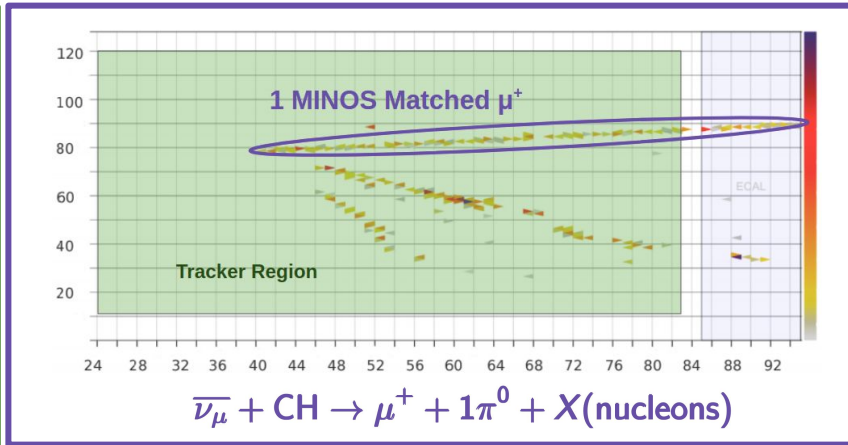
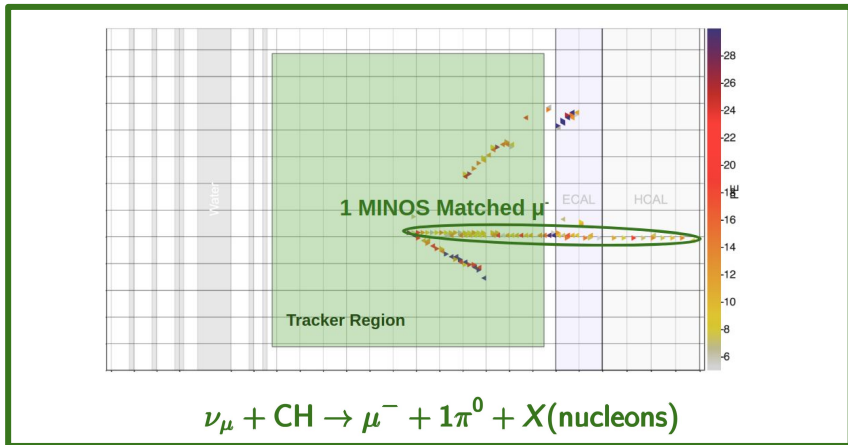
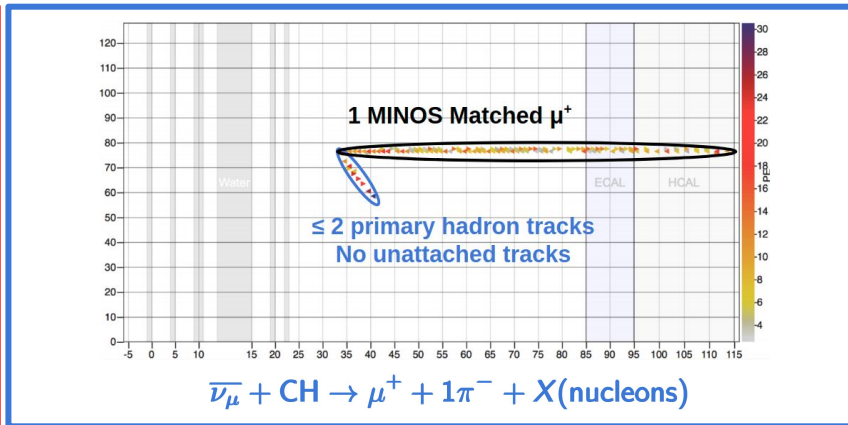
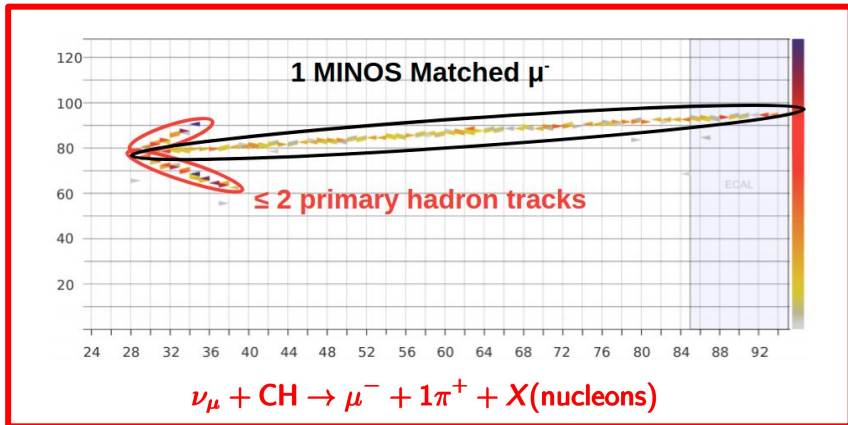


- Easier to see additional nucleons due to clearer vertex
- Large energy thresholds $\sim [0, 1000]$ MeV

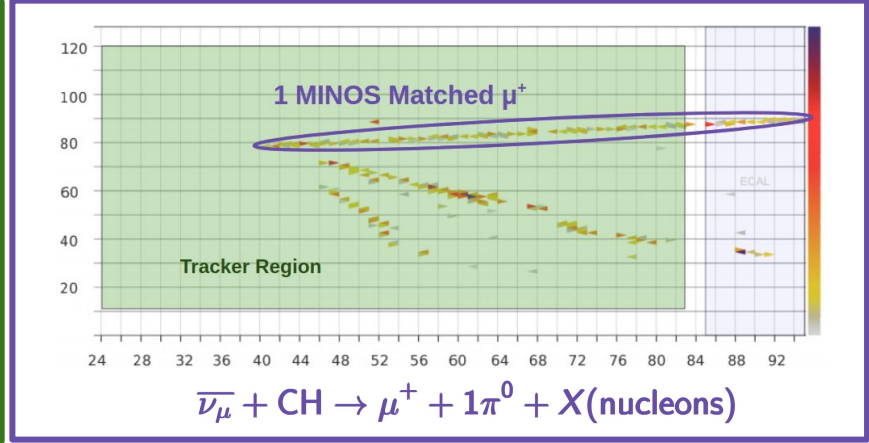
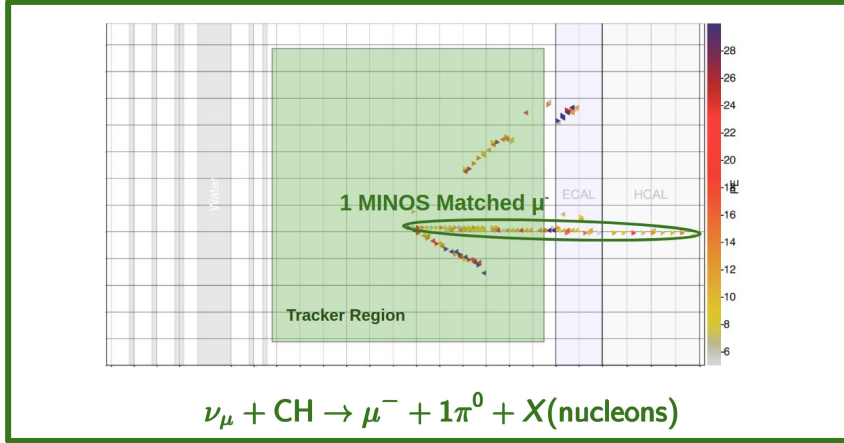
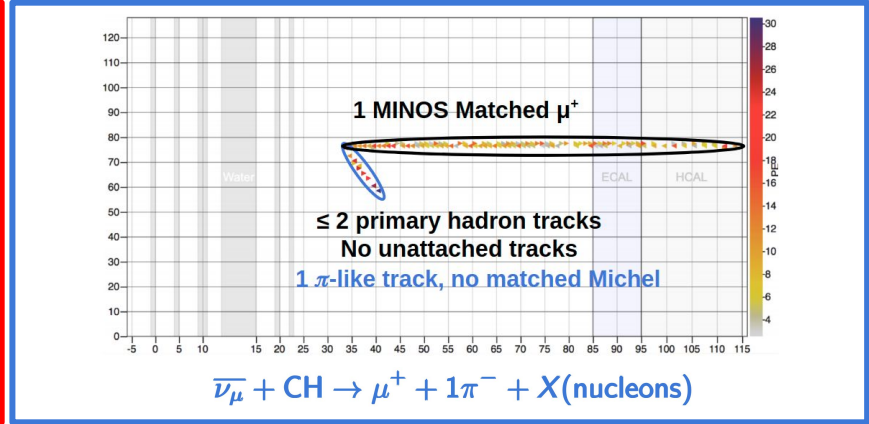
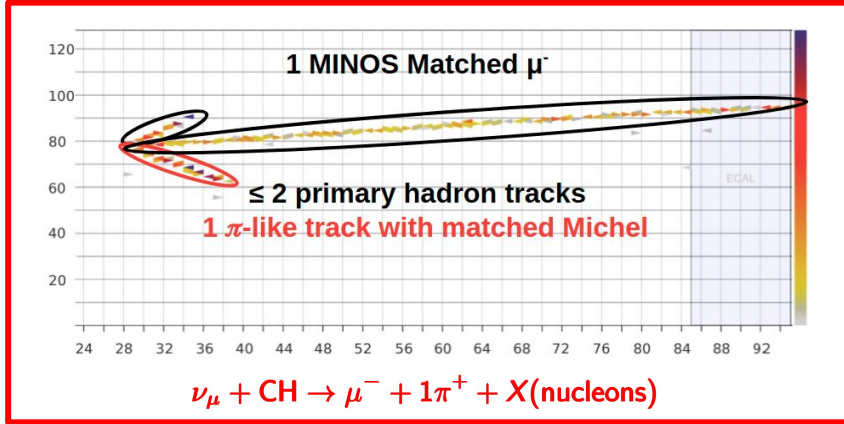
Event Selection



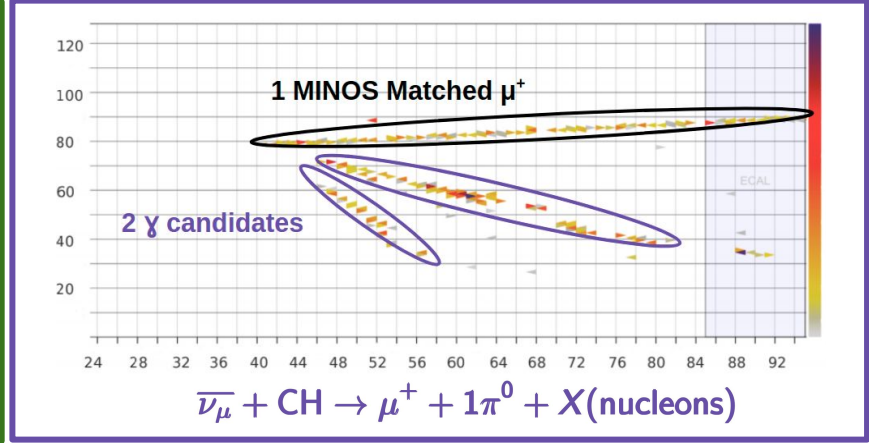
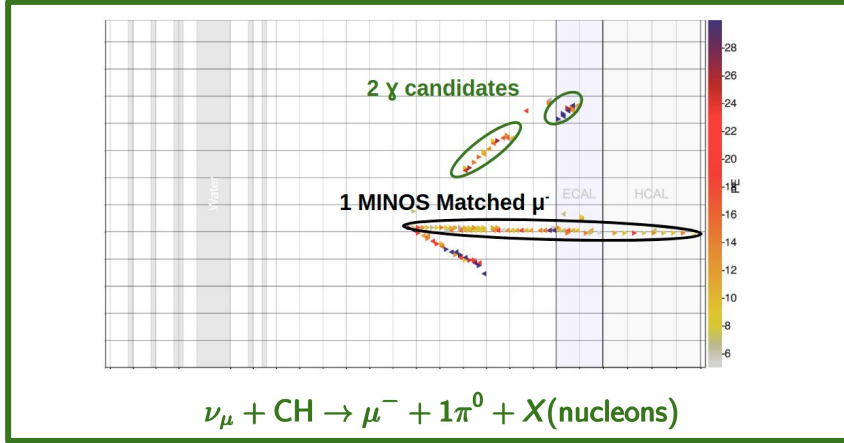
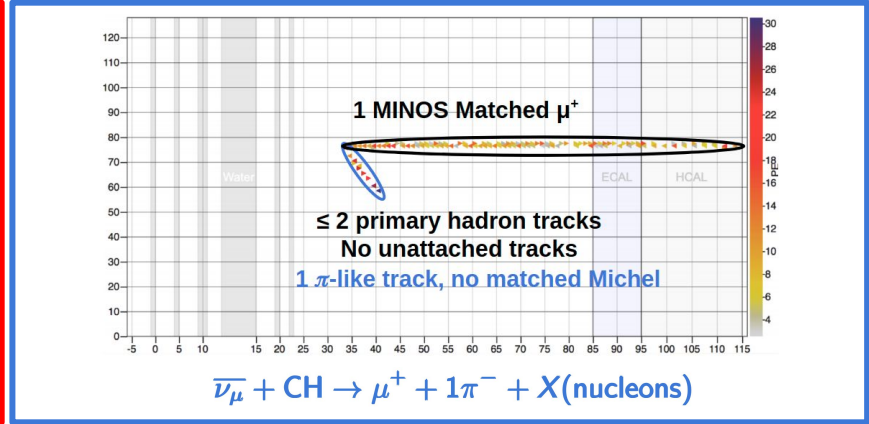
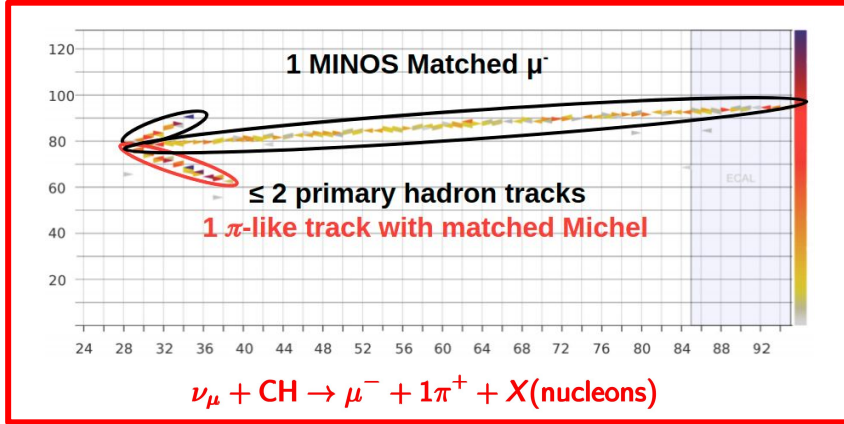
Event Selection



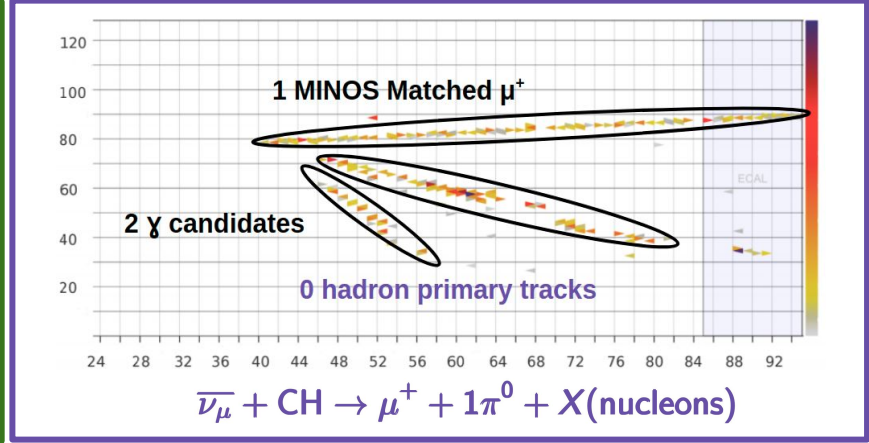
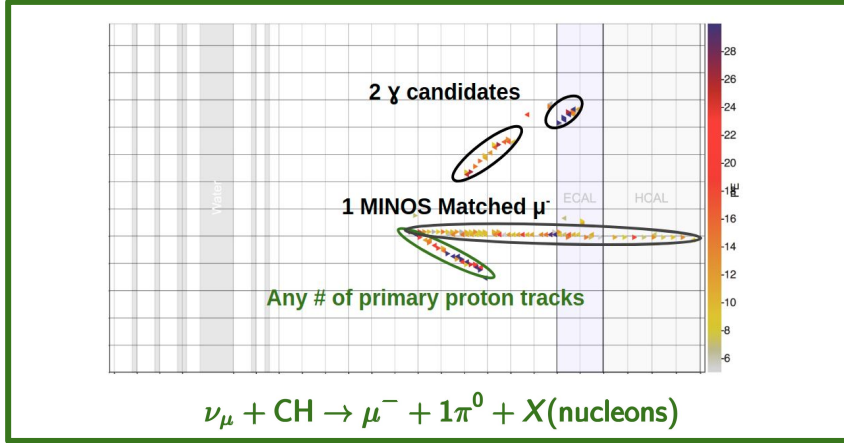
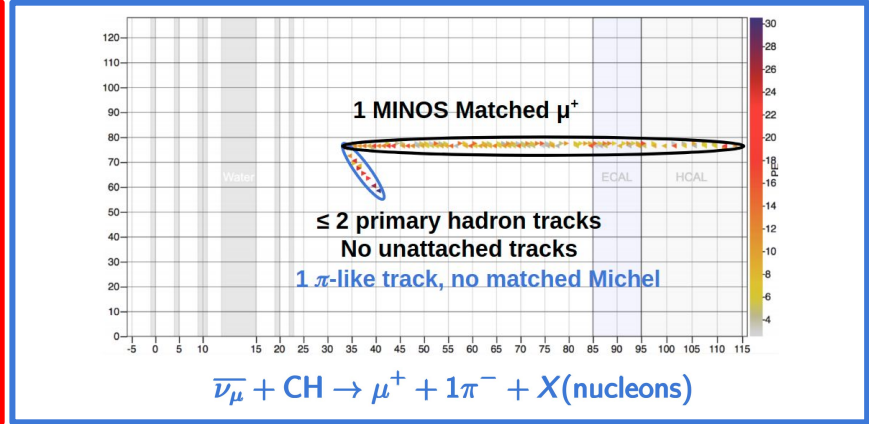
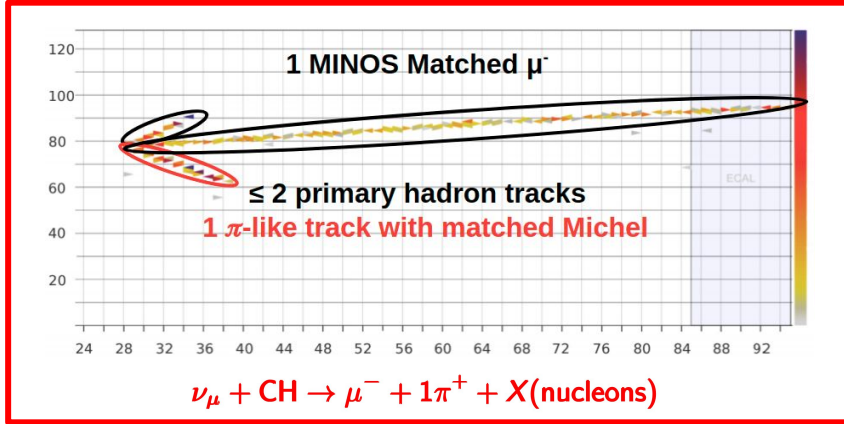
Event Selection



Event Selection



Event Selection



Kinematic Definition

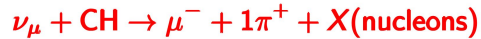
$$Q^2 = 2E_\nu(E_\mu - p_\mu \cos(\theta_{\mu\nu})) - m_\mu^2$$

- Four momentum transfer
- Primarily relies on muon kinematics
- Neutrino energy is typically the muon energy + calorimetric hadronic energy

$$W_{exp}^2 = -Q^2 + m_N^2 + 2m_N E_H \quad (m_N = \text{nucleon mass})$$

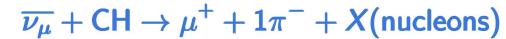
- Invariant hadronic mass

Event Selection and Phase Space



$$E_\nu = E_\mu + E_H \text{ (} E_H \text{ determined calorimetrically)}$$

- Reconstructed $E_\nu \in [1.5, 10]$ GeV
- $1\pi^+$: $W < 1.4$ GeV
- $N\pi^+$: $W < 1.8$ GeV



$$E_\nu = E_\mu + E_H \text{ (} E_H \text{ determined calorimetrically)}$$

- Reconstructed $E_\nu \in [1.5, 10]$ GeV
- $W < 1.8$ GeV



$$E_\nu = E_\mu + E_{\pi^0} + \Sigma T_p + E_{\text{vtx}} + E_{\text{extra}}$$

- Reconstructed $E_\nu \in [1.5, 20]$ GeV
- Invariant π^0 mass $\in [60, 200]$ MeV/ c^2
- $W < 1.8$ GeV



$$E_\nu = E_\mu + E_H \text{ (} E_H \text{ determined calorimetrically)}$$

- Reconstructed $E_\nu \in [1.5, 10]$ GeV
- Invariant π^0 mass $\in [75, 195]$ MeV/ c^2
- $W < 1.8$ GeV

$$Q^2 = 2E_\nu(E_\mu - p_\mu \cos(\theta_{\mu\nu})) - m_\mu^2$$

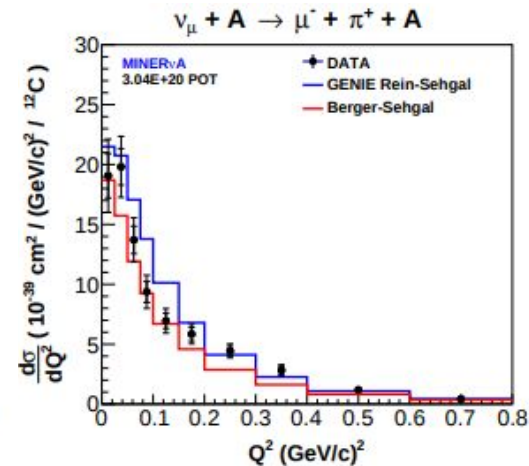
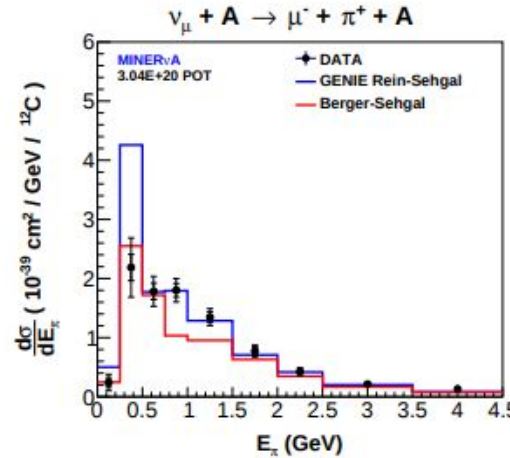
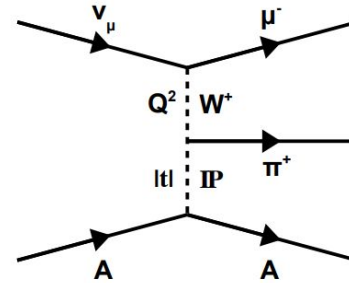
$$W_{\text{exp}}^2 = -Q^2 + m_N^2 + 2m_N E_H \text{ (} m_N = \text{nucleon mass)}$$

Coherent Pion Production

- MINERvA has also published results on neutrino and antineutrino induced coherent pion production
- Data shows preference for the Berger-Sehgal over GENIE's Rein-Sehgal model
 - Rein-Sehgal:** $A^{1/3}$ scaling, uses elastic π -nucleon scattering data
 - Berger-Sehgal:** $A^{2/3}$ scaling, uses elastic π -C scattering data
 - NEUT RS prediction was poor at low E_{π}
- Berger-Sehgal has been implemented in GENIE
- Coherent pion production provides an important contribution at low Q^2 for π^{\pm} channels
 - MINERvA has that scaled simulation at low E_{π} to MINERvA data, which suppresses $Q^2 \sim 0.1 \text{ GeV}^2$

Phys. Rev. D97 (2018) 032014

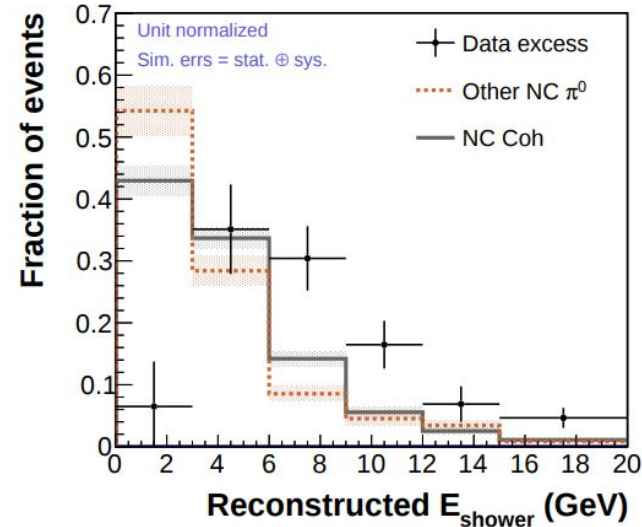
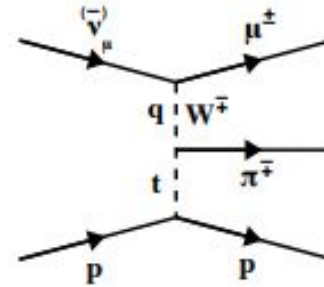
Phys. Rev. Lett. 13 (2014) 261802



High Energy Diffractive π^0

Phys. Rev. Lett. 117 (2016) 111801

- In electron neutrino analyses, found an excess of events with photon like showers
- These events are most consistent with high energy charged current diffractive π^0 production
- Production channel is missing in GENIE
 - Therefore, our GENIE predictions are missing a sample of neutral pions with $E_{\pi} > 2$ GeV at low Q^2



Simulation Software

- **GENIE Event Generator v2.8.4 with Tuning** simulates neutrino interactions
- **GEANT v9.4.2p02** simulates particle propagation through matter

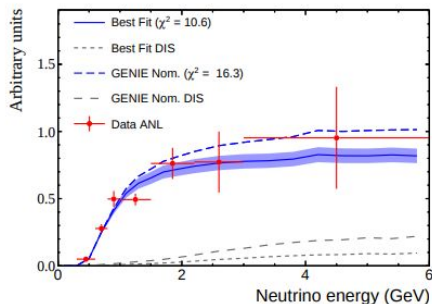
	GENIE 2.8.4	NuWro v17.01
Resonance	Rein Seghal ($W < 1.7$ GeV)	$\Delta(1232)$ w/ Rarita Schwinger
Non-Resonant	Scaled Bodek-Yang	Scaled Bodek-Yang
Nuclear Model	Relativistic Fermi Gas No Pauli blocking for res production	Local Fermi Gas
FSI Model	Effective Cascade	Salcedo-Oset, Full Cascade

- **Resonance Models**
 - **GENIE:** Rein-Seghal with modern baryon-resonance parameters
 - No resonance interference, no lepton mass
 - **NuWro:** Explicit $\Delta(1232)$ with background added incoherently as a fraction of DIS

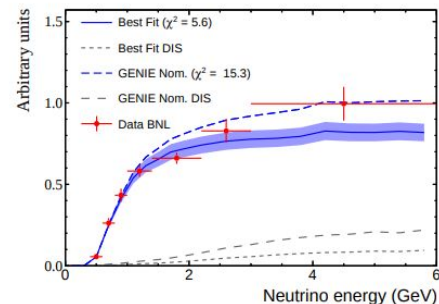
Refinements to GENIE v2.8.4

- Recent analyses have led to tuning that improves GENIE v2.8.4 ability to fit to MINERvA and external data
 - MnvGENIE-v1: Tune to Deuterium Data** (C. Wilkinson, P. Rodrigues & K. McFarland Eur.Phys.J. C76 (2016) no.8, 474.)
 - Reanalysis of ANL/BNL data on single pion production on deuterium
 - Primary effect is a reduction of non-resonant pion production
 - Leads to better agreement for CCInclusive analyses
 - For MINERvA tunes, we scaled the default GENIE $\nu_{\mu}(\bar{\nu}_{\mu})$ non-resonant pion production rate by 0.43 ± 0.04 (0.50 ± 0.50)

Model	GENIE default	ANL/BNL Tune
M_A^{RES} [GeV]	1.12 ± 0.22	0.94 ± 0.05
NormRES [%]	100 ± 20	115 ± 30
NonRES1 π [%]	100 ± 50	43 ± 4



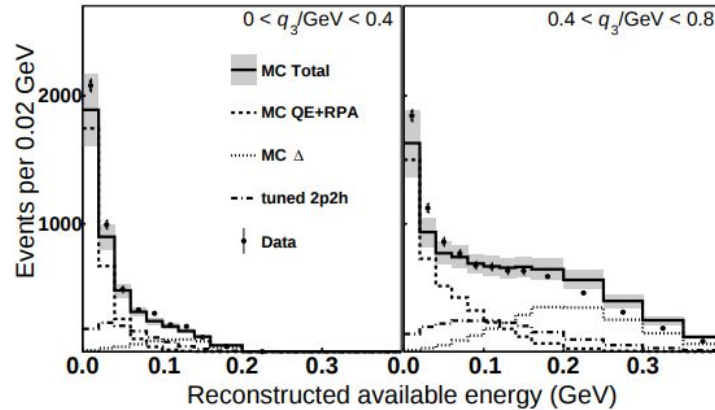
(a) ANL E_{ν}



(b) BNL E_{ν}

Further Refinements to GENIE v2.8.4

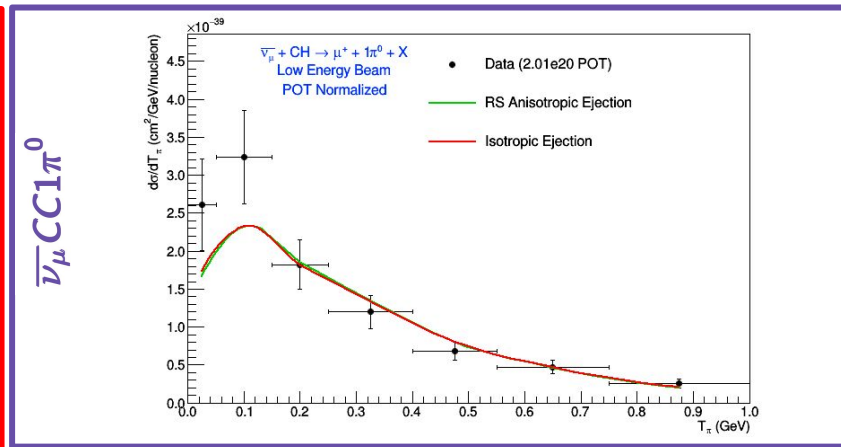
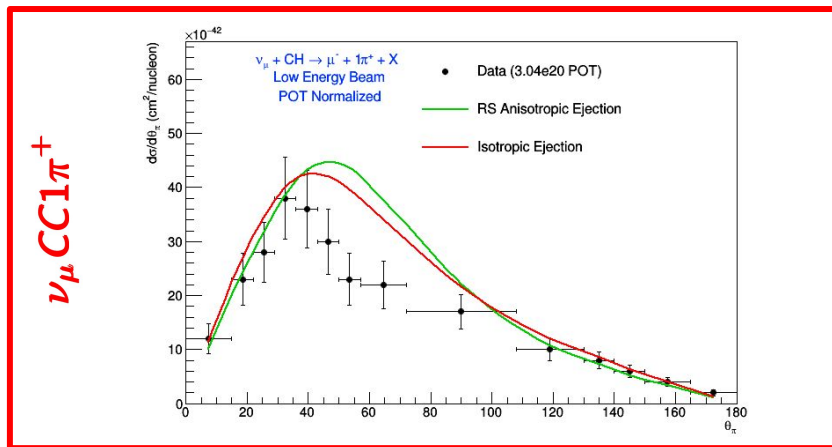
- **MnvGENIE-v1:** Added a sample of CCQE-like 2p2h events tuned to MINERvA data
 - Using the Valencia model prediction for 2p2h, reweighted events in (q_0, q_3) space to MINERvA data
 - Negligible effect on the pion production analyses
 - Valencia model, Phys. Rev. D **88**, 113007 (2013).
 - MINERvA-Tuned, Phys. Rev. Lett. **120**, 221805 (2018).



- **MnvGENIE-v1.1:** Reweighted CC coherent scattering model to match with MINERvA measurements
 - The outline of this tune was previously described in slide 19
 - Phys. Rev. D 97, 032014 (2018).

Important Caveat

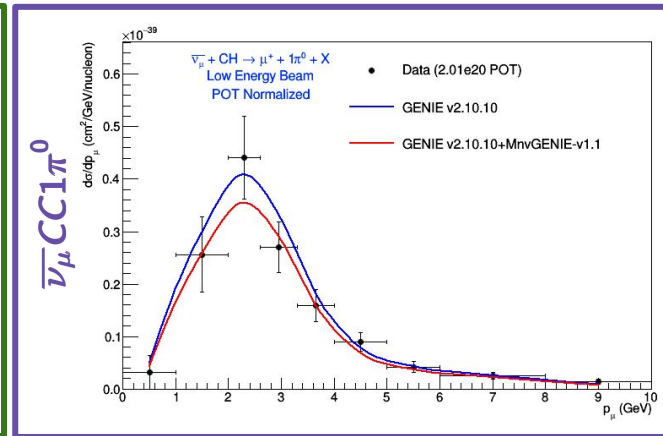
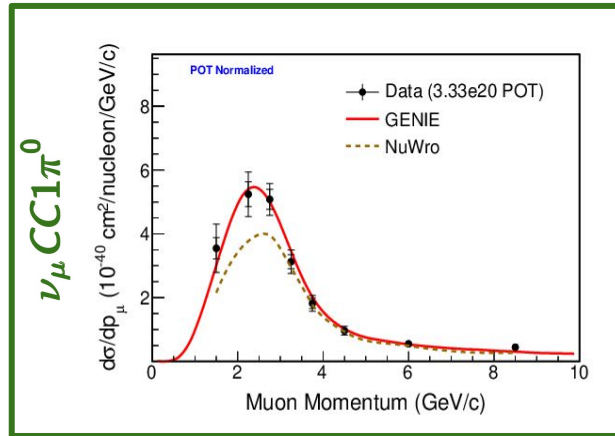
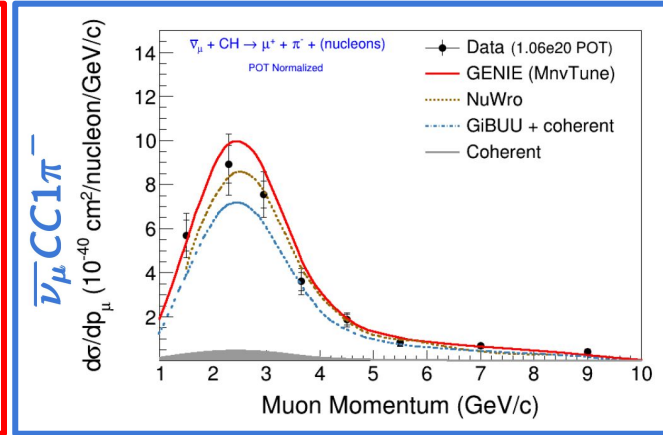
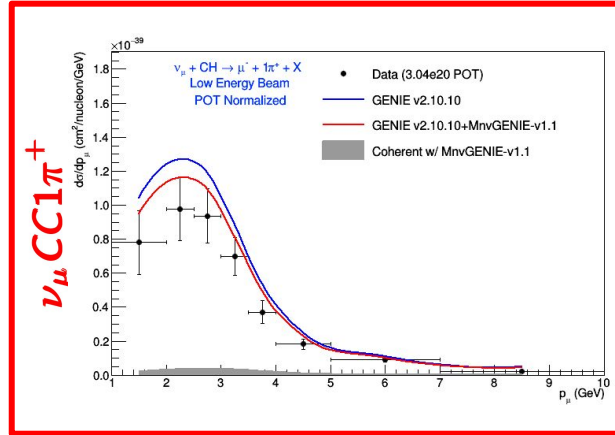
- Two channels: $\nu_\mu CC1\pi^+$ and $\bar{\nu}_\mu CC1\pi^0$ use GENIE v2.10.10 w/ MnvGENIE-v1.1
 - Only difference between v2.8.4 and v2.10.10 is the anisotropy of the $\Delta(1232)$ decay
 - v2.8.4: Δ^{++} decays anisotropically using 50% of the RS prediction, all other Δ decay isotropically
 - v2.10.10: all Δ decay isotropically
 - The difference between angular decay is small
 - Deuterium data prefers anisotropic decay, but all models shown today try to use isotropic decay



Muon Momentum

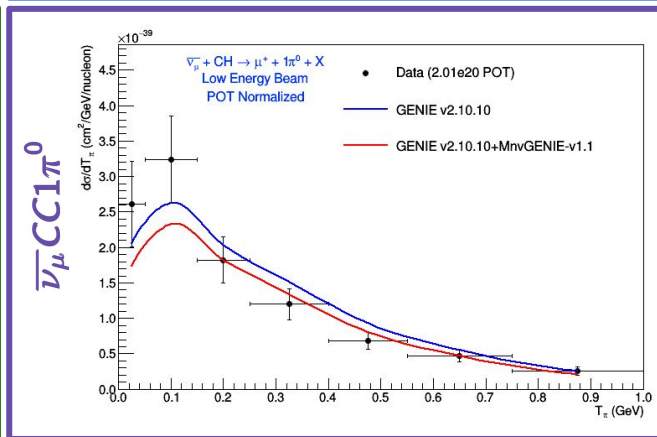
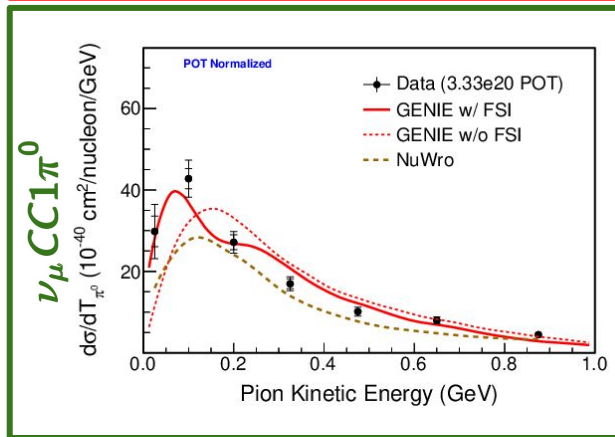
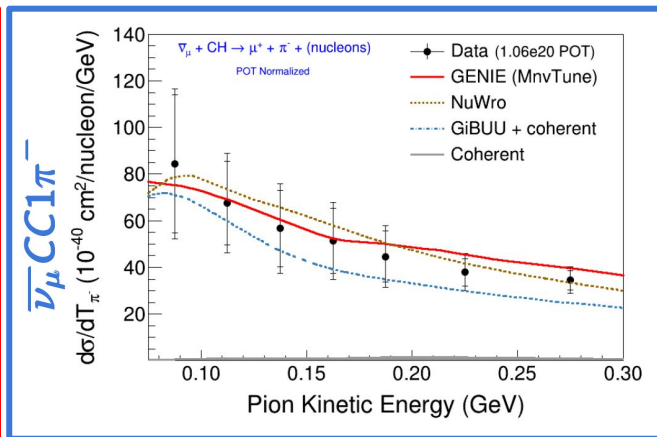
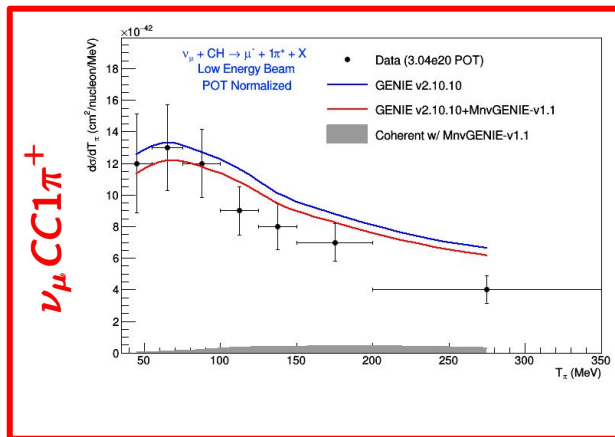
- For the following slides, the key curve is the solid red line, which is **GENIE+MnvGENIE-v1.1**
- MINERvA data has good shape agreement to the GENIE prediction in muon momentum
- GENIE overpredicts **CC1 π^{\pm}** production, but it seems to do well for **CC1 π^0** production
- NuWro has better agreement for charged pion production, but it underestimates neutral pion production
- FSI causes normalization losses in π^{\pm} , gains in π^0

References for plots on slide 3



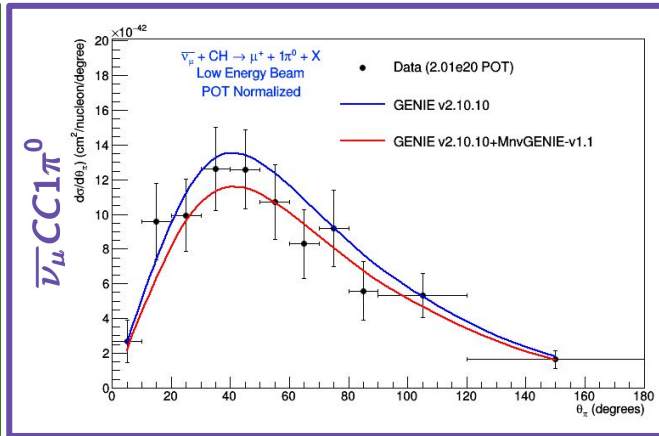
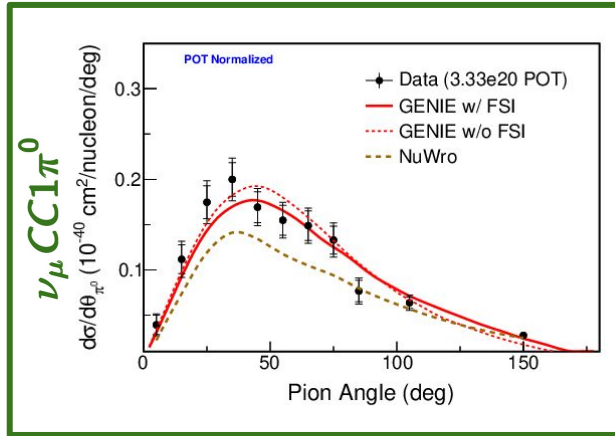
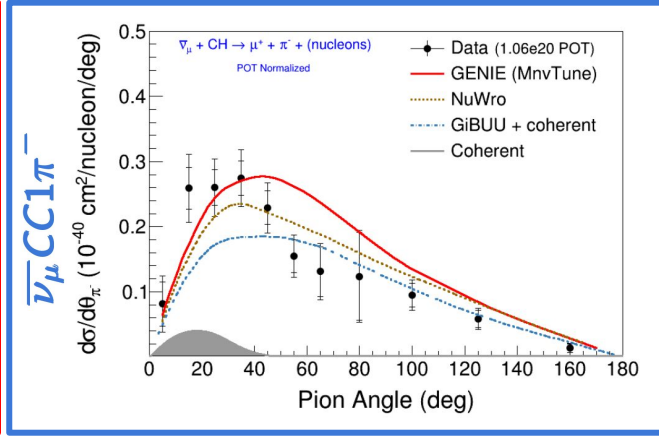
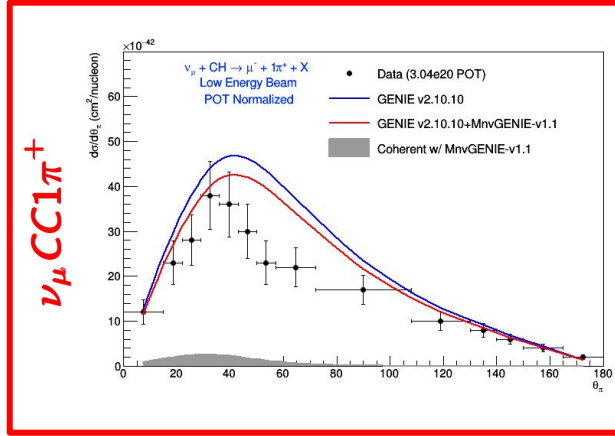
Pion Kinetic Energy

- GENIE w/ FSI has better agreement here than GENIE w/o FSI
 - Inelastic scattering is the primary cause of the hump at low kinetic energy
- Overall, GENIE underestimates at low T_{π} , and overestimates at higher T_{π}



Pion Production Angle

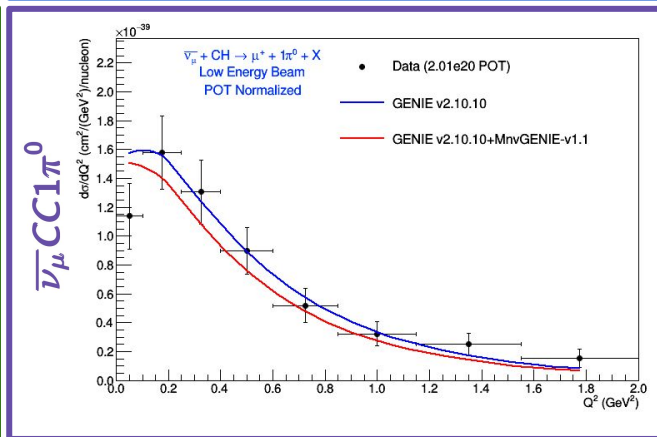
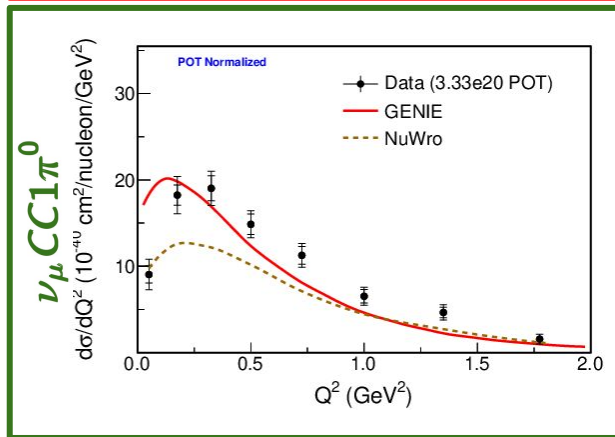
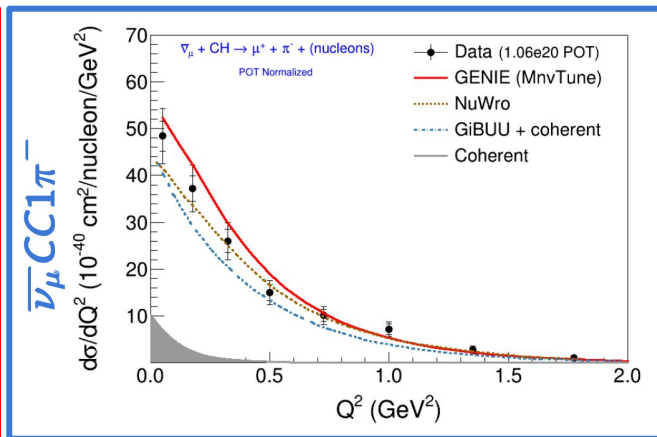
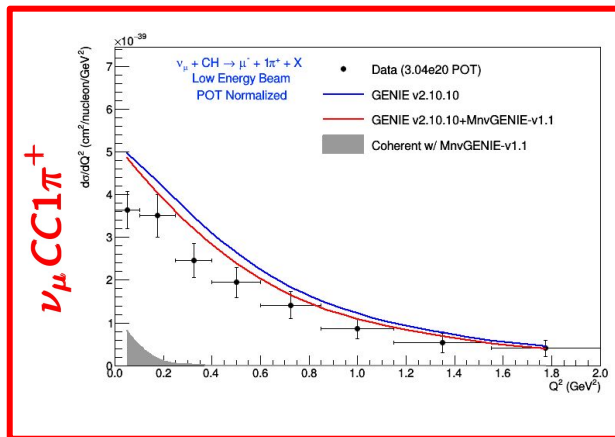
- All channels show a preference for more forward going pions
 - Acceptance for charged and neutral pions is very different



Q^2

$$Q^2 = 2E_\nu(E_\mu - p_\mu \cos(\theta_{\mu\nu})) - m_\mu^2$$

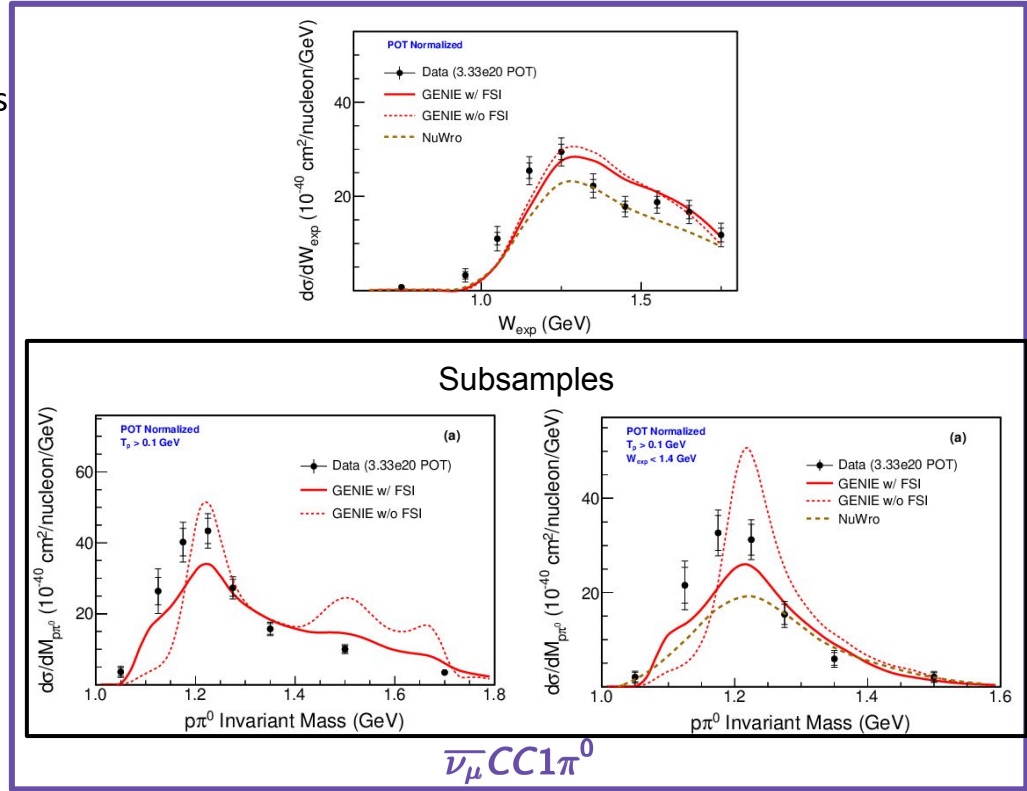
- Deficit of pion production at low Q^2 in $\overline{\nu}_\mu CC1\pi^0$ channels
- Due to the low Q^2 contribution from coherent pion production, there doesn't seem to be a significant deficit for the $\overline{\nu}_\mu CC1\pi^\pm$ channels
- The lack of a turnover in data for the $\overline{\nu}_\mu CC1\pi^\pm$ channels points to a process that provides events at low Q^2
 - This process is missing in neutral pion channels, i.e. a reaction where the target nucleon keeps its identity
 - Could diffractive scattering contribute to this?



$\Delta(1232)$ Peak

$$W_{exp}^2 = -Q^2 + m_N^2 + 2m_N E_H \quad (m_N = \text{nucleon mass})$$

- Currently, we only have differential cross sections for W in the $\bar{\nu}_\mu CC1\pi^0$ mode
 - We are working on equivalent cross sections for other channels
- MINERvA sees a shift in the pion spectra to lower values, which looks to be consistent with a shift to higher W in the delta peak
 - Shifts in Q^2 may induce a shift of 10 MeV
 - Possibly resonant/non-resonant interference that is absent from model
- In the $\bar{\nu}_\mu CC1\pi^0$ analysis, we had two enriched subsamples, where it is more accurate to call W the invariant mass of $p\pi^0$
 - $p\pi^0$ ($W < 1.8$ GeV)
 - Data shows a $\Delta(1232)$ peak more pronounced than either generator
 - $\Delta^+(1232)$ enhanced region ($W < 1.4$ GeV)
 - Low prediction at Δ peak with both generators



So what have we done to account for these discrepancies?

- Simulating pion production correctly requires that we improve our nucleon and nuclear models
 - Data on “free nucleons” (meaning deuterium) is limited
 - GENIE’s resonance model has a deficiency: there is no interference between non-resonant and resonant production
 - Electron scattering community believes Rein-Seghal does not describe resonance production well
 - Electron scattering uses MAID model which gets satisfactory results for non-resonant production.
 - Difficult to test versions with an axial piece with neutrinos
 - When discrepancies are found, it will be hard to disentangle whether these effects are **nucleon** or **nuclear** (FSI, initial state nuclear model)
- Still, in order to understand the discrepancy between model and MINERvA data, NUISANCE was used to tune pion production parameters in GENIE
 - This tune only looked at muon and pion kinematics
 - No Q^2 fit
 - Unfortunately, the recent $\bar{\nu}_\mu CC1\pi^-$ result could not be included in the fit in time for this talk

NUISANCE is a flexible framework in which neutrino interaction generators can be validated and/or empirically tuned to data

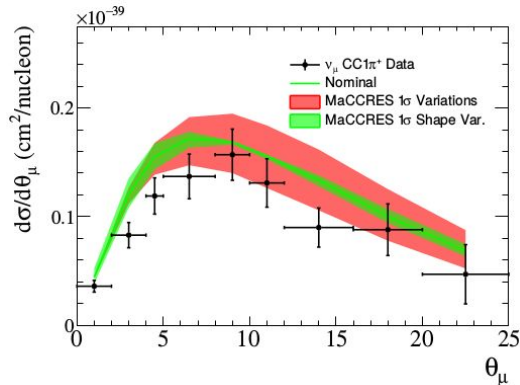
The following tunes come from Patrick Stowell’s work with the MINERvA collaboration.

(Publication in progress)

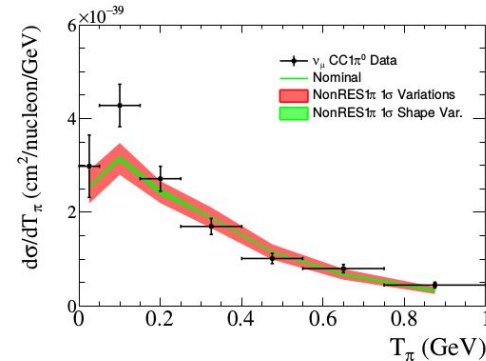


Rein-Seghal Tune

- GENIE has several parameters related to the pion production models
 - Form factors related to the Rein Seghal model (M_A^{RES} , Normalization)
 - Scaling for non-resonant 1π or 2π production
- Studied the effects of different nuclear and resonance models in GENIE (ie. LFG, Berger-Seghal, among others)
 - Changes normalization, but little change in shape
 - Used default Rein-Seghal implementation in GENIE for this study
- Fitted the model to both MINERvA and deuterium data
- The joint fit of all data sets does not produce a strong pull in either form factors or non-resonant terms



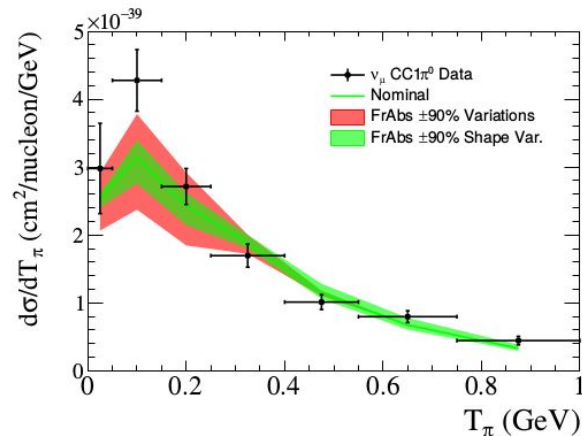
Effect of varying M_A^{RES} on the $\nu_\mu \text{CC}1\pi^+ \theta_\mu$



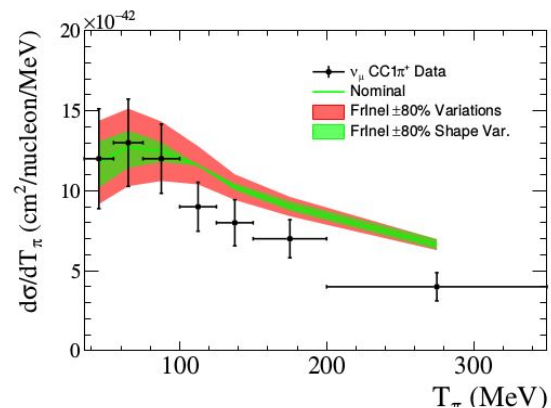
Effect of varying non-resonant 1π on the $\nu_\mu \text{CC}1\pi^0 T_\pi$

FSI Tuning

- While GENIE has several FSI related parameters, the study focused on two in particular due to the other having either strong prior constraints or little effect on the MINERvA MC
 - **FrAbs:** Pion absorption
 - **FrInel:** Inelastic scattering
- These parameters could not be varied at the same time
 - Varied individually and studied their behaviors
- In the joint fit of all data sets, both FSI parameters prefer the nominal
- To get any significant shape difference, the parameters need to be varied by $\pm 3\sigma$
- Points to either the need for strong FSI pulls or the need for other parameters that have more freedom to cover discrepancies in T_π and θ_μ



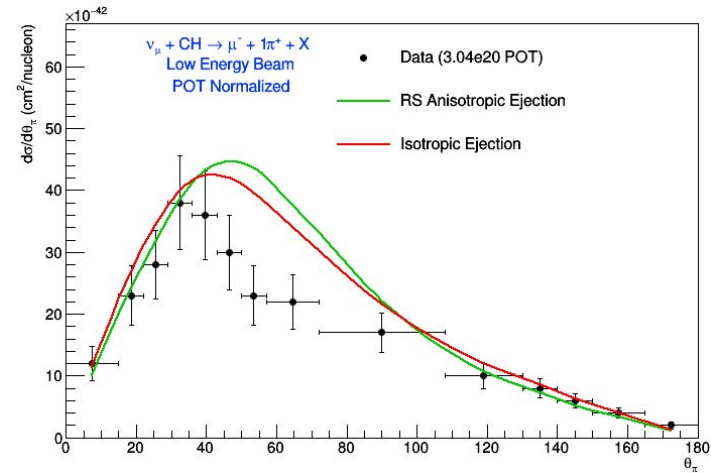
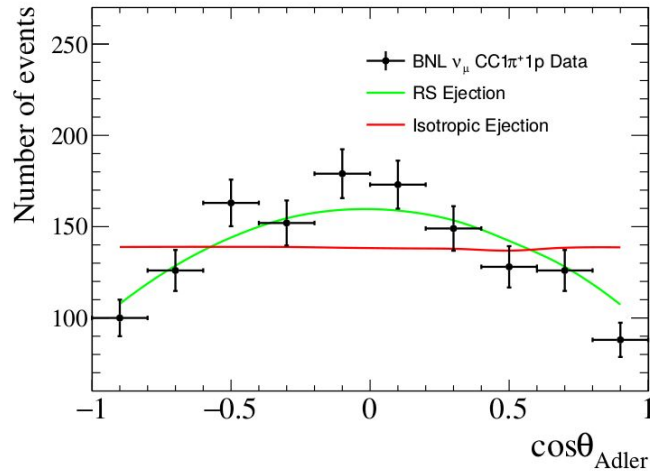
Effect of varying FrAbs on the T_π $\nu_\mu CC1\pi^0$ distribution



Effect of varying FrAbs on the T_π $\nu_\mu CC1\pi^+$ distribution

Tuning the Angular Ejection

- Can adjust pion angular ejection with **Theta- π** parameter
 - Allows for continuous reweighting between default anisotropic RS prediction and isotropic decay in rest frame
- Deuterium data prefers anisotropic emission
- Joint fit prefers isotropic emission, which improves agreement with MINERvA data
 - This perhaps says more about FSI than emission



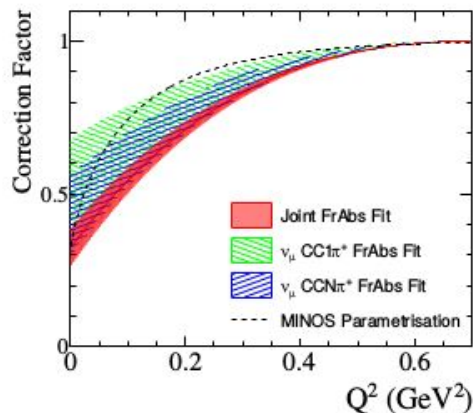
Ad-hoc Q² Suppression

- Several other experiments see discrepancies in low Q² and θ_μ distributions (MiniBooNE, MINOS)
 - MINOS introduced a suppression function tuned to Δ enriched sidebands that improved agreement to RES and RES-to-DIS enhanced samples
- MINERvA tried something similar with a different suppression function
 - π^0 production prefers strong low Q² suppression
 - π^+ production prefers smaller suppression
 - Charged coherent pion production partly fills the suppression at low Q²

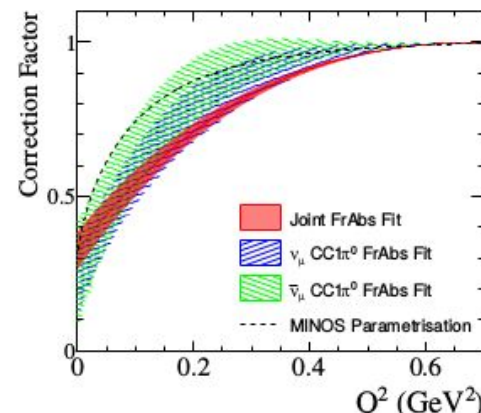
$$R(Q^2 < x_m) = \frac{R_1(x - x_2)(x - x_m)}{(x_1 - x_2)(x_1 - x_m)} + \frac{R_2(x - x_1)(x - x_m)}{(x_2 - x_1)(x_2 - x_m)} + \frac{(x - x_1)(x - x_2)}{(x_m - x_1)(x_m - x_2)}$$

$$W(Q^2) = 1 - (1 + R_1 + R(Q^2))(1 - R_1)^2$$

$$x_1 = 0.0 \text{ GeV}^2; x_2 = 0.35 \text{ GeV}^2; x_m = 0.7 \text{ GeV}^2$$



CC1 π^+ Suppression Curve

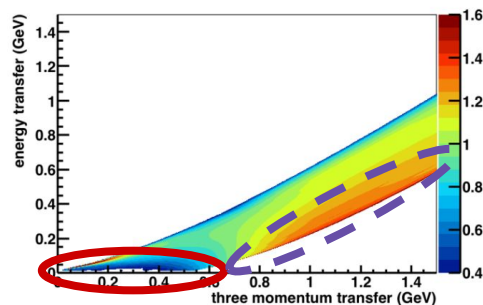


CC1 π^0 Suppression Curve

Should we use RPA predictions for pion interactions?

- There is a similar deficiency at low Q^2 in $CC0\pi$
 - Apply the Valencia RPA multinucleon effect to CCQE events
 - Primarily a suppression in energy transfer, not Q^2
- Seems flawed to shift Valencia model to Δ region
 - Includes several nuclear corrections
 - Also, fundamentally flawed to move a low q_0 process into the pion region, which requires a significant q_0 transfer
 - But, the real effect could have interesting shape in (q_0, q_3) space
 - Prediction helpful

RPA Correction

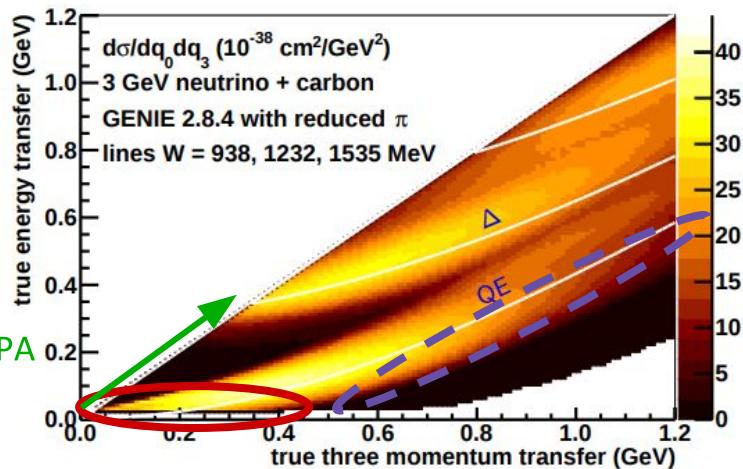


Energy Transfer q_0

$$q_0 = E_\nu - E_\mu$$

Momentum Transfer q_3

$$q_3 = \sqrt{Q^2 + q_0^2}$$



Shifting the RPA predictions

Conclusions

- MINERvA has measured all four charged current pion production channels in $\Delta(1232)$ dominated samples off of carbon in our low energy beam
 - We have uncovered important discrepancies between simulation and data in charged pion and neutral pion production, particularly in low Q^2
 - Current framework does not give enough tools to cover these discrepancies
- It seems that MINERvA has a decent understanding of coherent pion production. Working on effective A scaling
 - Diffraction pion production is tougher, will require more effort
- Theory can improve the situation with the nucleon model
 - We are happy to work with generator developers
 - Many thanks to NUISANCE developers
- Medium energy beam results coming soon!
 - More POT, more flux, higher statistics
 - Will allow us to look at heavier nuclei

This material is based upon work supported by U.S. Department of Energy award DE-SC-0008475



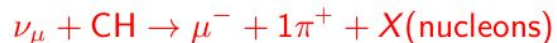
Thank you



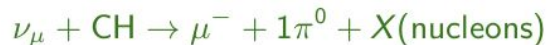
Backup Slides



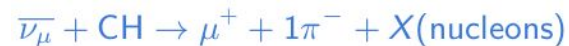
Additional Event Selection



- No matched michels
 - Interaction vertex, showers, track endpoints
- E_{vtx} = unassociated hits <9cm of interaction vertex
- Photon conversion distance > 14 cm
- Energy in nuclear targets < 20 MeV
- Non-track + vertex energy \in [50 MeV, 2.5 GeV]



- No matched michels
 - Interaction vertex, showers, track endpoints
- Photon conversion distance > 15 cm
- $E_{\text{vis}} \in$ [80 MeV, 2 GeV]

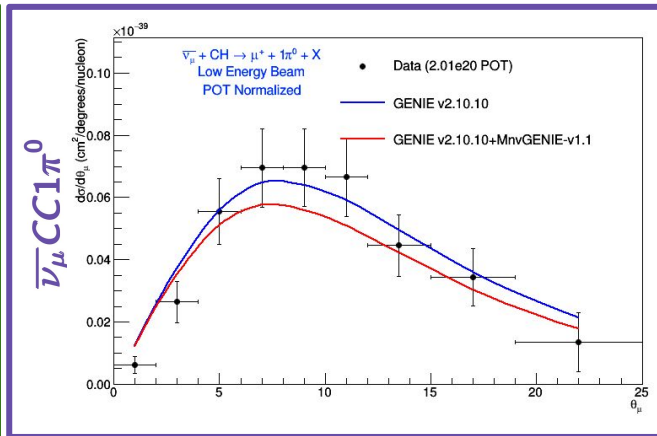
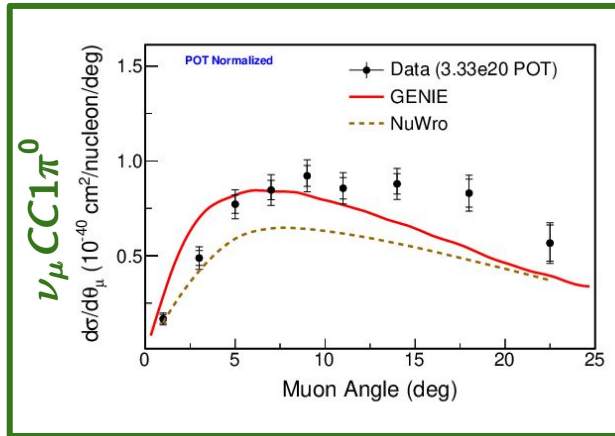
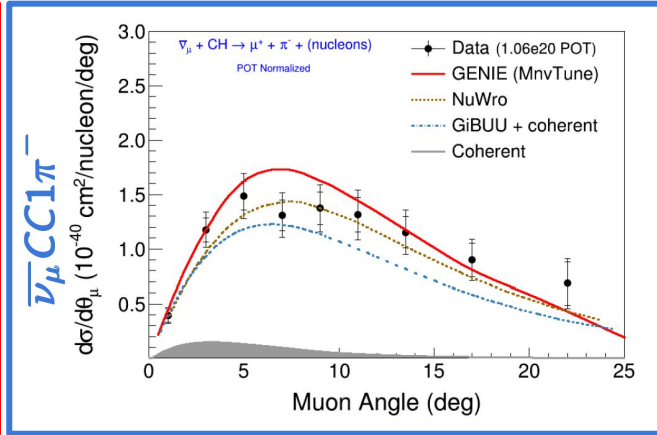
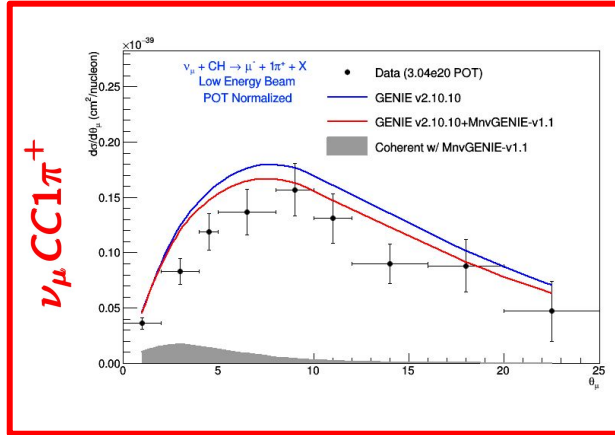


- No matched michels
 - Interaction vertex, showers, track endpoints



Muon Production Angle

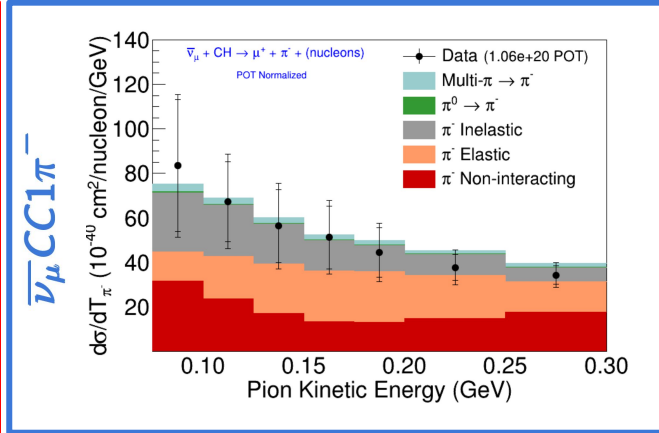
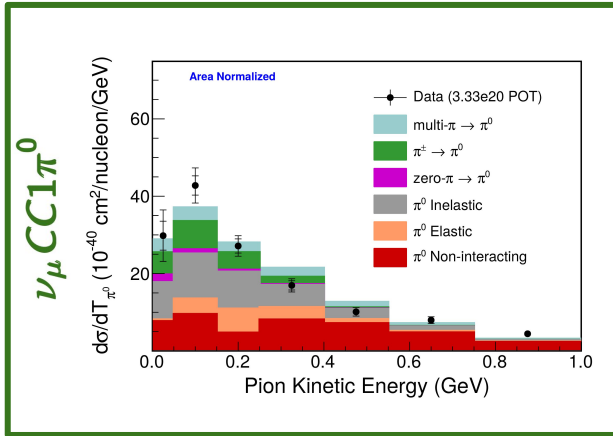
- Due to the MINOS match requirement, MINERvA acceptance for $\theta_{\mu} < 25^{\circ}$
- For $\overline{\nu}_{\mu} \text{CC}1\pi^0$ production, GENIE overestimates $\theta_{\mu} < 5^{\circ}$



Pion Kinetic Energy with FSI

- There are no FSI plots using GENIE 2.8.4 + MnvGENIE-v1.1 for the $\nu_\mu CC1\pi^+$ and $\bar{\nu}_\mu CC1\pi^0$ channels
- Neutral pion result is area normalized, charged pion result is POT normalized

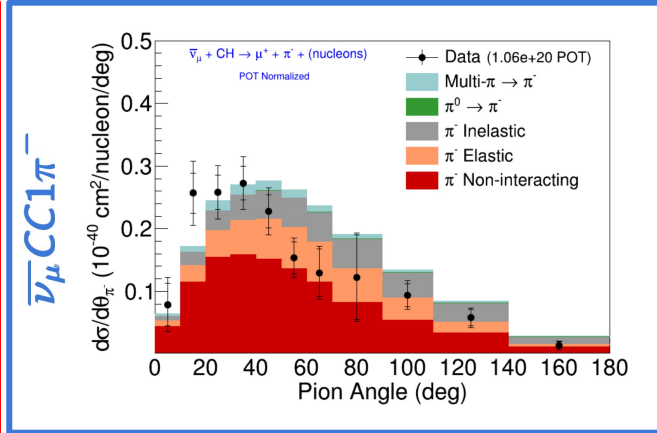
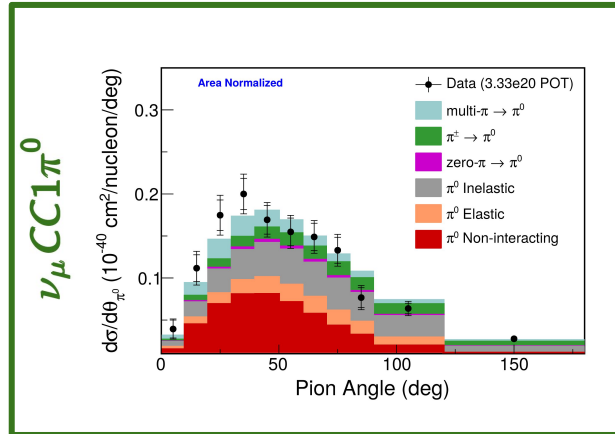
$\nu_\mu CC1\pi^+$



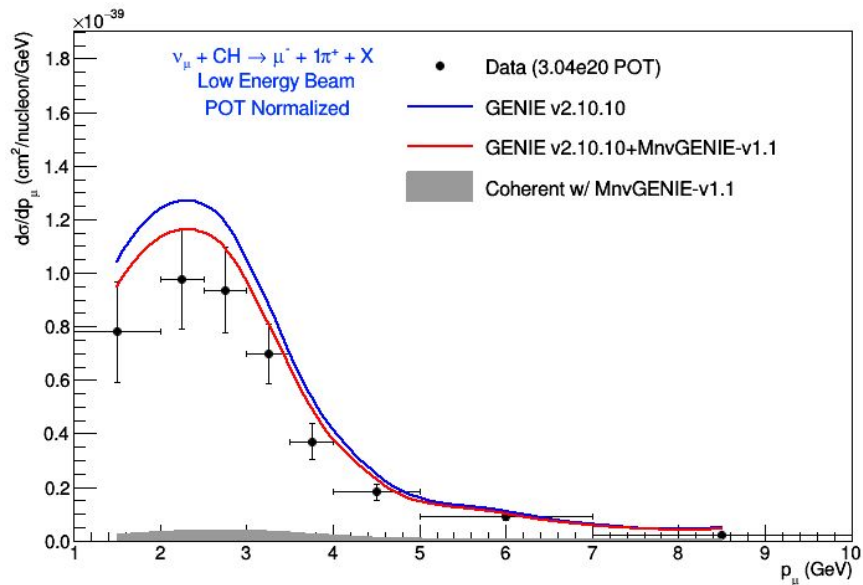
Pion Angle with FSI

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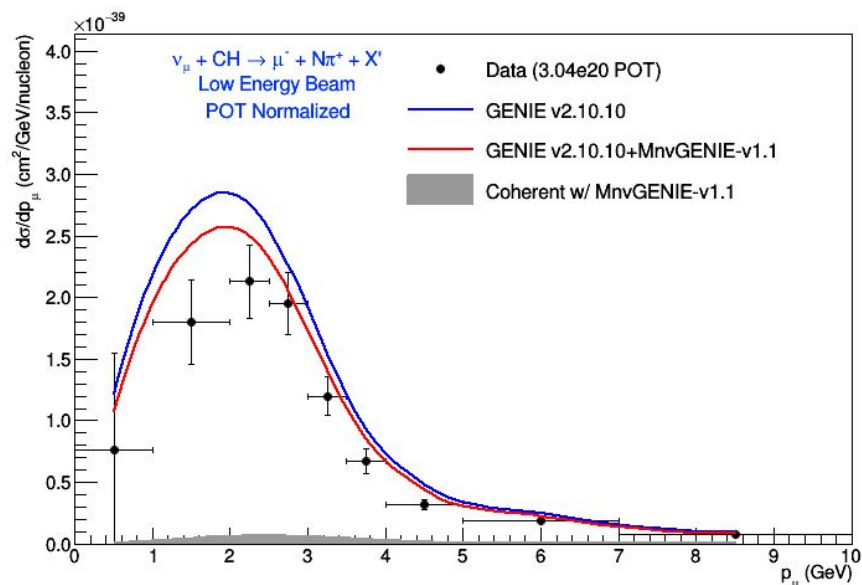
$\nu_\mu CC1\pi^+$



$\nu_{\mu} CC1/N\pi^{+}$ Muon Momentum



$\nu_{\mu} CC1\pi^{+}$

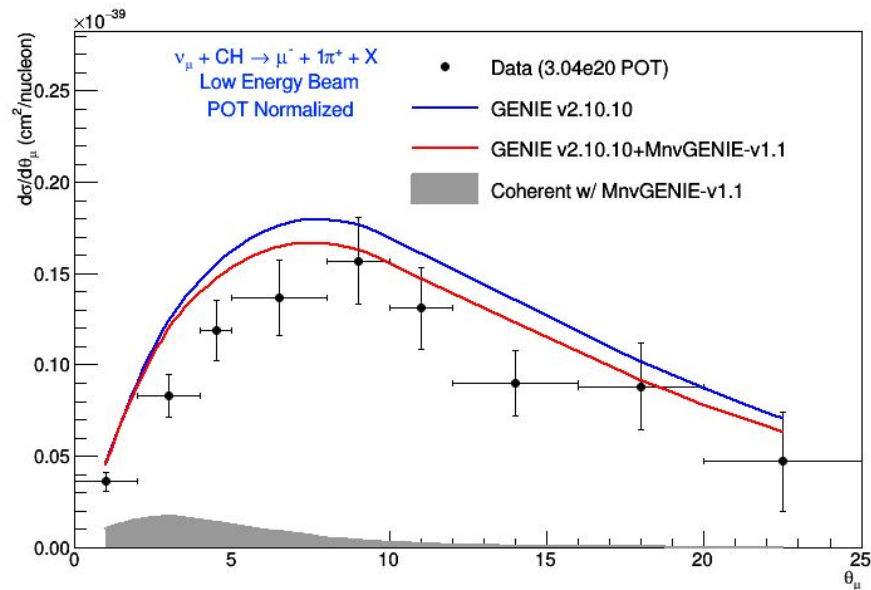


$\nu_{\mu} CCN\pi^{+}$

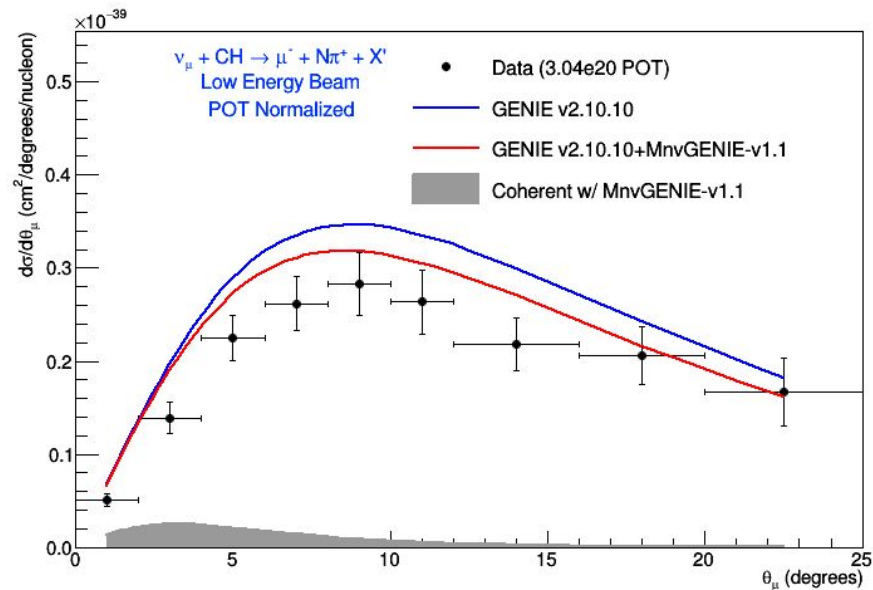
References for plots on slide 3



$\nu_{\mu} CC1/N\pi^+$ Muon Production Angle

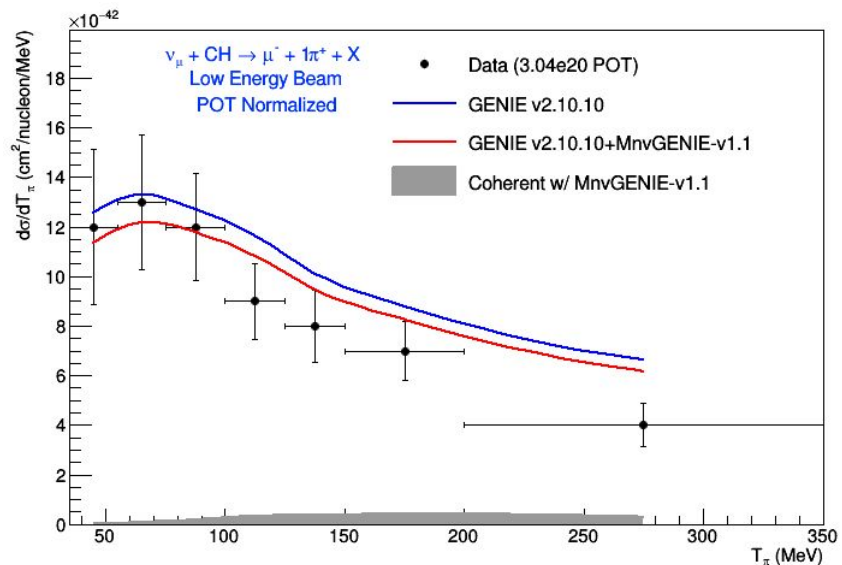


$\nu_{\mu} CC1\pi^+$

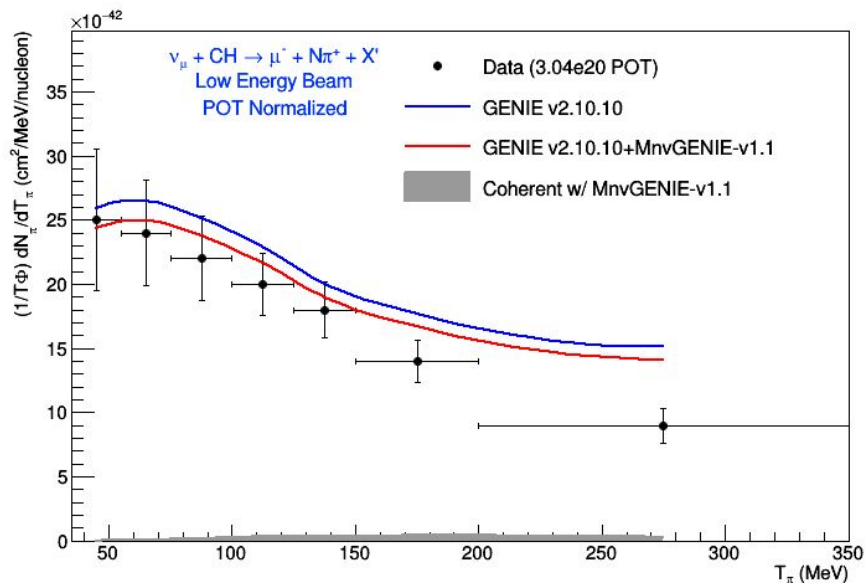


$\nu_{\mu} CCN\pi^+$

$\nu_{\mu} CC1/N\pi^+$ Pion Kinetic Energy

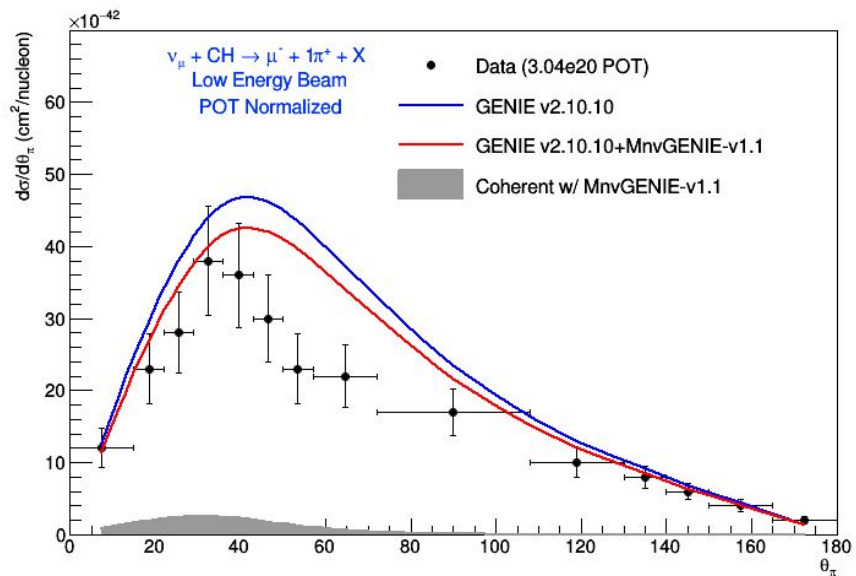


$\nu_{\mu} CC1\pi^+$

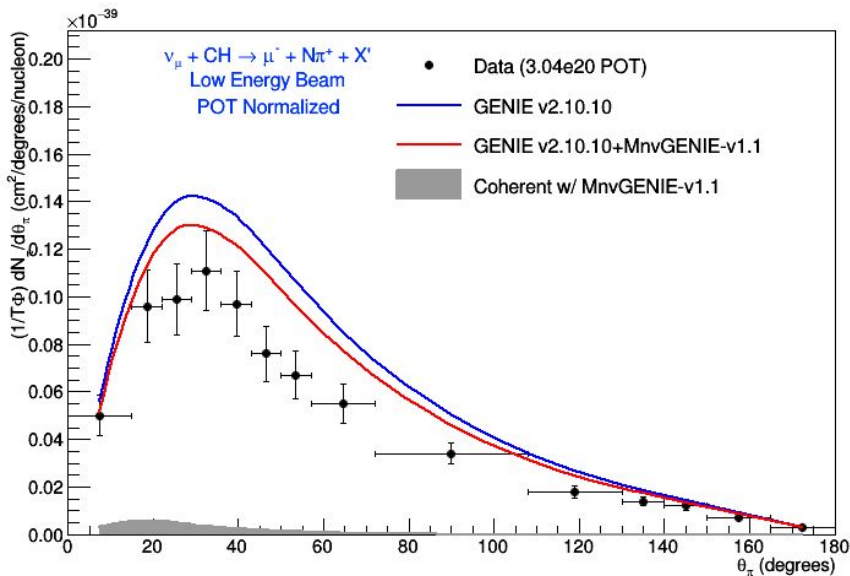


$\nu_{\mu} CCN\pi^+$

$\nu_{\mu} CC1/N\pi^{+}$ Pion Production Angle



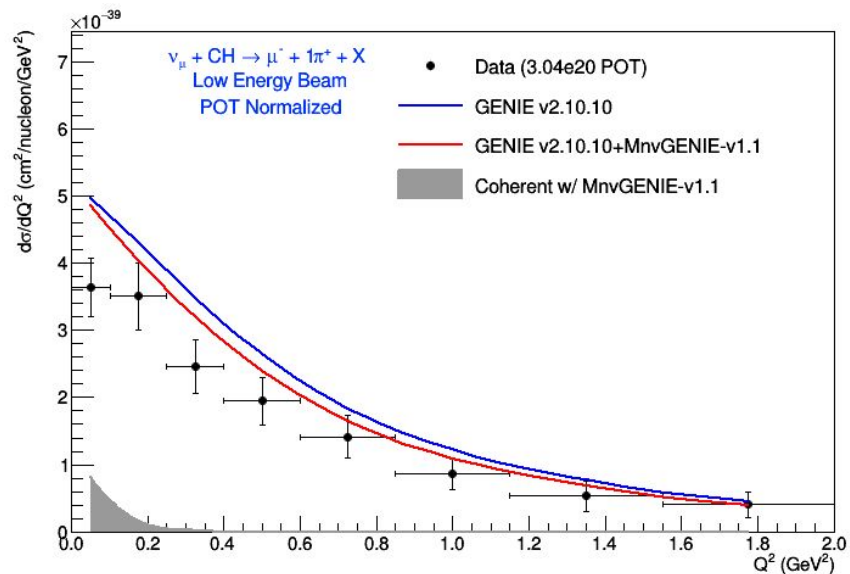
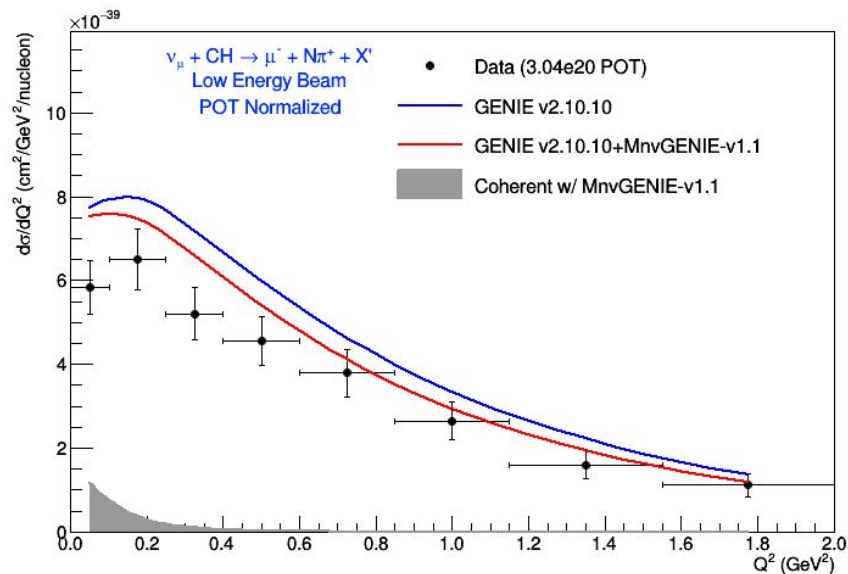
$\nu_{\mu} CC1\pi^{+}$



$\nu_{\mu} CCN\pi^{+}$

Q^2

$$Q^2 = 2E_\nu(E_\mu - p_\mu \cos(\theta_{\mu\nu})) - m_\mu^2$$

 $\nu_\mu CC 1\pi^+$  $\nu_\mu CC N\pi^+$

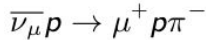
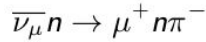
$\chi^2/\#$ of bins of Data vs GENIE 2.10.10 + MnvGENIEv1.1

Kinematic	$\nu_\mu CC1\pi^+$	$\nu_\mu CCN\pi^+$	$\nu_\mu CC1\pi^0$	$\bar{\nu}_\mu CC1\pi^0$	$\bar{\nu}_\mu CC1\pi^-$
p_μ	17.08/8	24.54/9	35.86/8	6.59/9	8.9/8
θ_μ	30.95/9	18.74/9	160.06/9	13.31/9	9.9/9
T_π	18.07/7	31.55/7	173.28/7	12.56/7	1.8/7
θ_π	80.11/14	29.76/14	130.25/11	5.45/11	32.0/11
Q^2	15.36/8	14.57/8	111.52/8	12.15/8	8.3/8
W_{exp}	N/A	N/A	141.88/10	N/A	N/A

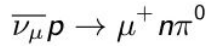
- Caution when using these χ^2 to determine goodness of fit. The covariance matrices may not be fully complete

Measurement of isospin amplitudes

- The recent $\bar{\nu}_\mu CC1\pi^-$ result can decompose its signal sample into nucleon level cross sections using a vertex energy fit to find overall rate



- Also, the $\bar{\nu}_\mu CC1\pi^0$ result has an exclusive nucleon reaction channel



- It is then possible to find the relative strength and phase between the amplitudes of isospin $I = 1/2, 3/2$

$$\langle |A_3|^2 \rangle = \frac{1}{2} \sigma(\mu^+ n\pi^-)$$

$$\langle |A_1|^2 \rangle = \frac{3}{4} \left(\sigma(\mu^+ n\pi^0) + \sigma(\mu^+ p\pi^-) - \frac{1}{3} \sigma(\mu^+ n\pi^-) \right)$$

$$\langle \Re(A_3^* A_1) \rangle = \frac{3}{8} \left(\sigma(\mu^+ p\pi^-) - 2\sigma(\mu^+ n\pi^0) + \frac{1}{3} \sigma(\mu^+ n\pi^-) \right)$$

$$R^{\bar{\nu}} = \left(\langle |A_1|^2 \rangle / \langle |A_3|^2 \rangle \right)^{1/2}$$

$$\cos \phi^{\bar{\nu}} = \langle \Re(A_3^* A_1) \rangle / \langle |A_1|^2 \rangle^{1/2} \langle |A_3|^2 \rangle^{1/2}$$

